Computer Algebra
Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.4-Improper/31-1.1.4.3-e-x-^m-a-x^j+b-x^k-^p-c+d-x^n-^q

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September 6, 2023 Compiled on September 6, 2023 at 4:01am

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [298]. This is test number [31].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (298)	0.00 (0)
Mathematica	99.33 (296)	0.67 (2)
Fricas	92.95 (277)	7.05 (21)
Maple	92.28 (275)	7.72 (23)
Giac	76.17 (227)	23.83 (71)
Maxima	71.14 (212)	28.86 (86)
Mupad	66.11 (197)	33.89 (101)
Sympy	46.31 (138)	53.69 (160)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

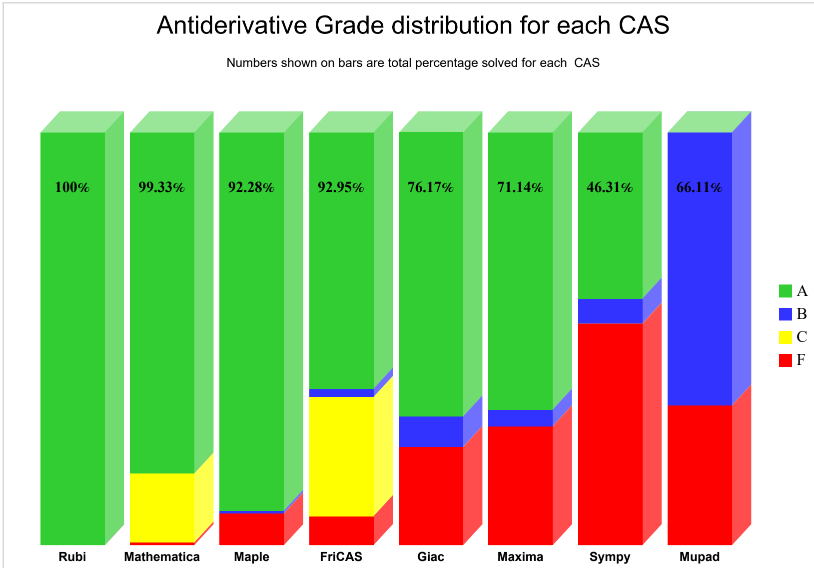
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

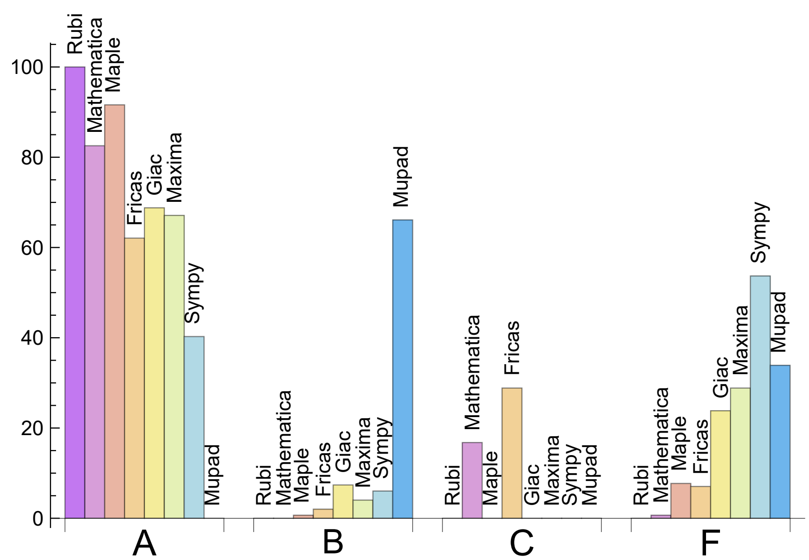
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Maple	91.611	0.671	0.000	7.718
Mathematica	82.550	0.000	16.779	0.671
Giac	68.792	7.383	0.000	23.826
Maxima	67.114	4.027	0.000	28.859
Fricas	62.081	2.013	28.859	7.047
Sympy	40.268	6.040	0.000	53.691
Mupad	0.000	66.107	0.000	33.893

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	2	100.00	0.00	0.00
Fricas	21	100.00	0.00	0.00
Maple	23	100.00	0.00	0.00
Giac	71	100.00	0.00	0.00
Maxima	86	100.00	0.00	0.00
Mupad	101	0.00	100.00	0.00
Sympy	160	69.38	30.62	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.12
Maxima	0.24
Fricas	0.28
Giac	0.36
Maple	1.80
Mathematica	1.88
Sympy	2.81
Mupad	5.66

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	97.26	0.83	87.50	0.90
Maple	122.09	0.90	95.00	0.89
Maxima	124.24	1.08	84.00	0.99
Rubi	145.03	1.00	103.50	1.00
Sympy	145.19	1.66	80.00	1.18
Giac	145.42	1.27	103.00	1.04
Mupad	151.83	1.14	76.00	0.96
Fricas	211.97	1.44	106.00	1.10

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

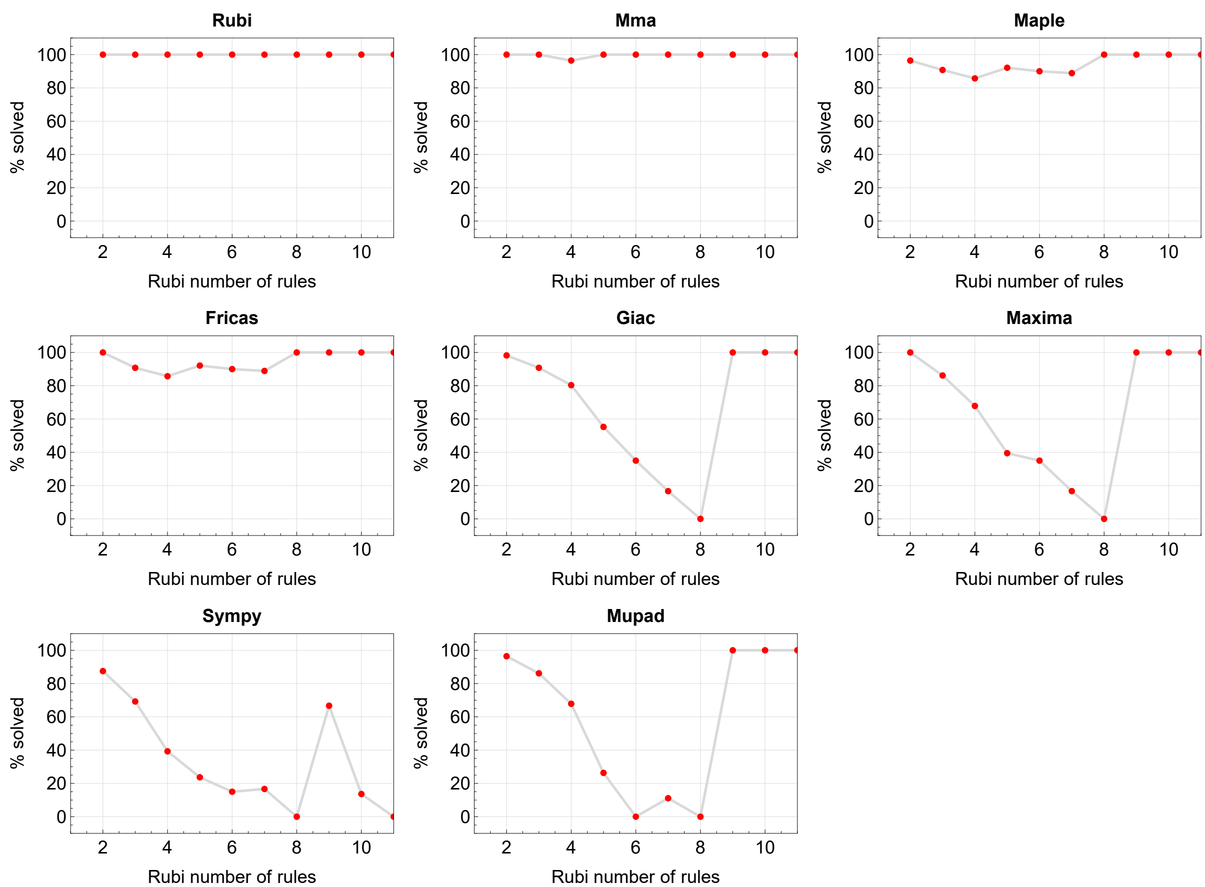


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

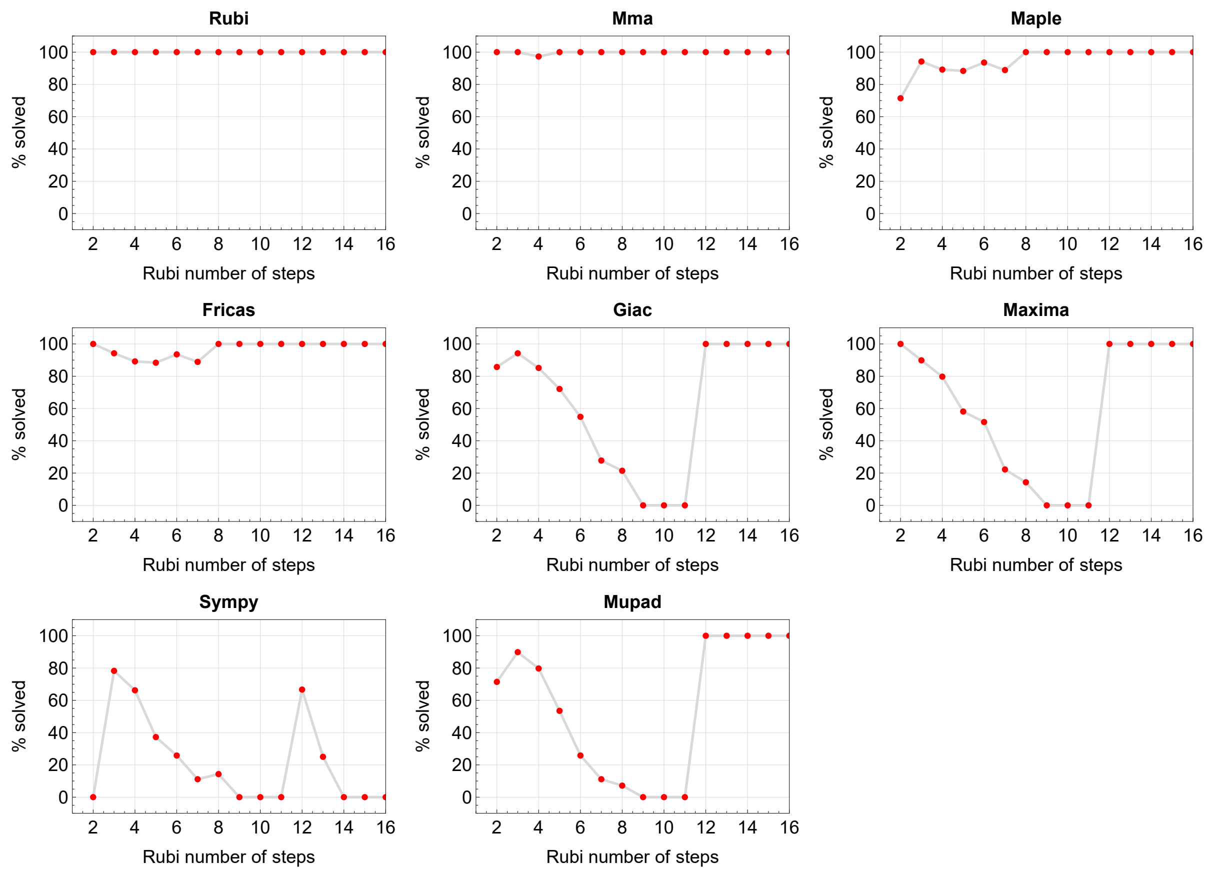


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

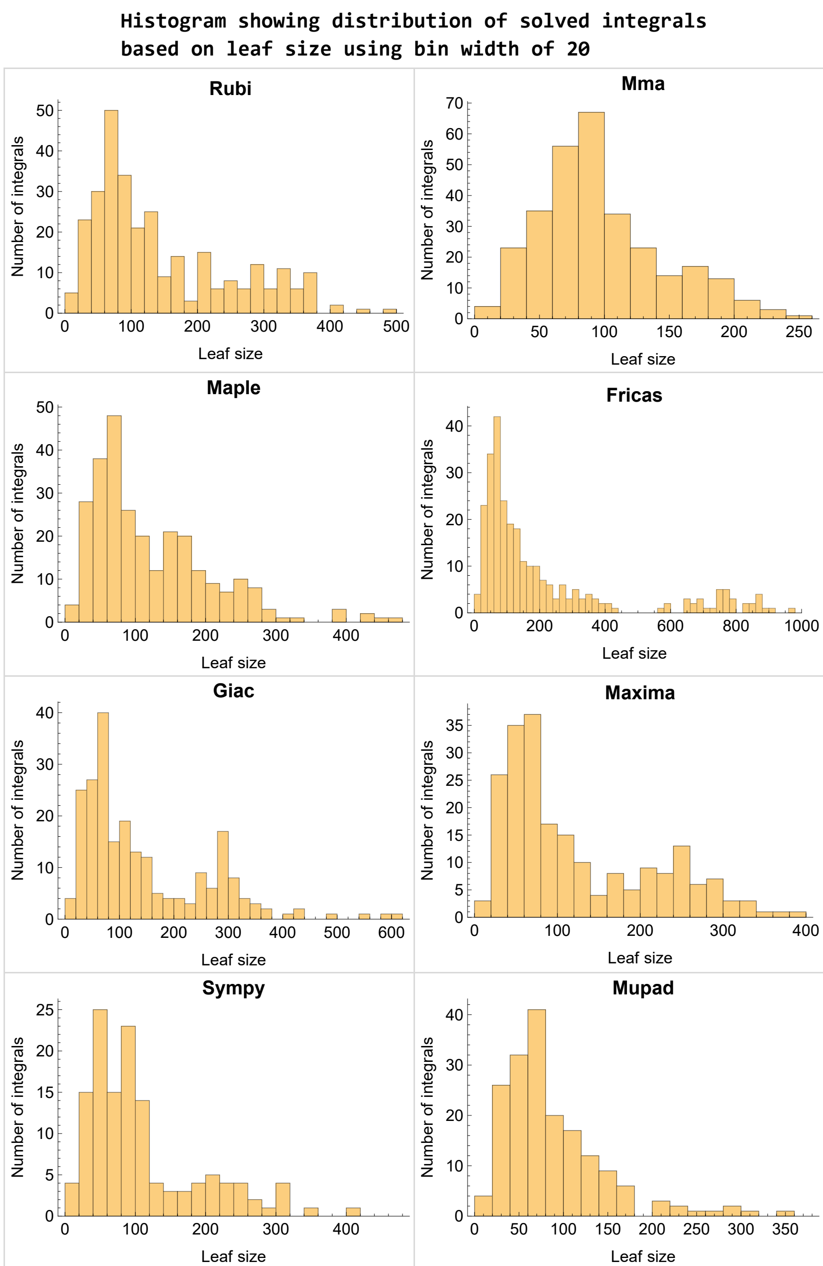


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

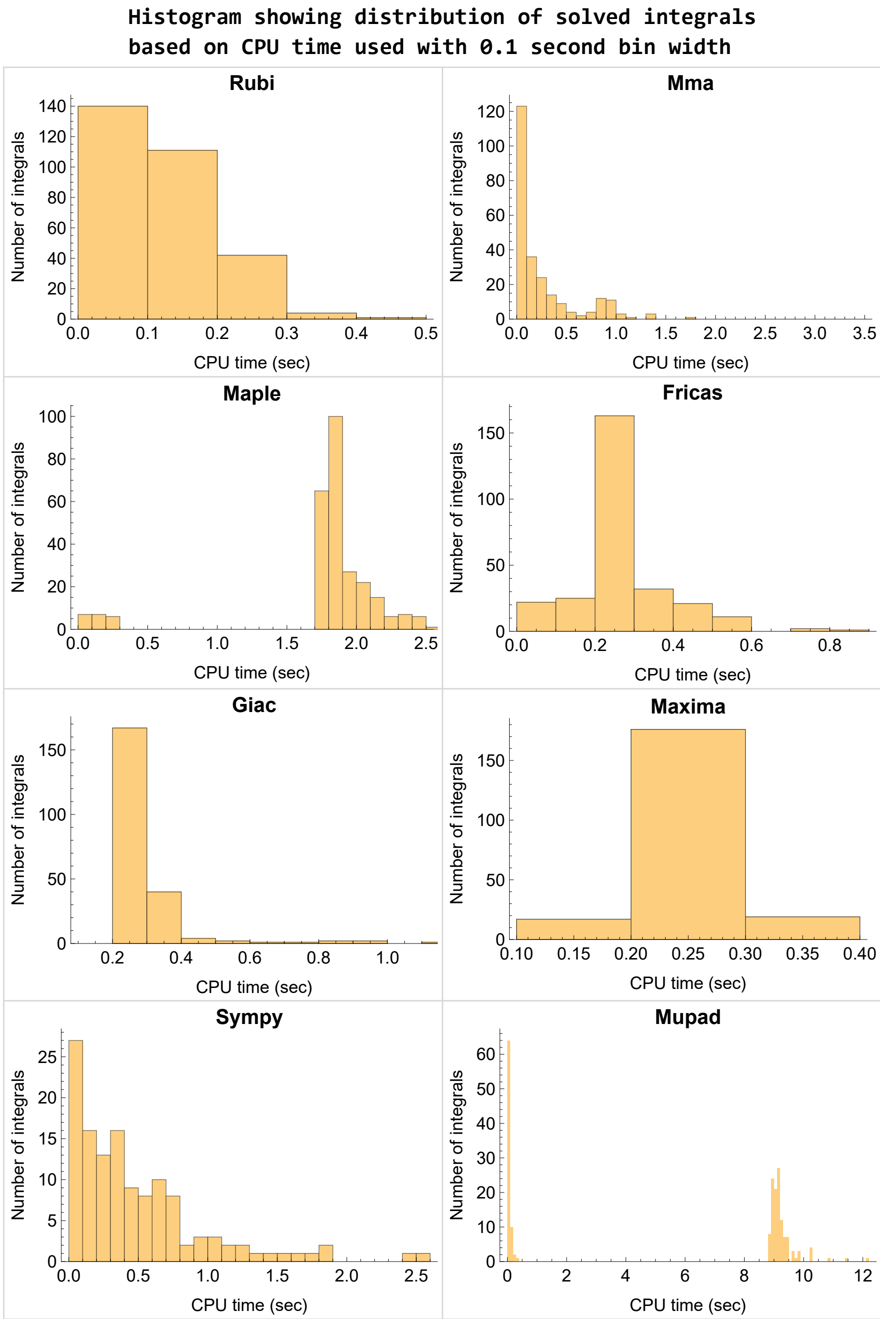


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

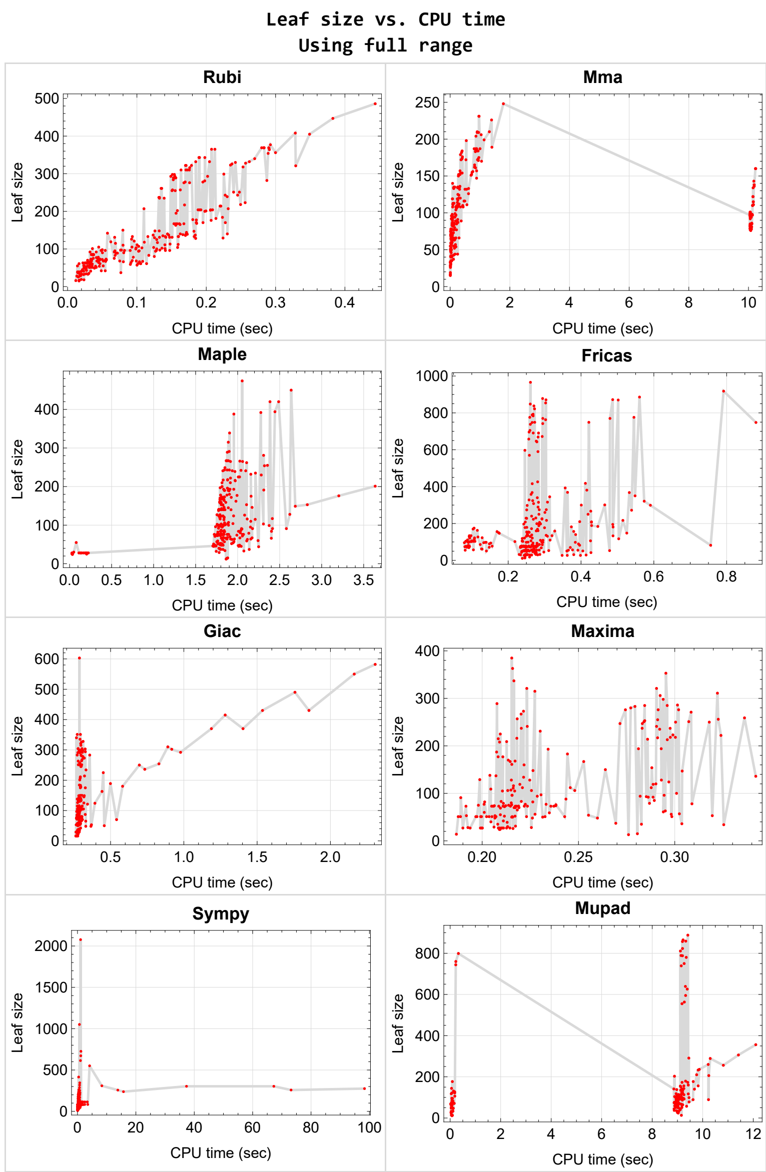


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

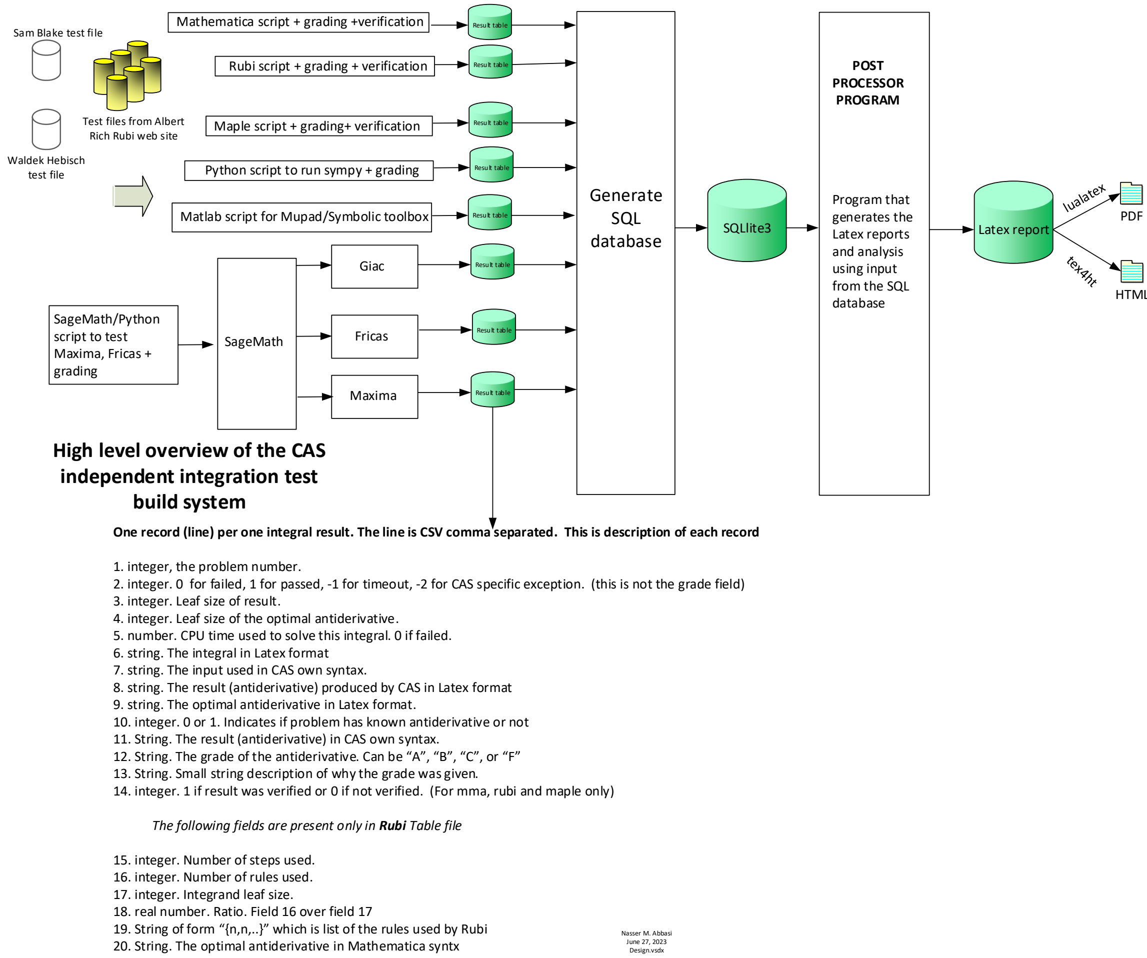
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 296, 297, 298 }

B grade { }

C grade { 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 283 }

F normal fail { 284, 292 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 271, 278, 279, 280, 281 }

B grade { 269, 270 }

C grade { }

F normal fail { 272, 273, 274, 275, 276, 277, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 275, 278, 279, 280, 281 }

B grade { 84, 86, 269, 270, 271, 283 }

C grade { 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268 }

F normal fail { 272, 273, 274, 276, 277, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 93, 94, 95, 97, 98, 99, 100, 101, 102, 106, 109, 110, 111, 112, 119, 120, 121, 122, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 269, 270, 271, 275, 278, 280, 281 }

B grade { 91, 96, 107, 108, 113, 114, 115, 116, 117, 118, 279, 283 }

C grade { }

F normal fail { 103, 104, 105, 123, 124, 125, 126, 127, 128, 129, 142, 143, 144, 156, 157, 158, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 274, 276, 277, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 278, 279, 280, 281 }

B grade { 28, 95, 96, 97, 98, 99, 113, 114, 115, 116, 117, 118, 135, 136, 137, 150, 151, 152, 269, 270, 271, 283 }

C grade { }

F normal fail { 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 274, 275, 276, 277, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 96, 97, 98, 99, 100, 101, 102, 103, 108, 114, 115, 116, 117, 118, 119, 120, 121, 122, 133, 134, 135, 136, 137, 138, 139, 140, 141, 148, 149, 150, 151, 152, 153, 154, 155, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 269, 270, 271, 278, 279, 280, 281 }

C grade { }

F normal fail { }

F(-1) timedout fail { 94, 95, 104, 105, 106, 107, 109, 110, 111, 112, 113, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 142, 143, 144, 145, 146, 147, 156, 157, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 274, 275, 276, 277, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 46, 47, 48, 50, 53, 55, 57, 58, 59, 61, 62, 63, 64, 65, 67, 68, 69, 71, 73, 74, 75, 76, 78, 79, 80, 81, 82, 84, 86, 87, 88, 89, 90, 91, 92, 106, 107, 108, 109, 130, 131, 132, 133, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 186, 187, 188, 189, 190, 191, 192, 278, 279, 280, 281 }

B grade { 28, 43, 45, 49, 51, 52, 54, 56, 60, 66, 70, 72, 77, 83, 85, 269, 270, 271 }

C grade { }

F normal fail { 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 234, 235, 236, 237, 238, 239, 240, 241, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 262, 263, 264, 265, 266, 267, 272, 273, 274, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

F(-1) timedout fail { 183, 184, 185, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 230, 231, 232, 233, 242, 243, 244, 245, 246, 257, 258, 259, 260, 261, 268, 275, 276, 277, 283 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.85
time (sec)	N/A	0.020	0.009	0.166	0.211	0.398	0.015	0.276	0.045

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.85
time (sec)	N/A	0.030	0.009	0.145	0.199	0.362	0.016	0.276	0.040

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.85
time (sec)	N/A	0.017	0.006	0.133	0.214	0.348	0.017	0.269	9.028

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.85
time (sec)	N/A	0.024	0.009	0.129	0.210	0.244	0.017	0.276	0.039

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	25
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	0.89
time (sec)	N/A	0.013	0.006	0.203	0.209	0.246	0.017	0.272	0.038

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	28	25	27	30	26
N.S.	1	1.00	1.00	0.97	0.97	0.86	0.93	1.03	0.90
time (sec)	N/A	0.019	0.011	0.111	0.193	0.246	0.046	0.283	9.003

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	24	28	20	23	24
N.S.	1	1.00	1.00	0.92	0.92	1.08	0.77	0.88	0.92
time (sec)	N/A	0.015	0.010	0.034	0.209	0.245	0.044	0.279	9.031

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	28	30	26	42	25
N.S.	1	1.00	1.00	0.90	0.97	1.03	0.90	1.45	0.86
time (sec)	N/A	0.018	0.014	0.036	0.226	0.252	0.093	0.295	0.044

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	27	25	26	29	27	28	26
N.S.	1	1.00	1.04	0.96	1.00	1.12	1.04	1.08	1.00
time (sec)	N/A	0.016	0.013	0.033	0.216	0.246	0.099	0.274	0.036

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	31	26	30	31	29	39	29
N.S.	1	1.00	1.07	0.90	1.03	1.07	1.00	1.34	1.00
time (sec)	N/A	0.018	0.019	0.027	0.213	0.380	0.196	0.286	0.053

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	33	28	29	29	32	31	29
N.S.	1	1.00	1.06	0.90	0.94	0.94	1.03	1.00	0.94
time (sec)	N/A	0.019	0.013	0.028	0.206	0.414	0.188	0.286	0.039

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	56	53	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	1.02	0.96	0.93
time (sec)	N/A	0.035	0.012	1.787	0.207	0.405	0.024	0.278	0.061

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	53	53	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	0.96	0.96	0.93
time (sec)	N/A	0.051	0.009	1.772	0.216	0.361	0.022	0.279	0.049

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	56	53	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	1.02	0.96	0.93
time (sec)	N/A	0.029	0.012	1.797	0.204	0.258	0.025	0.272	0.051

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	51	52	51	51	53	53	51
N.S.	1	1.00	1.21	1.24	1.21	1.21	1.26	1.26	1.21
time (sec)	N/A	0.054	0.014	1.764	0.233	0.245	0.022	0.265	0.048

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	0.96
time (sec)	N/A	0.024	0.010	1.793	0.260	0.251	0.022	0.457	0.047

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	51	51	52	49	49	53	48
N.S.	1	1.00	1.19	1.19	1.21	1.14	1.14	1.23	1.12
time (sec)	N/A	0.027	0.019	1.714	0.212	0.237	0.061	0.370	0.042

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	49	48	53	48	48	48
N.S.	1	1.00	1.00	1.02	1.00	1.10	1.00	1.00	1.00
time (sec)	N/A	0.025	0.021	1.724	0.207	0.242	0.059	0.366	0.051

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	49	48	52	54	48	70	48
N.S.	1	1.00	0.96	0.94	1.02	1.06	0.94	1.37	0.94
time (sec)	N/A	0.035	0.028	1.807	0.217	0.245	0.114	0.541	0.048

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	50	46	50	52	51	50	50
N.S.	1	1.00	1.04	0.96	1.04	1.08	1.06	1.04	1.04
time (sec)	N/A	0.026	0.022	1.726	0.231	0.254	0.123	0.277	0.050

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	50	46	54	55	51	72	51
N.S.	1	1.00	0.98	0.90	1.06	1.08	1.00	1.41	1.00
time (sec)	N/A	0.033	0.032	1.743	0.255	0.391	0.281	0.284	9.035

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	51	53	54	53	50
N.S.	1	1.00	1.00	0.94	1.06	1.10	1.12	1.10	1.04
time (sec)	N/A	0.026	0.023	1.713	0.202	0.478	0.313	0.289	9.035

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	53	46	55	55	56	66	51
N.S.	1	1.00	1.04	0.90	1.08	1.08	1.10	1.29	1.00
time (sec)	N/A	0.031	0.030	1.705	0.209	0.365	0.529	0.289	9.174

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	59	48	53	53	58	55	52
N.S.	1	1.00	1.11	0.91	1.00	1.00	1.09	1.04	0.98
time (sec)	N/A	0.026	0.020	1.729	0.207	0.311	0.570	0.291	9.001

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	76	73	73	80	77	69
N.S.	1	1.00	1.00	1.01	0.97	0.97	1.07	1.03	0.92
time (sec)	N/A	0.043	0.016	1.781	0.219	0.262	0.026	0.291	0.039

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	69	76	73	73	82	77	69
N.S.	1	1.00	1.01	1.12	1.07	1.07	1.21	1.13	1.01
time (sec)	N/A	0.099	0.021	1.767	0.224	0.251	0.028	0.286	0.032

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	76	73	73	82	77	69
N.S.	1	1.00	1.00	1.01	0.97	0.97	1.09	1.03	0.92
time (sec)	N/A	0.037	0.013	1.792	0.215	0.260	0.028	0.282	0.032

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	69	76	73	73	80	77	69
N.S.	1	1.00	1.64	1.81	1.74	1.74	1.90	1.83	1.64
time (sec)	N/A	0.049	0.021	1.773	0.219	0.270	0.028	0.288	0.032

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	73	70	70	76	73	65
N.S.	1	1.00	1.00	1.04	1.00	1.00	1.09	1.04	0.93
time (sec)	N/A	0.029	0.013	1.794	0.217	0.233	0.024	0.281	0.031

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	71	76	74	71	80	78	67
N.S.	1	1.00	1.18	1.27	1.23	1.18	1.33	1.30	1.12
time (sec)	N/A	0.036	0.023	1.722	0.206	0.248	0.079	0.292	0.037

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	71	69	75	68	70	65
N.S.	1	1.00	1.00	1.09	1.06	1.15	1.05	1.08	1.00
time (sec)	N/A	0.031	0.026	1.730	0.209	0.245	0.077	0.286	0.034

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	73	73	74	77	78	97	67
N.S.	1	1.00	1.03	1.03	1.04	1.08	1.10	1.37	0.94
time (sec)	N/A	0.056	0.033	1.764	0.212	0.238	0.128	0.269	0.041

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	71	70	73	75	75	74	68
N.S.	1	1.00	1.03	1.01	1.06	1.09	1.09	1.07	0.99
time (sec)	N/A	0.034	0.026	1.909	0.206	0.256	0.138	0.267	0.058

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	73	68	76	76	75	98	76
N.S.	1	1.00	1.01	0.94	1.06	1.06	1.04	1.36	1.06
time (sec)	N/A	0.050	0.034	1.879	0.238	0.243	0.306	0.276	8.875

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	64	73	75	78	75	73
N.S.	1	1.00	1.00	0.94	1.07	1.10	1.15	1.10	1.07
time (sec)	N/A	0.035	0.028	1.916	0.206	0.238	0.339	0.279	0.057

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	71	64	77	77	78	99	75
N.S.	1	1.00	1.00	0.90	1.08	1.08	1.10	1.39	1.06
time (sec)	N/A	0.045	0.040	2.157	0.233	0.236	0.667	0.284	0.067

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	66	63	73	75	80	77	71
N.S.	1	1.00	1.00	0.95	1.11	1.14	1.21	1.17	1.08
time (sec)	N/A	0.034	0.031	2.116	0.237	0.234	0.769	0.283	8.912

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	77	64	77	77	82	90	75
N.S.	1	1.00	1.22	1.02	1.22	1.22	1.30	1.43	1.19
time (sec)	N/A	0.033	0.036	1.873	0.222	0.236	1.272	0.276	0.082

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	66	75	75	83	79	74
N.S.	1	1.00	1.00	0.90	1.03	1.03	1.14	1.08	1.01
time (sec)	N/A	0.034	0.031	1.862	0.214	0.243	2.469	0.276	8.867

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	78	66	75	75	83	79	76
N.S.	1	1.00	1.59	1.35	1.53	1.53	1.69	1.61	1.55
time (sec)	N/A	0.027	0.021	1.856	0.198	0.235	3.521	0.291	8.907

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	119	123	124	274	204	133	144
N.S.	1	1.00	1.00	1.03	1.04	2.30	1.71	1.12	1.21
time (sec)	N/A	0.062	0.081	1.781	0.297	0.251	0.237	0.297	0.055

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	92	92	97	98	94	101	100
N.S.	1	1.00	0.96	0.96	1.01	1.02	0.98	1.05	1.04
time (sec)	N/A	0.090	0.045	1.772	0.231	0.250	0.190	0.292	8.933

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	98	99	100	228	180	108	118
N.S.	1	1.00	1.00	1.01	1.02	2.33	1.84	1.10	1.20
time (sec)	N/A	0.050	0.068	1.767	0.301	0.251	0.218	0.301	0.042

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	71	73	74	75	70	77	76
N.S.	1	1.00	0.95	0.97	0.99	1.00	0.93	1.03	1.01
time (sec)	N/A	0.067	0.032	1.754	0.238	0.244	0.179	0.276	0.060

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	75	78	178	153	85	96
N.S.	1	1.00	1.00	0.97	1.01	2.31	1.99	1.10	1.25
time (sec)	N/A	0.042	0.055	1.821	0.309	0.246	0.196	0.271	0.067

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	47	50	50	51	46	52	52
N.S.	1	1.00	0.87	0.93	0.93	0.94	0.85	0.96	0.96
time (sec)	N/A	0.046	0.023	1.821	0.216	0.240	0.154	0.271	0.071

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	57	51	53	129	90	57	70
N.S.	1	1.00	0.98	0.88	0.91	2.22	1.55	0.98	1.21
time (sec)	N/A	0.032	0.041	1.781	0.319	0.261	0.178	0.289	8.986

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	31	32	31	30	27	32	31
N.S.	1	1.00	0.89	0.91	0.89	0.86	0.77	0.91	0.89
time (sec)	N/A	0.031	0.014	1.745	0.217	0.234	0.130	0.273	0.057

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	34	34	99	82	34	31
N.S.	1	1.00	1.00	0.85	0.85	2.48	2.05	0.85	0.78
time (sec)	N/A	0.019	0.023	1.849	0.325	0.255	0.144	0.274	0.054

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	35	32	26	34	32
N.S.	1	1.00	1.00	0.97	1.03	0.94	0.76	1.00	0.94
time (sec)	N/A	0.027	0.016	2.074	0.208	0.230	0.392	0.288	8.984

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	36	105	82	36	35
N.S.	1	1.00	1.00	0.88	0.86	2.50	1.95	0.86	0.83
time (sec)	N/A	0.021	0.029	1.800	0.304	0.238	0.169	0.280	8.966

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	39	51	103	75	38	33
N.S.	1	1.00	1.00	0.95	1.24	2.51	1.83	0.93	0.80
time (sec)	N/A	0.023	0.028	1.789	0.298	0.240	0.183	0.286	0.112

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	46	48	47	41	71	46
N.S.	1	1.00	1.00	0.94	0.98	0.96	0.84	1.45	0.94
time (sec)	N/A	0.039	0.025	1.729	0.226	0.268	0.401	0.272	0.108

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	60	54	56	135	129	57	53
N.S.	1	1.00	0.98	0.89	0.92	2.21	2.11	0.93	0.87
time (sec)	N/A	0.033	0.058	1.946	0.298	0.254	0.212	0.273	8.991

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	64	70	73	61	100	70
N.S.	1	1.00	1.00	0.91	1.00	1.04	0.87	1.43	1.00
time (sec)	N/A	0.054	0.033	1.962	0.217	0.237	0.454	0.293	8.940

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	78	74	79	184	163	81	70
N.S.	1	1.00	1.00	0.95	1.01	2.36	2.09	1.04	0.90
time (sec)	N/A	0.044	0.055	1.991	0.287	0.446	0.250	0.279	8.942

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	96	86	96	98	88	126	92
N.S.	1	1.00	1.04	0.93	1.04	1.07	0.96	1.37	1.00
time (sec)	N/A	0.067	0.042	1.840	0.216	0.384	0.496	0.275	8.944

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	134	123	136	350	238	139	203
N.S.	1	1.00	1.01	0.92	1.02	2.63	1.79	1.05	1.53
time (sec)	N/A	0.116	0.113	1.802	0.342	0.548	0.419	0.280	8.881

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	93	103	107	148	104	135	121
N.S.	1	1.00	0.89	0.98	1.02	1.41	0.99	1.29	1.15
time (sec)	N/A	0.093	0.074	1.804	0.215	0.526	0.392	0.278	0.066

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	111	100	112	298	211	115	141
N.S.	1	1.00	1.01	0.91	1.02	2.71	1.92	1.05	1.28
time (sec)	N/A	0.080	0.092	1.827	0.288	0.252	0.387	0.273	8.857

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	72	76	82	121	78	106	86
N.S.	1	1.00	0.87	0.92	0.99	1.46	0.94	1.28	1.04
time (sec)	N/A	0.069	0.061	1.881	0.201	0.245	0.366	0.282	0.074

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	75	85	240	129	88	104
N.S.	1	1.00	1.00	0.84	0.96	2.70	1.45	0.99	1.17
time (sec)	N/A	0.070	0.078	1.779	0.289	0.264	0.350	0.286	8.888

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	50	59	60	81	56	70	62
N.S.	1	1.00	0.82	0.97	0.98	1.33	0.92	1.15	1.02
time (sec)	N/A	0.047	0.043	1.763	0.201	0.279	0.311	0.288	0.075

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	57	61	208	114	59	59
N.S.	1	1.00	1.00	0.84	0.90	3.06	1.68	0.87	0.87
time (sec)	N/A	0.042	0.056	1.843	0.294	0.247	0.274	0.277	8.893

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	38	40	44	36	37	37
N.S.	1	1.00	1.00	0.93	0.98	1.07	0.88	0.90	0.90
time (sec)	N/A	0.033	0.015	1.789	0.205	0.237	0.189	0.276	8.872

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	57	57	182	112	57	51
N.S.	1	1.00	1.00	0.90	0.90	2.89	1.78	0.90	0.81
time (sec)	N/A	0.022	0.049	1.956	0.302	0.374	0.214	0.295	8.921

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	46	48	51	70	46	52	47
N.S.	1	1.00	0.90	0.94	1.00	1.37	0.90	1.02	0.92
time (sec)	N/A	0.041	0.035	2.024	0.200	0.371	0.227	0.271	0.126

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	62	63	210	114	62	63
N.S.	1	1.00	1.00	0.89	0.90	3.00	1.63	0.89	0.90
time (sec)	N/A	0.046	0.036	1.822	0.295	0.396	0.262	0.287	9.029

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	64	76	76	117	70	80	78
N.S.	1	1.00	0.88	1.04	1.04	1.60	0.96	1.10	1.07
time (sec)	N/A	0.056	0.058	1.754	0.215	0.504	0.532	0.303	8.937

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	78	93	250	184	85	83
N.S.	1	1.00	1.00	0.87	1.03	2.78	2.04	0.94	0.92
time (sec)	N/A	0.081	0.080	1.802	0.285	0.284	0.311	0.271	8.946

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	85	96	106	154	100	150	100
N.S.	1	1.00	0.88	0.99	1.09	1.59	1.03	1.55	1.03
time (sec)	N/A	0.076	0.107	1.737	0.213	0.256	0.587	0.293	8.952

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	112	99	119	308	218	112	104
N.S.	1	1.00	1.01	0.89	1.07	2.77	1.96	1.01	0.94
time (sec)	N/A	0.140	0.090	1.833	0.289	0.270	0.357	0.280	8.992

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	133	119	147	416	252	138	177
N.S.	1	1.00	0.95	0.85	1.05	2.97	1.80	0.99	1.26
time (sec)	N/A	0.163	0.119	1.788	0.304	0.266	0.758	0.273	0.087

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	94	102	116	179	119	132	118
N.S.	1	1.00	0.85	0.92	1.05	1.61	1.07	1.19	1.06
time (sec)	N/A	0.100	0.070	1.756	0.216	0.251	0.807	0.285	8.946

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	113	95	120	358	214	111	138
N.S.	1	1.00	0.96	0.81	1.02	3.03	1.81	0.94	1.17
time (sec)	N/A	0.111	0.096	1.817	0.290	0.263	0.683	0.281	8.973

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	92	82	94	142	94	93	95
N.S.	1	1.00	1.03	0.92	1.06	1.60	1.06	1.04	1.07
time (sec)	N/A	0.072	0.039	1.765	0.229	0.252	0.694	0.288	0.092

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	92	77	94	328	194	80	92
N.S.	1	1.00	0.97	0.81	0.99	3.45	2.04	0.84	0.97
time (sec)	N/A	0.068	0.076	1.767	0.282	0.257	0.584	0.304	9.016

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	64	57	72	89	70	55	70
N.S.	1	1.00	0.96	0.85	1.07	1.33	1.04	0.82	1.04
time (sec)	N/A	0.055	0.031	1.752	0.212	0.393	0.494	0.292	8.921

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	83	76	92	301	155	78	82
N.S.	1	1.00	0.92	0.84	1.02	3.34	1.72	0.87	0.91
time (sec)	N/A	0.050	0.088	1.773	0.289	0.465	0.408	0.290	0.112

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	30	29	42	42	42	28	44
N.S.	1	1.00	0.94	0.91	1.31	1.31	1.31	0.88	1.38
time (sec)	N/A	0.023	0.017	1.780	0.211	0.427	0.278	0.279	8.866

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	84	77	92	300	150	78	82
N.S.	1	1.00	0.91	0.84	1.00	3.26	1.63	0.85	0.89
time (sec)	N/A	0.032	0.064	1.836	0.286	0.401	0.302	0.286	8.967

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	59	61	77	119	75	76	71
N.S.	1	1.00	0.87	0.90	1.13	1.75	1.10	1.12	1.04
time (sec)	N/A	0.052	0.053	1.822	0.201	0.239	0.294	0.288	0.145

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	96	82	96	324	194	82	113
N.S.	1	1.00	1.00	0.85	1.00	3.38	2.02	0.85	1.18
time (sec)	N/A	0.082	0.066	1.836	0.294	0.267	0.358	0.282	8.984

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	86	100	109	197	107	105	107
N.S.	1	1.00	0.89	1.03	1.12	2.03	1.10	1.08	1.10
time (sec)	N/A	0.078	0.064	1.777	0.204	0.243	0.594	0.274	8.976

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	119	98	128	368	226	108	114
N.S.	1	1.00	1.02	0.84	1.09	3.15	1.93	0.92	0.97
time (sec)	N/A	0.130	0.075	1.789	0.293	0.266	0.401	0.281	9.045

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	108	123	137	229	136	132	131
N.S.	1	1.00	0.89	1.02	1.13	1.89	1.12	1.09	1.08
time (sec)	N/A	0.096	0.084	1.789	0.207	0.249	0.668	0.280	0.132

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	140	119	154	426	260	135	135
N.S.	1	1.00	1.00	0.85	1.10	3.04	1.86	0.96	0.96
time (sec)	N/A	0.230	0.082	1.839	0.288	0.281	0.447	0.299	9.128

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	135	143	170	267	165	201	155
N.S.	1	1.00	0.91	0.97	1.15	1.80	1.11	1.36	1.05
time (sec)	N/A	0.126	0.129	1.809	0.214	0.427	0.697	0.290	9.113

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	189	178	321	368	345	245	289
N.S.	1	1.00	0.87	0.82	1.47	1.69	1.58	1.12	1.33
time (sec)	N/A	0.250	1.399	1.873	0.223	0.533	0.768	0.297	10.298

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	170	159	273	321	318	211	233
N.S.	1	1.00	0.94	0.88	1.51	1.77	1.76	1.17	1.29
time (sec)	N/A	0.214	1.052	1.804	0.221	0.574	0.714	0.294	9.819

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	147	140	225	272	287	177	177
N.S.	1	1.00	1.18	1.12	1.80	2.18	2.30	1.42	1.42
time (sec)	N/A	0.129	0.886	1.809	0.209	0.539	0.680	0.296	9.624

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	126	116	177	223	255	140	140
N.S.	1	1.00	1.18	1.08	1.65	2.08	2.38	1.31	1.31
time (sec)	N/A	0.103	0.615	2.114	0.211	0.269	0.626	0.297	9.673

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	97	91	128	172	0	103	117
N.S.	1	1.00	0.97	0.91	1.28	1.72	0.00	1.03	1.17
time (sec)	N/A	0.129	0.189	2.046	0.215	0.264	0.000	0.284	9.428

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	87	83	105	161	0	87	0
N.S.	1	1.00	0.90	0.86	1.08	1.66	0.00	0.90	0.00
time (sec)	N/A	0.133	0.263	1.845	0.211	0.282	0.000	0.327	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	99	86	96	160	0	163	0
N.S.	1	1.00	1.24	1.08	1.20	2.00	0.00	2.04	0.00
time (sec)	N/A	0.125	0.164	2.038	0.215	0.277	0.000	0.442	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	44	47	111	59	0	250	113
N.S.	1	1.00	0.72	0.77	1.82	0.97	0.00	4.10	1.85
time (sec)	N/A	0.101	0.164	2.007	0.216	0.268	0.000	0.696	9.256

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	66	66	161	85	0	310	160
N.S.	1	1.00	0.69	0.69	1.68	0.89	0.00	3.23	1.67
time (sec)	N/A	0.141	0.191	1.807	0.215	0.294	0.000	0.891	9.428

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	88	84	209	109	0	370	210
N.S.	1	1.00	0.66	0.63	1.57	0.82	0.00	2.78	1.58
time (sec)	N/A	0.160	0.245	1.795	0.213	0.289	0.000	1.404	9.760

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	110	104	257	133	0	430	260
N.S.	1	1.00	0.65	0.61	1.51	0.78	0.00	2.53	1.53
time (sec)	N/A	0.195	0.260	1.944	0.218	0.488	0.000	1.854	10.218

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	82	91	106	106	0	140	103
N.S.	1	1.00	0.63	0.69	0.81	0.81	0.00	1.07	0.79
time (sec)	N/A	0.142	0.088	2.415	0.226	0.426	0.000	0.294	9.127

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	64	67	83	82	0	105	83
N.S.	1	1.00	0.68	0.71	0.88	0.87	0.00	1.12	0.88
time (sec)	N/A	0.102	0.070	2.172	0.220	0.396	0.000	0.315	9.026

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	41	45	51	57	0	72	60
N.S.	1	1.00	0.67	0.74	0.84	0.93	0.00	1.18	0.98
time (sec)	N/A	0.017	0.047	2.015	0.223	0.377	0.000	0.281	8.967

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	84	84	0	159	0	116	99
N.S.	1	1.00	1.08	1.08	0.00	2.04	0.00	1.49	1.27
time (sec)	N/A	0.095	0.132	2.010	0.000	0.328	0.000	0.286	9.457

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	94	95	0	169	0	76	0
N.S.	1	1.00	0.94	0.95	0.00	1.69	0.00	0.76	0.00
time (sec)	N/A	0.098	0.158	2.152	0.000	0.281	0.000	0.303	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	106	103	0	198	0	132	0
N.S.	1	1.00	1.03	1.00	0.00	1.92	0.00	1.28	0.00
time (sec)	N/A	0.099	0.211	2.307	0.000	0.289	0.000	0.298	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	248	197	363	418	726	280	0
N.S.	1	1.00	1.11	0.88	1.63	1.87	3.26	1.26	0.00
time (sec)	N/A	0.256	1.784	1.792	0.216	0.412	1.175	0.302	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	226	178	315	369	672	246	0
N.S.	1	1.00	1.35	1.07	1.89	2.21	4.02	1.47	0.00
time (sec)	N/A	0.158	1.388	1.797	0.227	0.363	1.126	0.300	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	199	159	267	316	614	207	236
N.S.	1	1.00	1.34	1.07	1.80	2.14	4.15	1.40	1.59
time (sec)	N/A	0.132	1.137	1.859	0.220	0.308	1.013	0.293	9.865

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	157	140	216	275	551	178	0
N.S.	1	1.00	1.09	0.97	1.50	1.91	3.83	1.24	0.00
time (sec)	N/A	0.171	0.887	1.830	0.218	0.285	4.144	0.297	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	124	116	168	224	0	142	0
N.S.	1	1.00	0.91	0.85	1.23	1.64	0.00	1.04	0.00
time (sec)	N/A	0.184	0.292	1.792	0.217	0.293	0.000	0.289	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	116	101	148	209	0	121	0
N.S.	1	1.00	0.91	0.79	1.16	1.63	0.00	0.95	0.00
time (sec)	N/A	0.185	0.490	1.834	0.226	0.429	0.000	0.343	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	120	101	167	189	0	225	0
N.S.	1	1.00	0.88	0.74	1.23	1.39	0.00	1.65	0.00
time (sec)	N/A	0.170	0.339	1.854	0.215	0.429	0.000	0.450	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	110	107	177	207	0	254	0
N.S.	1	1.00	1.06	1.03	1.70	1.99	0.00	2.44	0.00
time (sec)	N/A	0.159	0.278	1.892	0.208	0.429	0.000	0.830	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	44	48	193	82	0	370	156
N.S.	1	1.00	0.72	0.79	3.16	1.34	0.00	6.07	2.56
time (sec)	N/A	0.114	0.251	1.827	0.234	0.756	0.000	1.188	9.820

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	66	68	241	109	0	430	206
N.S.	1	1.00	0.69	0.71	2.51	1.14	0.00	4.48	2.15
time (sec)	N/A	0.147	0.299	1.832	0.223	0.296	0.000	1.537	10.241

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	89	87	289	134	0	490	256
N.S.	1	1.00	0.67	0.65	2.17	1.01	0.00	3.68	1.92
time (sec)	N/A	0.183	0.370	1.804	0.208	0.320	0.000	1.758	10.817

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	110	106	337	157	0	550	306
N.S.	1	1.00	0.65	0.62	1.98	0.92	0.00	3.24	1.80
time (sec)	N/A	0.228	0.422	1.845	0.216	0.380	0.000	2.163	11.415

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	132	124	385	181	0	582	356
N.S.	1	1.00	0.64	0.60	1.86	0.87	0.00	2.81	1.72
time (sec)	N/A	0.232	0.497	1.825	0.215	0.488	0.000	2.305	12.101

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	113	115	150	154	0	175	143
N.S.	1	1.00	0.67	0.68	0.89	0.92	0.00	1.04	0.85
time (sec)	N/A	0.186	0.129	2.209	0.264	0.251	0.000	0.294	9.254

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	94	91	128	131	0	140	124
N.S.	1	1.00	0.72	0.69	0.98	1.00	0.00	1.07	0.95
time (sec)	N/A	0.147	0.102	2.177	0.225	0.250	0.000	0.286	9.169

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	70	67	105	106	0	105	103
N.S.	1	1.00	0.73	0.70	1.09	1.10	0.00	1.09	1.07
time (sec)	N/A	0.048	0.079	2.027	0.217	0.263	0.000	0.296	9.059

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	41	45	80	80	0	72	83
N.S.	1	1.00	0.67	0.74	1.31	1.31	0.00	1.18	1.36
time (sec)	N/A	0.105	0.053	2.042	0.215	0.269	0.000	0.287	9.014

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	108	99	0	206	0	140	0
N.S.	1	1.00	1.06	0.97	0.00	2.02	0.00	1.37	0.00
time (sec)	N/A	0.136	0.190	2.375	0.000	0.286	0.000	0.288	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	109	149	0	195	0	115	0
N.S.	1	1.00	0.82	1.12	0.00	1.47	0.00	0.86	0.00
time (sec)	N/A	0.141	0.210	2.686	0.000	0.487	0.000	0.307	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	114	117	0	217	0	145	0
N.S.	1	1.00	0.84	0.87	0.00	1.61	0.00	1.07	0.00
time (sec)	N/A	0.137	0.240	2.413	0.000	0.515	0.000	0.318	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	132	128	0	250	0	175	0
N.S.	1	1.00	0.94	0.91	0.00	1.79	0.00	1.25	0.00
time (sec)	N/A	0.144	0.316	2.621	0.000	0.419	0.000	0.328	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	154	153	0	299	0	214	0
N.S.	1	1.00	0.87	0.86	0.00	1.69	0.00	1.21	0.00
time (sec)	N/A	0.186	0.403	2.829	0.000	0.591	0.000	0.322	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	172	176	0	345	0	234	0
N.S.	1	1.00	0.80	0.82	0.00	1.61	0.00	1.09	0.00
time (sec)	N/A	0.220	0.528	3.204	0.000	0.314	0.000	0.332	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	198	201	0	393	0	294	0
N.S.	1	1.00	0.79	0.80	0.00	1.57	0.00	1.17	0.00
time (sec)	N/A	0.239	0.541	3.637	0.000	0.357	0.000	0.328	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	155	140	231	275	224	184	0
N.S.	1	1.00	0.88	0.80	1.31	1.56	1.27	1.05	0.00
time (sec)	N/A	0.207	0.734	1.849	0.230	0.305	0.848	0.287	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	133	121	183	226	201	150	0
N.S.	1	1.00	0.96	0.87	1.32	1.63	1.45	1.08	0.00
time (sec)	N/A	0.173	0.618	1.839	0.244	0.293	0.795	0.304	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	108	100	134	177	170	112	0
N.S.	1	1.00	1.30	1.20	1.61	2.13	2.05	1.35	0.00
time (sec)	N/A	0.117	0.435	1.891	0.222	0.300	0.751	0.292	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	91	84	88	131	139	77	89
N.S.	1	1.00	1.38	1.27	1.33	1.98	2.11	1.17	1.35
time (sec)	N/A	0.080	0.254	1.833	0.243	0.308	0.705	0.291	9.618

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	77	67	56	136	0	66	57
N.S.	1	1.00	1.35	1.18	0.98	2.39	0.00	1.16	1.00
time (sec)	N/A	0.100	0.112	1.807	0.227	0.304	0.000	0.302	9.362

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	43	40	70	38	0	124	39
N.S.	1	1.00	0.70	0.66	1.15	0.62	0.00	2.03	0.64
time (sec)	N/A	0.106	0.127	1.777	0.219	0.273	0.000	0.393	9.091

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	64	63	119	62	0	180	62
N.S.	1	1.00	0.67	0.66	1.24	0.65	0.00	1.88	0.65
time (sec)	N/A	0.128	0.155	1.933	0.220	0.271	0.000	0.582	9.175

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	89	85	167	86	0	236	121
N.S.	1	1.00	0.67	0.64	1.26	0.65	0.00	1.77	0.91
time (sec)	N/A	0.160	0.195	1.808	0.253	0.286	0.000	0.734	9.206

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	110	104	215	110	0	292	156
N.S.	1	1.00	0.65	0.61	1.26	0.65	0.00	1.72	0.92
time (sec)	N/A	0.186	0.228	1.882	0.208	0.315	0.000	0.978	9.253

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	85	84	106	83	0	130	87
N.S.	1	1.00	0.65	0.64	0.81	0.63	0.00	0.99	0.66
time (sec)	N/A	0.151	0.096	2.403	0.248	0.265	0.000	0.289	9.192

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	63	60	83	59	0	98	64
N.S.	1	1.00	0.67	0.64	0.88	0.63	0.00	1.04	0.68
time (sec)	N/A	0.127	0.073	2.243	0.210	0.270	0.000	0.281	9.210

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	40	37	50	36	0	67	41
N.S.	1	1.00	0.68	0.63	0.85	0.61	0.00	1.14	0.69
time (sec)	N/A	0.091	0.050	2.138	0.215	0.277	0.000	0.287	9.115

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	73	72	0	138	0	80	0
N.S.	1	1.00	1.33	1.31	0.00	2.51	0.00	1.45	0.00
time (sec)	N/A	0.014	0.077	2.057	0.000	0.265	0.000	0.296	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	86	90	0	152	0	66	0
N.S.	1	1.00	1.26	1.32	0.00	2.24	0.00	0.97	0.00
time (sec)	N/A	0.075	0.132	2.051	0.000	0.258	0.000	0.299	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	104	110	0	199	0	125	0
N.S.	1	1.00	1.01	1.07	0.00	1.93	0.00	1.21	0.00
time (sec)	N/A	0.107	0.192	2.156	0.000	0.279	0.000	0.318	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	156	145	237	340	0	183	0
N.S.	1	1.00	0.85	0.79	1.29	1.85	0.00	0.99	0.00
time (sec)	N/A	0.223	0.761	1.889	0.220	0.280	0.000	0.295	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	132	140	187	289	0	145	0
N.S.	1	1.00	0.90	0.95	1.27	1.97	0.00	0.99	0.00
time (sec)	N/A	0.182	0.551	1.921	0.207	0.282	0.000	0.287	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	108	114	138	230	0	104	0
N.S.	1	1.00	0.96	1.02	1.23	2.05	0.00	0.93	0.00
time (sec)	N/A	0.159	0.344	1.837	0.204	0.276	0.000	0.305	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	77	75	79	188	0	70	78
N.S.	1	1.00	1.15	1.12	1.18	2.81	0.00	1.04	1.16
time (sec)	N/A	0.108	0.138	1.819	0.211	0.284	0.000	0.308	9.460

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	37	65	49	0	65	53
N.S.	1	1.00	1.00	1.00	1.76	1.32	0.00	1.76	1.43
time (sec)	N/A	0.077	0.112	1.825	0.218	0.274	0.000	0.331	8.975

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	64	59	112	72	0	189	70
N.S.	1	1.00	0.97	0.89	1.70	1.09	0.00	2.86	1.06
time (sec)	N/A	0.104	0.161	2.108	0.221	0.277	0.000	0.499	9.083

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	85	80	160	98	0	302	95
N.S.	1	1.00	0.84	0.79	1.58	0.97	0.00	2.99	0.94
time (sec)	N/A	0.141	0.217	2.032	0.222	0.295	0.000	0.917	9.246

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	108	97	208	121	0	415	173
N.S.	1	1.00	0.78	0.70	1.51	0.88	0.00	3.01	1.25
time (sec)	N/A	0.166	0.252	1.796	0.209	0.266	0.000	1.282	9.406

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	82	91	82	93	0	145	92
N.S.	1	1.00	0.59	0.65	0.59	0.67	0.00	1.04	0.66
time (sec)	N/A	0.171	0.095	2.580	0.226	0.279	0.000	0.285	9.263

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	60	66	59	68	0	107	67
N.S.	1	1.00	0.58	0.63	0.57	0.65	0.00	1.03	0.64
time (sec)	N/A	0.124	0.069	2.401	0.235	0.291	0.000	0.279	9.159

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	35	44	39	45	0	63	44
N.S.	1	1.00	0.51	0.64	0.57	0.65	0.00	0.91	0.64
time (sec)	N/A	0.093	0.054	2.250	0.213	0.298	0.000	0.289	9.097

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	73	79	0	199	0	109	0
N.S.	1	1.00	1.14	1.23	0.00	3.11	0.00	1.70	0.00
time (sec)	N/A	0.091	0.109	2.133	0.000	0.285	0.000	0.277	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	96	126	0	260	0	107	0
N.S.	1	1.00	0.68	0.89	0.00	1.83	0.00	0.75	0.00
time (sec)	N/A	0.058	0.182	2.083	0.000	0.301	0.000	0.297	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	117	153	0	315	0	149	89
N.S.	1	1.00	0.85	1.12	0.00	2.30	0.00	1.09	0.65
time (sec)	N/A	0.123	0.257	2.106	0.000	0.283	0.000	0.320	10.229

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.016	0.039	0.223	0.200	0.253	0.905	0.282	0.056

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.016	0.038	0.200	0.210	0.249	0.593	0.276	9.046

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	41	28	27	32	46	29	31
N.S.	1	1.00	1.05	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.016	0.035	0.200	0.213	0.247	0.396	0.280	0.044

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	37	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	0.95	0.74	0.79
time (sec)	N/A	0.016	0.035	0.203	0.190	0.276	0.579	0.265	0.045

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.015	0.032	0.205	0.209	0.287	0.247	0.264	9.004

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	30	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.77	1.18	0.74	0.79
time (sec)	N/A	0.014	0.030	0.185	0.194	0.281	0.270	0.286	0.045

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	28	27	29	44	29	31
N.S.	1	1.00	0.95	0.76	0.73	0.78	1.19	0.78	0.84
time (sec)	N/A	0.015	0.034	0.191	0.208	0.259	0.307	0.281	0.045

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	30	27	29	44	29	31
N.S.	1	1.00	0.95	0.81	0.73	0.78	1.19	0.78	0.84
time (sec)	N/A	0.015	0.038	0.050	0.200	0.264	0.444	0.276	9.014

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	61	52	51	56	80	53	51
N.S.	1	1.00	0.97	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.027	0.057	1.790	0.188	0.267	1.874	0.279	8.998

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	61	52	51	56	80	53	51
N.S.	1	1.00	0.97	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.029	0.058	1.813	0.205	0.250	1.294	0.285	0.050

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	61	52	51	56	80	53	51
N.S.	1	1.00	0.97	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.027	0.053	1.789	0.199	0.278	0.962	0.276	0.051

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	61	52	51	56	66	53	51
N.S.	1	1.00	0.97	0.83	0.81	0.89	1.05	0.84	0.81
time (sec)	N/A	0.027	0.051	1.801	0.197	0.264	1.056	0.272	0.050

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	61	52	51	56	80	53	51
N.S.	1	1.00	0.97	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.027	0.051	1.802	0.243	0.255	0.626	0.277	0.050

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	61	52	51	56	80	53	51
N.S.	1	1.00	0.97	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.026	0.054	1.793	0.197	0.248	0.664	0.286	0.050

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	61	52	51	56	80	53	51
N.S.	1	1.00	0.97	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.025	0.052	1.766	0.207	0.285	0.718	0.271	0.051

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	52	51	54	80	53	51
N.S.	1	1.00	0.95	0.83	0.81	0.86	1.27	0.84	0.81
time (sec)	N/A	0.028	0.050	1.928	0.189	0.254	0.972	0.276	0.056

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	76	73	78	114	77	69
N.S.	1	1.00	1.00	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.034	0.121	1.838	0.218	0.269	3.359	0.274	0.041

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	76	73	78	114	77	69
N.S.	1	1.00	1.00	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.035	0.113	2.058	0.211	0.257	2.512	0.268	0.034

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	97	76	73	78	114	77	69
N.S.	1	1.00	1.14	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.038	0.076	2.070	0.236	0.276	1.789	0.278	0.034

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	83	76	73	78	95	77	69
N.S.	1	1.00	0.98	0.89	0.86	0.92	1.12	0.91	0.81
time (sec)	N/A	0.036	0.076	1.893	0.222	0.280	1.607	0.282	0.034

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	83	76	73	78	114	77	69
N.S.	1	1.00	0.98	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.038	0.077	1.813	0.192	0.249	1.398	0.279	0.034

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	83	76	73	78	114	77	69
N.S.	1	1.00	0.98	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.037	0.069	1.875	0.209	0.242	1.413	0.273	0.034

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	83	76	73	78	114	77	69
N.S.	1	1.00	0.98	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.037	0.068	1.807	0.205	0.247	1.563	0.269	0.035

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	83	76	73	78	114	77	69
N.S.	1	1.00	0.98	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.036	0.065	1.773	0.217	0.266	1.801	0.279	0.037

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	172	164	237	770	0	298	115
N.S.	1	1.00	0.62	0.59	0.85	2.77	0.00	1.07	0.41
time (sec)	N/A	0.195	0.365	1.839	0.283	0.295	0.000	0.311	9.150

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	173	162	259	647	0	298	788
N.S.	1	1.00	0.63	0.59	0.94	2.34	0.00	1.08	2.86
time (sec)	N/A	0.176	0.391	1.980	0.336	0.277	0.000	0.297	9.199

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	152	140	214	749	0	264	92
N.S.	1	1.00	0.59	0.54	0.83	2.91	0.00	1.03	0.36
time (sec)	N/A	0.151	0.282	1.773	0.286	0.273	0.000	0.302	9.111

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	150	138	235	597	275	263	789
N.S.	1	1.00	0.59	0.54	0.92	2.34	1.08	1.03	3.09
time (sec)	N/A	0.151	0.276	1.773	0.292	0.246	98.313	0.279	9.143

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	135	124	194	691	303	251	71
N.S.	1	1.00	0.57	0.52	0.82	2.92	1.28	1.06	0.30
time (sec)	N/A	0.132	0.254	1.952	0.281	0.269	37.344	0.287	0.162

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	134	127	218	570	238	251	739
N.S.	1	1.00	0.57	0.54	0.93	2.43	1.01	1.07	3.14
time (sec)	N/A	0.133	0.251	1.790	0.298	0.258	15.751	0.298	9.150

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	135	127	194	705	309	251	71
N.S.	1	1.00	0.57	0.54	0.83	3.00	1.31	1.07	0.30
time (sec)	N/A	0.139	0.315	1.754	0.293	0.260	8.341	0.303	9.058

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	136	124	218	590	257	251	811
N.S.	1	1.00	0.57	0.52	0.92	2.49	1.08	1.06	3.42
time (sec)	N/A	0.132	0.327	1.776	0.300	0.282	13.820	0.280	9.117

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	152	140	213	738	258	268	90
N.S.	1	1.00	0.60	0.55	0.84	2.89	1.01	1.05	0.35
time (sec)	N/A	0.155	0.330	1.789	0.296	0.261	73.160	0.305	9.050

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	154	141	247	643	303	257	555
N.S.	1	1.00	0.60	0.55	0.96	2.50	1.18	1.00	2.16
time (sec)	N/A	0.163	0.348	1.777	0.283	0.279	67.256	0.290	9.180

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	174	158	237	784	0	291	107
N.S.	1	1.00	0.63	0.57	0.86	2.84	0.00	1.05	0.39
time (sec)	N/A	0.168	0.404	1.789	0.298	0.269	0.000	0.290	9.056

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	176	160	276	672	0	291	563
N.S.	1	1.00	0.63	0.58	0.99	2.42	0.00	1.05	2.03
time (sec)	N/A	0.170	0.411	1.785	0.274	0.293	0.000	0.277	9.273

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	206	193	298	748	0	335	857
N.S.	1	1.00	0.62	0.58	0.90	2.25	0.00	1.01	2.58
time (sec)	N/A	0.205	1.039	1.833	0.294	0.266	0.000	0.292	9.198

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	184	170	247	848	0	299	127
N.S.	1	1.00	0.59	0.55	0.80	2.74	0.00	0.96	0.41
time (sec)	N/A	0.176	0.852	1.821	0.272	0.261	0.000	0.287	0.180

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	183	169	271	696	0	298	823
N.S.	1	1.00	0.59	0.55	0.87	2.25	0.00	0.96	2.65
time (sec)	N/A	0.164	0.846	1.808	0.308	0.257	0.000	0.297	9.200

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	162	153	223	793	0	283	106
N.S.	1	1.00	0.56	0.53	0.77	2.74	0.00	0.98	0.37
time (sec)	N/A	0.156	0.831	1.823	0.299	0.268	0.000	0.298	0.185

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	161	152	250	669	0	283	744
N.S.	1	1.00	0.56	0.53	0.87	2.31	0.00	0.98	2.57
time (sec)	N/A	0.153	0.851	1.826	0.318	0.282	0.000	0.272	0.222

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	152	146	217	776	0	273	91
N.S.	1	1.00	0.58	0.56	0.83	2.97	0.00	1.05	0.35
time (sec)	N/A	0.135	0.719	1.839	0.291	0.259	0.000	0.315	9.187

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	151	146	241	658	0	273	750
N.S.	1	1.00	0.58	0.56	0.92	2.52	0.00	1.05	2.87
time (sec)	N/A	0.136	0.700	2.172	0.291	0.257	0.000	0.321	9.265

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	162	153	222	789	0	278	104
N.S.	1	1.00	0.57	0.54	0.78	2.78	0.00	0.98	0.37
time (sec)	N/A	0.152	0.814	1.805	0.324	0.266	0.000	0.311	9.155

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	165	153	251	691	0	283	859
N.S.	1	1.00	0.57	0.53	0.87	2.39	0.00	0.98	2.97
time (sec)	N/A	0.155	0.819	1.807	0.307	0.283	0.000	0.358	9.329

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	187	170	250	838	0	303	121
N.S.	1	1.00	0.60	0.55	0.81	2.70	0.00	0.98	0.39
time (sec)	N/A	0.171	0.804	1.790	0.300	0.271	0.000	0.327	9.196

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	187	170	286	742	0	292	595
N.S.	1	1.00	0.60	0.55	0.92	2.39	0.00	0.94	1.92
time (sec)	N/A	0.173	0.861	1.804	0.301	0.295	0.000	0.308	9.322

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	209	190	276	886	0	328	142
N.S.	1	1.00	0.63	0.57	0.83	2.67	0.00	0.99	0.43
time (sec)	N/A	0.185	0.954	1.846	0.302	0.561	0.000	0.313	9.171

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	204	189	306	776	0	321	865
N.S.	1	1.00	0.59	0.55	0.89	2.26	0.00	0.94	2.52
time (sec)	N/A	0.190	0.865	1.898	0.292	0.545	0.000	0.296	9.223

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	184	173	256	872	0	304	138
N.S.	1	1.00	0.57	0.54	0.80	2.71	0.00	0.94	0.43
time (sec)	N/A	0.170	0.937	1.839	0.323	0.487	0.000	0.305	9.175

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	184	172	283	748	0	304	760
N.S.	1	1.00	0.57	0.53	0.88	2.32	0.00	0.94	2.36
time (sec)	N/A	0.179	0.917	1.924	0.279	0.880	0.000	0.279	0.225

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	171	166	251	871	0	293	122
N.S.	1	1.00	0.58	0.57	0.86	2.97	0.00	1.00	0.42
time (sec)	N/A	0.155	0.946	1.837	0.284	0.304	0.000	0.299	0.179

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	173	167	280	763	0	298	799
N.S.	1	1.00	0.58	0.56	0.94	2.56	0.00	1.00	2.68
time (sec)	N/A	0.151	0.938	1.806	0.277	0.303	0.000	0.289	0.327

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	175	168	253	878	0	298	124
N.S.	1	1.00	0.59	0.56	0.85	2.95	0.00	1.00	0.42
time (sec)	N/A	0.154	0.962	1.805	0.284	0.295	0.000	0.286	9.185

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	172	166	276	749	0	293	780
N.S.	1	1.00	0.59	0.57	0.94	2.56	0.00	1.00	2.66
time (sec)	N/A	0.149	0.917	1.789	0.291	0.422	0.000	0.285	9.348

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	187	173	255	870	0	300	133
N.S.	1	1.00	0.59	0.55	0.81	2.75	0.00	0.95	0.42
time (sec)	N/A	0.176	1.001	1.844	0.291	0.502	0.000	0.301	9.208

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	187	173	285	770	0	304	888
N.S.	1	1.00	0.58	0.54	0.89	2.39	0.00	0.94	2.76
time (sec)	N/A	0.171	0.889	1.810	0.284	0.480	0.000	0.279	9.411

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	209	190	285	918	0	326	152
N.S.	1	1.00	0.61	0.55	0.83	2.68	0.00	0.95	0.44
time (sec)	N/A	0.189	0.904	1.888	0.296	0.792	0.000	0.302	9.131

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	210	190	321	822	0	315	626
N.S.	1	1.00	0.61	0.55	0.94	2.40	0.00	0.92	1.83
time (sec)	N/A	0.198	0.906	1.841	0.291	0.272	0.000	0.297	9.387

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	231	210	311	966	0	351	173
N.S.	1	1.00	0.63	0.58	0.85	2.65	0.00	0.96	0.47
time (sec)	N/A	0.208	0.962	1.814	0.322	0.261	0.000	0.273	9.182

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	231	210	353	854	0	351	639
N.S.	1	1.00	0.63	0.58	0.97	2.34	0.00	0.96	1.75
time (sec)	N/A	0.212	0.970	1.913	0.295	0.303	0.000	0.294	9.315

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	136	241	0	123	0	0	0
N.S.	1	1.00	0.56	0.99	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.245	10.193	1.936	0.000	0.129	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	111	267	0	103	0	0	0
N.S.	1	1.00	0.30	0.72	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.283	10.160	1.868	0.000	0.109	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	111	217	0	98	0	0	0
N.S.	1	1.00	0.54	1.06	0.00	0.48	0.00	0.00	0.00
time (sec)	N/A	0.192	10.167	1.828	0.000	0.137	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	94	243	0	77	0	0	0
N.S.	1	1.00	0.29	0.75	0.00	0.24	0.00	0.00	0.00
time (sec)	N/A	0.237	10.094	1.875	0.000	0.156	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	94	193	0	74	0	0	0
N.S.	1	1.00	0.57	1.17	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.162	10.057	1.909	0.000	0.081	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	97	232	0	71	0	0	0
N.S.	1	1.00	0.30	0.72	0.00	0.22	0.00	0.00	0.00
time (sec)	N/A	0.235	10.047	2.157	0.000	0.087	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	97	180	0	61	0	0	0
N.S.	1	1.00	0.60	1.10	0.00	0.37	0.00	0.00	0.00
time (sec)	N/A	0.156	10.048	1.903	0.000	0.128	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	96	243	0	76	0	0	0
N.S.	1	1.00	0.29	0.74	0.00	0.23	0.00	0.00	0.00
time (sec)	N/A	0.257	10.048	1.901	0.000	0.087	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	98	194	0	71	0	0	0
N.S.	1	1.00	0.59	1.16	0.00	0.43	0.00	0.00	0.00
time (sec)	N/A	0.161	10.049	1.889	0.000	0.148	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	99	267	0	103	0	0	0
N.S.	1	1.00	0.27	0.72	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.280	10.061	1.890	0.000	0.104	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	98	218	0	96	0	0	0
N.S.	1	1.00	0.48	1.07	0.00	0.47	0.00	0.00	0.00
time (sec)	N/A	0.190	10.060	1.881	0.000	0.080	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	486	486	160	339	0	175	0	0	0
N.S.	1	1.00	0.33	0.70	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.444	10.245	1.904	0.000	0.106	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	160	289	0	170	0	0	0
N.S.	1	1.00	0.50	0.90	0.00	0.53	0.00	0.00	0.00
time (sec)	N/A	0.329	10.232	1.877	0.000	0.104	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	447	447	138	315	0	150	0	0	0
N.S.	1	1.00	0.31	0.70	0.00	0.34	0.00	0.00	0.00
time (sec)	N/A	0.383	10.194	1.889	0.000	0.174	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	138	265	0	147	0	0	0
N.S.	1	1.00	0.49	0.94	0.00	0.52	0.00	0.00	0.00
time (sec)	N/A	0.287	10.190	1.915	0.000	0.176	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	408	115	291	0	127	0	0	0
N.S.	1	1.00	0.28	0.71	0.00	0.31	0.00	0.00	0.00
time (sec)	N/A	0.328	10.161	1.899	0.000	0.117	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	115	241	0	121	0	0	0
N.S.	1	1.00	0.48	1.01	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.230	10.162	1.824	0.000	0.094	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	98	267	0	102	0	0	0
N.S.	1	1.00	0.27	0.72	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.283	10.124	1.878	0.000	0.099	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	97	217	0	97	0	0	0
N.S.	1	1.00	0.48	1.08	0.00	0.48	0.00	0.00	0.00
time (sec)	N/A	0.200	10.079	1.880	0.000	0.091	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	85	249	0	94	0	0	0
N.S.	1	1.00	0.24	0.70	0.00	0.26	0.00	0.00	0.00
time (sec)	N/A	0.300	10.078	1.849	0.000	0.152	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	101	200	0	86	0	0	0
N.S.	1	1.00	0.50	1.00	0.00	0.43	0.00	0.00	0.00
time (sec)	N/A	0.198	10.064	1.815	0.000	0.123	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	99	243	0	81	0	0	0
N.S.	1	1.00	0.28	0.69	0.00	0.23	0.00	0.00	0.00
time (sec)	N/A	0.289	10.051	1.842	0.000	0.090	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	101	196	0	75	0	0	0
N.S.	1	1.00	0.50	0.96	0.00	0.37	0.00	0.00	0.00
time (sec)	N/A	0.209	10.053	1.878	0.000	0.094	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	100	267	0	102	0	0	0
N.S.	1	1.00	0.27	0.73	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.291	10.063	1.864	0.000	0.095	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	143	239	0	122	0	0	0
N.S.	1	1.00	0.59	0.98	0.00	0.50	0.00	0.00	0.00
time (sec)	N/A	0.227	10.179	1.968	0.000	0.108	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	122	265	0	102	0	0	0
N.S.	1	1.00	0.33	0.72	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.283	10.155	2.059	0.000	0.219	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	122	215	0	99	0	0	0
N.S.	1	1.00	0.60	1.05	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.188	10.147	1.972	0.000	0.292	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	97	241	0	79	0	0	0
N.S.	1	1.00	0.29	0.73	0.00	0.24	0.00	0.00	0.00
time (sec)	N/A	0.242	10.130	2.096	0.000	0.093	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	97	191	0	73	0	0	0
N.S.	1	1.00	0.58	1.14	0.00	0.44	0.00	0.00	0.00
time (sec)	N/A	0.153	10.131	2.311	0.000	0.086	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	81	225	0	55	0	0	0
N.S.	1	1.00	0.28	0.77	0.00	0.19	0.00	0.00	0.00
time (sec)	N/A	0.202	10.108	2.023	0.000	0.082	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	80	175	0	51	0	0	0
N.S.	1	1.00	0.62	1.35	0.00	0.39	0.00	0.00	0.00
time (sec)	N/A	0.125	10.067	2.160	0.000	0.140	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	82	225	0	62	0	0	0
N.S.	1	1.00	0.29	0.80	0.00	0.22	0.00	0.00	0.00
time (sec)	N/A	0.198	10.054	2.008	0.000	0.127	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	82	177	0	57	0	0	0
N.S.	1	1.00	0.63	1.35	0.00	0.44	0.00	0.00	0.00
time (sec)	N/A	0.133	10.048	1.993	0.000	0.104	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	83	242	0	76	0	0	0
N.S.	1	1.00	0.25	0.73	0.00	0.23	0.00	0.00	0.00
time (sec)	N/A	0.262	10.060	2.040	0.000	0.083	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	85	193	0	69	0	0	0
N.S.	1	1.00	0.51	1.16	0.00	0.41	0.00	0.00	0.00
time (sec)	N/A	0.156	10.056	1.954	0.000	0.092	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	84	266	0	104	0	0	0
N.S.	1	1.00	0.23	0.72	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.290	10.058	2.026	0.000	0.084	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	84	216	0	96	0	0	0
N.S.	1	1.00	0.41	1.06	0.00	0.47	0.00	0.00	0.00
time (sec)	N/A	0.206	10.057	1.970	0.000	0.133	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	134	281	0	156	0	0	0
N.S.	1	1.00	0.53	1.12	0.00	0.62	0.00	0.00	0.00
time (sec)	N/A	0.248	10.155	2.309	0.000	0.169	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	110	420	0	134	0	0	0
N.S.	1	1.00	0.29	1.11	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.293	10.136	2.488	0.000	0.098	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	110	255	0	129	0	0	0
N.S.	1	1.00	0.51	1.19	0.00	0.60	0.00	0.00	0.00
time (sec)	N/A	0.221	10.136	2.355	0.000	0.093	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	85	394	0	106	0	0	0
N.S.	1	1.00	0.25	1.16	0.00	0.31	0.00	0.00	0.00
time (sec)	N/A	0.270	10.115	2.444	0.000	0.100	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	86	230	0	102	0	0	0
N.S.	1	1.00	0.48	1.29	0.00	0.57	0.00	0.00	0.00
time (sec)	N/A	0.181	10.111	2.281	0.000	0.088	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	78	388	0	98	0	0	0
N.S.	1	1.00	0.26	1.30	0.00	0.33	0.00	0.00	0.00
time (sec)	N/A	0.225	10.101	1.955	0.000	0.140	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	76	222	0	90	0	0	0
N.S.	1	1.00	0.55	1.62	0.00	0.66	0.00	0.00	0.00
time (sec)	N/A	0.146	10.077	1.915	0.000	0.148	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	77	392	0	115	0	0	0
N.S.	1	1.00	0.24	1.23	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.253	10.057	2.276	0.000	0.107	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	92	235	0	111	0	0	0
N.S.	1	1.00	0.55	1.41	0.00	0.66	0.00	0.00	0.00
time (sec)	N/A	0.168	10.059	2.212	0.000	0.088	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	368	79	420	0	134	0	0	0
N.S.	1	1.00	0.21	1.14	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.290	10.053	2.385	0.000	0.093	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	79	254	0	126	0	0	0
N.S.	1	1.00	0.39	1.25	0.00	0.62	0.00	0.00	0.00
time (sec)	N/A	0.206	10.052	2.319	0.000	0.094	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	405	79	450	0	163	0	0	0
N.S.	1	1.00	0.20	1.11	0.00	0.40	0.00	0.00	0.00
time (sec)	N/A	0.349	10.061	2.637	0.000	0.115	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	89	474	129	381	2077	603	291
N.S.	1	1.00	0.93	4.94	1.34	3.97	21.64	6.28	3.03
time (sec)	N/A	0.054	0.180	2.054	0.199	0.416	1.056	0.288	9.450

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	66	262	91	217	1051	340	179
N.S.	1	1.00	0.93	3.69	1.28	3.06	14.80	4.79	2.52
time (sec)	N/A	0.035	0.113	2.107	0.189	0.305	0.667	0.267	9.321

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	55	53	94	415	149	97
N.S.	1	1.00	0.93	1.22	1.18	2.09	9.22	3.31	2.16
time (sec)	N/A	0.021	0.079	0.079	0.192	0.268	0.395	0.283	9.197

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	55	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.033	0.118	0.000	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	92	80	0	0	0	0	0	0
N.S.	1	0.94	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.041	0.151	0.000	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	126	135	0	0	0	0	0	0
N.S.	1	0.90	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.092	0.156	0.000	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	63	0	112	140	0	0	0
N.S.	1	1.00	0.66	0.00	1.18	1.47	0.00	0.00	0.00
time (sec)	N/A	0.103	0.143	0.000	0.246	0.385	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	113	113	111	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.148	0.561	0.000	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	210	0	0	0	0	0	0
N.S.	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	1.313	0.000	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	15	15	17	17	13
N.S.	1	1.00	1.00	0.74	0.79	0.79	0.89	0.89	0.68
time (sec)	N/A	0.020	0.008	1.876	0.281	0.246	0.043	0.266	0.090

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	35	11	10	16	11
N.S.	1	1.00	1.00	0.80	2.33	0.73	0.67	1.07	0.73
time (sec)	N/A	0.016	0.006	1.863	0.283	0.239	0.051	0.276	0.071

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	16	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.82	0.94	0.76
time (sec)	N/A	0.017	0.008	1.871	0.276	0.246	0.058	0.263	9.156

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	18	10	16	14
N.S.	1	1.00	1.00	0.94	0.88	1.12	0.62	1.00	0.88
time (sec)	N/A	0.012	0.005	1.882	0.187	0.259	0.045	0.269	0.047

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	52	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.040	0.144	0.000	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	116	0	37	43	0	48	0
N.S.	1	1.00	6.44	0.00	2.06	2.39	0.00	2.67	0.00
time (sec)	N/A	0.024	0.465	0.000	0.269	0.277	0.000	0.331	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	195	157	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.156	0.331	0.000	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	119	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.068	0.239	0.000	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	97	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	0.193	0.000	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.024	0.193	0.000	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	77	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.037	0.213	0.000	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	115	112	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.069	0.301	0.000	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	207	207	184	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.110	0.401	0.000	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.047	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	202	202	179	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.158	0.345	0.000	0.000	0.000	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	150	120	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.080	0.286	0.000	0.000	0.000	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.023	0.177	0.000	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	88	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	0.220	0.000	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	125	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.093	0.258	0.000	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	217	182	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.169	0.382	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [207] had the largest ratio of [.4230999999999976]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	22	0.091
2	A	4	3	1.00	20	0.150
3	A	3	2	1.00	19	0.105
4	A	4	3	1.00	22	0.136
5	A	3	2	1.00	22	0.091
6	A	4	3	1.00	22	0.136
7	A	3	2	1.00	22	0.091
8	A	4	3	1.00	22	0.136
9	A	3	2	1.00	22	0.091
10	A	4	3	1.00	22	0.136
11	A	3	2	1.00	22	0.091
12	A	3	2	1.00	21	0.095
13	A	4	3	1.00	24	0.125
14	A	3	2	1.00	24	0.083
15	A	4	3	1.00	24	0.125
16	A	3	2	1.00	24	0.083
17	A	5	4	1.00	24	0.167
18	A	3	2	1.00	24	0.083
19	A	4	3	1.00	24	0.125
20	A	3	2	1.00	24	0.083
21	A	4	3	1.00	24	0.125
22	A	3	2	1.00	24	0.083
23	A	4	3	1.00	24	0.125
24	A	3	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	3	2	1.00	24	0.083
26	A	4	3	1.00	24	0.125
27	A	3	2	1.00	24	0.083
28	A	4	3	1.00	24	0.125
29	A	3	2	1.00	24	0.083
30	A	5	4	1.00	24	0.167
31	A	3	2	1.00	24	0.083
32	A	4	3	1.00	24	0.125
33	A	3	2	1.00	24	0.083
34	A	4	3	1.00	24	0.125
35	A	3	2	1.00	24	0.083
36	A	4	3	1.00	24	0.125
37	A	3	2	1.00	24	0.083
38	A	5	4	1.00	24	0.167
39	A	3	2	1.00	24	0.083
40	A	4	4	1.00	24	0.167
41	A	5	4	1.00	24	0.167
42	A	4	3	1.00	24	0.125
43	A	5	4	1.00	24	0.167
44	A	4	3	1.00	24	0.125
45	A	5	4	1.00	24	0.167
46	A	4	3	1.00	24	0.125
47	A	4	4	1.00	24	0.167
48	A	4	3	1.00	24	0.125
49	A	3	3	1.00	24	0.125
50	A	4	3	1.00	22	0.136
51	A	3	3	1.00	21	0.143
52	A	3	3	1.00	22	0.136
53	A	4	3	1.00	24	0.125
54	A	4	4	1.00	24	0.167
55	A	4	3	1.00	24	0.125
56	A	5	4	1.00	24	0.167
57	A	4	3	1.00	24	0.125
58	A	5	4	1.00	24	0.167
59	A	4	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	5	4	1.00	24	0.167
61	A	4	3	1.00	24	0.125
62	A	5	4	1.00	24	0.167
63	A	4	3	1.00	24	0.125
64	A	4	4	1.00	24	0.167
65	A	4	3	1.00	24	0.125
66	A	3	3	1.00	24	0.125
67	A	4	3	1.00	24	0.125
68	A	4	4	1.00	24	0.167
69	A	4	3	1.00	22	0.136
70	A	5	4	1.00	21	0.190
71	A	4	3	1.00	24	0.125
72	A	5	4	1.00	24	0.167
73	A	6	5	1.00	24	0.208
74	A	4	3	1.00	24	0.125
75	A	6	5	1.00	24	0.208
76	A	4	3	1.00	24	0.125
77	A	5	5	1.00	24	0.208
78	A	4	3	1.00	24	0.125
79	A	4	4	1.00	24	0.167
80	A	3	3	1.00	24	0.125
81	A	4	4	1.00	24	0.167
82	A	4	3	1.00	24	0.125
83	A	5	4	1.00	24	0.167
84	A	4	3	1.00	24	0.125
85	A	6	5	1.00	24	0.208
86	A	4	3	1.00	22	0.136
87	A	6	5	1.00	21	0.238
88	A	4	3	1.00	24	0.125
89	A	8	7	1.00	26	0.269
90	A	7	7	1.00	26	0.269
91	A	5	5	1.00	26	0.192
92	A	5	5	1.00	24	0.208
93	A	5	5	1.00	26	0.192
94	A	5	5	1.00	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	5	5	1.00	26	0.192
96	A	3	3	1.00	26	0.115
97	A	4	4	1.00	26	0.154
98	A	5	4	1.00	26	0.154
99	A	6	4	1.00	26	0.154
100	A	4	3	1.00	26	0.115
101	A	3	3	1.00	26	0.115
102	A	2	2	1.00	23	0.087
103	A	4	4	1.00	26	0.154
104	A	4	4	1.00	26	0.154
105	A	4	4	1.00	26	0.154
106	A	8	7	1.00	26	0.269
107	A	6	5	1.00	26	0.192
108	A	6	5	1.00	24	0.208
109	A	6	6	1.00	26	0.231
110	A	6	6	1.00	26	0.231
111	A	6	5	1.00	26	0.192
112	A	6	6	1.00	26	0.231
113	A	6	5	1.00	26	0.192
114	A	3	3	1.00	26	0.115
115	A	4	4	1.00	26	0.154
116	A	5	4	1.00	26	0.154
117	A	6	4	1.00	26	0.154
118	A	7	4	1.00	26	0.154
119	A	5	4	1.00	26	0.154
120	A	4	4	1.00	26	0.154
121	A	3	3	1.00	23	0.130
122	A	2	2	1.00	26	0.077
123	A	5	4	1.00	26	0.154
124	A	5	4	1.00	26	0.154
125	A	5	5	1.00	26	0.192
126	A	5	4	1.00	26	0.154
127	A	6	5	1.00	26	0.192
128	A	7	5	1.00	26	0.192
129	A	8	5	1.00	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	7	6	1.00	26	0.231
131	A	6	6	1.00	26	0.231
132	A	4	4	1.00	26	0.154
133	A	4	4	1.00	24	0.167
134	A	4	4	1.00	26	0.154
135	A	3	3	1.00	26	0.115
136	A	4	4	1.00	26	0.154
137	A	5	4	1.00	26	0.154
138	A	6	4	1.00	26	0.154
139	A	4	3	1.00	26	0.115
140	A	3	3	1.00	26	0.115
141	A	2	2	1.00	26	0.077
142	A	3	3	1.00	23	0.130
143	A	3	3	1.00	26	0.115
144	A	4	4	1.00	26	0.154
145	A	7	6	1.00	26	0.231
146	A	6	6	1.00	26	0.231
147	A	5	5	1.00	26	0.192
148	A	4	4	1.00	26	0.154
149	A	2	2	1.00	24	0.083
150	A	3	3	1.00	26	0.115
151	A	4	4	1.00	26	0.154
152	A	5	4	1.00	26	0.154
153	A	4	3	1.00	26	0.115
154	A	3	3	1.00	26	0.115
155	A	2	2	1.00	26	0.077
156	A	3	3	1.00	26	0.115
157	A	5	5	1.00	23	0.217
158	A	5	5	1.00	26	0.192
159	A	3	2	1.00	24	0.083
160	A	3	2	1.00	24	0.083
161	A	3	2	1.00	24	0.083
162	A	3	2	1.00	24	0.083
163	A	3	2	1.00	24	0.083
164	A	3	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	3	2	1.00	24	0.083
166	A	3	2	1.00	24	0.083
167	A	3	2	1.00	26	0.077
168	A	3	2	1.00	26	0.077
169	A	3	2	1.00	26	0.077
170	A	3	2	1.00	26	0.077
171	A	3	2	1.00	26	0.077
172	A	3	2	1.00	26	0.077
173	A	3	2	1.00	26	0.077
174	A	3	2	1.00	26	0.077
175	A	3	2	1.00	26	0.077
176	A	3	2	1.00	26	0.077
177	A	3	2	1.00	26	0.077
178	A	3	2	1.00	26	0.077
179	A	3	2	1.00	26	0.077
180	A	3	2	1.00	26	0.077
181	A	3	2	1.00	26	0.077
182	A	3	2	1.00	26	0.077
183	A	14	10	1.00	26	0.385
184	A	14	10	1.00	26	0.385
185	A	13	10	1.00	26	0.385
186	A	13	10	1.00	26	0.385
187	A	12	9	1.00	26	0.346
188	A	12	9	1.00	26	0.346
189	A	12	9	1.00	26	0.346
190	A	12	9	1.00	26	0.346
191	A	13	10	1.00	26	0.385
192	A	13	10	1.00	26	0.385
193	A	14	10	1.00	26	0.385
194	A	14	10	1.00	26	0.385
195	A	15	10	1.00	26	0.385
196	A	14	10	1.00	26	0.385
197	A	14	10	1.00	26	0.385
198	A	13	10	1.00	26	0.385
199	A	13	10	1.00	26	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	12	9	1.00	26	0.346
201	A	12	9	1.00	26	0.346
202	A	13	10	1.00	26	0.385
203	A	13	10	1.00	26	0.385
204	A	14	10	1.00	26	0.385
205	A	14	10	1.00	26	0.385
206	A	15	10	1.00	26	0.385
207	A	15	11	1.00	26	0.423
208	A	14	11	1.00	26	0.423
209	A	14	11	1.00	26	0.423
210	A	13	10	1.00	26	0.385
211	A	13	10	1.00	26	0.385
212	A	13	10	1.00	26	0.385
213	A	13	10	1.00	26	0.385
214	A	14	11	1.00	26	0.423
215	A	14	11	1.00	26	0.423
216	A	15	11	1.00	26	0.423
217	A	15	11	1.00	26	0.423
218	A	16	11	1.00	26	0.423
219	A	16	11	1.00	26	0.423
220	A	7	6	1.00	28	0.214
221	A	8	8	1.00	28	0.286
222	A	6	6	1.00	28	0.214
223	A	7	7	1.00	28	0.250
224	A	5	5	1.00	28	0.179
225	A	7	7	1.00	28	0.250
226	A	5	5	1.00	28	0.179
227	A	7	7	1.00	28	0.250
228	A	5	5	1.00	28	0.179
229	A	8	8	1.00	28	0.286
230	A	6	6	1.00	28	0.214
231	A	11	8	1.00	28	0.286
232	A	9	6	1.00	28	0.214
233	A	10	8	1.00	28	0.286
234	A	8	6	1.00	28	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	9	8	1.00	28	0.286
236	A	7	6	1.00	28	0.214
237	A	8	7	1.00	28	0.250
238	A	6	5	1.00	28	0.179
239	A	8	7	1.00	28	0.250
240	A	6	5	1.00	28	0.179
241	A	8	8	1.00	28	0.286
242	A	6	6	1.00	28	0.214
243	A	8	7	1.00	28	0.250
244	A	7	5	1.00	28	0.179
245	A	8	7	1.00	28	0.250
246	A	6	5	1.00	28	0.179
247	A	7	7	1.00	28	0.250
248	A	5	5	1.00	28	0.179
249	A	6	6	1.00	28	0.214
250	A	4	4	1.00	28	0.143
251	A	6	6	1.00	28	0.214
252	A	4	4	1.00	28	0.143
253	A	7	7	1.00	28	0.250
254	A	5	5	1.00	28	0.179
255	A	8	7	1.00	28	0.250
256	A	6	5	1.00	28	0.179
257	A	7	5	1.00	28	0.179
258	A	8	7	1.00	28	0.250
259	A	6	5	1.00	28	0.179
260	A	7	7	1.00	28	0.250
261	A	5	5	1.00	28	0.179
262	A	6	6	1.00	28	0.214
263	A	4	4	1.00	28	0.143
264	A	7	7	1.00	28	0.250
265	A	5	5	1.00	28	0.179
266	A	8	8	1.00	28	0.286
267	A	6	6	1.00	28	0.214
268	A	9	8	1.00	28	0.286
269	A	3	2	1.00	24	0.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	3	2	1.00	24	0.083
271	A	3	2	1.00	22	0.091
272	A	3	3	1.00	24	0.125
273	A	3	3	0.94	24	0.125
274	A	4	4	0.90	24	0.167
275	A	2	2	1.00	32	0.062
276	A	4	3	1.00	30	0.100
277	A	4	3	1.00	34	0.088
278	A	4	3	1.00	19	0.158
279	A	4	3	1.00	15	0.200
280	A	4	3	1.00	19	0.158
281	A	3	2	1.00	17	0.118
282	A	4	4	1.00	25	0.160
283	A	2	2	1.00	39	0.051
284	A	4	4	1.00	20	0.200
285	A	7	7	1.00	20	0.350
286	A	6	6	1.00	18	0.333
287	A	5	5	1.00	17	0.294
288	A	3	3	1.00	20	0.150
289	A	4	4	1.00	20	0.200
290	A	5	4	1.00	20	0.200
291	A	5	4	1.00	20	0.200
292	A	4	4	1.00	20	0.200
293	A	7	7	1.00	20	0.350
294	A	6	6	1.00	18	0.333
295	A	3	3	1.00	17	0.176
296	A	4	4	1.00	20	0.200
297	A	5	5	1.00	20	0.250
298	A	5	5	1.00	20	0.250

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^2(A+Bx^2)\left(bx^2+cx^4\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 33

$$\int x^2\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx=\frac{1}{5}Abx^5+\frac{1}{7}(bB+Ac)x^7+\frac{1}{9}Bcx^9$$

```
[Out] 1/5*A*b*x^5+1/7*(A*c+B*b)*x^7+1/9*B*c*x^9
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1598, 459}

$$\int x^2\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx=\frac{1}{7}x^7(Ac+bB)+\frac{1}{5}Abx^5+\frac{1}{9}Bcx^9$$

```
[In] Int[x^2*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (A*b*x^5)/5 + ((b*B + A*c)*x^7)/7 + (B*c*x^9)/9
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^4\left(A+Bx^2\right)\left(b+cx^2\right)\,dx\\ &= \int \left(Abx^4+(bB+Ac)x^6+Bcx^8\right)\,dx\\ &= \frac{1}{5}Abx^5+\frac{1}{7}(bB+Ac)x^7+\frac{1}{9}Bcx^9\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x^2\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx=\frac{1}{5}Abx^5+\frac{1}{7}(bB+Ac)x^7+\frac{1}{9}Bcx^9$$

```
[In] Integrate[x^2*(A + B*x^2)*(b*x^2 + c*x^4),x]
[Out] (A*b*x^5)/5 + ((b*B + A*c)*x^7)/7 + (B*c*x^9)/9
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{Ab\,x^5}{5}+\frac{(Ac+Bb)x^7}{7}+\frac{Bc\,x^9}{9}$	28
norman	$\frac{Bc\,x^9}{9}+\left(\frac{Ac}{7}+\frac{Bb}{7}\right)x^7+\frac{Ab\,x^5}{5}$	29
risch	$\frac{1}{5}Ab\,x^5+\frac{1}{7}x^7Ac+\frac{1}{7}bB\,x^7+\frac{1}{9}Bc\,x^9$	30
parallelrisch	$\frac{1}{5}Ab\,x^5+\frac{1}{7}x^7Ac+\frac{1}{7}bB\,x^7+\frac{1}{9}Bc\,x^9$	30
gosper	$\frac{x^5\left(35Bc\,x^4+45Ac\,x^2+45bB\,x^2+63Ab\right)}{315}$	32

```
[In] int(x^2*(B*x^2+A)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] 1/5*A*b*x^5+1/7*(A*c+B*b)*x^7+1/9*B*c*x^9
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^2(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{1}{9}\,Bcx^9+\frac{1}{7}\,(Bb+Ac)x^7+\frac{1}{5}\,Abx^5$$

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/9*B*c*x^9 + 1/7*(B*b + A*c)*x^7 + 1/5*A*b*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^2(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{Abx^5}{5}+\frac{Bcx^9}{9}+x^7\left(\frac{Ac}{7}+\frac{Bb}{7}\right)$$

```
[In] integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2),x)
[Out] A*b*x**5/5 + B*c*x**9/9 + x**7*(A*c/7 + B*b/7)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^2(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{1}{9}\,Bcx^9+\frac{1}{7}\,(Bb+Ac)x^7+\frac{1}{5}\,Abx^5$$

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 1/9*B*c*x^9 + 1/7*(B*b + A*c)*x^7 + 1/5*A*b*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^2(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{1}{9}\,Bcx^9+\frac{1}{7}\,Bbx^7+\frac{1}{7}\,Acx^7+\frac{1}{5}\,Abx^5$$

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
[Out] 1/9*B*c*x^9 + 1/7*B*b*x^7 + 1/7*A*c*x^7 + 1/5*A*b*x^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x^2 (A + Bx^2) (bx^2 + cx^4) \, dx = \frac{B\,c\,x^9}{9} + \left(\frac{A\,c}{7} + \frac{B\,b}{7}\right) x^7 + \frac{A\,b\,x^5}{5}$$

```
[In] int(x^2*(A + B*x^2)*(b*x^2 + c*x^4),x)
[Out] x^7*((A*c)/7 + (B*b)/7) + (A*b*x^5)/5 + (B*c*x^9)/9
```

3.2 $\int x(A+Bx^2)\left(bx^2+cx^4\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 33

$$\int x\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx=\frac{1}{4}Abx^4+\frac{1}{6}(bB+Ac)x^6+\frac{1}{8}Bcx^8$$

```
[Out] 1/4*A*b*x^4+1/6*(A*c+B*b)*x^6+1/8*B*c*x^8
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1598, 457, 77}

$$\int x\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx=\frac{1}{6}x^6(Ac+bB)+\frac{1}{4}Abx^4+\frac{1}{8}Bcx^8$$

```
[In] Int[x*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (A*b*x^4)/4 + ((b*B + A*c)*x^6)/6 + (B*c*x^8)/8
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^3\left(A+Bx^2\right)\left(b+cx^2\right)\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int x(A+Bx)(b+cx)\,dx,x,x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(Abx+(bB+Ac)x^2+Bcx^3\right)\,dx,x,x^2\right) \\
&= \frac{1}{4}Abx^4+\frac{1}{6}(bB+Ac)x^6+\frac{1}{8}Bcx^8
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx=\frac{1}{4}Abx^4+\frac{1}{6}(bB+Ac)x^6+\frac{1}{8}Bcx^8$$

```
[In] Integrate[x*(A + B*x^2)*(b*x^2 + c*x^4),x]
[Out] (A*b*x^4)/4 + ((b*B + A*c)*x^6)/6 + (B*c*x^8)/8
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{Ab\,x^{4}}{4}+\frac{(Ac+Bb)x^{6}}{6}+\frac{Bc\,x^{8}}{8}$	28
norman	$\frac{Bc\,x^{8}}{8}+\left(\frac{Ac}{6}+\frac{Bb}{6}\right)x^{6}+\frac{Ab\,x^{4}}{4}$	29
risch	$\frac{1}{4}Ab\,x^{4}+\frac{1}{6}x^{6}Ac+\frac{1}{6}bB\,x^{6}+\frac{1}{8}Bc\,x^{8}$	30
parallelrisch	$\frac{1}{4}Ab\,x^{4}+\frac{1}{6}x^{6}Ac+\frac{1}{6}bB\,x^{6}+\frac{1}{8}Bc\,x^{8}$	30
gosper	$\frac{x^{4}\left(3Bc\,x^{4}+4Ac\,x^{2}+4bB\,x^{2}+6Ab\right)}{24}$	32

```
[In] int(x*(B*x^2+A)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] 1/4*A*b*x^4+1/6*(A*c+B*b)*x^6+1/8*B*c*x^8
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx=\frac{1}{8}\,Bcx^8+\frac{1}{6}\,(Bb+Ac)x^6+\frac{1}{4}\,Abx^4$$

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/8*B*c*x^8 + 1/6*(B*b + A*c)*x^6 + 1/4*A*b*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx=\frac{Abx^4}{4}+\frac{Bcx^8}{8}+x^6\left(\frac{Ac}{6}+\frac{Bb}{6}\right)$$

```
[In] integrate(x*(B*x**2+A)*(c*x**4+b*x**2),x)
[Out] A*b*x**4/4 + B*c*x**8/8 + x**6*(A*c/6 + B*b/6)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{1}{8}\,Bcx^8+\frac{1}{6}\,(Bb+Ac)x^6+\frac{1}{4}\,Abx^4$$

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 1/8*B*c*x^8 + 1/6*(B*b + A*c)*x^6 + 1/4*A*b*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{1}{8}\,Bcx^8+\frac{1}{6}\,Bbx^6+\frac{1}{6}\,Acx^6+\frac{1}{4}\,Abx^4$$

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
[Out] 1/8*B*c*x^8 + 1/6*B*b*x^6 + 1/6*A*c*x^6 + 1/4*A*b*x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{B\,c\,x^8}{8}+\left(\frac{A\,c}{6}+\frac{B\,b}{6}\right)\,x^6+\frac{A\,b\,x^4}{4}$$

```
[In] int(x*(A + B*x^2)*(b*x^2 + c*x^4),x)
[Out] x^6*((A*c)/6 + (B*b)/6) + (A*b*x^4)/4 + (B*c*x^8)/8
```

3.3 $\int (A+Bx^2)(bx^2+cx^4)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 33

$$\int (A+Bx^2)(bx^2+cx^4)\,dx=\frac{1}{3}Abx^3+\frac{1}{5}(bB+Ac)x^5+\frac{1}{7}Bcx^7$$

```
[Out] 1/3*A*b*x^3+1/5*(A*c+B*b)*x^5+1/7*B*c*x^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1607, 459}

$$\int (A+Bx^2)(bx^2+cx^4)\,dx=\frac{1}{5}x^5(Ac+bB)+\frac{1}{3}Abx^3+\frac{1}{7}Bcx^7$$

```
[In] Int[(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (A*b*x^3)/3 + ((b*B + A*c)*x^5)/5 + (B*c*x^7)/7
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^2\left(A+Bx^2\right)\left(b+cx^2\right)\,dx\\ &= \int \left(Abx^2+(bB+Ac)x^4+Bcx^6\right)\,dx\\ &= \frac{1}{3}Abx^3+\frac{1}{5}(bB+Ac)x^5+\frac{1}{7}Bcx^7\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx=\frac{1}{3}Abx^3+\frac{1}{5}(bB+Ac)x^5+\frac{1}{7}Bcx^7$$

```
[In] Integrate[(A + B*x^2)*(b*x^2 + c*x^4),x]
[Out] (A*b*x^3)/3 + ((b*B + A*c)*x^5)/5 + (B*c*x^7)/7
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{Ab\,x^3}{3}+\frac{(Ac+Bb)x^5}{5}+\frac{Bc\,x^7}{7}$	28
norman	$\frac{Bc\,x^7}{7}+\left(\frac{Ac}{5}+\frac{Bb}{5}\right)x^5+\frac{Ab\,x^3}{3}$	29
risch	$\frac{1}{3}Ab\,x^3+\frac{1}{5}x^5Ac+\frac{1}{5}bB\,x^5+\frac{1}{7}Bc\,x^7$	30
parallelrisch	$\frac{1}{3}Ab\,x^3+\frac{1}{5}x^5Ac+\frac{1}{5}bB\,x^5+\frac{1}{7}Bc\,x^7$	30
gosper	$\frac{x^3\left(15Bc\,x^4+21Ac\,x^2+21bB\,x^2+35Ab\right)}{105}$	32

```
[In] int((B*x^2+A)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] 1/3*A*b*x^3+1/5*(A*c+B*b)*x^5+1/7*B*c*x^7
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \left(A+Bx^2\right) \left(bx^2+cx^4\right) \, dx = \frac{1}{7} \, Bcx^7 + \frac{1}{5} \, (Bb+Ac)x^5 + \frac{1}{3} \, Abx^3$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/7*B*c*x^7 + 1/5*(B*b + A*c)*x^5 + 1/3*A*b*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \left(A+Bx^2\right) \left(bx^2+cx^4\right) \, dx = \frac{Abx^3}{3} + \frac{Bcx^7}{7} + x^5\left(\frac{Ac}{5} + \frac{Bb}{5}\right)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2),x)
[Out] A*b*x**3/3 + B*c*x**7/7 + x**5*(A*c/5 + B*b/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \left(A+Bx^2\right) \left(bx^2+cx^4\right) \, dx = \frac{1}{7} \, Bcx^7 + \frac{1}{5} \, (Bb+Ac)x^5 + \frac{1}{3} \, Abx^3$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 1/7*B*c*x^7 + 1/5*(B*b + A*c)*x^5 + 1/3*A*b*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \left(A+Bx^2\right) \left(bx^2+cx^4\right) \, dx = \frac{1}{7} \, Bcx^7 + \frac{1}{5} \, Bbx^5 + \frac{1}{5} \, Acx^5 + \frac{1}{3} \, Abx^3$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
[Out] 1/7*B*c*x^7 + 1/5*B*b*x^5 + 1/5*A*c*x^5 + 1/3*A*b*x^3
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int (A+Bx^2)(bx^2+cx^4)\,dx=\frac{B\,c\,x^7}{7}+\left(\frac{A\,c}{5}+\frac{B\,b}{5}\right)x^5+\frac{A\,b\,x^3}{3}$$

```
[In] int((A + B*x^2)*(b*x^2 + c*x^4),x)
[Out] x^5*((A*c)/5 + (B*b)/5) + (A*b*x^3)/3 + (B*c*x^7)/7
```

3.4 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x} dx$

Optimal result

Integrand size = 22, antiderivative size = 33

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x} dx = \frac{1}{2}Abx^2 + \frac{1}{4}(bB+Ac)x^4 + \frac{1}{6}Bcx^6$$

```
[Out] 1/2*A*b*x^2+1/4*(A*c+B*b)*x^4+1/6*B*c*x^6
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1598, 455, 45}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x} dx = \frac{1}{4}x^4(Ac+bB) + \frac{1}{2}Abx^2 + \frac{1}{6}Bcx^6$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x,x]
```

```
[Out] (A*b*x^2)/2 + ((b*B + A*c)*x^4)/4 + (B*c*x^6)/6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x(A+Bx^2)\left(b+cx^2\right)\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int (A+Bx)(b+cx)\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(Ab+(bB+Ac)x+Bcx^2\right)\,dx, x, x^2\right) \\
&= \frac{1}{2}Abx^2+\frac{1}{4}(bB+Ac)x^4+\frac{1}{6}Bcx^6
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x}\,dx = \frac{1}{2}Abx^2+\frac{1}{4}(bB+Ac)x^4+\frac{1}{6}Bcx^6$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x,x]
[Out] (A*b*x^2)/2 + ((b*B + A*c)*x^4)/4 + (B*c*x^6)/6
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{Ab\,x^2}{2}+\frac{(Ac+Bb)x^4}{4}+\frac{Bc\,x^6}{6}$	28
norman	$\frac{Bc\,x^6}{6}+\left(\frac{Ac}{4}+\frac{Bb}{4}\right)x^4+\frac{Ab\,x^2}{2}$	29
risch	$\frac{1}{2}Ab\,x^2+\frac{1}{4}x^4Ac+\frac{1}{4}bB\,x^4+\frac{1}{6}Bc\,x^6$	30
parallelrisch	$\frac{1}{2}Ab\,x^2+\frac{1}{4}x^4Ac+\frac{1}{4}bB\,x^4+\frac{1}{6}Bc\,x^6$	30
gosper	$\frac{x^2(2Bc\,x^4+3Ac\,x^2+3bB\,x^2+6Ab)}{12}$	32

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*A*b*x^2+1/4*(A*c+B*b)*x^4+1/6*B*c*x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x}\,dx = \frac{1}{6}\,Bcx^6 + \frac{1}{4}\,(Bb+Ac)x^4 + \frac{1}{2}\,Abx^2$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x,x, algorithm="fricas")
```

```
[Out] 1/6*B*c*x^6 + 1/4*(B*b + A*c)*x^4 + 1/2*A*b*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x}\,dx = \frac{Abx^2}{2} + \frac{Bcx^6}{6} + x^4\left(\frac{Ac}{4} + \frac{Bb}{4}\right)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x,x)
```

```
[Out] A*b*x**2/2 + B*c*x**6/6 + x**4*(A*c/4 + B*b/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x}\,dx = \frac{1}{6}\,Bcx^6 + \frac{1}{4}\,(Bb+Ac)x^4 + \frac{1}{2}\,Abx^2$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x,x, algorithm="maxima")
```

```
[Out] 1/6*B*c*x^6 + 1/4*(B*b + A*c)*x^4 + 1/2*A*b*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x}\,dx = \frac{1}{6}\,Bcx^6 + \frac{1}{4}\,Bbx^4 + \frac{1}{4}\,Acx^4 + \frac{1}{2}\,Abx^2$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x,x, algorithm="giac")
[Out] 1/6*B*c*x^6 + 1/4*B*b*x^4 + 1/4*A*c*x^4 + 1/2*A*b*x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x}\,dx = \frac{B\,c\,x^6}{6} + \left(\frac{A\,c}{4} + \frac{B\,b}{4}\right)\,x^4 + \frac{A\,b\,x^2}{2}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x,x)
[Out] x^4*((A*c)/4 + (B*b)/4) + (A*b*x^2)/2 + (B*c*x^6)/6
```

3.5 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 28

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2}\,dx = Abx+\frac{1}{3}(bB+Ac)x^3+\frac{1}{5}Bcx^5$$

```
[Out] A*b*x+1/3*(A*c+B*b)*x^3+1/5*B*c*x^5
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1598, 380}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2}\,dx = \frac{1}{3}x^3(Ac+bB)+Abx+\frac{1}{5}Bcx^5$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^2,x]
```

```
[Out] A*b*x + ((b*B + A*c)*x^3)/3 + (B*c*x^5)/5
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(A+Bx^2\right)\left(b+cx^2\right)\,dx\\ &= \int \left(Ab+(bB+Ac)x^2+Bcx^4\right)\,dx\\ &= Abx+\frac{1}{3}(bB+Ac)x^3+\frac{1}{5}Bcx^5\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2}\,dx = Abx+\frac{1}{3}(bB+Ac)x^3+\frac{1}{5}Bcx^5$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^2,x]
[Out] A*b*x + ((b*B + A*c)*x^3)/3 + (B*c*x^5)/5
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$Abx+\frac{(Ac+Bb)x^3}{3}+\frac{Bc\,x^5}{5}$	25
risch	$Abx+\frac{1}{3}x^3Ac+\frac{1}{3}bB\,x^3+\frac{1}{5}Bc\,x^5$	27
parallelrisch	$Abx+\frac{1}{3}x^3Ac+\frac{1}{3}bB\,x^3+\frac{1}{5}Bc\,x^5$	27
gosper	$\frac{x(3Bc\,x^4+5Ac\,x^2+5bB\,x^2+15Ab)}{15}$	30
norman	$\frac{\left(\frac{Ac}{3}+\frac{Bb}{3}\right)x^4+Ab\,x^2+\frac{Bc\,x^6}{5}}{x}$	32

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^2,x,method=_RETURNVERBOSE)
[Out] A*b*x+1/3*(A*c+B*b)*x^3+1/5*B*c*x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2}\,dx = \frac{1}{5}\,Bcx^5 + \frac{1}{3}\,(Bb+Ac)x^3 + Abx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^2,x, algorithm="fricas")
[Out] 1/5*B*c*x^5 + 1/3*(B*b + A*c)*x^3 + A*b*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2}\,dx = Abx + \frac{Bcx^5}{5} + x^3\left(\frac{Ac}{3} + \frac{Bb}{3}\right)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**2,x)
[Out] A*b*x + B*c*x**5/5 + x**3*(A*c/3 + B*b/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2}\,dx = \frac{1}{5}\,Bcx^5 + \frac{1}{3}\,(Bb+Ac)x^3 + Abx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^2,x, algorithm="maxima")
[Out] 1/5*B*c*x^5 + 1/3*(B*b + A*c)*x^3 + A*b*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2}\,dx = \frac{1}{5}\,Bcx^5 + \frac{1}{3}\,Bbx^3 + \frac{1}{3}\,Acx^3 + Abx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^2,x, algorithm="giac")
[Out] 1/5*B*c*x^5 + 1/3*B*b*x^3 + 1/3*A*c*x^3 + A*b*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^2}\,dx = \frac{B\,c\,x^5}{5} + \left(\frac{A\,c}{3} + \frac{B\,b}{3}\right)x^3 + A\,b\,x$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^2,x)
[Out] x^3*((A*c)/3 + (B*b)/3) + A*b*x + (B*c*x^5)/5
```

3.6 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 29

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^3} dx = \frac{1}{2}(bB+Ac)x^2 + \frac{1}{4}Bcx^4 + Ab\log(x)$$

```
[Out] 1/2*(A*c+B*b)*x^2+1/4*B*c*x^4+A*b*ln(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1598, 457, 77}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^3} dx = \frac{1}{2}x^2(Ac+bB) + Ab\log(x) + \frac{1}{4}Bcx^4$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^3,x]
```

```
[Out] ((b*B + A*c)*x^2)/2 + (B*c*x^4)/4 + A*b*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(A+Bx^2)(b+cx^2)}{x}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)}{x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(bB+Ac+\frac{Ab}{x}+Bcx\right)\,dx, x, x^2\right)\\ &= \frac{1}{2}(bB+Ac)x^2+\frac{1}{4}Bcx^4+Ab\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^3}\,dx = \frac{1}{2}(bB+Ac)x^2+\frac{1}{4}Bcx^4+Ab\log(x)$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^3,x]
[Out] ((b*B + A*c)*x^2)/2 + (B*c*x^4)/4 + A*b*Log[x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{Bc\,x^{4}}{4}+\frac{Ac\,x^{2}}{2}+\frac{bB\,x^{2}}{2}+Ab\ln\left(x\right)$	28
parallelrisch	$\frac{Bc\,x^{4}}{4}+\frac{Ac\,x^{2}}{2}+\frac{bB\,x^{2}}{2}+Ab\ln\left(x\right)$	28
norman	$\frac{\left(\frac{Ac}{2}+\frac{Bb}{2}\right)x^{4}+\frac{Bc\,x^{6}}{4}}{x^{2}}+Ab\ln\left(x\right)$	32
risch	$\frac{Bc\,x^{4}}{4}+\frac{Ac\,x^{2}}{2}+\frac{bB\,x^{2}}{2}+\frac{c\,A^{2}}{4B}+\frac{Ab}{2}+\frac{B\,b^{2}}{4c}+Ab\ln\left(x\right)$	50

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/4*B*c*x^4+1/2*A*c*x^2+1/2*b*B*x^2+A*b*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)}{x^3}\,dx=\frac{1}{4}\,Bcx^4+\frac{1}{2}\,(Bb+Ac)x^2+Ab\log\left(x\right)$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^3,x, algorithm="fricas")
[Out] 1/4*B*c*x^4 + 1/2*(B*b + A*c)*x^2 + A*b*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)}{x^3}\,dx=Ab\log\left(x\right)+\frac{Bcx^4}{4}+x^2\left(\frac{Ac}{2}+\frac{Bb}{2}\right)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**3,x)
[Out] A*b*log(x) + B*c*x**4/4 + x**2*(A*c/2 + B*b/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)}{x^3}\,dx=\frac{1}{4}\,Bcx^4+\frac{1}{2}\,(Bb+Ac)x^2+\frac{1}{2}\,Ab\log\left(x^2\right)$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^3,x, algorithm="maxima")
[Out] 1/4*B*c*x^4 + 1/2*(B*b + A*c)*x^2 + 1/2*A*b*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^3}\,dx = \frac{1}{4}\,Bcx^4 + \frac{1}{2}\,Bbx^2 + \frac{1}{2}\,Acx^2 + \frac{1}{2}\,Ab\log\left(x^2\right)$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^3,x, algorithm="giac")
[Out] 1/4*B*c*x^4 + 1/2*B*b*x^2 + 1/2*A*c*x^2 + 1/2*A*b*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^3}\,dx = x^2\left(\frac{A\,c}{2}+\frac{B\,b}{2}\right)+\frac{B\,c\,x^4}{4}+A\,b\,\ln\left(x\right)$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^3,x)
[Out] x^2*((A*c)/2 + (B*b)/2) + (B*c*x^4)/4 + A*b*log(x)
```

3.7 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 26

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4}\,dx = -\frac{Ab}{x}+(bB+Ac)x+\frac{1}{3}Bcx^3$$

```
[Out] -A*b/x+(A*c+B*b)*x+1/3*B*c*x^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4}\,dx = x(Ac+bB)-\frac{Ab}{x}+\frac{1}{3}Bcx^3$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^4,x]
```

```
[Out] -((A*b)/x) + (b*B + A*c)*x + (B*c*x^3)/3
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(A+Bx^2)(b+cx^2)}{x^2}\,dx\\ &= \int \left(bB\left(1+\frac{Ac}{bB}\right)+\frac{Ab}{x^2}+Bcx^2\right)dx\\ &= -\frac{Ab}{x}+(bB+Ac)x+\frac{1}{3}Bcx^3\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4}\,dx = -\frac{Ab}{x}+(bB+Ac)x+\frac{1}{3}Bcx^3$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^4,x]
[Out] -((A*b)/x) + (b*B + A*c)*x + (B*c*x^3)/3
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{Bc\,x^3}{3}+Acx+bBx-\frac{Ab}{x}$	24
risch	$\frac{Bc\,x^3}{3}+Acx+bBx-\frac{Ab}{x}$	24
norman	$\frac{(Ac+Bb)x^4-Ab\,x^2+\frac{Bc\,x^6}{3}}{x^3}$	31
parallelrisch	$\frac{Bc\,x^4+3Ac\,x^2+3bB\,x^2-3Ab}{3x}$	31
gosper	$-\frac{-Bc\,x^4-3Ac\,x^2-3bB\,x^2+3Ab}{3x}$	32

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^4,x,method=_RETURNVERBOSE)
[Out] 1/3*B*c*x^3+A*c*x+b*B*x-A*b/x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4}\,dx = \frac{Bcx^4+3\,(Bb+Ac)x^2-3\,Ab}{3\,x}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^4,x, algorithm="fricas")
[Out] 1/3*(B*c*x^4 + 3*(B*b + A*c)*x^2 - 3*A*b)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4}\,dx = -\frac{Ab}{x}+\frac{Bcx^3}{3}+x(Ac+Bb)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**4,x)
[Out] -A*b/x + B*c*x**3/3 + x*(A*c + B*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4}\,dx = \frac{1}{3}\,Bcx^3+(Bb+Ac)x-\frac{Ab}{x}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^4,x, algorithm="maxima")
[Out] 1/3*B*c*x^3 + (B*b + A*c)*x - A*b/x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^4}\,dx = \frac{1}{3}\,Bcx^3+Bbx+Acx-\frac{Ab}{x}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^4,x, algorithm="giac")
[Out] 1/3*B*c*x^3 + B*b*x + A*c*x - A*b/x
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)}{x^4}\,dx = x\,(A\,c+B\,b)-\frac{A\,b}{x}+\frac{B\,c\,x^3}{3}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^4,x)
[Out] x*(A*c + B*b) - (A*b)/x + (B*c*x^3)/3
```

3.8 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^5} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 29

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^5} \, dx = -\frac{Ab}{2x^2}+\frac{1}{2}Bcx^2+(bB+Ac)\log(x)$$

```
[Out] -1/2*A*b/x^2+1/2*B*c*x^2+(A*c+B*b)*ln(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1598, 457, 77}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^5} \, dx = \log(x)(Ac+bB)-\frac{Ab}{2x^2}+\frac{1}{2}Bcx^2$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^5,x]
```

```
[Out] -1/2*(A*b)/x^2 + (B*c*x^2)/2 + (b*B + A*c)*Log[x]
```

<u>Rule 77</u>

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

<u>Rule 457</u>

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(A+Bx^2)\,(b+cx^2)}{x^3}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)}{x^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(Bc+\frac{Ab}{x^2}+\frac{bB+Ac}{x}\right)\,dx, x, x^2\right) \\
&= -\frac{Ab}{2x^2}+\frac{1}{2}Bcx^2+(bB+Ac)\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)}{x^5}\,dx = -\frac{Ab}{2x^2}+\frac{1}{2}Bcx^2+(bB+Ac)\log(x)$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^5,x]
[Out] -1/2*(A*b)/x^2 + (B*c*x^2)/2 + (b*B + A*c)*Log[x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{Ab}{2x^2}+\frac{Bc\,x^2}{2}+(Ac+Bb)\ln(x)$	26
risch	$-\frac{Ab}{2x^2}+\frac{Bc\,x^2}{2}+A\ln(x)\,c+bB\ln(x)$	26
norman	$\frac{-\frac{1}{2}Ab\,x^2+\frac{1}{2}Bc\,x^6}{x^4}+(Ac+Bb)\ln(x)$	31
parallelrisch	$\frac{Bc\,x^4+2A\ln(x)x^2c+2B\ln(x)x^2b-Ab}{2x^2}$	35

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/2*A*b/x^2+1/2*B*c*x^2+(A*c+B*b)*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)}{x^5}\,dx = \frac{Bcx^4+2\,(Bb+Ac)x^2\log(x)-Ab}{2\,x^2}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^5,x, algorithm="fricas")
[Out] 1/2*(B*c*x^4 + 2*(B*b + A*c)*x^2*log(x) - A*b)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)}{x^5}\,dx = -\frac{Ab}{2x^2}+\frac{Bcx^2}{2}+(Ac+Bb)\log(x)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**5,x)
[Out] -A*b/(2*x**2) + B*c*x**2/2 + (A*c + B*b)*log(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)}{x^5}\,dx = \frac{1}{2}\,Bcx^2+\frac{1}{2}\,(Bb+Ac)\log\left(x^2\right)-\frac{Ab}{2\,x^2}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^5,x, algorithm="maxima")
[Out] 1/2*B*c*x^2 + 1/2*(B*b + A*c)*log(x^2) - 1/2*A*b/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.45

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^5}\,dx = \frac{1}{2}\,Bcx^2 + \frac{1}{2}\,(Bb+Ac)\log\left(x^2\right) - \frac{Bbx^2+Acx^2+Ab}{2\,x^2}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^5,x, algorithm="giac")
[Out] 1/2*B*c*x^2 + 1/2*(B*b + A*c)*log(x^2) - 1/2*(B*b*x^2 + A*c*x^2 + A*b)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^5}\,dx = \ln(x)\,\left(A\,c+B\,b\right) - \frac{A\,b}{2\,x^2} + \frac{B\,c\,x^2}{2}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^5,x)
[Out] log(x)*(A*c + B*b) - (A*b)/(2*x^2) + (B*c*x^2)/2
```

3.9 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 26

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^6}\,dx = -\frac{Ab}{3x^3} - \frac{bB+Ac}{x} + Bcx$$

```
[Out] -1/3*A*b/x^3+(-A*c-B*b)/x+B*c*x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^6}\,dx = -\frac{Ac+bB}{x} - \frac{Ab}{3x^3} + Bcx$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^6,x]
```

```
[Out] -1/3*(A*b)/x^3 - (b*B + A*c)/x + B*c*x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(A+Bx^2)(b+cx^2)}{x^4}\,dx \\
&= \int \left(Bc+\frac{Ab}{x^4}+\frac{bB+Ac}{x^2}\right)dx \\
&= -\frac{Ab}{3x^3}-\frac{bB+Ac}{x}+Bcx
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^6}\,dx = -\frac{Ab}{3x^3}+\frac{-bB-Ac}{x}+Bcx$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^6,x]
[Out] -1/3*(A*b)/x^3 + (-(b*B) - A*c)/x + B*c*x
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

method	result	size
default	$Bcx-\frac{Ab}{3x^3}-\frac{Ac+Bb}{x}$	25
risch	$Bcx+\frac{(-Ac-Bb)x^2-\frac{Ab}{3}}{x^3}$	28
gosper	$-\frac{-3Bc\,x^4+3Ac\,x^2+3bB\,x^2+Ab}{3x^3}$	31
parallelrisch	$-\frac{-3Bc\,x^4+3Ac\,x^2+3bB\,x^2+Ab}{3x^3}$	31
norman	$\frac{(-Ac-Bb)x^4+Bc\,x^6-\frac{Ab\,x^2}{3}}{x^5}$	32

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^6,x,method=_RETURNVERBOSE)
[Out] B*c*x-1/3*A*b/x^3-(A*c+B*b)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)}{x^6}\,dx = \frac{3\,Bcx^4-3\,(Bb+Ac)x^2-Ab}{3\,x^3}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^6,x, algorithm="fricas")
[Out] 1/3*(3*B*c*x^4 - 3*(B*b + A*c)*x^2 - A*b)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)}{x^6}\,dx = Bcx + \frac{-Ab+x^2(-3Ac-3Bb)}{3x^3}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**6,x)
[Out] B*c*x + (-A*b + x**2*(-3*A*c - 3*B*b))/(3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)}{x^6}\,dx = Bcx - \frac{3\,(Bb+Ac)x^2+Ab}{3\,x^3}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^6,x, algorithm="maxima")
[Out] B*c*x - 1/3*(3*(B*b + A*c)*x^2 + A*b)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)}{x^6}\,dx = Bcx - \frac{3\,Bbx^2+3\,Acx^2+Ab}{3\,x^3}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^6,x, algorithm="giac")
[Out] B*c*x - 1/3*(3*B*b*x^2 + 3*A*c*x^2 + A*b)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^6}\,dx = B\,c\,x - \frac{(A\,c+B\,b)\;x^2+\frac{A\,b}{3}}{x^3}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^6,x)
[Out] B*c*x - ((A*b)/3 + x^2*(A*c + B*b))/x^3
```

3.10 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^7} dx$

Optimal result

Integrand size = 22, antiderivative size = 29

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^7} dx = -\frac{Ab}{4x^4} - \frac{bB+Ac}{2x^2} + Bc\log(x)$$

```
[Out] -1/4*A*b/x^4+1/2*(-A*c-B*b)/x^2+B*c*ln(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1598, 457, 77}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^7} dx = -\frac{Ac+bB}{2x^2} - \frac{Ab}{4x^4} + Bc\log(x)$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^7,x]
```

```
[Out] -1/4*(A*b)/x^4 - (b*B + A*c)/(2*x^2) + B*c*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(A+Bx^2)\,(b+cx^2)}{x^5}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)}{x^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{Ab}{x^3}+\frac{bB+Ac}{x^2}+\frac{Bc}{x}\right)\,dx, x, x^2\right) \\
&= -\frac{Ab}{4x^4}-\frac{bB+Ac}{2x^2}+Bc\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)}{x^7}\,dx = -\frac{Ab}{4x^4}+\frac{-bB-Ac}{2x^2}+Bc\log(x)$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^7,x]
[Out] -1/4*(A*b)/x^4 + (-(b*B) - A*c)/(2*x^2) + B*c*Log[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

method	result	size
default	$Bc\ln\left(x\right)-\frac{Ac+Bb}{2x^{2}}-\frac{Ab}{4x^{4}}$	26
risch	$\frac{\left(-\frac{Ac}{2}-\frac{Bb}{2}\right)x^{2}-\frac{Ab}{4}}{x^{4}}+Bc\ln\left(x\right)$	29
norman	$\frac{\left(-\frac{Ac}{2}-\frac{Bb}{2}\right)x^{4}-\frac{Ab\,x^{2}}{4}}{x^{6}}+Bc\ln\left(x\right)$	32
parallelrisch	$-\frac{-4Bc\ln(x)x^{4}+2Ac\,x^{2}+2bB\,x^{2}+Ab}{4x^{4}}$	33

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^7,x,method=_RETURNVERBOSE)
[Out] B*c*ln(x)-1/2*(A*c+B*b)/x^2-1/4*A*b/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int\frac{(A+Bx^2)\left(bx^2+cx^4\right)}{x^7}\,dx=\frac{4\,Bcx^4\log\left(x\right)-2\,(Bb+Ac)x^2-Ab}{4\,x^4}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^7,x, algorithm="fricas")
[Out] 1/4*(4*B*c*x^4*log(x) - 2*(B*b + A*c)*x^2 - A*b)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int\frac{(A+Bx^2)\left(bx^2+cx^4\right)}{x^7}\,dx=Bc\log\left(x\right)+\frac{-Ab+x^2(-2Ac-2Bb)}{4x^4}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**7,x)
[Out] B*c*log(x) + (-A*b + x**2*(-2*A*c - 2*B*b))/(4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^7}\,dx = \frac{1}{2}\,Bc\log\left(x^2\right) - \frac{2\,(Bb+Ac)x^2+Ab}{4\,x^4}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^7,x, algorithm="maxima")
[Out] 1/2*B*c*log(x^2) - 1/4*(2*(B*b + A*c)*x^2 + A*b)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^7}\,dx = \frac{1}{2}\,Bc\log\left(x^2\right) - \frac{3\,Bcx^4+2\,Bbx^2+2\,Acx^2+Ab}{4\,x^4}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^7,x, algorithm="giac")
[Out] 1/2*B*c*log(x^2) - 1/4*(3*B*c*x^4 + 2*B*b*x^2 + 2*A*c*x^2 + A*b)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^7}\,dx = B\,c\,\ln\left(x\right) - \frac{\left(\frac{A\,c}{2}+\frac{B\,b}{2}\right)\,x^2+\frac{A\,b}{4}}{x^4}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^7,x)
[Out] B*c*log(x) - ((A*b)/4 + x^2*((A*c)/2 + (B*b)/2))/x^4
```

3.11 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 31

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8}\,dx = -\frac{Ab}{5x^5}-\frac{bB+Ac}{3x^3}-\frac{Bc}{x}$$

```
[Out] -1/5*A*b/x^5+1/3*(-A*c-B*b)/x^3-B*c/x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8}\,dx = -\frac{Ac+bB}{3x^3}-\frac{Ab}{5x^5}-\frac{Bc}{x}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^8,x]
```

```
[Out] -1/5*(A*b)/x^5 - (b*B + A*c)/(3*x^3) - (B*c)/x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(A+Bx^2)(b+cx^2)}{x^6}\,dx \\
&= \int \left(\frac{Ab}{x^6}+\frac{bB+Ac}{x^4}+\frac{Bc}{x^2}\right)dx \\
&= -\frac{Ab}{5x^5}-\frac{bB+Ac}{3x^3}-\frac{Bc}{x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8}\,dx = -\frac{Ab}{5x^5}+\frac{-bB-Ac}{3x^3}-\frac{Bc}{x}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^8,x]
[Out] -1/5*(A*b)/x^5 + (-(b*B) - A*c)/(3*x^3) - (B*c)/x
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{Ac+Bb}{3x^3}-\frac{Bc}{x}-\frac{Ab}{5x^5}$	28
risch	$\frac{-Bc\,x^4+\left(-\frac{Ac}{3}-\frac{Bb}{3}\right)x^2-\frac{Ab}{5}}{x^5}$	30
gosper	$-\frac{15Bc\,x^4+5Ac\,x^2+5bB\,x^2+3Ab}{15x^5}$	32
parallelrisch	$-\frac{15Bc\,x^4+5Ac\,x^2+5bB\,x^2+3Ab}{15x^5}$	32
norman	$\frac{\left(-\frac{Ac}{3}-\frac{Bb}{3}\right)x^4-\frac{Ab\,x^2}{5}-Bc\,x^6}{x^7}$	33

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^8,x,method=_RETURNVERBOSE)
[Out] -1/3*(A*c+B*b)/x^3-B*c/x-1/5*A*b/x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8}\,dx = -\frac{15\,Bcx^4+5\,(Bb+Ac)x^2+3\,Ab}{15\,x^5}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^8,x, algorithm="fricas")
[Out] -1/15*(15*B*c*x^4 + 5*(B*b + A*c)*x^2 + 3*A*b)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8}\,dx = \frac{-3Ab-15Bcx^4+x^2(-5Ac-5Bb)}{15x^5}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**8,x)
[Out] (-3*A*b - 15*B*c*x**4 + x**2*(-5*A*c - 5*B*b))/(15*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8}\,dx = -\frac{15\,Bcx^4+5\,(Bb+Ac)x^2+3\,Ab}{15\,x^5}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^8,x, algorithm="maxima")
[Out] -1/15*(15*B*c*x^4 + 5*(B*b + A*c)*x^2 + 3*A*b)/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8}\,dx = -\frac{15\,Bcx^4+5\,Bbx^2+5\,Acx^2+3\,Ab}{15\,x^5}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^8,x, algorithm="giac")
[Out] -1/15*(15*B*c*x^4 + 5*B*b*x^2 + 5*A*c*x^2 + 3*A*b)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^8}\,dx = -\frac{B\,c\,x^4+\left(\frac{A\,c}{3}+\frac{B\,b}{3}\right)\,x^2+\frac{A\,b}{5}}{x^5}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^8,x)
```

```
[Out] -((A*b)/5 + x^2*((A*c)/3 + (B*b)/3) + B*c*x^4)/x^5
```

3.12 $\int (A+Bx^2)(bx^2+cx^4)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int (A+Bx^2)(bx^2+cx^4)^2\,dx=\frac{1}{5}Ab^2x^5+\frac{1}{7}b(bB+2Ac)x^7+\frac{1}{9}c(2bB+Ac)x^9+\frac{1}{11}Bc^2x^{11}$$

```
[Out] 1/5*A*b^2*x^5+1/7*b*(2*A*c+B*b)*x^7+1/9*c*(A*c+2*B*b)*x^9+1/11*B*c^2*x^11
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1607, 459}

$$\int (A+Bx^2)(bx^2+cx^4)^2\,dx=\frac{1}{5}Ab^2x^5+\frac{1}{9}cx^9(Ac+2bB)+\frac{1}{7}bx^7(2Ac+bB)+\frac{1}{11}Bc^2x^{11}$$

```
[In] Int[(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (A*b^2*x^5)/5 + (b*(b*B + 2*A*c)*x^7)/7 + (c*(2*b*B + A*c)*x^9)/9 + (B*c^2*
x^11)/11
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^4\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx\\ &= \int \left(Ab^2x^4+b(bB+2Ac)x^6+c(2bB+Ac)x^8+Bc^2x^{10}\right)\,dx\\ &= \frac{1}{5}Ab^2x^5+\frac{1}{7}b(bB+2Ac)x^7+\frac{1}{9}c(2bB+Ac)x^9+\frac{1}{11}Bc^2x^{11}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx = \frac{1}{5}Ab^2x^5+\frac{1}{7}b(bB+2Ac)x^7+\frac{1}{9}c(2bB+Ac)x^9+\frac{1}{11}Bc^2x^{11}$$

```
[In] Integrate[(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
[Out] (A*b^2*x^5)/5 + (b*(b*B + 2*A*c)*x^7)/7 + (c*(2*b*B + A*c)*x^9)/9 + (B*c^2*
x^11)/11
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{B\,c^2x^{11}}{11}+\frac{(A\,c^2+2Bbc)x^9}{9}+\frac{(2Abc+B\,b^2)x^7}{7}+\frac{A\,b^2x^5}{5}$	52
norman	$\frac{B\,c^2x^{11}}{11}+\left(\frac{1}{9}A\,c^2+\frac{2}{9}Bbc\right)x^9+\left(\frac{2}{7}Abc+\frac{1}{7}B\,b^2\right)x^7+\frac{A\,b^2x^5}{5}$	52
risch	$\frac{1}{11}B\,c^2x^{11}+\frac{1}{9}x^9A\,c^2+\frac{2}{9}x^9Bbc+\frac{2}{7}x^7Abc+\frac{1}{7}b^2B\,x^7+\frac{1}{5}A\,b^2x^5$	54
parallelrisch	$\frac{1}{11}B\,c^2x^{11}+\frac{1}{9}x^9A\,c^2+\frac{2}{9}x^9Bbc+\frac{2}{7}x^7Abc+\frac{1}{7}b^2B\,x^7+\frac{1}{5}A\,b^2x^5$	54
gosper	$\frac{x^5\left(315B\,c^2x^6+385A\,c^2x^4+770x^4Bbc+990Abc\,x^2+495b^2B\,x^2+693b^2A\right)}{3465}$	56

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/11*B*c^2*x^11+1/9*(A*c^2+2*B*b*c)*x^9+1/7*(2*A*b*c+B*b^2)*x^7+1/5*A*b^2*x
^5
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int (A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{1}{11}\,Bc^2x^{11}+\frac{1}{9}\,(2\,Bbc+Ac^2)x^9+\frac{1}{5}\,Ab^2x^5+\frac{1}{7}\,(Bb^2+2\,Abc)x^7$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/11*B*c^2*x^11 + 1/9*(2*B*b*c + A*c^2)*x^9 + 1/5*A*b^2*x^5 + 1/7*(B*b^2 +
2*A*b*c)*x^7
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int (A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{Ab^2x^5}{5}+\frac{Bc^2x^{11}}{11}+x^9\left(\frac{Ac^2}{9}+\frac{2Bbc}{9}\right)+x^7\cdot\left(\frac{2Abc}{7}+\frac{Bb^2}{7}\right)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

```
[Out] A*b**2*x**5/5 + B*c**2*x**11/11 + x**9*(A*c**2/9 + 2*B*b*c/9) + x**7*(2*A*b
*c/7 + B*b**2/7)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int (A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{1}{11}\,Bc^2x^{11}+\frac{1}{9}\,(2\,Bbc+Ac^2)x^9+\frac{1}{5}\,Ab^2x^5+\frac{1}{7}\,(Bb^2+2\,Abc)x^7$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/11*B*c^2*x^11 + 1/9*(2*B*b*c + A*c^2)*x^9 + 1/5*A*b^2*x^5 + 1/7*(B*b^2 +
2*A*b*c)*x^7
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx=\frac{1}{11}\,Bc^2x^{11}+\frac{2}{9}\,Bbcx^9+\frac{1}{9}\,Ac^2x^9+\frac{1}{7}\,Bb^2x^7+\frac{2}{7}\,Abcx^7+\frac{1}{5}\,Ab^2x^5$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/11*B*c^2*x^11 + 2/9*B*b*c*x^9 + 1/9*A*c^2*x^9 + 1/7*B*b^2*x^7 + 2/7*A*b*c
*x^7 + 1/5*A*b^2*x^5
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx=x^7\left(\frac{B\,b^2}{7}+\frac{2\,A\,c\,b}{7}\right)+x^9\left(\frac{A\,c^2}{9}+\frac{2\,B\,b\,c}{9}\right)+\frac{A\,b^2\,x^5}{5}+\frac{B\,c^2\,x^{11}}{11}$$

```
[In] int((A + B*x^2)*(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^7*((B*b^2)/7 + (2*A*b*c)/7) + x^9*((A*c^2)/9 + (2*B*b*c)/9) + (A*b^2*x^5)
/5 + (B*c^2*x^11)/11
```

3.13 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x} \, dx = \frac{1}{4}Ab^2x^4 + \frac{1}{6}b(bB+2Ac)x^6 + \frac{1}{8}c(2bB+Ac)x^8 + \frac{1}{10}Bc^2x^{10}$$

```
[Out] 1/4*A*b^2*x^4+1/6*b*(2*A*c+B*b)*x^6+1/8*c*(A*c+2*B*b)*x^8+1/10*B*c^2*x^10
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 457, 77}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x} \, dx = \frac{1}{4}Ab^2x^4 + \frac{1}{8}cx^8(Ac+2bB) + \frac{1}{6}bx^6(2Ac+bB) + \frac{1}{10}Bc^2x^{10}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x,x]
```

```
[Out] (A*b^2*x^4)/4 + (b*(b*B + 2*A*c)*x^6)/6 + (c*(2*b*B + A*c)*x^8)/8 + (B*c^2*
x^10)/10
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^3\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int x(A+Bx)(b+cx)^2\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(Ab^2x+b(bB+2Ac)x^2+c(2bB+Ac)x^3+Bc^2x^4\right)\,dx,x,x^2\right)\\ &= \frac{1}{4}Ab^2x^4+\frac{1}{6}b(bB+2Ac)x^6+\frac{1}{8}c(2bB+Ac)x^8+\frac{1}{10}Bc^2x^{10}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x}\,dx = \frac{1}{4}Ab^2x^4+\frac{1}{6}b(bB+2Ac)x^6+\frac{1}{8}c(2bB+Ac)x^8+\frac{1}{10}Bc^2x^{10}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x,x]
[Out] (A*b^2*x^4)/4 + (b*(b*B + 2*A*c)*x^6)/6 + (c*(2*b*B + A*c)*x^8)/8 + (B*c^2*
x^10)/10
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{B\,c^{2} x^{10}}{10}+\frac{\left(A\,c^{2}+2Bbc\right) x^{8}}{8}+\frac{\left(2Abc+B\,b^{2}\right) x^{6}}{6}+\frac{A\,b^{2} x^{4}}{4}$	52
norman	$\frac{B\,c^{2} x^{10}}{10}+\left(\frac{1}{8}A\,c^{2}+\frac{1}{4}Bbc\right) x^{8}+\left(\frac{1}{3}Abc+\frac{1}{6}B\,b^{2}\right) x^{6}+\frac{A\,b^{2} x^{4}}{4}$	52
risch	$\frac{1}{10}B\,c^{2} x^{10}+\frac{1}{8}x^{8} A\,c^{2}+\frac{1}{4}x^{8} Bbc+\frac{1}{3}x^{6} Abc+\frac{1}{6}b^{2} B\,x^{6}+\frac{1}{4}A\,b^{2} x^{4}$	54
parallelrisch	$\frac{1}{10}B\,c^{2} x^{10}+\frac{1}{8}x^{8} A\,c^{2}+\frac{1}{4}x^{8} Bbc+\frac{1}{3}x^{6} Abc+\frac{1}{6}b^{2} B\,x^{6}+\frac{1}{4}A\,b^{2} x^{4}$	54
gosper	$\frac{x^{4} \left(12B\,c^{2} x^{6}+15A\,c^{2} x^{4}+30x^{4} Bbc+40Abc\,x^{2}+20b^{2} B\,x^{2}+30b^{2} A\right)}{120}$	56

[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x,x,method=_RETURNVERBOSE)

[Out] 1/10*B*c^2*x^10+1/8*(A*c^2+2*B*b*c)*x^8+1/6*(2*A*b*c+B*b^2)*x^6+1/4*A*b^2*x^4

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x}\,dx=\frac{1}{10}\,Bc^2x^{10}+\frac{1}{8}\,(2\,Bbc+Ac^2)x^8+\frac{1}{4}\,Ab^2x^4+\frac{1}{6}\,(Bb^2+2\,Abc)x^6$$

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x,x, algorithm="fricas")

[Out] 1/10*B*c^2*x^10 + 1/8*(2*B*b*c + A*c^2)*x^8 + 1/4*A*b^2*x^4 + 1/6*(B*b^2 + 2*A*b*c)*x^6

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x}\,dx=\frac{Ab^2x^4}{4}+\frac{Bc^2x^{10}}{10}+x^8\left(\frac{Ac^2}{8}+\frac{Bbc}{4}\right)+x^6\left(\frac{Abc}{3}+\frac{Bb^2}{6}\right)$$

[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x,x)

[Out] A*b**2*x**4/4 + B*c**2*x**10/10 + x**8*(A*c**2/8 + B*b*c/4) + x**6*(A*b*c/3 + B*b**2/6)

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x}\,dx = \frac{1}{10}\,Bc^2x^{10} + \frac{1}{8}\,(2\,Bbc+Ac^2)x^8 + \frac{1}{4}\,Ab^2x^4 + \frac{1}{6}\,(Bb^2+2\,Abc)x^6$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x,x, algorithm="maxima")
[Out] 1/10*B*c^2*x^10 + 1/8*(2*B*b*c + A*c^2)*x^8 + 1/4*A*b^2*x^4 + 1/6*(B*b^2 +
2*A*b*c)*x^6
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x}\,dx = \frac{1}{10}\,Bc^2x^{10} + \frac{1}{4}\,Bbcx^8 + \frac{1}{8}\,Ac^2x^8 + \frac{1}{6}\,Bb^2x^6 + \frac{1}{3}\,Abcx^6 + \frac{1}{4}\,Ab^2x^4$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x,x, algorithm="giac")
[Out] 1/10*B*c^2*x^10 + 1/4*B*b*c*x^8 + 1/8*A*c^2*x^8 + 1/6*B*b^2*x^6 + 1/3*A*b*c
*x^6 + 1/4*A*b^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x}\,dx = x^6\left(\frac{B\,b^2}{6}+\frac{A\,c\,b}{3}\right) + x^8\left(\frac{A\,c^2}{8}+\frac{B\,b\,c}{4}\right) + \frac{A\,b^2\,x^4}{4} + \frac{B\,c^2\,x^{10}}{10}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x,x)
[Out] x^6*((B*b^2)/6 + (A*b*c)/3) + x^8*((A*c^2)/8 + (B*b*c)/4) + (A*b^2*x^4)/4 +
 (B*c^2*x^10)/10
```

3.14 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^2} dx = \frac{1}{3}Ab^2x^3 + \frac{1}{5}b(bB+2Ac)x^5 + \frac{1}{7}c(2bB+Ac)x^7 + \frac{1}{9}Bc^2x^9$$

```
[Out] 1/3*A*b^2*x^3+1/5*b*(2*A*c+B*b)*x^5+1/7*c*(A*c+2*B*b)*x^7+1/9*B*c^2*x^9
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^2} dx = \frac{1}{3}Ab^2x^3 + \frac{1}{7}cx^7(Ac+2bB) + \frac{1}{5}bx^5(2Ac+bB) + \frac{1}{9}Bc^2x^9$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^2,x]
```

```
[Out] (A*b^2*x^3)/3 + (b*(b*B + 2*A*c)*x^5)/5 + (c*(2*b*B + A*c)*x^7)/7 + (B*c^2*
x^9)/9
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^2(A+Bx^2)\left(b+cx^2\right)^2\,dx\\ &= \int \left(Ab^2x^2+b(bB+2Ac)x^4+c(2bB+Ac)x^6+Bc^2x^8\right)\,dx\\ &= \frac{1}{3}Ab^2x^3+\frac{1}{5}b(bB+2Ac)x^5+\frac{1}{7}c(2bB+Ac)x^7+\frac{1}{9}Bc^2x^9\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^2}\,dx = \frac{1}{3}Ab^2x^3+\frac{1}{5}b(bB+2Ac)x^5+\frac{1}{7}c(2bB+Ac)x^7+\frac{1}{9}Bc^2x^9$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^2,x]
[Out] (A*b^2*x^3)/3 + (b*(b*B + 2*A*c)*x^5)/5 + (c*(2*b*B + A*c)*x^7)/7 + (B*c^2*
x^9)/9
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{B\,c^2x^9}{9}+\frac{(A\,c^2+2Bbc)x^7}{7}+\frac{(2Abc+B\,b^2)x^5}{5}+\frac{A\,b^2x^3}{3}$	52
risch	$\frac{1}{9}B\,c^2x^9+\frac{1}{7}x^7A\,c^2+\frac{2}{7}x^7Bbc+\frac{2}{5}x^5Abc+\frac{1}{5}b^2B\,x^5+\frac{1}{3}A\,b^2x^3$	54
parallelrisch	$\frac{1}{9}B\,c^2x^9+\frac{1}{7}x^7A\,c^2+\frac{2}{7}x^7Bbc+\frac{2}{5}x^5Abc+\frac{1}{5}b^2B\,x^5+\frac{1}{3}A\,b^2x^3$	54
gosper	$\frac{x^3(35B\,c^2x^6+45A\,c^2x^4+90x^4Bbc+126Abc\,x^2+63b^2B\,x^2+105b^2A)}{315}$	56
norman	$\frac{(\frac{1}{7}A\,c^2+\frac{2}{7}Bbc)x^8+(\frac{2}{5}Abc+\frac{1}{5}B\,b^2)x^6+\frac{A\,b^2x^4}{3}+\frac{B\,c^2x^{10}}{9}}{x}$	56

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^2,x,method=_RETURNVERBOSE)
[Out] 1/9*B*c^2*x^9+1/7*(A*c^2+2*B*b*c)*x^7+1/5*(2*A*b*c+B*b^2)*x^5+1/3*A*b^2*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^2}\,dx = \frac{1}{9}\,Bc^2x^9 + \frac{1}{7}\left(2\,Bbc + Ac^2\right)x^7 + \frac{1}{3}\,Ab^2x^3 + \frac{1}{5}\left(Bb^2 + 2\,Abc\right)x^5$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/9*B*c^2*x^9 + 1/7*(2*B*b*c + A*c^2)*x^7 + 1/3*A*b^2*x^3 + 1/5*(B*b^2 + 2*
A*b*c)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^2}\,dx = \frac{Ab^2x^3}{3} + \frac{Bc^2x^9}{9} + x^7\left(\frac{Ac^2}{7} + \frac{2Bbc}{7}\right) + x^5\cdot\left(\frac{2Abc}{5} + \frac{Bb^2}{5}\right)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**2,x)
```

```
[Out] A*b**2*x**3/3 + B*c**2*x**9/9 + x**7*(A*c**2/7 + 2*B*b*c/7) + x**5*(2*A*b*c
/5 + B*b**2/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^2}\,dx = \frac{1}{9}\,Bc^2x^9 + \frac{1}{7}\left(2\,Bbc + Ac^2\right)x^7 + \frac{1}{3}\,Ab^2x^3 + \frac{1}{5}\left(Bb^2 + 2\,Abc\right)x^5$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*B*c^2*x^9 + 1/7*(2*B*b*c + A*c^2)*x^7 + 1/3*A*b^2*x^3 + 1/5*(B*b^2 + 2*
A*b*c)*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^2}\,dx = \frac{1}{9}\,Bc^2x^9 + \frac{2}{7}\,Bbcx^7 + \frac{1}{7}\,Ac^2x^7 + \frac{1}{5}\,Bb^2x^5 + \frac{2}{5}\,Abcx^5 + \frac{1}{3}\,Ab^2x^3$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/9*B*c^2*x^9 + 2/7*B*b*c*x^7 + 1/7*A*c^2*x^7 + 1/5*B*b^2*x^5 + 2/5*A*b*c*x
^5 + 1/3*A*b^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^2}\,dx = x^5\left(\frac{B\,b^2}{5}+\frac{2\,A\,c\,b}{5}\right)+x^7\left(\frac{A\,c^2}{7}+\frac{2\,B\,b\,c}{7}\right)+\frac{A\,b^2\,x^3}{3}+\frac{B\,c^2\,x^9}{9}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^2,x)
```

```
[Out] x^5*((B*b^2)/5 + (2*A*b*c)/5) + x^7*((A*c^2)/7 + (2*B*b*c)/7) + (A*b^2*x^3)
/3 + (B*c^2*x^9)/9
```

3.15 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 42

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^3}\,dx = -\frac{(bB-Ac)(b+cx^2)^3}{6c^2}+\frac{B(b+cx^2)^4}{8c^2}$$

```
[Out] -1/6*(-A*c+B*b)*(c*x^2+b)^3/c^2+1/8*B*(c*x^2+b)^4/c^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 455, 45}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^3}\,dx = \frac{B(b+cx^2)^4}{8c^2}-\frac{(b+cx^2)^3(bB-Ac)}{6c^2}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^3,x]
```

```
[Out] -1/6*((b*B - A*c)*(b + c*x^2)^3)/c^2 + (B*(b + c*x^2)^4)/(8*c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int (A+Bx)(b+cx)^2\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(-bB+Ac)(b+cx)^2}{c}+\frac{B(b+cx)^3}{c}\right)\,dx,x,x^2\right)\\ &= -\frac{(bB-Ac)\left(b+cx^2\right)^3}{6c^2}+\frac{B\left(b+cx^2\right)^4}{8c^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x^3}\,dx=\frac{1}{24}x^2\left(12Ab^2+6b(bB+2Ac)x^2+4c(2bB+Ac)x^4+3Bc^2x^6\right)$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^3,x]
[Out] (x^2*(12*A*b^2 + 6*b*(b*B + 2*A*c)*x^2 + 4*c*(2*b*B + A*c)*x^4 + 3*B*c^2*x^
6))/24
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{B\,c^{2}x^{8}}{8}+\frac{(A\,c^{2}+2Bbc)x^{6}}{6}+\frac{(2Abc+B\,b^{2})x^{4}}{4}+\frac{A\,b^{2}x^{2}}{2}$	52
risch	$\frac{1}{8}B\,c^{2}x^{8}+\frac{1}{6}x^{6}A\,c^{2}+\frac{1}{3}x^{6}Bbc+\frac{1}{2}x^{4}Abc+\frac{1}{4}b^{2}B\,x^{4}+\frac{1}{2}A\,b^{2}x^{2}$	54
parallelrisch	$\frac{1}{8}B\,c^{2}x^{8}+\frac{1}{6}x^{6}A\,c^{2}+\frac{1}{3}x^{6}Bbc+\frac{1}{2}x^{4}Abc+\frac{1}{4}b^{2}B\,x^{4}+\frac{1}{2}A\,b^{2}x^{2}$	54
gosper	$\frac{x^{2}(3B\,c^{2}x^{6}+4A\,c^{2}x^{4}+8x^{4}Bbc+12Abc\,x^{2}+6b^{2}B\,x^{2}+12b^{2}A)}{24}$	56
norman	$\frac{(\frac{1}{6}A\,c^{2}+\frac{1}{3}Bbc)x^{8}+(\frac{1}{2}Abc+\frac{1}{4}B\,b^{2})x^{6}+\frac{A\,b^{2}x^{4}}{2}+\frac{B\,c^{2}x^{10}}{8}}{x^{2}}$	56

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^3,x,method=_RETURNVERBOSE)
[Out] 1/8*B*c^2*x^8+1/6*(A*c^2+2*B*b*c)*x^6+1/4*(2*A*b*c+B*b^2)*x^4+1/2*A*b^2*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^3}\,dx=\frac{1}{8}\,Bc^2x^8+\frac{1}{6}\,(2\,Bbc+Ac^2)x^6+\frac{1}{2}\,Ab^2x^2+\frac{1}{4}\,(Bb^2+2\,Abc)x^4$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^3,x, algorithm="fricas")
[Out] 1/8*B*c^2*x^8 + 1/6*(2*B*b*c + A*c^2)*x^6 + 1/2*A*b^2*x^2 + 1/4*(B*b^2 + 2*
A*b*c)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^3}\,dx=\frac{Ab^2x^2}{2}+\frac{Bc^2x^8}{8}+x^6\left(\frac{Ac^2}{6}+\frac{Bbc}{3}\right)+x^4\left(\frac{Abc}{2}+\frac{Bb^2}{4}\right)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**3,x)
[Out] A*b**2*x**2/2 + B*c**2*x**8/8 + x**6*(A*c**2/6 + B*b*c/3) + x**4*(A*b*c/2 +
 B*b**2/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^3}\,dx = \frac{1}{8}\,Bc^2x^8 + \frac{1}{6}\,(2\,Bbc+Ac^2)x^6 + \frac{1}{2}\,Ab^2x^2 + \frac{1}{4}\,(Bb^2+2\,Abc)x^4$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*B*c^2*x^8 + 1/6*(2*B*b*c + A*c^2)*x^6 + 1/2*A*b^2*x^2 + 1/4*(B*b^2 + 2*
A*b*c)*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^3}\,dx = \frac{1}{8}\,Bc^2x^8 + \frac{1}{3}\,Bbcx^6 + \frac{1}{6}\,Ac^2x^6 + \frac{1}{4}\,Bb^2x^4 + \frac{1}{2}\,Abcx^4 + \frac{1}{2}\,Ab^2x^2$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/8*B*c^2*x^8 + 1/3*B*b*c*x^6 + 1/6*A*c^2*x^6 + 1/4*B*b^2*x^4 + 1/2*A*b*c*x
^4 + 1/2*A*b^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^3}\,dx = x^4\left(\frac{B\,b^2}{4}+\frac{A\,c\,b}{2}\right)+x^6\left(\frac{A\,c^2}{6}+\frac{B\,b\,c}{3}\right)+\frac{A\,b^2\,x^2}{2}+\frac{B\,c^2\,x^8}{8}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^3,x)
```

```
[Out] x^4*((B*b^2)/4 + (A*b*c)/2) + x^6*((A*c^2)/6 + (B*b*c)/3) + (A*b^2*x^2)/2 +
 (B*c^2*x^8)/8
```

3.16 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 50

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^4}\,dx = Ab^2x + \frac{1}{3}b(bB+2Ac)x^3 + \frac{1}{5}c(2bB+Ac)x^5 + \frac{1}{7}Bc^2x^7$$

```
[Out] A*b^2*x+1/3*b*(2*A*c+B*b)*x^3+1/5*c*(A*c+2*B*b)*x^5+1/7*B*c^2*x^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 380}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^4}\,dx = Ab^2x + \frac{1}{5}cx^5(Ac+2bB) + \frac{1}{3}bx^3(2Ac+bB) + \frac{1}{7}Bc^2x^7$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^4,x]
```

```
[Out] A*b^2*x + (b*(b*B + 2*A*c)*x^3)/3 + (c*(2*b*B + A*c)*x^5)/5 + (B*c^2*x^7)/7
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(A+Bx^2\right)\left(b+cx^2\right)^2 \, dx \\
&= \int \left(Ab^2+b(bB+2Ac)x^2+c(2bB+Ac)x^4+Bc^2x^6\right) dx \\
&= Ab^2x+\frac{1}{3}b(bB+2Ac)x^3+\frac{1}{5}c(2bB+Ac)x^5+\frac{1}{7}Bc^2x^7
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x^4} \, dx = Ab^2x+\frac{1}{3}b(bB+2Ac)x^3+\frac{1}{5}c(2bB+Ac)x^5+\frac{1}{7}Bc^2x^7$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^4,x]
[Out] A*b^2*x + (b*(b*B + 2*A*c)*x^3)/3 + (c*(2*b*B + A*c)*x^5)/5 + (B*c^2*x^7)/7
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{B\,c^2x^7}{7}+\frac{(A\,c^2+2Bbc)x^5}{5}+\frac{(2Abc+B\,b^2)x^3}{3}+A\,b^2x$	49
risch	$\frac{1}{7}B\,c^2x^7+\frac{1}{5}x^5A\,c^2+\frac{2}{5}x^5Bbc+\frac{2}{3}x^3Abc+\frac{1}{3}b^2B\,x^3+A\,b^2x$	51
parallelrisch	$\frac{1}{7}B\,c^2x^7+\frac{1}{5}x^5A\,c^2+\frac{2}{5}x^5Bbc+\frac{2}{3}x^3Abc+\frac{1}{3}b^2B\,x^3+A\,b^2x$	51
gosper	$\frac{x(15B\,c^2x^6+21A\,c^2x^4+42x^4Bbc+70Abc\,x^2+35b^2B\,x^2+105b^2A)}{105}$	54
norman	$\frac{(\frac{1}{5}A\,c^2+\frac{2}{5}Bbc)x^8+(\frac{2}{3}Abc+\frac{1}{3}B\,b^2)x^6+A\,b^2x^4+\frac{B\,c^2x^{10}}{7}}{x^3}$	55

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^4,x,method=_RETURNVERBOSE)
[Out] 1/7*B*c^2*x^7+1/5*(A*c^2+2*B*b*c)*x^5+1/3*(2*A*b*c+B*b^2)*x^3+A*b^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^4}\,dx = \frac{1}{7}\,Bc^2x^7 + \frac{1}{5}\,(2\,Bbc + Ac^2)x^5 + Ab^2x + \frac{1}{3}\,(Bb^2 + 2\,Abc)x^3$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/7*B*c^2*x^7 + 1/5*(2*B*b*c + A*c^2)*x^5 + A*b^2*x + 1/3*(B*b^2 + 2*A*b*c)
*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^4}\,dx = Ab^2x + \frac{Bc^2x^7}{7} + x^5\left(\frac{Ac^2}{5} + \frac{2Bbc}{5}\right) + x^3\cdot\left(\frac{2Abc}{3} + \frac{Bb^2}{3}\right)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**4,x)
```

```
[Out] A*b**2*x + B*c**2*x**7/7 + x**5*(A*c**2/5 + 2*B*b*c/5) + x**3*(2*A*b*c/3 +
B*b**2/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^4}\,dx = \frac{1}{7}\,Bc^2x^7 + \frac{1}{5}\,(2\,Bbc + Ac^2)x^5 + Ab^2x + \frac{1}{3}\,(Bb^2 + 2\,Abc)x^3$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/7*B*c^2*x^7 + 1/5*(2*B*b*c + A*c^2)*x^5 + A*b^2*x + 1/3*(B*b^2 + 2*A*b*c)
*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^4}\,dx = \frac{1}{7}\,Bc^2x^7 + \frac{2}{5}\,Bbcx^5 + \frac{1}{5}\,Ac^2x^5 + \frac{1}{3}\,Bb^2x^3 + \frac{2}{3}\,Abcx^3 + Ab^2x$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/7*B*c^2*x^7 + 2/5*B*b*c*x^5 + 1/5*A*c^2*x^5 + 1/3*B*b^2*x^3 + 2/3*A*b*c*x
^3 + A*b^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^4}\,dx = x^3\,\left(\frac{B\,b^2}{3}+\frac{2\,A\,c\,b}{3}\right)+x^5\,\left(\frac{A\,c^2}{5}+\frac{2\,B\,b\,c}{5}\right)+\frac{B\,c^2\,x^7}{7}+A\,b^2\,x$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^4,x)
```

```
[Out] x^3*((B*b^2)/3 + (2*A*b*c)/3) + x^5*((A*c^2)/5 + (2*B*b*c)/5) + (B*c^2*x^7)
/7 + A*b^2*x
```

3.17 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 43

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^5}\,dx = Abcx^2+\frac{1}{4}Ac^2x^4+\frac{B(b+cx^2)^3}{6c}+Ab^2\log(x)$$

```
[Out] A*b*c*x^2+1/4*A*c^2*x^4+1/6*B*(c*x^2+b)^3/c+A*b^2*ln(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 457, 81, 45}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^5}\,dx = Ab^2\log(x)+Abcx^2+\frac{1}{4}Ac^2x^4+\frac{B(b+cx^2)^3}{6c}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^5,x]
```

```
[Out] A*b*c*x^2 + (A*c^2*x^4)/4 + (B*(b + c*x^2)^3)/(6*c) + A*b^2*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(A+Bx^2)\,(b+cx^2)^2}{x}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^2}{x}\,dx, x, x^2\right) \\
&= \frac{B(b+cx^2)^3}{6c} + \frac{1}{2}A\text{Subst}\left(\int \frac{(b+cx)^2}{x}\,dx, x, x^2\right) \\
&= \frac{B(b+cx^2)^3}{6c} + \frac{1}{2}A\text{Subst}\left(\int \left(2bc+\frac{b^2}{x}+c^2x\right)\,dx, x, x^2\right) \\
&= Abcx^2 + \frac{1}{4}Ac^2x^4 + \frac{B(b+cx^2)^3}{6c} + Ab^2\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^5}\,dx = \frac{1}{2}b(bB+2Ac)x^2 + \frac{1}{4}c(2bB+Ac)x^4 + \frac{1}{6}Bc^2x^6 + Ab^2\log(x)$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^5,x]
[Out] (b*(b*B + 2*A*c)*x^2)/2 + (c*(2*b*B + A*c)*x^4)/4 + (B*c^2*x^6)/6 + A*b^2*L
og[x]
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

method	result	size
default	$\frac{B\,c^{2} x^{6}}{6}+\frac{A\,c^{2} x^{4}}{4}+\frac{x^{4} B b c}{2}+A b c\,x^{2}+\frac{b^{2} B\,x^{2}}{2}+A\,b^{2} \ln \left(x \right)$	51
risch	$\frac{B\,c^{2} x^{6}}{6}+\frac{A\,c^{2} x^{4}}{4}+\frac{x^{4} B b c}{2}+A b c\,x^{2}+\frac{b^{2} B\,x^{2}}{2}+A\,b^{2} \ln \left(x \right)$	51
parallelrisch	$\frac{B\,c^{2} x^{6}}{6}+\frac{A\,c^{2} x^{4}}{4}+\frac{x^{4} B b c}{2}+A b c\,x^{2}+\frac{b^{2} B\,x^{2}}{2}+A\,b^{2} \ln \left(x \right)$	51
norman	$\frac{\left(\frac{1}{4} A\,c^{2}+\frac{1}{2} B b c \right) x^{8}+\left(A b c +\frac{1}{2} B\,b^{2}\right) x^{6}+\frac{B\,c^{2} x^{10}}{6}}{x^{4}}+A\,b^{2} \ln \left(x \right)$	54

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*B*c^2*x^6+1/4*A*c^2*x^4+1/2*x^4*B*b*c+A*b*c*x^2+1/2*b^2*B*x^2+A*b^2*ln(
x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^5}\,dx = \frac{1}{6}\,Bc^2x^6 + \frac{1}{4}\,(2\,Bbc + Ac^2)x^4 + Ab^2\log(x) + \frac{1}{2}\,(Bb^2 + 2\,Abc)x^2$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/6*B*c^2*x^6 + 1/4*(2*B*b*c + A*c^2)*x^4 + A*b^2*log(x) + 1/2*(B*b^2 + 2*A
*b*c)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^5}\,dx = Ab^2\log(x) + \frac{Bc^2x^6}{6} + x^4\left(\frac{Ac^2}{4} + \frac{Bbc}{2}\right) + x^2\left(Abc + \frac{Bb^2}{2}\right)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**5,x)
```

```
[Out] A*b**2*log(x) + B*c**2*x**6/6 + x**4*(A*c**2/4 + B*b*c/2) + x**2*(A*b*c + B
*b**2/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^5}\,dx = \frac{1}{6}\,Bc^2x^6 + \frac{1}{4}\,\left(2\,Bbc + Ac^2\right)x^4 + \frac{1}{2}\,Ab^2\log\left(x^2\right) + \frac{1}{2}\,\left(Bb^2 + 2\,Abc\right)x^2$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*B*c^2*x^6 + 1/4*(2*B*b*c + A*c^2)*x^4 + 1/2*A*b^2*log(x^2) + 1/2*(B*b^2
 + 2*A*b*c)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^5}\,dx = \frac{1}{6}\,Bc^2x^6 + \frac{1}{2}\,Bbcx^4 + \frac{1}{4}\,Ac^2x^4 + \frac{1}{2}\,Bb^2x^2 + Abcx^2 + \frac{1}{2}\,Ab^2\log\left(x^2\right)$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/6*B*c^2*x^6 + 1/2*B*b*c*x^4 + 1/4*A*c^2*x^4 + 1/2*B*b^2*x^2 + A*b*c*x^2 +
 1/2*A*b^2*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^5}\,dx = x^2\left(\frac{B\,b^2}{2}+A\,c\,b\right)+x^4\left(\frac{A\,c^2}{4}+\frac{B\,b\,c}{2}\right)+\frac{B\,c^2\,x^6}{6}+A\,b^2\,\ln\left(x\right)$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^5,x)
```

```
[Out] x^2*((B*b^2)/2 + A*b*c) + x^4*((A*c^2)/4 + (B*b*c)/2) + (B*c^2*x^6)/6 + A*b
^2*log(x)
```

3.18 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^6} dx$

Optimal result

Integrand size = 24, antiderivative size = 48

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^6} dx = -\frac{Ab^2}{x} + b(bB+2Ac)x + \frac{1}{3}c(2bB+Ac)x^3 + \frac{1}{5}Bc^2x^5$$

```
[Out] -A*b^2/x+b*(2*A*c+B*b)*x+1/3*c*(A*c+2*B*b)*x^3+1/5*B*c^2*x^5
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^6} dx = -\frac{Ab^2}{x} + \frac{1}{3}cx^3(Ac+2bB) + bx(2Ac+bB) + \frac{1}{5}Bc^2x^5$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^6,x]
```

```
[Out] -((A*b^2)/x) + b*(b*B + 2*A*c)*x + (c*(2*b*B + A*c)*x^3)/3 + (B*c^2*x^5)/5
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(A+Bx^2)\,(b+cx^2)^2}{x^2}\,dx \\
&= \int \left(b(bB+2Ac)+\frac{Ab^2}{x^2}+c(2bB+Ac)x^2+Bc^2x^4\right)\,dx \\
&= -\frac{Ab^2}{x}+b(bB+2Ac)x+\frac{1}{3}c(2bB+Ac)x^3+\frac{1}{5}Bc^2x^5
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^6}\,dx = -\frac{Ab^2}{x}+b(bB+2Ac)x+\frac{1}{3}c(2bB+Ac)x^3+\frac{1}{5}Bc^2x^5$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^6,x]
[Out] -((A*b^2)/x) + b*(b*B + 2*A*c)*x + (c*(2*b*B + A*c)*x^3)/3 + (B*c^2*x^5)/5
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{B\,c^2x^5}{5}+\frac{A\,c^2x^3}{3}+\frac{2Bbc\,x^3}{3}+2Abcx+b^2Bx-\frac{A\,b^2}{x}$	49
risch	$\frac{B\,c^2x^5}{5}+\frac{A\,c^2x^3}{3}+\frac{2Bbc\,x^3}{3}+2Abcx+b^2Bx-\frac{A\,b^2}{x}$	49
norman	$\frac{\left(\frac{1}{3}A\,c^2+\frac{2}{3}Bbc\right)x^8+\left(2Abc+B\,b^2\right)x^6-A\,b^2x^4+\frac{B\,c^2x^{10}}{5}}{x^5}$	55
gosper	$-\frac{-3B\,c^2x^6-5A\,c^2x^4-10x^4Bbc-30Abc\,x^2-15b^2B\,x^2+15b^2A}{15x}$	56
parallelrisch	$\frac{3B\,c^2x^6+5A\,c^2x^4+10x^4Bbc+30Abc\,x^2+15b^2B\,x^2-15b^2A}{15x}$	56

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^6,x,method=_RETURNVERBOSE)
[Out] 1/5*B*c^2*x^5+1/3*A*c^2*x^3+2/3*B*b*c*x^3+2*A*b*c*x+b^2*B*x-A*b^2/x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^6}\,dx=\frac{3\,Bc^2x^6+5\,(2\,Bbc+Ac^2)x^4-15\,Ab^2+15\,(Bb^2+2\,Abc)x^2}{15\,x}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^6,x, algorithm="fricas")
[Out] 1/15*(3*B*c^2*x^6 + 5*(2*B*b*c + A*c^2)*x^4 - 15*A*b^2 + 15*(B*b^2 + 2*A*b*
c)*x^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^6}\,dx=-\frac{Ab^2}{x}+\frac{Bc^2x^5}{5}+x^3\left(\frac{Ac^2}{3}+\frac{2Bbc}{3}\right)+x\left(2Abc+Bb^2\right)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**6,x)
[Out] -A*b**2/x + B*c**2*x**5/5 + x**3*(A*c**2/3 + 2*B*b*c/3) + x*(2*A*b*c + B*b*
*2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^6}\,dx=\frac{1}{5}\,Bc^2x^5+\frac{1}{3}\,\left(2\,Bbc+Ac^2\right)x^3-\frac{Ab^2}{x}+\left(Bb^2+2\,Abc\right)x$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^6,x, algorithm="maxima")
[Out] 1/5*B*c^2*x^5 + 1/3*(2*B*b*c + A*c^2)*x^3 - A*b^2/x + (B*b^2 + 2*A*b*c)*x
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^6}\,dx = \frac{1}{5}\,Bc^2x^5 + \frac{2}{3}\,Bbcx^3 + \frac{1}{3}\,Ac^2x^3 + Bb^2x + 2\,Abcx - \frac{Ab^2}{x}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/5*B*c^2*x^5 + 2/3*B*b*c*x^3 + 1/3*A*c^2*x^3 + B*b^2*x + 2*A*b*c*x - A*b^2
/x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^6}\,dx = x^3\left(\frac{A\,c^2}{3}+\frac{2\,B\,b\,c}{3}\right)+x\left(B\,b^2+2\,A\,c\,b\right)-\frac{A\,b^2}{x}+\frac{B\,c^2\,x^5}{5}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^6,x)
```

```
[Out] x^3*((A*c^2)/3 + (2*B*b*c)/3) + x*(B*b^2 + 2*A*b*c) - (A*b^2)/x + (B*c^2*x^
5)/5
```

3.19 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7} dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7} dx = -\frac{Ab^2}{2x^2}+\frac{1}{2}c(2bB+Ac)x^2+\frac{1}{4}Bc^2x^4+b(bB+2Ac)\log(x)$$

```
[Out] -1/2*A*b^2/x^2+1/2*c*(A*c+2*B*b)*x^2+1/4*B*c^2*x^4+b*(2*A*c+B*b)*ln(x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 77}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7} dx = -\frac{Ab^2}{2x^2}+\frac{1}{2}cx^2(Ac+2bB)+b\log(x)(2Ac+bB)+\frac{1}{4}Bc^2x^4$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^7,x]
```

```
[Out] -1/2*(A*b^2)/x^2 + (c*(2*b*B + A*c)*x^2)/2 + (B*c^2*x^4)/4 + b*(b*B + 2*A*c
)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(A+Bx^2)(b+cx^2)^2}{x^3}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^2}{x^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(c(2bB+Ac)+\frac{Ab^2}{x^2}+\frac{b(bB+2Ac)}{x}+Bc^2x\right)\,dx, x, x^2\right)\\
&= -\frac{Ab^2}{2x^2}+\frac{1}{2}c(2bB+Ac)x^2+\frac{1}{4}Bc^2x^4+b(bB+2Ac)\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7}\,dx = \frac{1}{4}\left(-\frac{2Ab^2}{x^2}+2c(2bB+Ac)x^2+Bc^2x^4+4b(bB+2Ac)\log(x)\right)$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^7,x]
[Out] ((-2*A*b^2)/x^2 + 2*c*(2*b*B + A*c)*x^2 + B*c^2*x^4 + 4*b*(b*B + 2*A*c)*Log
[x])/4
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{B c^2 x^4}{4}+\frac{A c^2 x^2}{2}+Bbc\, x^2+b(2Ac+Bb)\ln(x)-\frac{A b^2}{2x^2}$	48
norman	$\frac{\left(\frac{1}{2}A c^2+Bbc\right)x^8-\frac{A b^2 x^4}{2}+\frac{B c^2 x^{10}}{4}}{x^6}+\left(2Abc+B b^2\right)\ln(x)$	54
parallelrisch	$\frac{B c^2 x^6+2A c^2 x^4+4x^4 Bbc+8A\ln(x)x^2 bc+4B\ln(x)x^2 b^2-2b^2 A}{4x^2}$	59
risch	$\frac{B c^2 x^4}{4}+\frac{A c^2 x^2}{2}+Bbc\, x^2+\frac{A^2 c^2}{4B}+Abc+B b^2-\frac{A b^2}{2x^2}+2A\ln(x)\, bc+b^2 B\ln(x)$	70

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^7,x,method=_RETURNVERBOSE)
[Out] 1/4*B*c^2*x^4+1/2*A*c^2*x^2+B*b*c*x^2+b*(2*A*c+B*b)*ln(x)-1/2*A*b^2/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7}\,dx = \frac{Bc^2x^6+2(2Bbc+Ac^2)x^4+4(Bb^2+2Abc)x^2\log(x)-2Ab^2}{4x^2}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^7,x, algorithm="fricas")
[Out] 1/4*(B*c^2*x^6 + 2*(2*B*b*c + A*c^2)*x^4 + 4*(B*b^2 + 2*A*b*c)*x^2*log(x) -
 2*A*b^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7}\,dx = -\frac{Ab^2}{2x^2}+\frac{Bc^2x^4}{4}+b(2Ac+Bb)\log(x)+x^2\left(\frac{Ac^2}{2}+Bbc\right)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**7,x)
[Out] -A*b**2/(2*x**2) + B*c**2*x**4/4 + b*(2*A*c + B*b)*log(x) + x**2*(A*c**2/2
+ B*b*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7}\,dx = \frac{1}{4}\,Bc^2x^4 + \frac{1}{2}\,(2\,Bbc + Ac^2)x^2 + \frac{1}{2}\,(Bb^2 + 2\,Abc)\log\left(x^2\right) - \frac{Ab^2}{2\,x^2}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*B*c^2*x^4 + 1/2*(2*B*b*c + A*c^2)*x^2 + 1/2*(B*b^2 + 2*A*b*c)*log(x^2)
- 1/2*A*b^2/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7}\,dx = \frac{1}{4}\,Bc^2x^4 + Bbcx^2 + \frac{1}{2}\,Ac^2x^2 + \frac{1}{2}\,(Bb^2 + 2\,Abc)\log\left(x^2\right) - \frac{Bb^2x^2 + 2\,Abcx^2 + Ab^2}{2\,x^2}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/4*B*c^2*x^4 + B*b*c*x^2 + 1/2*A*c^2*x^2 + 1/2*(B*b^2 + 2*A*b*c)*log(x^2)
- 1/2*(B*b^2*x^2 + 2*A*b*c*x^2 + A*b^2)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^7}\,dx = x^2\left(\frac{A\,c^2}{2} + B\,b\,c\right) + \ln\left(x\right)\,\left(B\,b^2 + 2\,A\,c\,b\right) - \frac{A\,b^2}{2\,x^2} + \frac{B\,c^2\,x^4}{4}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^7,x)
```

```
[Out] x^2*((A*c^2)/2 + B*b*c) + log(x)*(B*b^2 + 2*A*b*c) - (A*b^2)/(2*x^2) + (B*c
^2*x^4)/4
```

3.20 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^8} dx$

Optimal result

Integrand size = 24, antiderivative size = 48

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^8} dx = -\frac{Ab^2}{3x^3} - \frac{b(bB+2Ac)}{x} + c(2bB+Ac)x + \frac{1}{3}Bc^2x^3$$

```
[Out] -1/3*A*b^2/x^3-b*(2*A*c+B*b)/x+c*(A*c+2*B*b)*x+1/3*B*c^2*x^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^8} dx = -\frac{Ab^2}{3x^3} + cx(Ac+2bB) - \frac{b(2Ac+bB)}{x} + \frac{1}{3}Bc^2x^3$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^8,x]
```

```
[Out] -1/3*(A*b^2)/x^3 - (b*(b*B + 2*A*c))/x + c*(2*b*B + A*c)*x + (B*c^2*x^3)/3
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(A+Bx^2)(b+cx^2)^2}{x^4}\,dx\\
&= \int \left(c(2bB+Ac)+\frac{Ab^2}{x^4}+\frac{b(bB+2Ac)}{x^2}+Bc^2x^2\right)dx\\
&= -\frac{Ab^2}{3x^3}-\frac{b(bB+2Ac)}{x}+c(2bB+Ac)x+\frac{1}{3}Bc^2x^3
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^8}\,dx = -\frac{Ab^2}{3x^3}+\frac{-b^2B-2Abc}{x}+c(2bB+Ac)x+\frac{1}{3}Bc^2x^3$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^8,x]
[Out] -1/3*(A*b^2)/x^3 + (-(b^2*B) - 2*A*b*c)/x + c*(2*b*B + A*c)*x + (B*c^2*x^3)
/3
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{B\,c^2x^3}{3}+A\,c^2x+2Bbcx-\frac{A\,b^2}{3x^3}-\frac{b(2Ac+Bb)}{x}$	46
risch	$\frac{B\,c^2x^3}{3}+A\,c^2x+2Bbcx+\frac{(-2Abc-B\,b^2)x^2-\frac{b^2A}{3}}{x^3}$	50
gosper	$-\frac{-B\,c^2x^6-3A\,c^2x^4-6x^4Bbc+6Abc\,x^2+3b^2B\,x^2+b^2A}{3x^3}$	55
norman	$\frac{(A\,c^2+2Bbc)x^8+(-2Abc-B\,b^2)x^6-\frac{A\,b^2x^4}{3}+\frac{B\,c^2x^{10}}{3}}{x^7}$	55
parallelrisch	$\frac{B\,c^2x^6+3A\,c^2x^4+6x^4Bbc-6Abc\,x^2-3b^2B\,x^2-b^2A}{3x^3}$	55

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^8,x,method=_RETURNVERBOSE)
[Out] 1/3*B*c^2*x^3+A*c^2*x+2*B*b*c*x-1/3*A*b^2/x^3-b*(2*A*c+B*b)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^8}\,dx = \frac{Bc^2x^6+3\,(2\,Bbc+Ac^2)x^4-Ab^2-3\,(Bb^2+2\,Abc)x^2}{3\,x^3}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^8,x, algorithm="fricas")
```

```
[Out] 1/3*(B*c^2*x^6 + 3*(2*B*b*c + A*c^2)*x^4 - A*b^2 - 3*(B*b^2 + 2*A*b*c)*x^2)
/x^3
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^8}\,dx = \frac{Bc^2x^3}{3}+x\big(Ac^2+2Bbc\big)+\frac{-Ab^2+x^2(-6Abc-3Bb^2)}{3x^3}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**8,x)
```

```
[Out] B*c**2*x**3/3 + x*(A*c**2 + 2*B*b*c) + (-A*b**2 + x**2*(-6*A*b*c - 3*B*b**2
))/(3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^8}\,dx = \frac{1}{3}\,Bc^2x^3+\big(2\,Bbc+Ac^2\big)x-\frac{Ab^2+3\,(Bb^2+2\,Abc)x^2}{3\,x^3}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^8,x, algorithm="maxima")
```

```
[Out] 1/3*B*c^2*x^3 + (2*B*b*c + A*c^2)*x - 1/3*(A*b^2 + 3*(B*b^2 + 2*A*b*c)*x^2)
/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^8}\,dx = \frac{1}{3}Bc^2x^3 + 2Bbcx + Ac^2x - \frac{3Bb^2x^2 + 6Abcx^2 + Ab^2}{3x^3}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^8,x, algorithm="giac")
```

```
[Out] 1/3*B*c^2*x^3 + 2*B*b*c*x + A*c^2*x - 1/3*(3*B*b^2*x^2 + 6*A*b*c*x^2 + A*b^
2)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^8}\,dx = x\left(A\,c^2 + 2\,B\,b\,c\right) - \frac{x^2\,(B\,b^2 + 2\,A\,c\,b) + \frac{A\,b^2}{3}}{x^3} + \frac{B\,c^2\,x^3}{3}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^8,x)
```

```
[Out] x*(A*c^2 + 2*B*b*c) - (x^2*(B*b^2 + 2*A*b*c) + (A*b^2)/3)/x^3 + (B*c^2*x^3)
/3
```

3.21 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^9} dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^9} dx = -\frac{Ab^2}{4x^4} - \frac{b(bB+2Ac)}{2x^2} + \frac{1}{2}Bc^2x^2 + c(2bB+Ac)\log(x)$$

```
[Out] -1/4*A*b^2/x^4-1/2*b*(2*A*c+B*b)/x^2+1/2*B*c^2*x^2+c*(A*c+2*B*b)*ln(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 457, 77}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^9} dx = -\frac{Ab^2}{4x^4} - \frac{b(2Ac+bB)}{2x^2} + c\log(x)(Ac+2bB) + \frac{1}{2}Bc^2x^2$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^9,x]
```

```
[Out] -1/4*(A*b^2)/x^4 - (b*(b*B + 2*A*c))/(2*x^2) + (B*c^2*x^2)/2 + c*(2*b*B + A
*c)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(A+Bx^2)\,(b+cx^2)^2}{x^5}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^2}{x^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(Bc^2+\frac{Ab^2}{x^3}+\frac{b(bB+2Ac)}{x^2}+\frac{c(2bB+Ac)}{x}\right)\,dx, x, x^2\right) \\
&= -\frac{Ab^2}{4x^4}-\frac{b(bB+2Ac)}{2x^2}+\frac{1}{2}Bc^2x^2+c(2bB+Ac)\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^9}\,dx = -\frac{Ab(b+4cx^2)+2Bx^2(b^2-c^2x^4)}{4x^4}+c(2bB+Ac)\log(x)$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^9,x]
[Out] -1/4*(A*b*(b + 4*c*x^2) + 2*B*x^2*(b^2 - c^2*x^4))/x^4 + c*(2*b*B + A*c)*Lo
g[x]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{A\,b^{2}}{4x^{4}}-\frac{b(2Ac+Bb)}{2x^{2}}+\frac{B\,c^{2}x^{2}}{2}+c(Ac+2Bb)\ln\left(x\right)$	46
risch	$\frac{B\,c^{2}x^{2}}{2}+\frac{\left(-Abc-\frac{1}{2}B\,b^{2}\right)x^{2}-\frac{b^{2}A}{4}}{x^{4}}+A\ln\left(x\right)c^{2}+2B\ln\left(x\right)bc$	52
norman	$\frac{\left(-Abc-\frac{1}{2}B\,b^{2}\right)x^{6}-\frac{A\,b^{2}x^{4}}{4}+\frac{B\,c^{2}x^{10}}{2}}{x^{8}}+\left(A\,c^{2}+2Bbc\right)\ln\left(x\right)$	55
parallelrisch	$\frac{2B\,c^{2}x^{6}+4A\ln(x)x^{4}c^{2}+8B\ln(x)x^{4}bc-4Abc\,x^{2}-2b^{2}B\,x^{2}-b^{2}A}{4x^{4}}$	60

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^9,x,method=_RETURNVERBOSE)
[Out] -1/4*A*b^2/x^4-1/2*b*(2*A*c+B*b)/x^2+1/2*B*c^2*x^2+c*(A*c+2*B*b)*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int\frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^9}\,dx=\frac{2\,Bc^2x^6+4\,(2\,Bbc+Ac^2)x^4\log\left(x\right)-Ab^2-2\,(Bb^2+2\,Abc)x^2}{4\,x^4}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^9,x, algorithm="fricas")
[Out] 1/4*(2*B*c^2*x^6 + 4*(2*B*b*c + A*c^2)*x^4*log(x) - A*b^2 - 2*(B*b^2 + 2*A*
b*c)*x^2)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int\frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^9}\,dx=\frac{Bc^2x^2}{2}+c(Ac+2Bb)\log\left(x\right)+\frac{-Ab^2+x^2(-4Abc-2Bb^2)}{4x^4}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**9,x)
[Out] B*c**2*x**2/2 + c*(A*c + 2*B*b)*log(x) + (-A*b**2 + x**2*(-4*A*b*c - 2*B*b*
*2))/(4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^9}\,dx = \frac{1}{2}\,Bc^2x^2 + \frac{1}{2}\left(2\,Bbc + Ac^2\right)\log\left(x^2\right) - \frac{Ab^2 + 2\,(Bb^2 + 2\,Abc)x^2}{4\,x^4}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*B*c^2*x^2 + 1/2*(2*B*b*c + A*c^2)*log(x^2) - 1/4*(A*b^2 + 2*(B*b^2 + 2*
A*b*c)*x^2)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.41

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^9}\,dx = \frac{1}{2}\,Bc^2x^2 + \frac{1}{2}\left(2\,Bbc + Ac^2\right)\log\left(x^2\right) - \frac{6\,Bbcx^4 + 3\,Ac^2x^4 + 2\,Bb^2x^2 + 4\,Abcx^2 + Ab^2}{4\,x^4}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^9,x, algorithm="giac")
```

```
[Out] 1/2*B*c^2*x^2 + 1/2*(2*B*b*c + A*c^2)*log(x^2) - 1/4*(6*B*b*c*x^4 + 3*A*c^2
*x^4 + 2*B*b^2*x^2 + 4*A*b*c*x^2 + A*b^2)/x^4
```

Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^9}\,dx = \ln(x)\ \left(A\,c^2 + 2\,B\,b\,c\right) - \frac{x^2\left(\frac{B\,b^2}{2} + A\,c\,b\right) + \frac{A\,b^2}{4}}{x^4} + \frac{B\,c^2\,x^2}{2}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^9,x)
```

```
[Out] log(x)*(A*c^2 + 2*B*b*c) - (x^2*((B*b^2)/2 + A*b*c) + (A*b^2)/4)/x^4 + (B*c
^2*x^2)/2
```

3.22 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}} dx$

Optimal result

Integrand size = 24, antiderivative size = 48

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}} dx = -\frac{Ab^2}{5x^5} - \frac{b(bB+2Ac)}{3x^3} - \frac{c(2bB+Ac)}{x} + Bc^2x$$

```
[Out] -1/5*A*b^2/x^5-1/3*b*(2*A*c+B*b)/x^3-c*(A*c+2*B*b)/x+B*c^2*x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}} dx = -\frac{Ab^2}{5x^5} - \frac{b(2Ac+bB)}{3x^3} - \frac{c(Ac+2bB)}{x} + Bc^2x$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^10,x]
```

```
[Out] -1/5*(A*b^2)/x^5 - (b*(b*B + 2*A*c))/(3*x^3) - (c*(2*b*B + A*c))/x + B*c^2*
x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(A+Bx^2)(b+cx^2)^2}{x^6}\,dx\\ &= \int \left(Bc^2+\frac{Ab^2}{x^6}+\frac{b(bB+2Ac)}{x^4}+\frac{c(2bB+Ac)}{x^2}\right)dx\\ &= -\frac{Ab^2}{5x^5}-\frac{b(bB+2Ac)}{3x^3}-\frac{c(2bB+Ac)}{x}+Bc^2x\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}}\,dx = -\frac{Ab^2}{5x^5}-\frac{b(bB+2Ac)}{3x^3}-\frac{c(2bB+Ac)}{x}+Bc^2x$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^10,x]
[Out] -1/5*(A*b^2)/x^5 - (b*(b*B + 2*A*c))/(3*x^3) - (c*(2*b*B + A*c))/x + B*c^2*
x
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{A\,b^2}{5x^5}-\frac{b(2Ac+Bb)}{3x^3}-\frac{c(Ac+2Bb)}{x}+B\,c^2x$	45
risch	$B\,c^2x+\frac{(-A\,c^2-2Bbc)x^4+(-\frac{2}{3}Abc-\frac{1}{3}B\,b^2)x^2-\frac{b^2A}{5}}{x^5}$	51
norman	$\frac{(-\frac{2}{3}Abc-\frac{1}{3}B\,b^2)x^6+(-A\,c^2-2Bbc)x^8+B\,c^2x^{10}-\frac{A\,b^2x^4}{5}}{x^9}$	55
gosper	$-\frac{-15B\,c^2x^6+15A\,c^2x^4+30x^4Bbc+10Abc\,x^2+5b^2B\,x^2+3b^2A}{15x^5}$	56
parallelrisch	$-\frac{-15B\,c^2x^6+15A\,c^2x^4+30x^4Bbc+10Abc\,x^2+5b^2B\,x^2+3b^2A}{15x^5}$	56

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^10,x,method=_RETURNVERBOSE)
[Out] -1/5*A*b^2/x^5-1/3*b*(2*A*c+B*b)/x^3-c*(A*c+2*B*b)/x+B*c^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}}\,dx = \frac{15\,Bc^2x^6 - 15\,(2\,Bbc + Ac^2)x^4 - 3\,Ab^2 - 5\,(Bb^2 + 2\,Abc)x^2}{15\,x^5}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^10,x, algorithm="fricas")
```

```
[Out] 1/15*(15*B*c^2*x^6 - 15*(2*B*b*c + A*c^2)*x^4 - 3*A*b^2 - 5*(B*b^2 + 2*A*b*
c)*x^2)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}}\,dx = Bc^2x + \frac{-3Ab^2 + x^4(-15Ac^2 - 30Bbc) + x^2(-10Abc - 5Bb^2)}{15x^5}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**10,x)
```

```
[Out] B*c**2*x + (-3*A*b**2 + x**4*(-15*A*c**2 - 30*B*b*c) + x**2*(-10*A*b*c - 5*
B*b**2))/(15*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}}\,dx = Bc^2x - \frac{15\,(2\,Bbc + Ac^2)x^4 + 3\,Ab^2 + 5\,(Bb^2 + 2\,Abc)x^2}{15\,x^5}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^10,x, algorithm="maxima")
```

```
[Out] B*c^2*x - 1/15*(15*(2*B*b*c + A*c^2)*x^4 + 3*A*b^2 + 5*(B*b^2 + 2*A*b*c)*x^
2)/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}}\,dx = Bc^2x - \frac{30\,Bbcx^4 + 15\,Ac^2x^4 + 5\,Bb^2x^2 + 10\,Abcx^2 + 3\,Ab^2}{15\,x^5}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^10,x, algorithm="giac")
```

```
[Out] B*c^2*x - 1/15*(30*B*b*c*x^4 + 15*A*c^2*x^4 + 5*B*b^2*x^2 + 10*A*b*c*x^2 +
3*A*b^2)/x^5
```

Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{10}}\,dx = B\,c^2\,x - \frac{x^2\left(\frac{B\,b^2}{3} + \frac{2\,A\,c\,b}{3}\right) + x^4\,(A\,c^2 + 2\,B\,b\,c) + \frac{A\,b^2}{5}}{x^5}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^10,x)
```

```
[Out] B*c^2*x - (x^2*((B*b^2)/3 + (2*A*b*c)/3) + x^4*(A*c^2 + 2*B*b*c) + (A*b^2)/
5)/x^5
```

3.23 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{11}} dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{11}} dx = -\frac{Ab^2}{6x^6} - \frac{b(bB+2Ac)}{4x^4} - \frac{c(2bB+Ac)}{2x^2} + Bc^2\log(x)$$

```
[Out] -1/6*A*b^2/x^6-1/4*b*(2*A*c+B*b)/x^4-1/2*c*(A*c+2*B*b)/x^2+B*c^2*ln(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 77}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{11}} dx = -\frac{Ab^2}{6x^6} - \frac{b(2Ac+bB)}{4x^4} - \frac{c(Ac+2bB)}{2x^2} + Bc^2\log(x)$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^11,x]
```

```
[Out] -1/6*(A*b^2)/x^6 - (b*(b*B + 2*A*c))/(4*x^4) - (c*(2*b*B + A*c))/(2*x^2) +
B*c^2*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(A+Bx^2)\,(b+cx^2)^2}{x^7}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^2}{x^4}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{Ab^2}{x^4}+\frac{b(bB+2Ac)}{x^3}+\frac{c(2bB+Ac)}{x^2}+\frac{Bc^2}{x}\right)\,dx, x, x^2\right) \\
&= -\frac{Ab^2}{6x^6}-\frac{b(bB+2Ac)}{4x^4}-\frac{c(2bB+Ac)}{2x^2}+Bc^2\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^{11}}\,dx = -\frac{3bBx^2(b+4cx^2)+2A(b^2+3bcx^2+3c^2x^4)}{12x^6}+Bc^2\log(x)$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^11,x]
```

```
[Out] -1/12*(3*b*B*x^2*(b + 4*c*x^2) + 2*A*(b^2 + 3*b*c*x^2 + 3*c^2*x^4))/x^6 + B
*c^2*Log[x]
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{A\,b^{2}}{6x^{6}}-\frac{b(2Ac+Bb)}{4x^{4}}-\frac{c(Ac+2Bb)}{2x^{2}}+B\,c^{2}\ln\left(x\right)$	46
risch	$\frac{\left(-\frac{1}{2}A\,c^{2}-Bbc\right)x^{4}+\left(-\frac{1}{2}Abc-\frac{1}{4}B\,b^{2}\right)x^{2}-\frac{b^{2}A}{6}}{x^{6}}+B\,c^{2}\ln\left(x\right)$	52
norman	$\frac{\left(-\frac{1}{2}A\,c^{2}-Bbc\right)x^{8}+\left(-\frac{1}{2}Abc-\frac{1}{4}B\,b^{2}\right)x^{6}-\frac{A\,b^{2}x^{4}}{6}}{x^{10}}+B\,c^{2}\ln\left(x\right)$	55
parallelrisch	$-\frac{-12B\,c^{2}\ln\left(x\right)x^{6}+6A\,c^{2}x^{4}+12x^{4}Bbc+6Abc\,x^{2}+3b^{2}B\,x^{2}+2b^{2}A}{12x^{6}}$	58

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*A*b^2/x^6-1/4*b*(2*A*c+B*b)/x^4-1/2*c*(A*c+2*B*b)/x^2+B*c^2*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^{11}}\,dx = \frac{12\,Bc^2x^6\log\left(x\right)-6\,(2\,Bbc+Ac^2)x^4-2\,Ab^2-3\,(Bb^2+2\,Abc)x^2}{12\,x^6}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^11,x, algorithm="fricas")
```

```
[Out] 1/12*(12*B*c^2*x^6*log(x) - 6*(2*B*b*c + A*c^2)*x^4 - 2*A*b^2 - 3*(B*b^2 +
2*A*b*c)*x^2)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^{11}}\,dx = Bc^2\log\left(x\right) + \frac{-2Ab^2+x^4(-6Ac^2-12Bbc)+x^2(-6Abc-3Bb^2)}{12x^6}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**11,x)
```

```
[Out] B*c**2*log(x) + (-2*A*b**2 + x**4*(-6*A*c**2 - 12*B*b*c) + x**2*(-6*A*b*c -
 3*B*b**2))/(12*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{11}}\,dx = \frac{1}{2}\,Bc^2\log\left(x^2\right) - \frac{6\,(2\,Bbc+Ac^2)x^4+2\,Ab^2+3\,(Bb^2+2\,Abc)x^2}{12\,x^6}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^11,x, algorithm="maxima")
```

```
[Out] 1/2*B*c^2*log(x^2) - 1/12*(6*(2*B*b*c + A*c^2)*x^4 + 2*A*b^2 + 3*(B*b^2 + 2
*A*b*c)*x^2)/x^6
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{11}}\,dx = \frac{1}{2}\,Bc^2\log\left(x^2\right) - \frac{11\,Bc^2x^6+12\,Bbcx^4+6\,Ac^2x^4+3\,Bb^2x^2+6\,Abcx^2+2\,Ab^2}{12\,x^6}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^11,x, algorithm="giac")
```

```
[Out] 1/2*B*c^2*log(x^2) - 1/12*(11*B*c^2*x^6 + 12*B*b*c*x^4 + 6*A*c^2*x^4 + 3*B*
b^2*x^2 + 6*A*b*c*x^2 + 2*A*b^2)/x^6
```

Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{11}}\,dx = B\,c^2\,\ln\left(x\right) - \frac{x^2\left(\frac{B\,b^2}{4}+\frac{A\,c\,b}{2}\right)+x^4\left(\frac{A\,c^2}{2}+B\,b\,c\right)+\frac{A\,b^2}{6}}{x^6}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^11,x)
```

```
[Out] B*c^2*log(x) - (x^2*((B*b^2)/4 + (A*b*c)/2) + x^4*((A*c^2)/2 + B*b*c) + (A*
b^2)/6)/x^6
```

3.24 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{12}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 53

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{12}}\,dx = -\frac{Ab^2}{7x^7}-\frac{b(bB+2Ac)}{5x^5}-\frac{c(2bB+Ac)}{3x^3}-\frac{Bc^2}{x}$$

```
[Out] -1/7*A*b^2/x^7-1/5*b*(2*A*c+B*b)/x^5-1/3*c*(A*c+2*B*b)/x^3-B*c^2/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{12}}\,dx = -\frac{Ab^2}{7x^7}-\frac{b(2Ac+bB)}{5x^5}-\frac{c(Ac+2bB)}{3x^3}-\frac{Bc^2}{x}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^12,x]
```

```
[Out] -1/7*(A*b^2)/x^7 - (b*(b*B + 2*A*c))/(5*x^5) - (c*(2*b*B + A*c))/(3*x^3) -
(B*c^2)/x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(A+Bx^2)\,(b+cx^2)^2}{x^8}\,dx \\
&= \int \left(\frac{Ab^2}{x^8}+\frac{b(bB+2Ac)}{x^6}+\frac{c(2bB+Ac)}{x^4}+\frac{Bc^2}{x^2}\right)dx \\
&= -\frac{Ab^2}{7x^7}-\frac{b(bB+2Ac)}{5x^5}-\frac{c(2bB+Ac)}{3x^3}-\frac{Bc^2}{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^{12}}\,dx = -\frac{Ab^2}{7x^7}+\frac{-b^2B-2Abc}{5x^5}+\frac{-2bBc-Ac^2}{3x^3}-\frac{Bc^2}{x}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^12,x]
[Out] -1/7*(A*b^2)/x^7 + (-(b^2*B) - 2*A*b*c)/(5*x^5) + (-2*b*B*c - A*c^2)/(3*x^3
) - (B*c^2)/x
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{A\,b^2}{7x^7}-\frac{b(2Ac+Bb)}{5x^5}-\frac{c(Ac+2Bb)}{3x^3}-\frac{B\,c^2}{x}$	48
risch	$\frac{-B\,c^2x^6+\left(-\frac{1}{3}A\,c^2-\frac{2}{3}Bbc\right)x^4+\left(-\frac{2}{5}Abc-\frac{1}{5}B\,b^2\right)x^2-\frac{b^2A}{7}}{x^7}$	53
gosper	$-\frac{105B\,c^2x^6+35A\,c^2x^4+70x^4Bbc+42Abc\,x^2+21b^2B\,x^2+15b^2A}{105x^7}$	56
norman	$\frac{\left(-\frac{1}{3}A\,c^2-\frac{2}{3}Bbc\right)x^8+\left(-\frac{2}{5}Abc-\frac{1}{5}B\,b^2\right)x^6-\frac{A\,b^2x^4}{7}-B\,c^2x^{10}}{x^{11}}$	56
parallelrisch	$-\frac{105B\,c^2x^6+35A\,c^2x^4+70x^4Bbc+42Abc\,x^2+21b^2B\,x^2+15b^2A}{105x^7}$	56

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^12,x,method=_RETURNVERBOSE)
[Out] -1/7*A*b^2/x^7-1/5*b*(2*A*c+B*b)/x^5-1/3*c*(A*c+2*B*b)/x^3-B*c^2/x
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{12}}\,dx = -\frac{105\,Bc^2x^6+35\,(2\,Bbc+Ac^2)x^4+15\,Ab^2+21\,(Bb^2+2\,Abc)x^2}{105\,x^7}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^12,x, algorithm="fricas")
[Out] -1/105*(105*B*c^2*x^6 + 35*(2*B*b*c + A*c^2)*x^4 + 15*A*b^2 + 21*(B*b^2 + 2
*A*b*c)*x^2)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{12}}\,dx = \frac{-15Ab^2-105Bc^2x^6+x^4(-35Ac^2-70Bbc)+x^2(-42Abc-21Bb^2)}{105x^7}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**12,x)
[Out] (-15*A*b**2 - 105*B*c**2*x**6 + x**4*(-35*A*c**2 - 70*B*b*c) + x**2*(-42*A*
b*c - 21*B*b**2))/(105*x**7)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{12}}\,dx = -\frac{105\,Bc^2x^6+35\,(2\,Bbc+Ac^2)x^4+15\,Ab^2+21\,(Bb^2+2\,Abc)x^2}{105\,x^7}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^12,x, algorithm="maxima")
[Out] -1/105*(105*B*c^2*x^6 + 35*(2*B*b*c + A*c^2)*x^4 + 15*A*b^2 + 21*(B*b^2 + 2
*A*b*c)*x^2)/x^7
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{12}}\,dx$$
$$= -\frac{105\,Bc^2x^6+70\,Bbcx^4+35\,Ac^2x^4+21\,Bb^2x^2+42\,Abcx^2+15\,Ab^2}{105\,x^7}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^12,x, algorithm="giac")
```

```
[Out] -1/105*(105*B*c^2*x^6 + 70*B*b*c*x^4 + 35*A*c^2*x^4 + 21*B*b^2*x^2 + 42*A*b
*c*x^2 + 15*A*b^2)/x^7
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{12}}\,dx = -\frac{x^2\left(\frac{B\,b^2}{5}+\frac{2\,A\,c\,b}{5}\right)+x^4\left(\frac{A\,c^2}{3}+\frac{2\,B\,b\,c}{3}\right)+\frac{A\,b^2}{7}+B\,c^2\,x^6}{x^7}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^12,x)
```

```
[Out] -(x^2*((B*b^2)/5 + (2*A*b*c)/5) + x^4*((A*c^2)/3 + (2*B*b*c)/3) + (A*b^2)/7
 + B*c^2*x^6)/x^7
```

3.25 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 75

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^2} dx = \frac{1}{5}Ab^3x^5 + \frac{1}{7}b^2(bB+3Ac)x^7 + \frac{1}{3}bc(bB+Ac)x^9 + \frac{1}{11}c^2(3bB+Ac)x^{11} + \frac{1}{13}Bc^3x^{13}$$

```
[Out] 1/5*A*b^3*x^5+1/7*b^2*(3*A*c+B*b)*x^7+1/3*b*c*(A*c+B*b)*x^9+1/11*c^2*(A*c+3
*B*b)*x^11+1/13*B*c^3*x^13
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^2} dx = \frac{1}{5}Ab^3x^5 + \frac{1}{7}b^2x^7(3Ac+bB) + \frac{1}{11}c^2x^{11}(Ac+3bB) + \frac{1}{3}bcx^9(Ac+bB) + \frac{1}{13}Bc^3x^{13}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^2,x]
```

```
[Out] (A*b^3*x^5)/5 + (b^2*(b*B + 3*A*c)*x^7)/7 + (b*c*(b*B + A*c)*x^9)/3 + (c^2*
(3*b*B + A*c)*x^11)/11 + (B*c^3*x^13)/13
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
```

```
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^4 (A + Bx^2) \left(b + cx^2\right)^3 \, dx \\ &= \int \left(Ab^3x^4 + b^2(bB + 3Ac)x^6 + 3bc(bB + Ac)x^8 + c^2(3bB + Ac)x^{10} + Bc^3x^{12}\right) \, dx \\ &= \frac{1}{5}Ab^3x^5 + \frac{1}{7}b^2(bB + 3Ac)x^7 + \frac{1}{3}bc(bB + Ac)x^9 + \frac{1}{11}c^2(3bB + Ac)x^{11} + \frac{1}{13}Bc^3x^{13}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(A + Bx^2)\left(bx^2 + cx^4\right)^3}{x^2} \, dx = \frac{1}{5}Ab^3x^5 + \frac{1}{7}b^2(bB + 3Ac)x^7 + \frac{1}{3}bc(bB + Ac)x^9 + \frac{1}{11}c^2(3bB + Ac)x^{11} + \frac{1}{13}Bc^3x^{13}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^2,x]
[Out] (A*b^3*x^5)/5 + (b^2*(b*B + 3*A*c)*x^7)/7 + (b*c*(b*B + A*c)*x^9)/3 + (c^2*
(3*b*B + A*c)*x^11)/11 + (B*c^3*x^13)/13
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

method	result
default	$\frac{B\,c^{3}x^{13}}{13}+\frac{\left(A\,c^{3}+3Bb\,c^{2}\right)x^{11}}{11}+\frac{\left(3Ab\,c^{2}+3B\,b^{2}c\right)x^{9}}{9}+\frac{\left(3b^{2}Ac+B\,b^{3}\right)x^{7}}{7}+\frac{A\,b^{3}x^{5}}{5}$
risch	$\frac{1}{13}B\,c^{3}x^{13}+\frac{1}{11}x^{11}A\,c^{3}+\frac{3}{11}x^{11}Bb\,c^{2}+\frac{1}{3}x^{9}Ab\,c^{2}+\frac{1}{3}x^{9}B\,b^{2}c+\frac{3}{7}x^{7}b^{2}Ac+\frac{1}{7}b^{3}B\,x^{7}+\frac{1}{5}A\,b^{3}x^{5}$
parallelrisch	$\frac{1}{13}B\,c^{3}x^{13}+\frac{1}{11}x^{11}A\,c^{3}+\frac{3}{11}x^{11}Bb\,c^{2}+\frac{1}{3}x^{9}Ab\,c^{2}+\frac{1}{3}x^{9}B\,b^{2}c+\frac{3}{7}x^{7}b^{2}Ac+\frac{1}{7}b^{3}B\,x^{7}+\frac{1}{5}A\,b^{3}x^{5}$
norman	$\frac{\left(\frac{1}{11}A\,c^{3}+\frac{3}{11}Bb\,c^{2}\right)x^{12}+\left(\frac{1}{3}Ab\,c^{2}+\frac{1}{3}B\,b^{2}c\right)x^{10}+\left(\frac{3}{7}b^{2}Ac+\frac{1}{7}B\,b^{3}\right)x^{8}+\frac{B\,c^{3}x^{14}}{13}+\frac{x^{6}b^{3}A}{5}}{x}$
gosper	$\frac{x^{5}\left(1155B\,c^{3}x^{8}+1365A\,c^{3}x^{6}+4095x^{6}Bb\,c^{2}+5005Ab\,c^{2}x^{4}+5005x^{4}B\,b^{2}c+6435A\,b^{2}c\,x^{2}+2145b^{3}B\,x^{2}+3003b^{3}A\right)}{15015}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*B*c^3*x^13+1/11*(A*c^3+3*B*b*c^2)*x^11+1/9*(3*A*b*c^2+3*B*b^2*c)*x^9+1
/7*(3*A*b^2*c+B*b^3)*x^7+1/5*A*b^3*x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^3}{x^2}\,dx=\frac{1}{13}\,Bc^3x^{13}+\frac{1}{11}\left(3\,Bbc^2+Ac^3\right)x^{11}+\frac{1}{3}\left(Bb^2c+Abc^2\right)x^9+\frac{1}{5}\,Ab^3x^5+\frac{1}{7}\left(Bb^3+3\,Ab^2c\right)x^7$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/13*B*c^3*x^13 + 1/11*(3*B*b*c^2 + A*c^3)*x^11 + 1/3*(B*b^2*c + A*b*c^2)*x
^9 + 1/5*A*b^3*x^5 + 1/7*(B*b^3 + 3*A*b^2*c)*x^7
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^3}{x^2}\,dx=\frac{Ab^3x^5}{5}+\frac{Bc^3x^{13}}{13}+x^{11}\left(\frac{Ac^3}{11}+\frac{3Bbc^2}{11}\right)+x^9\left(\frac{Abc^2}{3}+\frac{Bb^2c}{3}\right)+x^7\cdot\left(\frac{3Ab^2c}{7}+\frac{Bb^3}{7}\right)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**2,x)
```

```
[Out] A*b**3*x**5/5 + B*c**3*x**13/13 + x**11*(A*c**3/11 + 3*B*b*c**2/11) + x**9*
(A*b*c**2/3 + B*b**2*c/3) + x**7*(3*A*b**2*c/7 + B*b**3/7)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^2}\,dx = \frac{1}{13}\,Bc^3x^{13} + \frac{1}{11}\,(3\,Bbc^2 + Ac^3)x^{11} + \frac{1}{3}\,(Bb^2c + Abc^2)x^9 + \frac{1}{5}\,Ab^3x^5 + \frac{1}{7}\,(Bb^3 + 3\,Ab^2c)x^7$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/13*B*c^3*x^13 + 1/11*(3*B*b*c^2 + A*c^3)*x^11 + 1/3*(B*b^2*c + A*b*c^2)*x
^9 + 1/5*A*b^3*x^5 + 1/7*(B*b^3 + 3*A*b^2*c)*x^7
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^2}\,dx = \frac{1}{13}\,Bc^3x^{13} + \frac{3}{11}\,Bbc^2x^{11} + \frac{1}{11}\,Ac^3x^{11} + \frac{1}{3}\,Bb^2cx^9 + \frac{1}{3}\,Abc^2x^9 + \frac{1}{7}\,Bb^3x^7 + \frac{3}{7}\,Ab^2cx^7 + \frac{1}{5}\,Ab^3x^5$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/13*B*c^3*x^13 + 3/11*B*b*c^2*x^11 + 1/11*A*c^3*x^11 + 1/3*B*b^2*c*x^9 + 1
/3*A*b*c^2*x^9 + 1/7*B*b^3*x^7 + 3/7*A*b^2*c*x^7 + 1/5*A*b^3*x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^2}\,dx = x^7\left(\frac{B\,b^3}{7} + \frac{3\,A\,c\,b^2}{7}\right) + x^{11}\left(\frac{A\,c^3}{11} + \frac{3\,B\,b\,c^2}{11}\right) + \frac{A\,b^3\,x^5}{5} + \frac{B\,c^3\,x^{13}}{13} + \frac{b\,c\,x^9\,(A\,c + B\,b)}{3}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^2,x)
```

```
[Out] x^7*((B*b^3)/7 + (3*A*b^2*c)/7) + x^11*((A*c^3)/11 + (3*B*b*c^2)/11) + (A*b
^3*x^5)/5 + (B*c^3*x^13)/13 + (b*c*x^9*(A*c + B*b))/3
```

3.26 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^3}\,dx=\frac{b(bB-Ac)(b+cx^2)^4}{8c^3}-\frac{(2bB-Ac)(b+cx^2)^5}{10c^3}+\frac{B(b+cx^2)^6}{12c^3}$$

```
[Out] 1/8*b*(-A*c+B*b)*(c*x^2+b)^4/c^3-1/10*(-A*c+2*B*b)*(c*x^2+b)^5/c^3+1/12*B*(
c*x^2+b)^6/c^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 77}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^3}\,dx=-\frac{(b+cx^2)^5(2bB-Ac)}{10c^3}+\frac{b(b+cx^2)^4(bB-Ac)}{8c^3}+\frac{B(b+cx^2)^6}{12c^3}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^3,x]
```

```
[Out] (b*(b*B - A*c)*(b + c*x^2)^4)/(8*c^3) - ((2*b*B - A*c)*(b + c*x^2)^5)/(10*c
^3) + (B*(b + c*x^2)^6)/(12*c^3)
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
```

```
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^3 (A + Bx^2) \left(b + cx^2\right)^3 \, dx \\
&= \frac{1}{2}\text{Subst}\left(\int x(A + Bx)(b + cx)^3 \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b(bB - Ac)(b + cx)^3}{c^2} + \frac{(-2bB + Ac)(b + cx)^4}{c^2} + \frac{B(b + cx)^5}{c^2}\right) dx, x, x^2\right) \\
&= \frac{b(bB - Ac)\left(b + cx^2\right)^4}{8c^3} - \frac{(2bB - Ac)\left(b + cx^2\right)^5}{10c^3} + \frac{B\left(b + cx^2\right)^6}{12c^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \frac{(A + Bx^2)\left(bx^2 + cx^4\right)^3}{x^3} dx = \frac{1}{120}x^4\left(30Ab^3 + 20b^2(bB + 3Ac)x^2 + 45bc(bB + Ac)x^4 + 12c^2(3bB + Ac)x^6 + 10Bc^3x^8\right)$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^3,x]
[Out] (x^4*(30*A*b^3 + 20*b^2*(b*B + 3*A*c)*x^2 + 45*b*c*(b*B + A*c)*x^4 + 12*c^2
*(3*b*B + A*c)*x^6 + 10*B*c^3*x^8))/120
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

method	result
default	$\frac{B\,c^3x^{12}}{12}+\frac{(A\,c^3+3Bb\,c^2)x^{10}}{10}+\frac{(3Ab\,c^2+3B\,b^2c)x^8}{8}+\frac{(3b^2Ac+B\,b^3)x^6}{6}+\frac{A\,b^3x^4}{4}$
risch	$\frac{1}{12}B\,c^3x^{12}+\frac{1}{10}x^{10}A\,c^3+\frac{3}{10}x^{10}Bb\,c^2+\frac{3}{8}x^8Ab\,c^2+\frac{3}{8}x^8B\,b^2c+\frac{1}{2}x^6b^2Ac+\frac{1}{6}b^3B\,x^6+\frac{1}{4}A\,b^3x^4$
parallelrisch	$\frac{1}{12}B\,c^3x^{12}+\frac{1}{10}x^{10}A\,c^3+\frac{3}{10}x^{10}Bb\,c^2+\frac{3}{8}x^8Ab\,c^2+\frac{3}{8}x^8B\,b^2c+\frac{1}{2}x^6b^2Ac+\frac{1}{6}b^3B\,x^6+\frac{1}{4}A\,b^3x^4$
norman	$\frac{(\frac{1}{10}A\,c^3+\frac{3}{10}Bb\,c^2)x^{12}+(\frac{3}{8}Ab\,c^2+\frac{3}{8}B\,b^2c)x^{10}+(\frac{1}{2}b^2Ac+\frac{1}{6}B\,b^3)x^8+\frac{B\,c^3x^{14}}{12}+\frac{x^6b^3A}{4}}{x^2}$
gosper	$\frac{x^4(10B\,c^3x^8+12A\,c^3x^6+36x^6Bb\,c^2+45Ab\,c^2x^4+45x^4B\,b^2c+60A\,b^2c\,x^2+20b^3B\,x^2+30b^3A)}{120}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*B*c^3*x^12+1/10*(A*c^3+3*B*b*c^2)*x^10+1/8*(3*A*b*c^2+3*B*b^2*c)*x^8+1
/6*(3*A*b^2*c+B*b^3)*x^6+1/4*A*b^3*x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^3}\,dx = \frac{1}{12}\,Bc^3x^{12}+\frac{1}{10}\,(3\,Bbc^2+Ac^3)x^{10}+\frac{3}{8}\,(Bb^2c+Abc^2)x^8+\frac{1}{4}\,Ab^3x^4+\frac{1}{6}\,(Bb^3+3\,Ab^2c)x^6$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*B*c^3*x^12 + 1/10*(3*B*b*c^2 + A*c^3)*x^10 + 3/8*(B*b^2*c + A*b*c^2)*x
^8 + 1/4*A*b^3*x^4 + 1/6*(B*b^3 + 3*A*b^2*c)*x^6
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^3}\,dx = \frac{Ab^3x^4}{4}+\frac{Bc^3x^{12}}{12}+x^{10}\left(\frac{Ac^3}{10}+\frac{3Bbc^2}{10}\right)+x^8\cdot\left(\frac{3Abc^2}{8}+\frac{3Bb^2c}{8}\right)+x^6\left(\frac{Ab^2c}{2}+\frac{Bb^3}{6}\right)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**3,x)
```

```
[Out] A*b**3*x**4/4 + B*c**3*x**12/12 + x**10*(A*c**3/10 + 3*B*b*c**2/10) + x**8*
(3*A*b*c**2/8 + 3*B*b**2*c/8) + x**6*(A*b**2*c/2 + B*b**3/6)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^3}\,dx = \frac{1}{12}\,Bc^3x^{12} + \frac{1}{10}\,(3\,Bbc^2 + Ac^3)x^{10} + \frac{3}{8}\,(Bb^2c + Abc^2)x^8 + \frac{1}{4}\,Ab^3x^4 + \frac{1}{6}\,(Bb^3 + 3\,Ab^2c)x^6$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*B*c^3*x^12 + 1/10*(3*B*b*c^2 + A*c^3)*x^10 + 3/8*(B*b^2*c + A*b*c^2)*x
^8 + 1/4*A*b^3*x^4 + 1/6*(B*b^3 + 3*A*b^2*c)*x^6
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.13

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^3}\,dx = \frac{1}{12}\,Bc^3x^{12} + \frac{3}{10}\,Bbc^2x^{10} + \frac{1}{10}\,Ac^3x^{10} + \frac{3}{8}\,Bb^2cx^8 + \frac{3}{8}\,Abc^2x^8 + \frac{1}{6}\,Bb^3x^6 + \frac{1}{2}\,Ab^2cx^6 + \frac{1}{4}\,Ab^3x^4$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/12*B*c^3*x^12 + 3/10*B*b*c^2*x^10 + 1/10*A*c^3*x^10 + 3/8*B*b^2*c*x^8 + 3
/8*A*b*c^2*x^8 + 1/6*B*b^3*x^6 + 1/2*A*b^2*c*x^6 + 1/4*A*b^3*x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^3}\,dx = x^6\left(\frac{B\,b^3}{6} + \frac{A\,c\,b^2}{2}\right) + x^{10}\left(\frac{A\,c^3}{10} + \frac{3\,B\,b\,c^2}{10}\right) + \frac{A\,b^3\,x^4}{4} + \frac{B\,c^3\,x^{12}}{12} + \frac{3\,b\,c\,x^8\,(A\,c + B\,b)}{8}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^3,x)
```

```
[Out] x^6*((B*b^3)/6 + (A*b^2*c)/2) + x^10*((A*c^3)/10 + (3*B*b*c^2)/10) + (A*b^3
*x^4)/4 + (B*c^3*x^12)/12 + (3*b*c*x^8*(A*c + B*b))/8
```

3.27 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4} dx$

Optimal result

Integrand size = 24, antiderivative size = 75

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4} dx = \frac{1}{3}Ab^3x^3 + \frac{1}{5}b^2(bB+3Ac)x^5 + \frac{3}{7}bc(bB+Ac)x^7 + \frac{1}{9}c^2(3bB+Ac)x^9 + \frac{1}{11}Bc^3x^{11}$$

```
[Out] 1/3*A*b^3*x^3+1/5*b^2*(3*A*c+B*b)*x^5+3/7*b*c*(A*c+B*b)*x^7+1/9*c^2*(A*c+3*
B*b)*x^9+1/11*B*c^3*x^11
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4} dx = \frac{1}{3}Ab^3x^3 + \frac{1}{5}b^2x^5(3Ac+bB) + \frac{1}{9}c^2x^9(Ac+3bB) + \frac{3}{7}bcx^7(Ac+bB) + \frac{1}{11}Bc^3x^{11}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^4,x]
```

```
[Out] (A*b^3*x^3)/3 + (b^2*(b*B + 3*A*c)*x^5)/5 + (3*b*c*(b*B + A*c)*x^7)/7 + (c^
2*(3*b*B + A*c)*x^9)/9 + (B*c^3*x^11)/11
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
```

```
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^2(A+Bx^2)\left(b+cx^2\right)^3\,dx\\ &= \int \left(Ab^3x^2+b^2(bB+3Ac)x^4+3bc(bB+Ac)x^6+c^2(3bB+Ac)x^8+Bc^3x^{10}\right)\,dx\\ &= \frac{1}{3}Ab^3x^3+\frac{1}{5}b^2(bB+3Ac)x^5+\frac{3}{7}bc(bB+Ac)x^7+\frac{1}{9}c^2(3bB+Ac)x^9+\frac{1}{11}Bc^3x^{11}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^3}{x^4}\,dx = \frac{1}{3}Ab^3x^3+\frac{1}{5}b^2(bB+3Ac)x^5+\frac{3}{7}bc(bB+Ac)x^7 + \frac{1}{9}c^2(3bB+Ac)x^9+\frac{1}{11}Bc^3x^{11}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^4,x]
[Out] (A*b^3*x^3)/3 + (b^2*(b*B + 3*A*c)*x^5)/5 + (3*b*c*(b*B + A*c)*x^7)/7 + (c^
2*(3*b*B + A*c)*x^9)/9 + (B*c^3*x^11)/11
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

method	result	s
default	$\frac{B\,c^{3} x^{11}}{11}+\frac{\left(A\,c^{3}+3 B b\,c^{2}\right) x^{9}}{9}+\frac{\left(3 A b\,c^{2}+3 B\,b^{2} c\right) x^{7}}{7}+\frac{\left(3 b^{2} A c+B\,b^{3}\right) x^{5}}{5}+\frac{A\,b^{3} x^{3}}{3}$	7
risch	$\frac{1}{11} B\,c^{3} x^{11}+\frac{1}{9} x^{9} A\,c^{3}+\frac{1}{3} x^{9} B b\,c^{2}+\frac{3}{7} x^{7} A b\,c^{2}+\frac{3}{7} x^{7} B\,b^{2} c+\frac{3}{5} x^{5} b^{2} A c+\frac{1}{5} b^{3} B\,x^{5}+\frac{1}{3} A\,b^{3} x^{3}$	7
parallelrisch	$\frac{1}{11} B\,c^{3} x^{11}+\frac{1}{9} x^{9} A\,c^{3}+\frac{1}{3} x^{9} B b\,c^{2}+\frac{3}{7} x^{7} A b\,c^{2}+\frac{3}{7} x^{7} B\,b^{2} c+\frac{3}{5} x^{5} b^{2} A c+\frac{1}{5} b^{3} B\,x^{5}+\frac{1}{3} A\,b^{3} x^{3}$	7
norman	$\frac{\left(\frac{1}{9} A\,c^{3}+\frac{1}{3} B b\,c^{2}\right) x^{12}+\left(\frac{3}{7} A b\,c^{2}+\frac{3}{7} B\,b^{2} c\right) x^{10}+\left(\frac{3}{5} b^{2} A c+\frac{1}{5} B\,b^{3}\right) x^{8}+\frac{B\,c^{3} x^{14}}{11}+\frac{x^{6} b^{3} A}{3}}{x^{3}}$	7
gosper	$\frac{x^{3} \left(315 B\,c^{3} x^{8}+385 A\,c^{3} x^{6}+1155 x^{6} B b\,c^{2}+1485 A b\,c^{2} x^{4}+1485 x^{4} B\,b^{2} c+2079 A\,b^{2} c\,x^{2}+693 b^{3} B\,x^{2}+1155 b^{3} A\right)}{3465}$	8

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*B*c^3*x^11+1/9*(A*c^3+3*B*b*c^2)*x^9+1/7*(3*A*b*c^2+3*B*b^2*c)*x^7+1/5
*(3*A*b^2*c+B*b^3)*x^5+1/3*A*b^3*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4}\,dx = \frac{1}{11}\,Bc^3x^{11}+\frac{1}{9}\left(3\,Bbc^2+Ac^3\right)x^9+\frac{3}{7}\left(Bb^2c+Abc^2\right)x^7 + \frac{1}{3}\,Ab^3x^3+\frac{1}{5}\left(Bb^3+3\,Ab^2c\right)x^5$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/11*B*c^3*x^11 + 1/9*(3*B*b*c^2 + A*c^3)*x^9 + 3/7*(B*b^2*c + A*b*c^2)*x^7
 + 1/3*A*b^3*x^3 + 1/5*(B*b^3 + 3*A*b^2*c)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4}\,dx = \frac{Ab^3x^3}{3}+\frac{Bc^3x^{11}}{11}+x^9\left(\frac{Ac^3}{9}+\frac{Bbc^2}{3}\right)+x^7 \cdot\left(\frac{3Abc^2}{7}+\frac{3Bb^2c}{7}\right)+x^5\cdot\left(\frac{3Ab^2c}{5}+\frac{Bb^3}{5}\right)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**4,x)
```

```
[Out] A*b**3*x**3/3 + B*c**3*x**11/11 + x**9*(A*c**3/9 + B*b*c**2/3) + x**7*(3*A*
b*c**2/7 + 3*B*b**2*c/7) + x**5*(3*A*b**2*c/5 + B*b**3/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4}\,dx = \frac{1}{11}\,Bc^3x^{11} + \frac{1}{9}\left(3\,Bbc^2 + Ac^3\right)x^9 + \frac{3}{7}\left(Bb^2c + Abc^2\right)x^7 + \frac{1}{3}\,Ab^3x^3 + \frac{1}{5}\left(Bb^3 + 3\,Ab^2c\right)x^5$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/11*B*c^3*x^11 + 1/9*(3*B*b*c^2 + A*c^3)*x^9 + 3/7*(B*b^2*c + A*b*c^2)*x^7
 + 1/3*A*b^3*x^3 + 1/5*(B*b^3 + 3*A*b^2*c)*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4}\,dx = \frac{1}{11}\,Bc^3x^{11} + \frac{1}{3}\,Bbc^2x^9 + \frac{1}{9}\,Ac^3x^9 + \frac{3}{7}\,Bb^2cx^7 + \frac{3}{7}\,Abc^2x^7 + \frac{1}{5}\,Bb^3x^5 + \frac{3}{5}\,Ab^2cx^5 + \frac{1}{3}\,Ab^3x^3$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/11*B*c^3*x^11 + 1/3*B*b*c^2*x^9 + 1/9*A*c^3*x^9 + 3/7*B*b^2*c*x^7 + 3/7*A
*b*c^2*x^7 + 1/5*B*b^3*x^5 + 3/5*A*b^2*c*x^5 + 1/3*A*b^3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^4}\,dx = x^5\left(\frac{B\,b^3}{5} + \frac{3\,A\,c\,b^2}{5}\right) + x^9\left(\frac{A\,c^3}{9} + \frac{B\,b\,c^2}{3}\right) + \frac{A\,b^3\,x^3}{3} + \frac{B\,c^3\,x^{11}}{11} + \frac{3\,b\,c\,x^7\,(A\,c + B\,b)}{7}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^4,x)
```

```
[Out] x^5*((B*b^3)/5 + (3*A*b^2*c)/5) + x^9*((A*c^3)/9 + (B*b*c^2)/3) + (A*b^3*x^
3)/3 + (B*c^3*x^11)/11 + (3*b*c*x^7*(A*c + B*b))/7
```

3.28 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 42

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^5}\,dx = -\frac{(bB-Ac)(b+cx^2)^4}{8c^2}+\frac{B(b+cx^2)^5}{10c^2}$$

```
[Out] -1/8*(-A*c+B*b)*(c*x^2+b)^4/c^2+1/10*B*(c*x^2+b)^5/c^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 455, 45}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^5}\,dx = \frac{B(b+cx^2)^5}{10c^2}-\frac{(b+cx^2)^4(bB-Ac)}{8c^2}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^5,x]
```

```
[Out] -1/8*((b*B - A*c)*(b + c*x^2)^4)/c^2 + (B*(b + c*x^2)^5)/(10*c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x\left(A+Bx^2\right)\left(b+cx^2\right)^3\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int (A+Bx)(b+cx)^3\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(-bB+Ac)(b+cx)^3}{c}+\frac{B(b+cx)^4}{c}\right)\,dx,x,x^2\right)\\
&= -\frac{(bB-Ac)\left(b+cx^2\right)^4}{8c^2}+\frac{B\left(b+cx^2\right)^5}{10c^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.64

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3}{x^5}\,dx = \frac{1}{40}x^2\left(20Ab^3+10b^2(bB+3Ac)x^2+20bc(bB+Ac)x^4 + 5c^2(3bB+Ac)x^6+4Bc^3x^8\right)$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^5,x]
[Out] (x^2*(20*A*b^3 + 10*b^2*(b*B + 3*A*c)*x^2 + 20*b*c*(b*B + A*c)*x^4 + 5*c^2*
(3*b*B + A*c)*x^6 + 4*B*c^3*x^8))/40
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.81

method	result	s
default	$\frac{B c^3 x^{10}}{10}+\frac{(A c^3+3 B b c^2) x^8}{8}+\frac{(3 A b c^2+3 B b^2 c) x^6}{6}+\frac{(3 b^2 A c+B b^3) x^4}{4}+\frac{A b^3 x^2}{2}$	7
risch	$\frac{1}{10} B c^3 x^{10}+\frac{1}{8} x^8 A c^3+\frac{3}{8} x^8 B b c^2+\frac{1}{2} x^6 A b c^2+\frac{1}{2} x^6 B b^2 c+\frac{3}{4} x^4 b^2 A c+\frac{1}{4} b^3 B x^4+\frac{1}{2} A b^3 x^2$	7
parallelrisch	$\frac{1}{10} B c^3 x^{10}+\frac{1}{8} x^8 A c^3+\frac{3}{8} x^8 B b c^2+\frac{1}{2} x^6 A b c^2+\frac{1}{2} x^6 B b^2 c+\frac{3}{4} x^4 b^2 A c+\frac{1}{4} b^3 B x^4+\frac{1}{2} A b^3 x^2$	7
norman	$\frac{(\frac{1}{8} A c^3+\frac{3}{8} B b c^2) x^{12}+(\frac{1}{2} A b c^2+\frac{1}{2} B b^2 c) x^{10}+(\frac{3}{4} b^2 A c+\frac{1}{4} B b^3) x^8+\frac{B c^3 x^{14}}{10}+\frac{x^6 b^3 A}{2}}{x^4}$	7
gosper	$\frac{x^2 (4 B c^3 x^8+5 A c^3 x^6+15 x^6 B b c^2+20 A b c^2 x^4+20 x^4 B b^2 c+30 A b^2 c x^2+10 b^3 B x^2+20 b^3 A)}{40}$	8

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*B*c^3*x^10+1/8*(A*c^3+3*B*b*c^2)*x^8+1/6*(3*A*b*c^2+3*B*b^2*c)*x^6+1/4
*(3*A*b^2*c+B*b^3)*x^4+1/2*A*b^3*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.74

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^5}\,dx = \frac{1}{10}Bc^3x^{10}+\frac{1}{8}(3Bbc^2+Ac^3)x^8+\frac{1}{2}(Bb^2c+Abc^2)x^6 + \frac{1}{2}Ab^3x^2+\frac{1}{4}(Bb^3+3Ab^2c)x^4$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^5,x, algorithm="fricas")
```

```
[Out] 1/10*B*c^3*x^10 + 1/8*(3*B*b*c^2 + A*c^3)*x^8 + 1/2*(B*b^2*c + A*b*c^2)*x^6
 + 1/2*A*b^3*x^2 + 1/4*(B*b^3 + 3*A*b^2*c)*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(36) = 72.

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.90

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^5}\,dx = \frac{Ab^3x^2}{2}+\frac{Bc^3x^{10}}{10}+x^8\left(\frac{Ac^3}{8}+\frac{3Bbc^2}{8}\right) + x^6\left(\frac{Abc^2}{2}+\frac{Bb^2c}{2}\right)+x^4\cdot\left(\frac{3Ab^2c}{4}+\frac{Bb^3}{4}\right)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**5,x)
```

```
[Out] A*b**3*x**2/2 + B*c**3*x**10/10 + x**8*(A*c**3/8 + 3*B*b*c**2/8) + x**6*(A*
b*c**2/2 + B*b**2*c/2) + x**4*(3*A*b**2*c/4 + B*b**3/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.74

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^3}{x^5}\,dx = \frac{1}{10}\,Bc^3x^{10} + \frac{1}{8}\,\left(3\,Bbc^2+Ac^3\right)x^8 + \frac{1}{2}\,\left(Bb^2c+Abc^2\right)x^6 + \frac{1}{2}\,Ab^3x^2 + \frac{1}{4}\,\left(Bb^3+3\,Ab^2c\right)x^4$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^5,x, algorithm="maxima")
```

```
[Out] 1/10*B*c^3*x^10 + 1/8*(3*B*b*c^2 + A*c^3)*x^8 + 1/2*(B*b^2*c + A*b*c^2)*x^6
 + 1/2*A*b^3*x^2 + 1/4*(B*b^3 + 3*A*b^2*c)*x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(38) = 76.

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.83

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^3}{x^5}\,dx = \frac{1}{10}\,Bc^3x^{10} + \frac{3}{8}\,Bbc^2x^8 + \frac{1}{8}\,Ac^3x^8 + \frac{1}{2}\,Bb^2cx^6 + \frac{1}{2}\,Abc^2x^6 + \frac{1}{4}\,Bb^3x^4 + \frac{3}{4}\,Ab^2cx^4 + \frac{1}{2}\,Ab^3x^2$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^5,x, algorithm="giac")
```

```
[Out] 1/10*B*c^3*x^10 + 3/8*B*b*c^2*x^8 + 1/8*A*c^3*x^8 + 1/2*B*b^2*c*x^6 + 1/2*A
*b*c^2*x^6 + 1/4*B*b^3*x^4 + 3/4*A*b^2*c*x^4 + 1/2*A*b^3*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.64

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^3}{x^5}\,dx = x^4\left(\frac{B\,b^3}{4}+\frac{3\,A\,c\,b^2}{4}\right)+x^8\left(\frac{A\,c^3}{8}+\frac{3\,B\,b\,c^2}{8}\right) + \frac{A\,b^3\,x^2}{2}+\frac{B\,c^3\,x^{10}}{10}+\frac{b\,c\,x^6\,\left(A\,c+B\,b\right)}{2}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^5,x)
```

```
[Out] x^4*((B*b^3)/4 + (3*A*b^2*c)/4) + x^8*((A*c^3)/8 + (3*B*b*c^2)/8) + (A*b^3*
x^2)/2 + (B*c^3*x^10)/10 + (b*c*x^6*(A*c + B*b))/2
```

3.29 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6} dx$

Optimal result

Integrand size = 24, antiderivative size = 70

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6} dx = Ab^3x + \frac{1}{3}b^2(bB+3Ac)x^3 + \frac{3}{5}bc(bB+Ac)x^5 + \frac{1}{7}c^2(3bB+Ac)x^7 + \frac{1}{9}Bc^3x^9$$

```
[Out] A*b^3*x+1/3*b^2*(3*A*c+B*b)*x^3+3/5*b*c*(A*c+B*b)*x^5+1/7*c^2*(A*c+3*B*b)*x
^7+1/9*B*c^3*x^9
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 380}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6} dx = Ab^3x + \frac{1}{3}b^2x^3(3Ac+bB) + \frac{1}{7}c^2x^7(Ac+3bB) + \frac{3}{5}bcx^5(Ac+bB) + \frac{1}{9}Bc^3x^9$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^6,x]
```

```
[Out] A*b^3*x + (b^2*(b*B + 3*A*c)*x^3)/3 + (3*b*c*(b*B + A*c)*x^5)/5 + (c^2*(3*b
*B + A*c)*x^7)/7 + (B*c^3*x^9)/9
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
```

```
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int (A+Bx^2)\,(b+cx^2)^3\,dx\\ &= \int \left(Ab^3+b^2(bB+3Ac)x^2+3bc(bB+Ac)x^4+c^2(3bB+Ac)x^6+Bc^3x^8\right)dx\\ &= Ab^3x+\frac{1}{3}b^2(bB+3Ac)x^3+\frac{3}{5}bc(bB+Ac)x^5+\frac{1}{7}c^2(3bB+Ac)x^7+\frac{1}{9}Bc^3x^9\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^6}\,dx = Ab^3x+\frac{1}{3}b^2(bB+3Ac)x^3+\frac{3}{5}bc(bB+Ac)x^5 + \frac{1}{7}c^2(3bB+Ac)x^7+\frac{1}{9}Bc^3x^9$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^6,x]
[Out] A*b^3*x + (b^2*(b*B + 3*A*c)*x^3)/3 + (3*b*c*(b*B + A*c)*x^5)/5 + (c^2*(3*b
*B + A*c)*x^7)/7 + (B*c^3*x^9)/9
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{B\,c^{3}x^{9}}{9}+\frac{(A\,c^{3}+3Bb\,c^{2})x^{7}}{7}+\frac{(3Ab\,c^{2}+3B\,b^{2}c)x^{5}}{5}+\frac{(3b^{2}Ac+B\,b^{3})x^{3}}{3}+A\,b^{3}x$	73
risch	$\frac{1}{9}B\,c^{3}x^{9}+\frac{1}{7}x^{7}A\,c^{3}+\frac{3}{7}x^{7}Bb\,c^{2}+\frac{3}{5}x^{5}Ab\,c^{2}+\frac{3}{5}x^{5}B\,b^{2}c+x^{3}b^{2}Ac+\frac{1}{3}b^{3}B\,x^{3}+A\,b^{3}x$	74
parallelrisch	$\frac{1}{9}B\,c^{3}x^{9}+\frac{1}{7}x^{7}A\,c^{3}+\frac{3}{7}x^{7}Bb\,c^{2}+\frac{3}{5}x^{5}Ab\,c^{2}+\frac{3}{5}x^{5}B\,b^{2}c+x^{3}b^{2}Ac+\frac{1}{3}b^{3}B\,x^{3}+A\,b^{3}x$	74
norman	$\frac{(\frac{1}{7}A\,c^{3}+\frac{3}{7}Bb\,c^{2})x^{12}+(\frac{3}{5}Ab\,c^{2}+\frac{3}{5}B\,b^{2}c)x^{10}+(b^{2}Ac+\frac{1}{3}B\,b^{3})x^{8}+x^{6}b^{3}A+\frac{B\,c^{3}x^{14}}{9}}{x^{5}}$	77
gosper	$\frac{x(35B\,c^{3}x^{8}+45A\,c^{3}x^{6}+135x^{6}Bb\,c^{2}+189Ab\,c^{2}x^{4}+189x^{4}B\,b^{2}c+315A\,b^{2}c\,x^{2}+105b^{3}B\,x^{2}+315b^{3}A)}{315}$	78

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*B*c^3*x^9+1/7*(A*c^3+3*B*b*c^2)*x^7+1/5*(3*A*b*c^2+3*B*b^2*c)*x^5+1/3*(
3*A*b^2*c+B*b^3)*x^3+A*b^3*x
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6}\,dx = \frac{1}{9}Bc^3x^9+\frac{1}{7}\left(3Bbc^2+Ac^3\right)x^7+\frac{3}{5}\left(Bb^2c+Abc^2\right)x^5+Ab^3x+\frac{1}{3}\left(Bb^3+3Ab^2c\right)x^3$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^6,x, algorithm="fricas")
```

```
[Out] 1/9*B*c^3*x^9 + 1/7*(3*B*b*c^2 + A*c^3)*x^7 + 3/5*(B*b^2*c + A*b*c^2)*x^5 +
 A*b^3*x + 1/3*(B*b^3 + 3*A*b^2*c)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6}\,dx = Ab^3x+\frac{Bc^3x^9}{9}+x^7\left(\frac{Ac^3}{7}+\frac{3Bbc^2}{7}\right)+x^5\cdot\left(\frac{3Abc^2}{5}+\frac{3Bb^2c}{5}\right)+x^3\left(Ab^2c+\frac{Bb^3}{3}\right)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**6,x)
```

```
[Out] A*b**3*x + B*c**3*x**9/9 + x**7*(A*c**3/7 + 3*B*b*c**2/7) + x**5*(3*A*b*c**
2/5 + 3*B*b**2*c/5) + x**3*(A*b**2*c + B*b**3/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6}\,dx = \frac{1}{9}\,Bc^3x^9 + \frac{1}{7}\left(3\,Bbc^2 + Ac^3\right)x^7 + \frac{3}{5}\left(Bb^2c + Abc^2\right)x^5 + Ab^3x + \frac{1}{3}\left(Bb^3 + 3\,Ab^2c\right)x^3$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^6,x, algorithm="maxima")
```

```
[Out] 1/9*B*c^3*x^9 + 1/7*(3*B*b*c^2 + A*c^3)*x^7 + 3/5*(B*b^2*c + A*b*c^2)*x^5 +
 A*b^3*x + 1/3*(B*b^3 + 3*A*b^2*c)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6}\,dx = \frac{1}{9}\,Bc^3x^9 + \frac{3}{7}\,Bbc^2x^7 + \frac{1}{7}\,Ac^3x^7 + \frac{3}{5}\,Bb^2cx^5 + \frac{3}{5}\,Abc^2x^5 + \frac{1}{3}\,Bb^3x^3 + Ab^2cx^3 + Ab^3x$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^6,x, algorithm="giac")
```

```
[Out] 1/9*B*c^3*x^9 + 3/7*B*b*c^2*x^7 + 1/7*A*c^3*x^7 + 3/5*B*b^2*c*x^5 + 3/5*A*b
*c^2*x^5 + 1/3*B*b^3*x^3 + A*b^2*c*x^3 + A*b^3*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^6}\,dx = x^3\left(\frac{B\,b^3}{3} + A\,c\,b^2\right) + x^7\left(\frac{A\,c^3}{7} + \frac{3\,B\,b\,c^2}{7}\right) + \frac{B\,c^3\,x^9}{9} + A\,b^3\,x + \frac{3\,b\,c\,x^5\,(A\,c + B\,b)}{5}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^6,x)
```

```
[Out] x^3*((B*b^3)/3 + A*b^2*c) + x^7*((A*c^3)/7 + (3*B*b*c^2)/7) + (B*c^3*x^9)/9
 + A*b^3*x + (3*b*c*x^5*(A*c + B*b))/5
```

3.30 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^7} dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^7} dx = \frac{3}{2}Ab^2cx^2 + \frac{3}{4}Abc^2x^4 + \frac{1}{6}Ac^3x^6 + \frac{B(b+cx^2)^4}{8c} + Ab^3\log(x)$$

```
[Out] 3/2*A*b^2*c*x^2+3/4*A*b*c^2*x^4+1/6*A*c^3*x^6+1/8*B*(c*x^2+b)^4/c+A*b^3*ln(
x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 457, 81, 45}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^7} dx = Ab^3\log(x) + \frac{3}{2}Ab^2cx^2 + \frac{3}{4}Abc^2x^4 + \frac{1}{6}Ac^3x^6 + \frac{B(b+cx^2)^4}{8c}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^7,x]
```

```
[Out] (3*A*b^2*c*x^2)/2 + (3*A*b*c^2*x^4)/4 + (A*c^3*x^6)/6 + (B*(b + c*x^2)^4)/(
8*c) + A*b^3*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(A+Bx^2)\,(b+cx^2)^3}{x}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x}\,dx,x,x^2\right) \\
&= \frac{B(b+cx^2)^4}{8c}+\frac{1}{2}A\text{Subst}\left(\int \frac{(b+cx)^3}{x}\,dx,x,x^2\right) \\
&= \frac{B(b+cx^2)^4}{8c}+\frac{1}{2}A\text{Subst}\left(\int \left(3b^2c+\frac{b^3}{x}+3bc^2x+c^3x^2\right)\,dx,x,x^2\right) \\
&= \frac{3}{2}Ab^2cx^2+\frac{3}{4}Abc^2x^4+\frac{1}{6}Ac^3x^6+\frac{B(b+cx^2)^4}{8c}+Ab^3\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\begin{aligned}
\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^7}\,dx &= \frac{1}{2}b^2(bB+3Ac)x^2+\frac{3}{4}bc(bB+Ac)x^4 \\
&\quad+\frac{1}{6}c^2(3bB+Ac)x^6+\frac{1}{8}Bc^3x^8+Ab^3\log(x)
\end{aligned}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^7,x]
[Out] (b^2*(b*B + 3*A*c)*x^2)/2 + (3*b*c*(b*B + A*c)*x^4)/4 + (c^2*(3*b*B + A*c)*
x^6)/6 + (B*c^3*x^8)/8 + A*b^3*Log[x]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.27

method	result	size
default	$\frac{B c^3 x^8}{8}+\frac{A c^3 x^6}{6}+\frac{x^6 B b c^2}{2}+\frac{3 A b c^2 x^4}{4}+\frac{3 x^4 B b^2 c}{4}+\frac{3 A b^2 c x^2}{2}+\frac{b^3 B x^2}{2}+A b^3 \ln \left(x\right)$	76
risch	$\frac{B c^3 x^8}{8}+\frac{A c^3 x^6}{6}+\frac{x^6 B b c^2}{2}+\frac{3 A b c^2 x^4}{4}+\frac{3 x^4 B b^2 c}{4}+\frac{3 A b^2 c x^2}{2}+\frac{b^3 B x^2}{2}+A b^3 \ln \left(x\right)$	76
parallelrisch	$\frac{B c^3 x^8}{8}+\frac{A c^3 x^6}{6}+\frac{x^6 B b c^2}{2}+\frac{3 A b c^2 x^4}{4}+\frac{3 x^4 B b^2 c}{4}+\frac{3 A b^2 c x^2}{2}+\frac{b^3 B x^2}{2}+A b^3 \ln \left(x\right)$	76
norman	$\frac{\left(\frac{1}{6} A c^3+\frac{1}{2} B b c^2\right) x^{12}+\left(\frac{3}{4} A b c^2+\frac{3}{4} B b^2 c\right) x^{10}+\left(\frac{3}{2} b^2 A c+\frac{1}{2} B b^3\right) x^8+\frac{B c^3 x^{14}}{8}}{x^6}+A b^3 \ln \left(x\right)$	78

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*B*c^3*x^8+1/6*A*c^3*x^6+1/2*x^6*B*b*c^2+3/4*A*b*c^2*x^4+3/4*x^4*B*b^2*c
+3/2*A*b^2*c*x^2+1/2*b^3*B*x^2+A*b^3*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3}{x^7}\,dx = \frac{1}{8}\,Bc^3x^8+\frac{1}{6}\left(3\,Bbc^2+Ac^3\right)x^6+\frac{3}{4}\left(Bb^2c+Abc^2\right)x^4 + Ab^3\log\left(x\right)+\frac{1}{2}\left(Bb^3+3\,Ab^2c\right)x^2$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^7,x, algorithm="fricas")
```

```
[Out] 1/8*B*c^3*x^8 + 1/6*(3*B*b*c^2 + A*c^3)*x^6 + 3/4*(B*b^2*c + A*b*c^2)*x^4 +
 A*b^3*log(x) + 1/2*(B*b^3 + 3*A*b^2*c)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.33

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3}{x^7}\,dx = Ab^3\log\left(x\right)+\frac{Bc^3x^8}{8}+x^6\left(\frac{Ac^3}{6}+\frac{Bbc^2}{2}\right)+x^4 \cdot\left(\frac{3Abc^2}{4}+\frac{3Bb^2c}{4}\right)+x^2\cdot\left(\frac{3Ab^2c}{2}+\frac{Bb^3}{2}\right)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**7,x)
```

```
[Out] A*b**3*log(x) + B*c**3*x**8/8 + x**6*(A*c**3/6 + B*b*c**2/2) + x**4*(3*A*b*
c**2/4 + 3*B*b**2*c/4) + x**2*(3*A*b**2*c/2 + B*b**3/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^7}\,dx = \frac{1}{8}\,Bc^3x^8 + \frac{1}{6}\,(3\,Bbc^2 + Ac^3)x^6 + \frac{3}{4}\,(Bb^2c + Abc^2)x^4 + \frac{1}{2}\,Ab^3\log\left(x^2\right) + \frac{1}{2}\,(Bb^3 + 3\,Ab^2c)x^2$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^7,x, algorithm="maxima")
```

```
[Out] 1/8*B*c^3*x^8 + 1/6*(3*B*b*c^2 + A*c^3)*x^6 + 3/4*(B*b^2*c + A*b*c^2)*x^4 +
 1/2*A*b^3*log(x^2) + 1/2*(B*b^3 + 3*A*b^2*c)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^7}\,dx = \frac{1}{8}\,Bc^3x^8 + \frac{1}{2}\,Bbc^2x^6 + \frac{1}{6}\,Ac^3x^6 + \frac{3}{4}\,Bb^2cx^4 + \frac{3}{4}\,Abc^2x^4 + \frac{1}{2}\,Bb^3x^2 + \frac{3}{2}\,Ab^2cx^2 + \frac{1}{2}\,Ab^3\log\left(x^2\right)$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^7,x, algorithm="giac")
```

```
[Out] 1/8*B*c^3*x^8 + 1/2*B*b*c^2*x^6 + 1/6*A*c^3*x^6 + 3/4*B*b^2*c*x^4 + 3/4*A*b
*c^2*x^4 + 1/2*B*b^3*x^2 + 3/2*A*b^2*c*x^2 + 1/2*A*b^3*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^7}\,dx = x^2\left(\frac{B\,b^3}{2} + \frac{3\,A\,c\,b^2}{2}\right) + x^6\left(\frac{A\,c^3}{6} + \frac{B\,b\,c^2}{2}\right) + \frac{B\,c^3\,x^8}{8} + A\,b^3\,\ln\left(x\right) + \frac{3\,b\,c\,x^4\,(A\,c + B\,b)}{4}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^7,x)
```

```
[Out] x^2*((B*b^3)/2 + (3*A*b^2*c)/2) + x^6*((A*c^3)/6 + (B*b*c^2)/2) + (B*c^3*x^
8)/8 + A*b^3*log(x) + (3*b*c*x^4*(A*c + B*b))/4
```

3.31 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8} dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8} dx = -\frac{Ab^3}{x} + b^2(bB+3Ac)x + bc(bB+Ac)x^3 + \frac{1}{5}c^2(3bB+Ac)x^5 + \frac{1}{7}Bc^3x^7$$

```
[Out] -A*b^3/x+b^2*(3*A*c+B*b)*x+b*c*(A*c+B*b)*x^3+1/5*c^2*(A*c+3*B*b)*x^5+1/7*B*
c^3*x^7
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8} dx = -\frac{Ab^3}{x} + b^2x(3Ac+bB) + \frac{1}{5}c^2x^5(Ac+3bB) + bcx^3(Ac+bB) + \frac{1}{7}Bc^3x^7$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^8,x]
```

```
[Out] -((A*b^3)/x) + b^2*(b*B + 3*A*c)*x + b*c*(b*B + A*c)*x^3 + (c^2*(3*b*B + A*
c)*x^5)/5 + (B*c^3*x^7)/7
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
```

```
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^2}\,dx \\
&= \int \left(b^2(bB+3Ac)+\frac{Ab^3}{x^2}+3bc(bB+Ac)x^2+c^2(3bB+Ac)x^4+Bc^3x^6\right)dx \\
&= -\frac{Ab^3}{x}+b^2(bB+3Ac)x+bc(bB+Ac)x^3+\frac{1}{5}c^2(3bB+Ac)x^5+\frac{1}{7}Bc^3x^7
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$
\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8}\,dx = -\frac{Ab^3}{x}+b^2(bB+3Ac)x+bc(bB+Ac)x^3 + \frac{1}{5}c^2(3bB+Ac)x^5+\frac{1}{7}Bc^3x^7
$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^8,x]
[Out] -((A*b^3)/x) + b^2*(b*B + 3*A*c)*x + b*c*(b*B + A*c)*x^3 + (c^2*(3*b*B + A*
c)*x^5)/5 + (B*c^3*x^7)/7
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{B\,c^3x^7}{7}+\frac{A\,c^3x^5}{5}+\frac{3Bb\,c^2x^5}{5}+Ab\,c^2x^3+B\,b^2c\,x^3+3A\,b^2cx+b^3Bx-\frac{A\,b^3}{x}$	71
risch	$\frac{B\,c^3x^7}{7}+\frac{A\,c^3x^5}{5}+\frac{3Bb\,c^2x^5}{5}+Ab\,c^2x^3+B\,b^2c\,x^3+3A\,b^2cx+b^3Bx-\frac{A\,b^3}{x}$	71
norman	$\frac{\left(\frac{1}{5}A\,c^3+\frac{3}{5}Bb\,c^2\right)x^{12}+\left(Ab\,c^2+B\,b^2c\right)x^{10}+\left(3b^2Ac+B\,b^3\right)x^8+\frac{B\,c^3x^{14}}{7}-x^6b^3A}{x^7}$	76
gosper	$-\frac{-5B\,c^3x^8-7A\,c^3x^6-21x^6Bb\,c^2-35Ab\,c^2x^4-35x^4B\,b^2c-105A\,b^2c\,x^2-35b^3B\,x^2+35b^3A}{35x}$	80
parallelrisch	$\frac{5B\,c^3x^8+7A\,c^3x^6+21x^6Bb\,c^2+35Ab\,c^2x^4+35x^4B\,b^2c+105A\,b^2c\,x^2+35b^3B\,x^2-35b^3A}{35x}$	80

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*B*c^3*x^7+1/5*A*c^3*x^5+3/5*B*b*c^2*x^5+A*b*c^2*x^3+B*b^2*c*x^3+3*A*b^2
*c*x+b^3*B*x-A*b^3/x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^8}\,dx = \frac{5\,Bc^3x^8+7\,(3\,Bbc^2+Ac^3)x^6+35\,(Bb^2c+Abc^2)x^4-35\,Ab^3+35\,(Bb^3+3\,Ab^2c)x^2}{35\,x}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^8,x, algorithm="fricas")
```

```
[Out] 1/35*(5*B*c^3*x^8 + 7*(3*B*b*c^2 + A*c^3)*x^6 + 35*(B*b^2*c + A*b*c^2)*x^4
- 35*A*b^3 + 35*(B*b^3 + 3*A*b^2*c)*x^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^8}\,dx = -\frac{Ab^3}{x}+\frac{Bc^3x^7}{7}+x^5\left(\frac{Ac^3}{5}+\frac{3Bbc^2}{5}\right)+x^3\left(Abc^2+Bb^2c\right)+x\left(3Ab^2c+Bb^3\right)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**8,x)
```

```
[Out] -A*b**3/x + B*c**3*x**7/7 + x**5*(A*c**3/5 + 3*B*b*c**2/5) + x**3*(A*b*c**2
 + B*b**2*c) + x*(3*A*b**2*c + B*b**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8}\,dx = \frac{1}{7}Bc^3x^7 + \frac{1}{5}\left(3\,Bbc^2 + Ac^3\right)x^5 + \left(Bb^2c + Abc^2\right)x^3 - \frac{Ab^3}{x} + \left(Bb^3 + 3\,Ab^2c\right)x$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^8,x, algorithm="maxima")
[Out] 1/7*B*c^3*x^7 + 1/5*(3*B*b*c^2 + A*c^3)*x^5 + (B*b^2*c + A*b*c^2)*x^3 - A*b
^3/x + (B*b^3 + 3*A*b^2*c)*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8}\,dx = \frac{1}{7}Bc^3x^7 + \frac{3}{5}Bbc^2x^5 + \frac{1}{5}Ac^3x^5 + Bb^2cx^3 + Abc^2x^3 + Bb^3x + 3\,Ab^2cx - \frac{Ab^3}{x}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^8,x, algorithm="giac")
[Out] 1/7*B*c^3*x^7 + 3/5*B*b*c^2*x^5 + 1/5*A*c^3*x^5 + B*b^2*c*x^3 + A*b*c^2*x^3
 + B*b^3*x + 3*A*b^2*c*x - A*b^3/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^8}\,dx = x\left(B\,b^3 + 3\,A\,c\,b^2\right) + x^5\left(\frac{A\,c^3}{5} + \frac{3\,B\,b\,c^2}{5}\right) - \frac{A\,b^3}{x} + \frac{B\,c^3\,x^7}{7} + b\,c\,x^3\,(A\,c + B\,b)$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^8,x)
[Out] x*(B*b^3 + 3*A*b^2*c) + x^5*((A*c^3)/5 + (3*B*b*c^2)/5) - (A*b^3)/x + (B*c^
3*x^7)/7 + b*c*x^3*(A*c + B*b)
```

3.32 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^9} dx$

Optimal result

Integrand size = 24, antiderivative size = 71

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^9} dx = -\frac{Ab^3}{2x^2} + \frac{3}{2}bc(bB+Ac)x^2 + \frac{1}{4}c^2(3bB+Ac)x^4 + \frac{1}{6}Bc^3x^6 + b^2(bB+3Ac)\log(x)$$

```
[Out] -1/2*A*b^3/x^2+3/2*b*c*(A*c+B*b)*x^2+1/4*c^2*(A*c+3*B*b)*x^4+1/6*B*c^3*x^6+
b^2*(3*A*c+B*b)*ln(x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 77}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^9} dx = -\frac{Ab^3}{2x^2} + b^2\log(x)(3Ac+bB) + \frac{1}{4}c^2x^4(Ac+3bB) + \frac{3}{2}bcx^2(Ac+bB) + \frac{1}{6}Bc^3x^6$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^9,x]
```

```
[Out] -1/2*(A*b^3)/x^2 + (3*b*c*(b*B + A*c)*x^2)/2 + (c^2*(3*b*B + A*c)*x^4)/4 +
(B*c^3*x^6)/6 + b^2*(b*B + 3*A*c)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
```

```
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(A+Bx^2)\,(b+cx^2)^3}{x^3}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(3bc(bB+Ac)+\frac{Ab^3}{x^2}+\frac{b^2(bB+3Ac)}{x}+c^2(3bB+Ac)x+Bc^3x^2\right)\,dx,x,x^2\right)\\
&= -\frac{Ab^3}{2x^2}+\frac{3}{2}bc(bB+Ac)x^2+\frac{1}{4}c^2(3bB+Ac)x^4+\frac{1}{6}Bc^3x^6+b^2(bB+3Ac)\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\begin{aligned}
\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^9}\,dx =& -\frac{Ab^3}{2x^2}+\frac{3}{2}bc(bB+Ac)x^2+\frac{1}{4}c^2(3bB+Ac)x^4\\
&+\frac{1}{6}Bc^3x^6+\left(b^3B+3Ab^2c\right)\log(x)
\end{aligned}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^9,x]
```

```
[Out] -1/2*(A*b^3)/x^2 + (3*b*c*(b*B + A*c)*x^2)/2 + (c^2*(3*b*B + A*c)*x^4)/4 +
(B*c^3*x^6)/6 + (b^3*B + 3*A*b^2*c)*Log[x]
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{B\,c^{3}x^{6}}{6}+\frac{A\,c^{3}x^{4}}{4}+\frac{3Bb\,c^{2}x^{4}}{4}+\frac{3Ab\,c^{2}x^{2}}{2}+\frac{3B\,b^{2}c\,x^{2}}{2}+b^{2}(3Ac+Bb)\ln\left(x\right)-\frac{A\,b^{3}}{2x^{2}}$	73
risch	$\frac{B\,c^{3}x^{6}}{6}+\frac{A\,c^{3}x^{4}}{4}+\frac{3Bb\,c^{2}x^{4}}{4}+\frac{3Ab\,c^{2}x^{2}}{2}+\frac{3B\,b^{2}c\,x^{2}}{2}-\frac{A\,b^{3}}{2x^{2}}+3A\ln\left(x\right)b^{2}c+b^{3}B\ln\left(x\right)$	75
norman	$\frac{\left(\frac{1}{4}A\,c^{3}+\frac{3}{4}Bb\,c^{2}\right)x^{12}+\left(\frac{3}{2}Ab\,c^{2}+\frac{3}{2}B\,b^{2}c\right)x^{10}+\frac{B\,c^{3}x^{14}}{6}-\frac{x^{6}b^{3}A}{2}}{x^{8}}+\left(3b^{2}Ac+B\,b^{3}\right)\ln\left(x\right)$	78
parallelrisch	$\frac{2B\,c^{3}x^{8}+3A\,c^{3}x^{6}+9x^{6}Bb\,c^{2}+18Ab\,c^{2}x^{4}+18x^{4}B\,b^{2}c+36A\ln(x)x^{2}b^{2}c+12B\ln(x)x^{2}b^{3}-6b^{3}A}{12x^{2}}$	84

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^9,x,method=_RETURNVERBOSE)
[Out] 1/6*B*c^3*x^6+1/4*A*c^3*x^4+3/4*B*b*c^2*x^4+3/2*A*b*c^2*x^2+3/2*B*b^2*c*x^2
+b^2*(3*A*c+B*b)*ln(x)-1/2*A*b^3/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^9}\,dx = \frac{2\,Bc^3x^8+3\,(3\,Bbc^2+Ac^3)x^6+18\,(Bb^2c+Abc^2)x^4-6\,Ab^3+12\,(Bb^3+3\,Ab^2c)x^2\log\left(x\right)}{12\,x^2}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^9,x, algorithm="fricas")
[Out] 1/12*(2*B*c^3*x^8 + 3*(3*B*b*c^2 + A*c^3)*x^6 + 18*(B*b^2*c + A*b*c^2)*x^4
- 6*A*b^3 + 12*(B*b^3 + 3*A*b^2*c)*x^2*log(x))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^9}\,dx = -\frac{Ab^3}{2x^2}+\frac{Bc^3x^6}{6}+b^2\cdot(3Ac+Bb)\log\left(x\right) + x^4\left(\frac{Ac^3}{4}+\frac{3Bbc^2}{4}\right)+x^2\cdot\left(\frac{3Abc^2}{2}+\frac{3Bb^2c}{2}\right)$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**9,x)
[Out] -A*b**3/(2*x**2) + B*c**3*x**6/6 + b**2*(3*A*c + B*b)*log(x) + x**4*(A*c**3
/4 + 3*B*b*c**2/4) + x**2*(3*A*b*c**2/2 + 3*B*b**2*c/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^9}\,dx = \frac{1}{6}Bc^3x^6 + \frac{1}{4}\left(3Bbc^2+Ac^3\right)x^4 + \frac{3}{2}\left(Bb^2c+Abc^2\right)x^2 - \frac{Ab^3}{2x^2} + \frac{1}{2}\left(Bb^3+3Ab^2c\right)\log\left(x^2\right)$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^9,x, algorithm="maxima")
```

```
[Out] 1/6*B*c^3*x^6 + 1/4*(3*B*b*c^2 + A*c^3)*x^4 + 3/2*(B*b^2*c + A*b*c^2)*x^2 -
 1/2*A*b^3/x^2 + 1/2*(B*b^3 + 3*A*b^2*c)*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.37

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^9}\,dx = \frac{1}{6}Bc^3x^6 + \frac{3}{4}Bbc^2x^4 + \frac{1}{4}Ac^3x^4 + \frac{3}{2}Bb^2cx^2 + \frac{3}{2}Abc^2x^2 + \frac{1}{2}\left(Bb^3+3Ab^2c\right)\log\left(x^2\right) - \frac{Bb^3x^2+3Ab^2cx^2+Ab^3}{2x^2}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^9,x, algorithm="giac")
```

```
[Out] 1/6*B*c^3*x^6 + 3/4*B*b*c^2*x^4 + 1/4*A*c^3*x^4 + 3/2*B*b^2*c*x^2 + 3/2*A*b
*c^2*x^2 + 1/2*(B*b^3 + 3*A*b^2*c)*log(x^2) - 1/2*(B*b^3*x^2 + 3*A*b^2*c*x^
2 + A*b^3)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^9}\,dx = x^4\left(\frac{A\,c^3}{4}+\frac{3\,B\,b\,c^2}{4}\right) + \ln(x)\,\left(B\,b^3+3\,A\,c\,b^2\right) - \frac{A\,b^3}{2\,x^2} + \frac{B\,c^3\,x^6}{6} + \frac{3\,b\,c\,x^2\,(A\,c+B\,b)}{2}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^9,x)
```

```
[Out] x^4*((A*c^3)/4 + (3*B*b*c^2)/4) + log(x)*(B*b^3 + 3*A*b^2*c) - (A*b^3)/(2*x
^2) + (B*c^3*x^6)/6 + (3*b*c*x^2*(A*c + B*b))/2
```

3.33 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}} dx$

Optimal result

Integrand size = 24, antiderivative size = 69

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}} dx = -\frac{Ab^3}{3x^3} - \frac{b^2(bB+3Ac)}{x} + 3bc(bB+Ac)x + \frac{1}{3}c^2(3bB+Ac)x^3 + \frac{1}{5}Bc^3x^5$$

```
[Out] -1/3*A*b^3/x^3-b^2*(3*A*c+B*b)/x+3*b*c*(A*c+B*b)*x+1/3*c^2*(A*c+3*B*b)*x^3+
1/5*B*c^3*x^5
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}} dx = -\frac{Ab^3}{3x^3} - \frac{b^2(3Ac+bB)}{x} + \frac{1}{3}c^2x^3(Ac+3bB) + 3bcx(Ac+bB) + \frac{1}{5}Bc^3x^5$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^10,x]
```

```
[Out] -1/3*(A*b^3)/x^3 - (b^2*(b*B + 3*A*c))/x + 3*b*c*(b*B + A*c)*x + (c^2*(3*b*
B + A*c)*x^3)/3 + (B*c^3*x^5)/5
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
```

```
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^4}\,dx\\ &= \int \left(3bc(bB+Ac)+\frac{Ab^3}{x^4}+\frac{b^2(bB+3Ac)}{x^2}+c^2(3bB+Ac)x^2+Bc^3x^4\right)dx\\ &= -\frac{Ab^3}{3x^3}-\frac{b^2(bB+3Ac)}{x}+3bc(bB+Ac)x+\frac{1}{3}c^2(3bB+Ac)x^3+\frac{1}{5}Bc^3x^5\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}}\,dx = -\frac{Ab^3}{3x^3}+\frac{-b^3B-3Ab^2c}{x}+3bc(bB+Ac)x + \frac{1}{3}c^2(3bB+Ac)x^3+\frac{1}{5}Bc^3x^5$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^10,x]
[Out] -1/3*(A*b^3)/x^3 + (-(b^3*B) - 3*A*b^2*c)/x + 3*b*c*(b*B + A*c)*x + (c^2*(3
*b*B + A*c)*x^3)/3 + (B*c^3*x^5)/5
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{B c^3 x^5}{5}+\frac{A c^3 x^3}{3}+B b c^2 x^3+3 A b c^2 x+3 B b^2 c x-\frac{A b^3}{3 x^3}-\frac{b^2 (3 A c+B b)}{x}$	70
risch	$\frac{B c^3 x^5}{5}+\frac{A c^3 x^3}{3}+B b c^2 x^3+3 A b c^2 x+3 B b^2 c x+\frac{(-3 b^2 A c-B b^3) x^2-\frac{b^3 A}{3}}{x^3}$	74
norman	$\frac{(\frac{1}{3} A c^3+B b c^2) x^{12}+(3 A b c^2+3 B b^2 c) x^{10}+(-3 b^2 A c-B b^3) x^8+\frac{B c^3 x^{14}}{5}-\frac{x^6 b^3 A}{3}}{x^9}$	78
gosper	$-\frac{-3 B c^3 x^8-5 A c^3 x^6-15 x^6 B b c^2-45 A b c^2 x^4-45 x^4 B b^2 c+45 A b^2 c x^2+15 b^3 B x^2+5 b^3 A}{15 x^3}$	80
parallelrisch	$\frac{3 B c^3 x^8+5 A c^3 x^6+15 x^6 B b c^2+45 A b c^2 x^4+45 x^4 B b^2 c-45 A b^2 c x^2-15 b^3 B x^2-5 b^3 A}{15 x^3}$	80

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*B*c^3*x^5+1/3*A*c^3*x^3+B*b*c^2*x^3+3*A*b*c^2*x+3*B*b^2*c*x-1/3*A*b^3/x
^3-b^2*(3*A*c+B*b)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}}\,dx = \frac{3\,Bc^3x^8+5\,(3\,Bbc^2+Ac^3)x^6+45\,(Bb^2c+Abc^2)x^4-5\,Ab^3-15\,(Bb^3+3\,Ab^2c)x^2}{15\,x^3}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^10,x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*c^3*x^8 + 5*(3*B*b*c^2 + A*c^3)*x^6 + 45*(B*b^2*c + A*b*c^2)*x^4
- 5*A*b^3 - 15*(B*b^3 + 3*A*b^2*c)*x^2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}}\,dx = \frac{Bc^3x^5}{5}+x^3\left(\frac{Ac^3}{3}+Bbc^2\right)+x\left(3Abc^2+3Bb^2c\right)+\frac{-Ab^3+x^2(-9Ab^2c-3Bb^3)}{3x^3}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**10,x)
```

```
[Out] B*c**3*x**5/5 + x**3*(A*c**3/3 + B*b*c**2) + x*(3*A*b*c**2 + 3*B*b**2*c) +
(-A*b**3 + x**2*(-9*A*b**2*c - 3*B*b**3))/(3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}}\,dx = \frac{1}{5}\,Bc^3x^5 + \frac{1}{3}\,(3\,Bbc^2 + Ac^3)x^3 + 3\,(Bb^2c + Abc^2)x - \frac{Ab^3 + 3\,(Bb^3 + 3\,Ab^2c)x^2}{3\,x^3}$$

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^10,x, algorithm="maxima")

[Out] 1/5*B*c^3*x^5 + 1/3*(3*B*b*c^2 + A*c^3)*x^3 + 3*(B*b^2*c + A*b*c^2)*x - 1/3
*(A*b^3 + 3*(B*b^3 + 3*A*b^2*c)*x^2)/x^3

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}}\,dx = \frac{1}{5}\,Bc^3x^5 + Bbc^2x^3 + \frac{1}{3}\,Ac^3x^3 + 3\,Bb^2cx + 3\,Abc^2x - \frac{3\,Bb^3x^2 + 9\,Ab^2cx^2 + Ab^3}{3\,x^3}$$

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^10,x, algorithm="giac")

[Out] 1/5*B*c^3*x^5 + B*b*c^2*x^3 + 1/3*A*c^3*x^3 + 3*B*b^2*c*x + 3*A*b*c^2*x - 1
/3*(3*B*b^3*x^2 + 9*A*b^2*c*x^2 + A*b^3)/x^3

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{10}}\,dx = x^3\left(\frac{A\,c^3}{3} + B\,b\,c^2\right) - \frac{\frac{A\,b^3}{3} + x^2\,(B\,b^3 + 3\,A\,c\,b^2)}{x^3} + \frac{B\,c^3\,x^5}{5} + 3\,b\,c\,x\,(A\,c + B\,b)$$

[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^10,x)

[Out] x^3*((A*c^3)/3 + B*b*c^2) - ((A*b^3)/3 + x^2*(B*b^3 + 3*A*b^2*c))/x^3 + (B*
c^3*x^5)/5 + 3*b*c*x*(A*c + B*b)

3.34 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{11}} dx$

Optimal result

Integrand size = 24, antiderivative size = 72

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{11}} dx = -\frac{Ab^3}{4x^4} - \frac{b^2(bB+3Ac)}{2x^2} + \frac{1}{2}c^2(3bB+Ac)x^2 + \frac{1}{4}Bc^3x^4 + 3bc(bB+Ac)\log(x)$$

```
[Out] -1/4*A*b^3/x^4-1/2*b^2*(3*A*c+B*b)/x^2+1/2*c^2*(A*c+3*B*b)*x^2+1/4*B*c^3*x^
4+3*b*c*(A*c+B*b)*ln(x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 77}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{11}} dx = -\frac{Ab^3}{4x^4} - \frac{b^2(3Ac+bB)}{2x^2} + \frac{1}{2}c^2x^2(Ac+3bB) + 3bc\log(x)(Ac+bB) + \frac{1}{4}Bc^3x^4$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^11,x]
```

```
[Out] -1/4*(A*b^3)/x^4 - (b^2*(b*B + 3*A*c))/(2*x^2) + (c^2*(3*b*B + A*c)*x^2)/2
+ (B*c^3*x^4)/4 + 3*b*c*(b*B + A*c)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
```

```
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(A+Bx^2)\,(b+cx^2)^3}{x^5}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(c^2(3bB+Ac)+\frac{Ab^3}{x^3}+\frac{b^2(bB+3Ac)}{x^2}+\frac{3bc(bB+Ac)}{x}+Bc^3x\right)\,dx, x, x^2\right) \\
&= -\frac{Ab^3}{4x^4}-\frac{b^2(bB+3Ac)}{2x^2}+\frac{1}{2}c^2(3bB+Ac)x^2+\frac{1}{4}Bc^3x^4+3bc(bB+Ac)\log(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$
\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^{11}}\,dx = \frac{-A(b^3+6b^2cx^2-2c^3x^6)+Bx^2(-2b^3+6bc^2x^4+c^3x^6)}{4x^4} + 3bc(bB+Ac)\log(x)
$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^11,x]
[Out] (-(A*(b^3 + 6*b^2*c*x^2 - 2*c^3*x^6)) + B*x^2*(-2*b^3 + 6*b*c^2*x^4 + c^3*x
^6))/(4*x^4) + 3*b*c*(b*B + A*c)*Log[x]
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

method	result
default	$\frac{B\,c^3x^4}{4}+\frac{A\,c^3x^2}{2}+\frac{3Bb\,c^2x^2}{2}+3bc(Ac+Bb)\ln(x)-\frac{b^2(3Ac+Bb)}{2x^2}-\frac{A\,b^3}{4x^4}$
norman	$\frac{\left(\frac{1}{2}A\,c^3+\frac{3}{2}Bb\,c^2\right)x^{12}+\left(-\frac{3}{2}b^2Ac-\frac{1}{2}B\,b^3\right)x^8+\frac{B\,c^3x^{14}}{4}-\frac{x^6b^3A}{4}}{x^{10}}+\left(3Ab\,c^2+3B\,b^2c\right)\ln(x)$
parallelrisch	$\frac{B\,c^3x^8+2A\,c^3x^6+6x^6Bb\,c^2+12A\ln(x)x^4b\,c^2+12B\ln(x)x^4b^2c-6A\,b^2c\,x^2-2b^3B\,x^2-b^3A}{4x^4}$
risch	$\frac{B\,c^3x^4}{4}+\frac{A\,c^3x^2}{2}+\frac{3Bb\,c^2x^2}{2}+\frac{c^3A^2}{4B}+\frac{3Ab\,c^2}{2}+\frac{9B\,b^2c}{4}+\frac{\left(-\frac{3}{2}b^2Ac-\frac{1}{2}B\,b^3\right)x^2-\frac{b^3A}{4}}{x^4}+3A\ln(x)\,b\,c^2+3B$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*B*c^3*x^4+1/2*A*c^3*x^2+3/2*B*b*c^2*x^2+3*b*c*(A*c+B*b)*ln(x)-1/2*b^2*(
3*A*c+B*b)/x^2-1/4*A*b^3/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^{11}}\,dx = \frac{Bc^3x^8+2\,(3\,Bbc^2+Ac^3)x^6+12\,(Bb^2c+Abc^2)x^4\log(x)-Ab^3-2\,(Bb^3+3\,Ab^2c)x^2}{4\,x^4}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^11,x, algorithm="fricas")
```

```
[Out] 1/4*(B*c^3*x^8 + 2*(3*B*b*c^2 + A*c^3)*x^6 + 12*(B*b^2*c + A*b*c^2)*x^4*log
(x) - A*b^3 - 2*(B*b^3 + 3*A*b^2*c)*x^2)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^{11}}\,dx = \frac{Bc^3x^4}{4}+3bc(Ac+Bb)\log(x)+x^2\left(\frac{Ac^3}{2}+\frac{3Bbc^2}{2}\right)+\frac{-Ab^3+x^2(-6Ab^2c-2Bb^3)}{4x^4}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**11,x)
```

```
[Out] B*c**3*x**4/4 + 3*b*c*(A*c + B*b)*log(x) + x**2*(A*c**3/2 + 3*B*b*c**2/2) +
 (-A*b**3 + x**2*(-6*A*b**2*c - 2*B*b**3))/(4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{11}}\,dx = \frac{1}{4}\,Bc^3x^4 + \frac{1}{2}\left(3\,Bbc^2+Ac^3\right)x^2 + \frac{3}{2}\left(Bb^2c+Abc^2\right)\log\left(x^2\right) - \frac{Ab^3+2\,(Bb^3+3\,Ab^2c)x^2}{4\,x^4}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^11,x, algorithm="maxima")
```

```
[Out] 1/4*B*c^3*x^4 + 1/2*(3*B*b*c^2 + A*c^3)*x^2 + 3/2*(B*b^2*c + A*b*c^2)*log(x
^2) - 1/4*(A*b^3 + 2*(B*b^3 + 3*A*b^2*c)*x^2)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.36

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{11}}\,dx = \frac{1}{4}\,Bc^3x^4 + \frac{3}{2}\,Bbc^2x^2 + \frac{1}{2}\,Ac^3x^2 + \frac{3}{2}\left(Bb^2c+Abc^2\right)\log\left(x^2\right) - \frac{9\,Bb^2cx^4+9\,Abc^2x^4+2\,Bb^3x^2+6\,Ab^2cx^2+Ab^3}{4\,x^4}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^11,x, algorithm="giac")
```

```
[Out] 1/4*B*c^3*x^4 + 3/2*B*b*c^2*x^2 + 1/2*A*c^3*x^2 + 3/2*(B*b^2*c + A*b*c^2)*l
og(x^2) - 1/4*(9*B*b^2*c*x^4 + 9*A*b*c^2*x^4 + 2*B*b^3*x^2 + 6*A*b^2*c*x^2
+ A*b^3)/x^4
```

Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{11}}\,dx = \ln(x)\,\left(3\,B\,b^2\,c+3\,A\,b\,c^2\right) - \frac{\frac{A\,b^3}{4}+x^2\left(\frac{B\,b^3}{2}+\frac{3\,A\,c\,b^2}{2}\right)}{x^4} + x^2\left(\frac{A\,c^3}{2}+\frac{3\,B\,b\,c^2}{2}\right) + \frac{B\,c^3\,x^4}{4}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^11,x)
```

```
[Out] log(x)*(3*A*b*c^2 + 3*B*b^2*c) - ((A*b^3)/4 + x^2*((B*b^3)/2 + (3*A*b^2*c)/
2))/x^4 + x^2*((A*c^3)/2 + (3*B*b*c^2)/2) + (B*c^3*x^4)/4
```

3.35 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}} dx$

Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}} dx = -\frac{Ab^3}{5x^5} - \frac{b^2(bB+3Ac)}{3x^3} - \frac{3bc(bB+Ac)}{x} + c^2(3bB+Ac)x + \frac{1}{3}Bc^3x^3$$

```
[Out] -1/5*A*b^3/x^5-1/3*b^2*(3*A*c+B*b)/x^3-3*b*c*(A*c+B*b)/x+c^2*(A*c+3*B*b)*x+
1/3*B*c^3*x^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}} dx = -\frac{Ab^3}{5x^5} - \frac{b^2(3Ac+bB)}{3x^3} + c^2x(Ac+3bB) - \frac{3bc(Ac+bB)}{x} + \frac{1}{3}Bc^3x^3$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^12,x]
```

```
[Out] -1/5*(A*b^3)/x^5 - (b^2*(b*B + 3*A*c))/(3*x^3) - (3*b*c*(b*B + A*c))/x + c^
2*(3*b*B + A*c)*x + (B*c^3*x^3)/3
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(A+Bx^2)\,(b+cx^2)^3}{x^6}\,dx \\
&= \int \left(c^2(3bB+Ac)+\frac{Ab^3}{x^6}+\frac{b^2(bB+3Ac)}{x^4}+\frac{3bc(bB+Ac)}{x^2}+Bc^3x^2\right)dx \\
&= -\frac{Ab^3}{5x^5}-\frac{b^2(bB+3Ac)}{3x^3}-\frac{3bc(bB+Ac)}{x}+c^2(3bB+Ac)x+\frac{1}{3}Bc^3x^3
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^{12}}\,dx = -\frac{Ab^3}{5x^5}-\frac{b^2(bB+3Ac)}{3x^3}-\frac{3bc(bB+Ac)}{x} + c^2(3bB+Ac)x+\frac{1}{3}Bc^3x^3$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^12,x]
[Out] -1/5*(A*b^3)/x^5 - (b^2*(b*B + 3*A*c))/(3*x^3) - (3*b*c*(b*B + A*c))/x + c^
2*(3*b*B + A*c)*x + (B*c^3*x^3)/3
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{B\,c^3 x^3}{3}+A\,c^3 x+3 B b\,c^2 x-\frac{b^2 (3 A c+B b)}{3 x^3}-\frac{3 b c (A c+B b)}{x}-\frac{A\,b^3}{5 x^5}$	64
risch	$\frac{B\,c^3 x^3}{3}+A\,c^3 x+3 B b\,c^2 x+\frac{\left(-3 A b\,c^2-3 B\,b^2 c\right) x^4+\left(-b^2 A c-\frac{1}{3} B\,b^3\right) x^2-\frac{b^3 A}{5}}{x^5}$	73
norman	$\frac{\left(-b^2 A c-\frac{1}{3} B\,b^3\right) x^8+\left(A\,c^3+3 B b\,c^2\right) x^{12}+\left(-3 A b\,c^2-3 B\,b^2 c\right) x^{10}+\frac{B\,c^3 x^{14}}{3}-\frac{x^6 b^3 A}{5}}{x^{11}}$	78
gosper	$-\frac{-5 B\,c^3 x^8-15 A\,c^3 x^6-45 x^6 B b\,c^2+45 A b\,c^2 x^4+45 x^4 B\,b^2 c+15 A\,b^2 c\,x^2+5 b^3 B\,x^2+3 b^3 A}{15 x^5}$	80
parallelrisch	$\frac{5 B\,c^3 x^8+15 A\,c^3 x^6+45 x^6 B b\,c^2-45 A b\,c^2 x^4-45 x^4 B\,b^2 c-15 A\,b^2 c\,x^2-5 b^3 B\,x^2-3 b^3 A}{15 x^5}$	80

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*B*c^3*x^3+A*c^3*x+3*B*b*c^2*x-1/3*b^2*(3*A*c+B*b)/x^3-3*b*c*(A*c+B*b)/x
-1/5*A*b^3/x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}}\,dx = \frac{5\,Bc^3x^8+15\,(3\,Bbc^2+Ac^3)x^6-45\,(Bb^2c+Abc^2)x^4-3\,Ab^3-5\,(Bb^3+3\,Ab^2c)x^2}{15\,x^5}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^12,x, algorithm="fricas")
```

```
[Out] 1/15*(5*B*c^3*x^8 + 15*(3*B*b*c^2 + A*c^3)*x^6 - 45*(B*b^2*c + A*b*c^2)*x^4
 - 3*A*b^3 - 5*(B*b^3 + 3*A*b^2*c)*x^2)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}}\,dx = \frac{Bc^3x^3}{3}+x\left(Ac^3+3Bbc^2\right)+\frac{-3Ab^3+x^4(-45Abc^2-45Bb^2c)+x^2(-15Ab^2c-5Bb^3)}{15x^5}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**12,x)
```

```
[Out] B*c**3*x**3/3 + x*(A*c**3 + 3*B*b*c**2) + (-3*A*b**3 + x**4*(-45*A*b*c**2 -
 45*B*b**2*c) + x**2*(-15*A*b**2*c - 5*B*b**3))/(15*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}}\,dx = \frac{1}{3}Bc^3x^3 + \left(3Bbc^2 + Ac^3\right)x - \frac{45(Bb^2c + Abc^2)x^4 + 3Ab^3 + 5(Bb^3 + 3Ab^2c)x^2}{15x^5}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^12,x, algorithm="maxima")
```

```
[Out] 1/3*B*c^3*x^3 + (3*B*b*c^2 + A*c^3)*x - 1/15*(45*(B*b^2*c + A*b*c^2)*x^4 +
3*A*b^3 + 5*(B*b^3 + 3*A*b^2*c)*x^2)/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}}\,dx = \frac{1}{3}Bc^3x^3 + 3Bbc^2x + Ac^3x - \frac{45Bb^2cx^4 + 45Abc^2x^4 + 5Bb^3x^2 + 15Ab^2cx^2 + 3Ab^3}{15x^5}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^12,x, algorithm="giac")
```

```
[Out] 1/3*B*c^3*x^3 + 3*B*b*c^2*x + A*c^3*x - 1/15*(45*B*b^2*c*x^4 + 45*A*b*c^2*x
^4 + 5*B*b^3*x^2 + 15*A*b^2*c*x^2 + 3*A*b^3)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{12}}\,dx = x\left(A\,c^3 + 3\,B\,b\,c^2\right) - \frac{x^4\left(3\,B\,b^2\,c + 3\,A\,b\,c^2\right) + \frac{A\,b^3}{5} + x^2\left(\frac{B\,b^3}{3} + A\,c\,b^2\right)}{x^5} + \frac{B\,c^3\,x^3}{3}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^12,x)
```

```
[Out] x*(A*c^3 + 3*B*b*c^2) - (x^4*(3*A*b*c^2 + 3*B*b^2*c) + (A*b^3)/5 + x^2*((B*
b^3)/3 + A*b^2*c))/x^5 + (B*c^3*x^3)/3
```

3.36 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{13}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 71

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{13}}\,dx = -\frac{Ab^3}{6x^6}-\frac{b^2(bB+3Ac)}{4x^4}-\frac{3bc(bB+Ac)}{2x^2}+\frac{1}{2}Bc^3x^2+c^2(3bB+Ac)\log(x)$$

```
[Out] -1/6*A*b^3/x^6-1/4*b^2*(3*A*c+B*b)/x^4-3/2*b*c*(A*c+B*b)/x^2+1/2*B*c^3*x^2+
c^2*(A*c+3*B*b)*ln(x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 77}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{13}}\,dx = -\frac{Ab^3}{6x^6}-\frac{b^2(3Ac+bB)}{4x^4}+c^2\log(x)(Ac+3bB)-\frac{3bc(Ac+bB)}{2x^2}+\frac{1}{2}Bc^3x^2$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^13,x]
```

```
[Out] -1/6*(A*b^3)/x^6 - (b^2*(b*B + 3*A*c))/(4*x^4) - (3*b*c*(b*B + A*c))/(2*x^2
) + (B*c^3*x^2)/2 + c^2*(3*b*B + A*c)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
```

```
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(A+Bx^2)\,(b+cx^2)^3}{x^7}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x^4}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(Bc^3+\frac{Ab^3}{x^4}+\frac{b^2(bB+3Ac)}{x^3}+\frac{3bc(bB+Ac)}{x^2}+\frac{c^2(3bB+Ac)}{x}\right)\,dx, x, x^2\right) \\
&= -\frac{Ab^3}{6x^6}-\frac{b^2(bB+3Ac)}{4x^4}-\frac{3bc(bB+Ac)}{2x^2}+\frac{1}{2}Bc^3x^2+c^2(3bB+Ac)\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^{13}}\,dx = -\frac{Ab^3}{6x^6}-\frac{b^2(bB+3Ac)}{4x^4}-\frac{3bc(bB+Ac)}{2x^2} + \frac{1}{2}Bc^3x^2+c^2(3bB+Ac)\log(x)$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^13,x]
```

```
[Out] -1/6*(A*b^3)/x^6 - (b^2*(b*B + 3*A*c))/(4*x^4) - (3*b*c*(b*B + A*c))/(2*x^2
) + (B*c^3*x^2)/2 + c^2*(3*b*B + A*c)*Log[x]
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{A\,b^{3}}{6x^{6}}-\frac{b^{2}(3Ac+Bb)}{4x^{4}}-\frac{3bc(Ac+Bb)}{2x^{2}}+\frac{B\,c^{3}x^{2}}{2}+c^{2}(Ac+3Bb)\ln\left(x\right)$	64
risch	$\frac{B\,c^{3}x^{2}}{2}+\frac{\left(-\frac{3}{2}Ab\,c^{2}-\frac{3}{2}B\,b^{2}c\right)x^{4}+\left(-\frac{3}{4}b^{2}Ac-\frac{1}{4}B\,b^{3}\right)x^{2}-\frac{b^{3}A}{6}}{x^{6}}+A\ln\left(x\right)c^{3}+3B\ln\left(x\right)b\,c^{2}$	75
norman	$\frac{\left(-\frac{3}{2}Ab\,c^{2}-\frac{3}{2}B\,b^{2}c\right)x^{10}+\left(-\frac{3}{4}b^{2}Ac-\frac{1}{4}B\,b^{3}\right)x^{8}+\frac{B\,c^{3}x^{14}}{2}-\frac{x^{6}b^{3}A}{6}}{x^{12}}+\left(A\,c^{3}+3Bb\,c^{2}\right)\ln\left(x\right)$	78
parallelrisch	$\frac{6B\,c^{3}x^{8}+12A\ln(x)x^{6}c^{3}+36B\ln(x)x^{6}b\,c^{2}-18Ab\,c^{2}x^{4}-18x^{4}B\,b^{2}c-9A\,b^{2}c\,x^{2}-3b^{3}B\,x^{2}-2b^{3}A}{12x^{6}}$	84

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*A*b^3/x^6-1/4*b^2*(3*A*c+B*b)/x^4-3/2*b*c*(A*c+B*b)/x^2+1/2*B*c^3*x^2+
c^2*(A*c+3*B*b)*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^3}{x^{13}}\,dx = \frac{6\,Bc^3x^8+12\,(3\,Bbc^2+Ac^3)x^6\log\left(x\right)-18\,(Bb^2c+Abc^2)x^4-2\,Ab^3-3\,(Bb^3+3\,Ab^2c)x^2}{12\,x^6}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^13,x, algorithm="fricas")
```

```
[Out] 1/12*(6*B*c^3*x^8 + 12*(3*B*b*c^2 + A*c^3)*x^6*log(x) - 18*(B*b^2*c + A*b*c
^2)*x^4 - 2*A*b^3 - 3*(B*b^3 + 3*A*b^2*c)*x^2)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^3}{x^{13}}\,dx = \frac{Bc^3x^2}{2}+c^2(Ac+3Bb)\log\left(x\right) + \frac{-2Ab^3+x^4(-18Abc^2-18Bb^2c)+x^2(-9Ab^2c-3Bb^3)}{12x^6}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**13,x)
```

```
[Out] B*c**3*x**2/2 + c**2*(A*c + 3*B*b)*log(x) + (-2*A*b**3 + x**4*(-18*A*b*c**2
 - 18*B*b**2*c) + x**2*(-9*A*b**2*c - 3*B*b**3))/(12*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{13}}\,dx = \frac{1}{2}\,Bc^3x^2 + \frac{1}{2}\left(3\,Bbc^2 + Ac^3\right)\log\left(x^2\right) - \frac{18\,(Bb^2c + Abc^2)x^4 + 2\,Ab^3 + 3\,(Bb^3 + 3\,Ab^2c)x^2}{12\,x^6}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^13,x, algorithm="maxima")
```

```
[Out] 1/2*B*c^3*x^2 + 1/2*(3*B*b*c^2 + A*c^3)*log(x^2) - 1/12*(18*(B*b^2*c + A*b*
c^2)*x^4 + 2*A*b^3 + 3*(B*b^3 + 3*A*b^2*c)*x^2)/x^6
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.39

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{13}}\,dx = \frac{1}{2}\,Bc^3x^2 + \frac{1}{2}\left(3\,Bbc^2 + Ac^3\right)\log\left(x^2\right) - \frac{33\,Bbc^2x^6 + 11\,Ac^3x^6 + 18\,Bb^2cx^4 + 18\,Abc^2x^4 + 3\,Bb^3x^2 + 9\,Ab^2cx^2 + 2\,Ab^3}{12\,x^6}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^13,x, algorithm="giac")
```

```
[Out] 1/2*B*c^3*x^2 + 1/2*(3*B*b*c^2 + A*c^3)*log(x^2) - 1/12*(33*B*b*c^2*x^6 + 1
1*A*c^3*x^6 + 18*B*b^2*c*x^4 + 18*A*b*c^2*x^4 + 3*B*b^3*x^2 + 9*A*b^2*c*x^2
 + 2*A*b^3)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{13}}\,dx = \ln\left(x\right)\,\left(A\,c^3 + 3\,B\,b\,c^2\right) - \frac{x^4\,\left(\frac{3\,B\,b^2\,c}{2} + \frac{3\,A\,b\,c^2}{2}\right) + \frac{A\,b^3}{6} + x^2\,\left(\frac{B\,b^3}{4} + \frac{3\,A\,c\,b^2}{4}\right)}{x^6} + \frac{B\,c^3\,x^2}{2}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^13,x)
```

```
[Out] log(x)*(A*c^3 + 3*B*b*c^2) - (x^4*((3*A*b*c^2)/2 + (3*B*b^2*c)/2) + (A*b^3)
/6 + x^2*((B*b^3)/4 + (3*A*b^2*c)/4))/x^6 + (B*c^3*x^2)/2
```

3.37 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 66

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}} \, dx = -\frac{Ab^3}{7x^7} - \frac{b^2(bB+3Ac)}{5x^5} - \frac{bc(bB+Ac)}{x^3} - \frac{c^2(3bB+Ac)}{x} + Bc^3x$$

```
[Out] -1/7*A*b^3/x^7-1/5*b^2*(3*A*c+B*b)/x^5-b*c*(A*c+B*b)/x^3-c^2*(A*c+3*B*b)/x+
B*c^3*x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}} \, dx = -\frac{Ab^3}{7x^7} - \frac{b^2(3Ac+bB)}{5x^5} - \frac{c^2(Ac+3bB)}{x} - \frac{bc(Ac+bB)}{x^3} + Bc^3x$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^14,x]
```

```
[Out] -1/7*(A*b^3)/x^7 - (b^2*(b*B + 3*A*c))/(5*x^5) - (b*c*(b*B + A*c))/x^3 - (c
^2*(3*b*B + A*c))/x + B*c^3*x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{(A+Bx^2)(b+cx^2)^3}{x^8}\,dx \\ &= \int \left(Bc^3+\frac{Ab^3}{x^8}+\frac{b^2(bB+3Ac)}{x^6}+\frac{3bc(bB+Ac)}{x^4}+\frac{c^2(3bB+Ac)}{x^2}\right)dx \\ &= -\frac{Ab^3}{7x^7}-\frac{b^2(bB+3Ac)}{5x^5}-\frac{bc(bB+Ac)}{x^3}-\frac{c^2(3bB+Ac)}{x}+Bc^3x \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}}\,dx = -\frac{Ab^3}{7x^7}-\frac{b^2(bB+3Ac)}{5x^5}-\frac{bc(bB+Ac)}{x^3}-\frac{c^2(3bB+Ac)}{x}+Bc^3x$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^14,x]
[Out] -1/7*(A*b^3)/x^7 - (b^2*(b*B + 3*A*c))/(5*x^5) - (b*c*(b*B + A*c))/x^3 - (c
^2*(3*b*B + A*c))/x + B*c^3*x
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{A\,b^3}{7x^7}-\frac{b^2(3Ac+Bb)}{5x^5}-\frac{bc(Ac+Bb)}{x^3}-\frac{c^2(Ac+3Bb)}{x}+B\,c^3x$	63
risch	$B\,c^3x+\frac{(-A\,c^3-3Bb\,c^2)x^6+(-Ab\,c^2-B\,b^2c)x^4+(-\frac{3}{5}b^2Ac-\frac{1}{5}B\,b^3)x^2-\frac{b^3A}{7}}{x^7}$	74
norman	$\frac{(-\frac{3}{5}b^2Ac-\frac{1}{5}B\,b^3)x^8+(-A\,c^3-3Bb\,c^2)x^{12}+(-Ab\,c^2-B\,b^2c)x^{10}+B\,c^3x^{14}-\frac{x^6b^3A}{7}}{x^{13}}$	78
gosper	$-\frac{-35B\,c^3x^8+35A\,c^3x^6+105x^6Bb\,c^2+35Ab\,c^2x^4+35x^4B\,b^2c+21A\,b^2c\,x^2+7b^3B\,x^2+5b^3A}{35x^7}$	80
parallelrisch	$-\frac{-35B\,c^3x^8+35A\,c^3x^6+105x^6Bb\,c^2+35Ab\,c^2x^4+35x^4B\,b^2c+21A\,b^2c\,x^2+7b^3B\,x^2+5b^3A}{35x^7}$	80

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^14,x,method=_RETURNVERBOSE)
[Out] -1/7*A*b^3/x^7-1/5*b^2*(3*A*c+B*b)/x^5-b*c*(A*c+B*b)/x^3-c^2*(A*c+3*B*b)/x+
B*c^3*x
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}}\,dx = \frac{35\,Bc^3x^8-35\,(3\,Bbc^2+Ac^3)x^6-35\,(Bb^2c+Abc^2)x^4-5\,Ab^3-7\,(Bb^3+3\,Ab^2c)x^2}{35\,x^7}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^14,x, algorithm="fricas")
```

```
[Out] 1/35*(35*B*c^3*x^8 - 35*(3*B*b*c^2 + A*c^3)*x^6 - 35*(B*b^2*c + A*b*c^2)*x^
4 - 5*A*b^3 - 7*(B*b^3 + 3*A*b^2*c)*x^2)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}}\,dx = Bc^3x + \frac{-5Ab^3+x^6(-35Ac^3-105Bbc^2)+x^4(-35Abc^2-35Bb^2c)+x^2(-21Ab^2c-7Bb^3)}{35x^7}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**14,x)
```

```
[Out] B*c**3*x + (-5*A*b**3 + x**6*(-35*A*c**3 - 105*B*b*c**2) + x**4*(-35*A*b*c*
*2 - 35*B*b**2*c) + x**2*(-21*A*b**2*c - 7*B*b**3))/(35*x**7)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}}\,dx = Bc^3x - \frac{35\,(3\,Bbc^2+Ac^3)x^6+35\,(Bb^2c+Abc^2)x^4+5\,Ab^3+7\,(Bb^3+3\,Ab^2c)x^2}{35\,x^7}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^14,x, algorithm="maxima")
```

```
[Out] B*c^3*x - 1/35*(35*(3*B*b*c^2 + A*c^3)*x^6 + 35*(B*b^2*c + A*b*c^2)*x^4 + 5
*A*b^3 + 7*(B*b^3 + 3*A*b^2*c)*x^2)/x^7
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}}\,dx = Bc^3x - \frac{105\,Bbc^2x^6 + 35\,Ac^3x^6 + 35\,Bb^2cx^4 + 35\,Abc^2x^4 + 7\,Bb^3x^2 + 21\,Ab^2cx^2 + 5\,Ab^3}{35\,x^7}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^14,x, algorithm="giac")
```

```
[Out] B*c^3*x - 1/35*(105*B*b*c^2*x^6 + 35*A*c^3*x^6 + 35*B*b^2*c*x^4 + 35*A*b*c^
2*x^4 + 7*B*b^3*x^2 + 21*A*b^2*c*x^2 + 5*A*b^3)/x^7
```

Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{14}}\,dx = B\,c^3\,x - \frac{x^4\,(B\,b^2\,c + A\,b\,c^2) + \frac{A\,b^3}{7} + x^2\left(\frac{B\,b^3}{5} + \frac{3\,A\,c\,b^2}{5}\right) + x^6\,(A\,c^3 + 3\,B\,b\,c^2)}{x^7}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^14,x)
```

```
[Out] B*c^3*x - (x^4*(A*b*c^2 + B*b^2*c) + (A*b^3)/7 + x^2*((B*b^3)/5 + (3*A*b^2*
c)/5) + x^6*(A*c^3 + 3*B*b*c^2))/x^7
```

3.38 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}} \, dx = -\frac{b^3B}{6x^6} - \frac{3b^2Bc}{4x^4} - \frac{3bBc^2}{2x^2} - \frac{A(b+cx^2)^4}{8bx^8} + Bc^3\log(x)$$

```
[Out] -1/6*b^3*B/x^6-3/4*b^2*B*c/x^4-3/2*b*B*c^2/x^2-1/8*A*(c*x^2+b)^4/b/x^8+B*c^
3*ln(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1598, 457, 79, 45}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}} \, dx = -\frac{A(b+cx^2)^4}{8bx^8} - \frac{b^3B}{6x^6} - \frac{3b^2Bc}{4x^4} - \frac{3bBc^2}{2x^2} + Bc^3\log(x)$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^15,x]
```

```
[Out] -1/6*(b^3*B)/x^6 - (3*b^2*B*c)/(4*x^4) - (3*b*B*c^2)/(2*x^2) - (A*(b + c*x^
2)^4)/(8*b*x^8) + B*c^3*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(A+Bx^2)\left(b+cx^2\right)^3}{x^9}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x^5}\,dx,x,x^2\right)\\
&= -\frac{A(b+cx^2)^4}{8bx^8}+\frac{1}{2}B\text{Subst}\left(\int \frac{(b+cx)^3}{x^4}\,dx,x,x^2\right)\\
&= -\frac{A(b+cx^2)^4}{8bx^8}+\frac{1}{2}B\text{Subst}\left(\int \left(\frac{b^3}{x^4}+\frac{3b^2c}{x^3}+\frac{3bc^2}{x^2}+\frac{c^3}{x}\right)\,dx,x,x^2\right)\\
&= -\frac{b^3B}{6x^6}-\frac{3b^2Bc}{4x^4}-\frac{3bBc^2}{2x^2}-\frac{A(b+cx^2)^4}{8bx^8}+Bc^3\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}}\,dx$$
$$= -\frac{2bBx^2(2b^2+9bcx^2+18c^2x^4)+3A(b^3+4b^2cx^2+6bc^2x^4+4c^3x^6)}{24x^8} + Bc^3\log(x)$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^15,x]
```

```
[Out] -1/24*(2*b*B*x^2*(2*b^2 + 9*b*c*x^2 + 18*c^2*x^4) + 3*A*(b^3 + 4*b^2*c*x^2
+ 6*b*c^2*x^4 + 4*c^3*x^6))/x^8 + B*c^3*Log[x]
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

method	result	size
default	$B\,c^3\ln(x)-\frac{b^2(3Ac+Bb)}{6x^6}-\frac{b^3A}{8x^8}-\frac{c^2(Ac+3Bb)}{2x^2}-\frac{3bc(Ac+Bb)}{4x^4}$	64
risch	$\frac{\left(-\frac{1}{2}A\,c^3-\frac{3}{2}Bb\,c^2\right)x^6+\left(-\frac{3}{4}Ab\,c^2-\frac{3}{4}B\,b^2c\right)x^4+\left(-\frac{1}{2}b^2Ac-\frac{1}{6}B\,b^3\right)x^2-\frac{b^3A}{8}}{x^8}+B\,c^3\ln(x)$	75
norman	$\frac{\left(-\frac{1}{2}A\,c^3-\frac{3}{2}Bb\,c^2\right)x^{12}+\left(-\frac{3}{4}Ab\,c^2-\frac{3}{4}B\,b^2c\right)x^{10}+\left(-\frac{1}{2}b^2Ac-\frac{1}{6}B\,b^3\right)x^8-\frac{x^6b^3A}{8}}{x^{14}}+B\,c^3\ln(x)$	78
parallelrisch	$-\frac{-24B\,c^3\ln(x)x^8+12A\,c^3x^6+36x^6Bb\,c^2+18Ab\,c^2x^4+18x^4B\,b^2c+12A\,b^2c\,x^2+4b^3B\,x^2+3b^3A}{24x^8}$	82

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^15,x,method=_RETURNVERBOSE)
```

```
[Out] B*c^3*ln(x)-1/6*b^2*(3*A*c+B*b)/x^6-1/8*b^3*A/x^8-1/2*c^2*(A*c+3*B*b)/x^2-3
/4*b*c*(A*c+B*b)/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}}\,dx$$
$$= \frac{24\,Bc^3x^8\log(x)-12\,(3\,Bbc^2+Ac^3)x^6-18\,(Bb^2c+Abc^2)x^4-3\,Ab^3-4\,(Bb^3+3\,Ab^2c)x^2}{24\,x^8}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^15,x, algorithm="fricas")
```

```
[Out] 1/24*(24*B*c^3*x^8*log(x) - 12*(3*B*b*c^2 + A*c^3)*x^6 - 18*(B*b^2*c + A*b*
c^2)*x^4 - 3*A*b^3 - 4*(B*b^3 + 3*A*b^2*c)*x^2)/x^8
```

Sympy [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.30

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}}\,dx = Bc^3\log(x) + \frac{-3Ab^3+x^6(-12Ac^3-36Bbc^2)+x^4(-18Abc^2-18Bb^2c)+x^2(-12Ab^2c-4Bb^3)}{24x^8}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**15,x)
[Out] B*c**3*log(x) + (-3*A*b**3 + x**6*(-12*A*c**3 - 36*B*b*c**2) + x**4*(-18*A*
b*c**2 - 18*B*b**2*c) + x**2*(-12*A*b**2*c - 4*B*b**3))/(24*x**8)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}}\,dx = \frac{1}{2}Bc^3\log(x^2) - \frac{12(3Bbc^2+Ac^3)x^6+18(Bb^2c+Abc^2)x^4+3Ab^3+4(Bb^3+3Ab^2c)x^2}{24x^8}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^15,x, algorithm="maxima")
[Out] 1/2*B*c^3*log(x^2) - 1/24*(12*(3*B*b*c^2 + A*c^3)*x^6 + 18*(B*b^2*c + A*b*c
^2)*x^4 + 3*A*b^3 + 4*(B*b^3 + 3*A*b^2*c)*x^2)/x^8
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.43

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{15}}\,dx = \frac{1}{2}Bc^3\log(x^2) - \frac{25Bc^3x^8+36Bbc^2x^6+12Ac^3x^6+18Bb^2cx^4+18Abc^2x^4+4Bb^3x^2+12Ab^2cx^2+3Ab^3}{24x^8}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^15,x, algorithm="giac")
[Out] 1/2*B*c^3*log(x^2) - 1/24*(25*B*c^3*x^8 + 36*B*b*c^2*x^6 + 12*A*c^3*x^6 + 1
8*B*b^2*c*x^4 + 18*A*b*c^2*x^4 + 4*B*b^3*x^2 + 12*A*b^2*c*x^2 + 3*A*b^3)/x^
8
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^{15}}\,dx$$
$$= B\,c^3\,\ln(x) - \frac{x^4\,\left(\frac{3\,B\,b^2\,c}{4}+\frac{3\,A\,b\,c^2}{4}\right)+\frac{A\,b^3}{8}+x^2\,\left(\frac{B\,b^3}{6}+\frac{A\,c\,b^2}{2}\right)+x^6\,\left(\frac{A\,c^3}{2}+\frac{3\,B\,b\,c^2}{2}\right)}{x^8}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^15,x)
```

```
[Out] B*c^3*log(x) - (x^4*((3*A*b*c^2)/4 + (3*B*b^2*c)/4) + (A*b^3)/8 + x^2*((B*b
^3)/6 + (A*b^2*c)/2) + x^6*((A*c^3)/2 + (3*B*b*c^2)/2))/x^8
```

3.39 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 73

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}}\,dx=-\frac{Ab^3}{9x^9}-\frac{b^2(bB+3Ac)}{7x^7}-\frac{3bc(bB+Ac)}{5x^5}-\frac{c^2(3bB+Ac)}{3x^3}-\frac{Bc^3}{x}$$

```
[Out] -1/9*A*b^3/x^9-1/7*b^2*(3*A*c+B*b)/x^7-3/5*b*c*(A*c+B*b)/x^5-1/3*c^2*(A*c+3
*B*b)/x^3-B*c^3/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}}\,dx=-\frac{Ab^3}{9x^9}-\frac{b^2(3Ac+bB)}{7x^7}-\frac{c^2(Ac+3bB)}{3x^3}-\frac{3bc(Ac+bB)}{5x^5}-\frac{Bc^3}{x}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^16,x]
```

```
[Out] -1/9*(A*b^3)/x^9 - (b^2*(b*B + 3*A*c))/(7*x^7) - (3*b*c*(b*B + A*c))/(5*x^5
) - (c^2*(3*b*B + A*c))/(3*x^3) - (B*c^3)/x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(A+Bx^2)\,(b+cx^2)^3}{x^{10}}\,dx\\ &= \int \left(\frac{Ab^3}{x^{10}}+\frac{b^2(bB+3Ac)}{x^8}+\frac{3bc(bB+Ac)}{x^6}+\frac{c^2(3bB+Ac)}{x^4}+\frac{Bc^3}{x^2}\right)dx\\ &= -\frac{Ab^3}{9x^9}-\frac{b^2(bB+3Ac)}{7x^7}-\frac{3bc(bB+Ac)}{5x^5}-\frac{c^2(3bB+Ac)}{3x^3}-\frac{Bc^3}{x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^{16}}\,dx = -\frac{Ab^3}{9x^9}-\frac{b^2(bB+3Ac)}{7x^7}-\frac{3bc(bB+Ac)}{5x^5}-\frac{c^2(3bB+Ac)}{3x^3}-\frac{Bc^3}{x}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^16,x]
[Out] -1/9*(A*b^3)/x^9 - (b^2*(b*B + 3*A*c))/(7*x^7) - (3*b*c*(b*B + A*c))/(5*x^5
) - (c^2*(3*b*B + A*c))/(3*x^3) - (B*c^3)/x
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{A\,b^3}{9x^9}-\frac{b^2(3Ac+Bb)}{7x^7}-\frac{3bc(Ac+Bb)}{5x^5}-\frac{c^2(Ac+3Bb)}{3x^3}-\frac{B\,c^3}{x}$	66
risch	$\frac{-B\,c^3x^8+(-\frac{1}{3}A\,c^3-Bb\,c^2)x^6+(-\frac{3}{5}Ab\,c^2-\frac{3}{5}B\,b^2c)x^4+(-\frac{3}{7}b^2Ac-\frac{1}{7}B\,b^3)x^2-\frac{b^3A}{9}}{x^9}$	76
norman	$\frac{(-\frac{1}{3}A\,c^3-Bb\,c^2)x^{12}+(-\frac{3}{5}Ab\,c^2-\frac{3}{5}B\,b^2c)x^{10}+(-\frac{3}{7}b^2Ac-\frac{1}{7}B\,b^3)x^8-B\,c^3x^{14}-\frac{x^6b^3A}{9}}{x^{15}}$	79
gosper	$-\frac{315B\,c^3x^8+105A\,c^3x^6+315x^6Bb\,c^2+189Ab\,c^2x^4+189x^4B\,b^2c+135A\,b^2c\,x^2+45b^3B\,x^2+35b^3A}{315x^9}$	80
parallelrisch	$-\frac{315B\,c^3x^8+105A\,c^3x^6+315x^6Bb\,c^2+189Ab\,c^2x^4+189x^4B\,b^2c+135A\,b^2c\,x^2+45b^3B\,x^2+35b^3A}{315x^9}$	80

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^16,x,method=_RETURNVERBOSE)
[Out] -1/9*A*b^3/x^9-1/7*b^2*(3*A*c+B*b)/x^7-3/5*b*c*(A*c+B*b)/x^5-1/3*c^2*(A*c+3
*B*b)/x^3-B*c^3/x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}}\,dx = -\frac{315\,Bc^3x^8+105\,(3\,Bbc^2+Ac^3)x^6+189\,(Bb^2c+Abc^2)x^4+35\,Ab^3+45\,(Bb^3+3\,Ab^2c)x^2}{315\,x^9}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^16,x, algorithm="fricas")
```

```
[Out] -1/315*(315*B*c^3*x^8 + 105*(3*B*b*c^2 + A*c^3)*x^6 + 189*(B*b^2*c + A*b*c^
2)*x^4 + 35*A*b^3 + 45*(B*b^3 + 3*A*b^2*c)*x^2)/x^9
```

Sympy [A] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}}\,dx = \frac{-35Ab^3-315Bc^3x^8+x^6(-105Ac^3-315Bbc^2)+x^4(-189Abc^2-189Bb^2c)+x^2(-135Ab^2c-45Bb^3)}{315x^9}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**16,x)
```

```
[Out] (-35*A*b**3 - 315*B*c**3*x**8 + x**6*(-105*A*c**3 - 315*B*b*c**2) + x**4*(-
189*A*b*c**2 - 189*B*b**2*c) + x**2*(-135*A*b**2*c - 45*B*b**3))/(315*x**9)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}}\,dx = -\frac{315\,Bc^3x^8+105\,(3\,Bbc^2+Ac^3)x^6+189\,(Bb^2c+Abc^2)x^4+35\,Ab^3+45\,(Bb^3+3\,Ab^2c)x^2}{315\,x^9}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^16,x, algorithm="maxima")
```

```
[Out] -1/315*(315*B*c^3*x^8 + 105*(3*B*b*c^2 + A*c^3)*x^6 + 189*(B*b^2*c + A*b*c^
2)*x^4 + 35*A*b^3 + 45*(B*b^3 + 3*A*b^2*c)*x^2)/x^9
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}}\,dx = -\frac{315\,Bc^3x^8+315\,Bbc^2x^6+105\,Ac^3x^6+189\,Bb^2cx^4+189\,Abc^2x^4+45\,Bb^3x^2+135\,Ab^2cx^2+35\,Ab^3}{315\,x^9}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^16,x, algorithm="giac")
[Out] -1/315*(315*B*c^3*x^8 + 315*B*b*c^2*x^6 + 105*A*c^3*x^6 + 189*B*b^2*c*x^4 +
 189*A*b*c^2*x^4 + 45*B*b^3*x^2 + 135*A*b^2*c*x^2 + 35*A*b^3)/x^9
```

Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{16}}\,dx = -\frac{x^4\left(\frac{3\,B\,b^2\,c}{5}+\frac{3\,A\,b\,c^2}{5}\right)+\frac{A\,b^3}{9}+x^2\left(\frac{B\,b^3}{7}+\frac{3\,A\,c\,b^2}{7}\right)+x^6\left(\frac{A\,c^3}{3}+B\,b\,c^2\right)+B\,c^3\,x^8}{x^9}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^16,x)
[Out] -(x^4*((3*A*b*c^2)/5 + (3*B*b^2*c)/5) + (A*b^3)/9 + x^2*((B*b^3)/7 + (3*A*b
^2*c)/7) + x^6*((A*c^3)/3 + B*b*c^2) + B*c^3*x^8)/x^9
```

3.40 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{17}} dx$

Optimal result

Integrand size = 24, antiderivative size = 49

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{17}} dx = -\frac{A(b+cx^2)^4}{10bx^{10}} - \frac{(5bB-Ac)(b+cx^2)^4}{40b^2x^8}$$

```
[Out] -1/10*A*(c*x^2+b)^4/b/x^10-1/40*(-A*c+5*B*b)*(c*x^2+b)^4/b^2/x^8
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 457, 79, 37}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{17}} dx = -\frac{(b+cx^2)^4(5bB-Ac)}{40b^2x^8} - \frac{A(b+cx^2)^4}{10bx^{10}}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^17,x]
```

```
[Out] -1/10*(A*(b + c*x^2)^4)/(b*x^10) - ((5*b*B - A*c)*(b + c*x^2)^4)/(40*b^2*x^
8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(A+Bx^2)\,(b+cx^2)^3}{x^{11}}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)(b+cx)^3}{x^6}\,dx, x, x^2\right) \\
&= -\frac{A(b+cx^2)^4}{10bx^{10}} + \frac{(5bB-Ac)\text{Subst}\left(\int \frac{(b+cx)^3}{x^5}\,dx, x, x^2\right)}{10b} \\
&= -\frac{A(b+cx^2)^4}{10bx^{10}} - \frac{(5bB-Ac)\,(b+cx^2)^4}{40b^2x^8}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^{17}}\,dx = -\frac{5Bx^2(b^3+4b^2cx^2+6bc^2x^4+4c^3x^6)+A(4b^3+15b^2cx^2+20bc^2x^4+10c^3x^6)}{40x^{10}}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^17,x]
[Out] -1/40*(5*B*x^2*(b^3 + 4*b^2*c*x^2 + 6*b*c^2*x^4 + 4*c^3*x^6) + A*(4*b^3 + 1
5*b^2*c*x^2 + 20*b*c^2*x^4 + 10*c^3*x^6))/x^10
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.35

method	result	size
default	$-\frac{bc(Ac+Bb)}{2x^{6}}-\frac{b^{2}(3Ac+Bb)}{8x^{8}}-\frac{b^{3}A}{10x^{10}}-\frac{B\,c^{3}}{2x^{2}}-\frac{c^{2}(Ac+3Bb)}{4x^{4}}$	66
risch	$\frac{-\frac{B\,c^{3}x^{8}}{2}+\left(-\frac{1}{4}A\,c^{3}-\frac{3}{4}Bb\,c^{2}\right)x^{6}+\left(-\frac{1}{2}Ab\,c^{2}-\frac{1}{2}B\,b^{2}c\right)x^{4}+\left(-\frac{3}{8}b^{2}Ac-\frac{1}{8}B\,b^{3}\right)x^{2}-\frac{b^{3}A}{10}}{x^{10}}$	76
norman	$\frac{\left(-\frac{1}{4}A\,c^{3}-\frac{3}{4}Bb\,c^{2}\right)x^{12}+\left(-\frac{1}{2}Ab\,c^{2}-\frac{1}{2}B\,b^{2}c\right)x^{10}+\left(-\frac{3}{8}b^{2}Ac-\frac{1}{8}B\,b^{3}\right)x^{8}-\frac{B\,c^{3}x^{14}}{2}-\frac{x^{6}b^{3}A}{10}}{x^{16}}$	79
gosper	$-\frac{20B\,c^{3}x^{8}+10A\,c^{3}x^{6}+30x^{6}Bb\,c^{2}+20Ab\,c^{2}x^{4}+20x^{4}B\,b^{2}c+15A\,b^{2}c\,x^{2}+5b^{3}B\,x^{2}+4b^{3}A}{40x^{10}}$	80
parallelrisch	$-\frac{20B\,c^{3}x^{8}+10A\,c^{3}x^{6}+30x^{6}Bb\,c^{2}+20Ab\,c^{2}x^{4}+20x^{4}B\,b^{2}c+15A\,b^{2}c\,x^{2}+5b^{3}B\,x^{2}+4b^{3}A}{40x^{10}}$	80

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^17,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*c*(A*c+B*b)/x^6-1/8*b^2*(3*A*c+B*b)/x^8-1/10*b^3*A/x^10-1/2*B*c^3/x^
2-1/4*c^2*(A*c+3*B*b)/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.53

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^3}{x^{17}}\,dx = -\frac{20\,Bc^3x^8+10\,(3\,Bbc^2+Ac^3)x^6+20\,(Bb^2c+Abc^2)x^4+4\,Ab^3+5\,(Bb^3+3\,Ab^2c)x^2}{40\,x^{10}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^17,x, algorithm="fricas")
```

```
[Out] -1/40*(20*B*c^3*x^8 + 10*(3*B*b*c^2 + A*c^3)*x^6 + 20*(B*b^2*c + A*b*c^2)*x
^4 + 4*A*b^3 + 5*(B*b^3 + 3*A*b^2*c)*x^2)/x^10
```

Sympy [A] (verification not implemented)

Time = 3.52 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^3}{x^{17}}\,dx = \frac{-4Ab^3-20Bc^3x^8+x^6(-10Ac^3-30Bbc^2)+x^4(-20Abc^2-20Bb^2c)+x^2(-15Ab^2c-5Bb^3)}{40x^{10}}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**17,x)
```

```
[Out] (-4*A*b**3 - 20*B*c**3*x**8 + x**6*(-10*A*c**3 - 30*B*b*c**2) + x**4*(-20*A
*b*c**2 - 20*B*b**2*c) + x**2*(-15*A*b**2*c - 5*B*b**3))/(40*x**10)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.53

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{17}}\,dx = -\frac{20\,Bc^3x^8+10\,(3\,Bbc^2+Ac^3)x^6+20\,(Bb^2c+Abc^2)x^4+4\,Ab^3+5\,(Bb^3+3\,Ab^2c)x^2}{40\,x^{10}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^17,x, algorithm="maxima")
[Out] -1/40*(20*B*c^3*x^8 + 10*(3*B*b*c^2 + A*c^3)*x^6 + 20*(B*b^2*c + A*b*c^2)*x
^4 + 4*A*b^3 + 5*(B*b^3 + 3*A*b^2*c)*x^2)/x^10
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.61

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{17}}\,dx = -\frac{20\,Bc^3x^8+30\,Bbc^2x^6+10\,Ac^3x^6+20\,Bb^2cx^4+20\,Abc^2x^4+5\,Bb^3x^2+15\,Ab^2cx^2+4\,Ab^3}{40\,x^{10}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^17,x, algorithm="giac")
[Out] -1/40*(20*B*c^3*x^8 + 30*B*b*c^2*x^6 + 10*A*c^3*x^6 + 20*B*b^2*c*x^4 + 20*A
*b*c^2*x^4 + 5*B*b^3*x^2 + 15*A*b^2*c*x^2 + 4*A*b^3)/x^10
```

Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.55

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{17}}\,dx = -\frac{x^4\left(\frac{B\,b^2\,c}{2}+\frac{A\,b\,c^2}{2}\right)+\frac{A\,b^3}{10}+x^2\left(\frac{B\,b^3}{8}+\frac{3\,A\,c\,b^2}{8}\right)+x^6\left(\frac{A\,c^3}{4}+\frac{3\,B\,b\,c^2}{4}\right)+\frac{B\,c^3\,x^8}{2}}{x^{10}}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^17,x)
[Out] -(x^4*((A*b*c^2)/2 + (B*b^2*c)/2) + (A*b^3)/10 + x^2*((B*b^3)/8 + (3*A*b^2*
c)/8) + x^6*((A*c^3)/4 + (3*B*b*c^2)/4) + (B*c^3*x^8)/2)/x^10
```

3.41 $\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4} dx$

Optimal result

Integrand size = 24, antiderivative size = 119

$$\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4} dx = \frac{b^3(bB-Ac)x}{c^5} - \frac{b^2(bB-Ac)x^3}{3c^4} + \frac{b(bB-Ac)x^5}{5c^3} - \frac{(bB-Ac)x^7}{7c^2} + \frac{Bx^9}{9c} - \frac{b^{7/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{11/2}}$$

```
[Out] b^3*(-A*c+B*b)*x/c^5-1/3*b^2*(-A*c+B*b)*x^3/c^4+1/5*b*(-A*c+B*b)*x^5/c^3-1/
7*(-A*c+B*b)*x^7/c^2+1/9*B*x^9/c-b^(7/2)*(-A*c+B*b)*arctan(x*c^(1/2)/b^(1/2
))/c^(11/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 470, 308, 211}

$$\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4} dx = -\frac{b^{7/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{11/2}} + \frac{b^3x(bB-Ac)}{c^5} - \frac{b^2x^3(bB-Ac)}{3c^4} + \frac{bx^5(bB-Ac)}{5c^3} - \frac{x^7(bB-Ac)}{7c^2} + \frac{Bx^9}{9c}$$

```
[In] Int[(x^10*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (b^3*(b*B - A*c)*x)/c^5 - (b^2*(b*B - A*c)*x^3)/(3*c^4) + (b*(b*B - A*c)*x^
5)/(5*c^3) - ((b*B - A*c)*x^7)/(7*c^2) + (B*x^9)/(9*c) - (b^(7/2)*(b*B - A*
c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(11/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^8(A+Bx^2)}{b+cx^2}\,dx \\
&= \frac{Bx^9}{9c} - \frac{(9bB-9Ac)\int \frac{x^8}{b+cx^2}\,dx}{9c} \\
&= \frac{Bx^9}{9c} - \frac{(9bB-9Ac)\int \left(-\frac{b^3}{c^4}+\frac{b^2x^2}{c^3}-\frac{bx^4}{c^2}+\frac{x^6}{c}+\frac{b^4}{c^4(b+cx^2)}\right)dx}{9c} \\
&= \frac{b^3(bB-Ac)x}{c^5} - \frac{b^2(bB-Ac)x^3}{3c^4} + \frac{b(bB-Ac)x^5}{5c^3} \\
&\quad - \frac{(bB-Ac)x^7}{7c^2} + \frac{Bx^9}{9c} - \frac{(b^4(bB-Ac))\int \frac{1}{b+cx^2}\,dx}{c^5} \\
&= \frac{b^3(bB-Ac)x}{c^5} - \frac{b^2(bB-Ac)x^3}{3c^4} + \frac{b(bB-Ac)x^5}{5c^3} \\
&\quad - \frac{(bB-Ac)x^7}{7c^2} + \frac{Bx^9}{9c} - \frac{b^{7/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{11/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{b^3(bB-Ac)x}{c^5}-\frac{b^2(bB-Ac)x^3}{3c^4}+\frac{b(bB-Ac)x^5}{5c^3}+\frac{(-bB+Ac)x^7}{7c^2}+\frac{Bx^9}{9c}-\frac{b^{7/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{11/2}}$$

```
[In] Integrate[(x^10*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (b^3*(b*B - A*c)*x)/c^5 - (b^2*(b*B - A*c)*x^3)/(3*c^4) + (b*(b*B - A*c)*x^
5)/(5*c^3) + ((-(b*B) + A*c)*x^7)/(7*c^2) + (B*x^9)/(9*c) - (b^(7/2)*(b*B -
 A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(11/2)
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03

method	result
default	$-\frac{-\frac{1}{9}B\,x^9c^4-\frac{1}{7}A\,c^4x^7+\frac{1}{7}Bb\,c^3x^7+\frac{1}{5}Ab\,c^3x^5-\frac{1}{5}B\,b^2c^2x^5-\frac{1}{3}A\,b^2c^2x^3+\frac{1}{3}B\,b^3c\,x^3+A\,b^3cx-B\,b^4x}{c^5}+\frac{b^4(Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{c^5\sqrt{bc}}$
risch	$\frac{B\,x^9}{9c}+\frac{A\,x^7}{7c}-\frac{Bb\,x^7}{7c^2}-\frac{Ab\,x^5}{5c^2}+\frac{B\,b^2x^5}{5c^3}+\frac{A\,b^2x^3}{3c^3}-\frac{B\,b^3x^3}{3c^4}-\frac{A\,b^3x}{c^4}+\frac{B\,b^4x}{c^5}+\frac{\sqrt{-bc}\,b^3\ln\left(-\sqrt{-bc}\,x+b\right)A}{2c^5}-\sqrt{-bc}\,b$

```
[In] int(x^10*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^5*(-1/9*B*x^9*c^4-1/7*A*c^4*x^7+1/7*B*b*c^3*x^7+1/5*A*b*c^3*x^5-1/5*B*
b^2*c^2*x^5-1/3*A*b^2*c^2*x^3+1/3*B*b^3*c*x^3+A*b^3*c*x-B*b^4*x)+b^4*(A*c-B
*b)/c^5/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.30

$$\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4}\,dx = \left[\frac{70\,Bc^4x^9-90\,(Bbc^3-Ac^4)x^7+126\,(Bb^2c^2-Abc^3)x^5-210\,(Bb^3c-Ab^2c^2)x^3-315\,(Bb^4-Ab^3c)\sqrt{-\frac{b}{c}}}{630\,c^5}\right.$$

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/630*(70*B*c^4*x^9 - 90*(B*b*c^3 - A*c^4)*x^7 + 126*(B*b^2*c^2 - A*b*c^3)
*x^5 - 210*(B*b^3*c - A*b^2*c^2)*x^3 - 315*(B*b^4 - A*b^3*c)*sqrt(-b/c)*log
((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 630*(B*b^4 - A*b^3*c)*x)/c^5
, 1/315*(35*B*c^4*x^9 - 45*(B*b*c^3 - A*c^4)*x^7 + 63*(B*b^2*c^2 - A*b*c^3)
*x^5 - 105*(B*b^3*c - A*b^2*c^2)*x^3 - 315*(B*b^4 - A*b^3*c)*sqrt(b/c)*arct
an(c*x*sqrt(b/c)/b) + 315*(B*b^4 - A*b^3*c)*x)/c^5]
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.71

$$\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bx^9}{9c} + x^7\left(\frac{A}{7c} - \frac{Bb}{7c^2}\right) + x^5\left(-\frac{Ab}{5c^2} + \frac{Bb^2}{5c^3}\right) + x^3\left(\frac{Ab^2}{3c^3} - \frac{Bb^3}{3c^4}\right) + x\left(-\frac{Ab^3}{c^4} + \frac{Bb^4}{c^5}\right) + \frac{\sqrt{-\frac{b^7}{c^{11}}}(-Ac+Bb)\log\left(-\frac{c^5\sqrt{-\frac{b^7}{c^{11}}}(-Ac+Bb)}{-Ab^3c+Bb^4} + x\right)}{2} - \frac{\sqrt{-\frac{b^7}{c^{11}}}(-Ac+Bb)\log\left(\frac{c^5\sqrt{-\frac{b^7}{c^{11}}}(-Ac+Bb)}{-Ab^3c+Bb^4} + x\right)}{2}$$

```
[In] integrate(x**10*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**9/(9*c) + x**7*(A/(7*c) - B*b/(7*c**2)) + x**5*(-A*b/(5*c**2) + B*b**2
/(5*c**3)) + x**3*(A*b**2/(3*c**3) - B*b**3/(3*c**4)) + x*(-A*b**3/c**4 + B
*b**4/c**5) + sqrt(-b**7/c**11)*(-A*c + B*b)*log(-c**5*sqrt(-b**7/c**11)*(-
A*c + B*b)/(-A*b**3*c + B*b**4) + x)/2 - sqrt(-b**7/c**11)*(-A*c + B*b)*log
(c**5*sqrt(-b**7/c**11)*(-A*c + B*b)/(-A*b**3*c + B*b**4) + x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04

$$\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{(Bb^5 - Ab^4c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^5} + \frac{35\,Bc^4x^9 - 45\,(Bbc^3 - Ac^4)x^7 + 63\,(Bb^2c^2 - Abc^3)x^5 - 105\,(Bb^3c - Ab^2c^2)x^3 + 315\,(Bb^4 - Ab^3c)x}{315\,c^5}$$

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -(B*b^5 - A*b^4*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^5) + 1/315*(35*B*c^4*
x^9 - 45*(B*b*c^3 - A*c^4)*x^7 + 63*(B*b^2*c^2 - A*b*c^3)*x^5 - 105*(B*b^3*
c - A*b^2*c^2)*x^3 + 315*(B*b^4 - A*b^3*c)*x)/c^5
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.12

$$\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{(Bb^5-Ab^4c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^5} + \frac{35\,Bc^8x^9-45\,Bbc^7x^7+45\,Ac^8x^7+63\,Bb^2c^6x^5-63\,Abc^7x^5-105\,Bb^3c^5x^3+105\,Ab^2c^6x^3+315\,Bb^4c^4x}{315\,c^9}$$

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -(B*b^5 - A*b^4*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^5) + 1/315*(35*B*c^8*
x^9 - 45*B*b*c^7*x^7 + 45*A*c^8*x^7 + 63*B*b^2*c^6*x^5 - 63*A*b*c^7*x^5 - 1
05*B*b^3*c^5*x^3 + 105*A*b^2*c^6*x^3 + 315*B*b^4*c^4*x - 315*A*b^3*c^5*x)/c
^9
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.21

$$\int \frac{x^{10}(A+Bx^2)}{bx^2+cx^4}\,dx = x^7\left(\frac{A}{7\,c}-\frac{B\,b}{7\,c^2}\right)+\frac{B\,x^9}{9\,c}+\frac{b^2\,x^3\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{3\,c^2} - \frac{b^{7/2}\,\mathrm{atan}\left(\frac{b^{7/2}\,\sqrt{c}\,x\,(A\,c-B\,b)}{B\,b^5-A\,b^4\,c}\right)\,(A\,c-B\,b)}{c^{11/2}} - \frac{b\,x^5\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{5\,c}-\frac{b^3\,x\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{c^3}$$

```
[In] int((x^10*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^7*(A/(7*c) - (B*b)/(7*c^2)) + (B*x^9)/(9*c) + (b^2*x^3*(A/c - (B*b)/c^2))
/(3*c^2) - (b^(7/2)*atan((b^(7/2)*c^(1/2)*x*(A*c - B*b))/(B*b^5 - A*b^4*c))
*(A*c - B*b))/c^(11/2) - (b*x^5*(A/c - (B*b)/c^2))/(5*c) - (b^3*x*(A/c - (B
*b)/c^2))/c^3
```

3.42 $\int \frac{x^9(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 96

$$\int \frac{x^9(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{b^2(bB-Ac)x^2}{2c^4}+\frac{b(bB-Ac)x^4}{4c^3}-\frac{(bB-Ac)x^6}{6c^2}+\frac{Bx^8}{8c}+\frac{b^3(bB-Ac)\log(b+cx^2)}{2c^5}$$

```
[Out] -1/2*b^2*(-A*c+B*b)*x^2/c^4+1/4*b*(-A*c+B*b)*x^4/c^3-1/6*(-A*c+B*b)*x^6/c^2
+1/8*B*x^8/c+1/2*b^3*(-A*c+B*b)*ln(c*x^2+b)/c^5
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 457, 78}

$$\int \frac{x^9(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{b^3(bB-Ac)\log(b+cx^2)}{2c^5}-\frac{b^2x^2(bB-Ac)}{2c^4}+\frac{bx^4(bB-Ac)}{4c^3}-\frac{x^6(bB-Ac)}{6c^2}+\frac{Bx^8}{8c}$$

```
[In] Int[(x^9*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] -1/2*(b^2*(b*B - A*c)*x^2)/c^4 + (b*(b*B - A*c)*x^4)/(4*c^3) - ((b*B - A*c)
*x^6)/(6*c^2) + (B*x^8)/(8*c) + (b^3*(b*B - A*c)*Log[b + c*x^2])/(2*c^5)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
```

```
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^7(A+Bx^2)}{b+cx^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(A+Bx)}{b+cx}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{b^2(bB-Ac)}{c^4}+\frac{b(bB-Ac)x}{c^3}+\frac{(-bB+Ac)x^2}{c^2}+\frac{Bx^3}{c}\right.\right. \\
&\qquad \left.\left. +\frac{b^3(bB-Ac)}{c^4(b+cx)}\right)\,dx, x, x^2\right) \\
&= -\frac{b^2(bB-Ac)x^2}{2c^4}+\frac{b(bB-Ac)x^4}{4c^3}-\frac{(bB-Ac)x^6}{6c^2}+\frac{Bx^8}{8c}+\frac{b^3(bB-Ac)\log(b+cx^2)}{2c^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

$$\int \frac{x^9(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{cx^2(-12b^3B+6b^2c(2A+Bx^2)-2bc^2x^2(3A+2Bx^2)+c^3x^4(4A+3Bx^2))+12b^3(bB-Ac)\log(b+cx^2)}{24c^5}$$

```
[In] Integrate[(x^9*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (c*x^2*(-12*b^3*B + 6*b^2*c*(2*A + B*x^2) - 2*b*c^2*x^2*(3*A + 2*B*x^2) + c
^3*x^4*(4*A + 3*B*x^2)) + 12*b^3*(b*B - A*c)*Log[b + c*x^2])/(24*c^5)
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

method	result
norman	$\frac{\frac{B x^9}{8c}+\frac{(Ac-Bb)x^7}{6c^2}-\frac{b(Ac-Bb)x^5}{4c^3}+\frac{b^2(Ac-Bb)x^3}{2c^4}}{x}-\frac{b^3(Ac-Bb)\ln(c x^2+b)}{2c^5}$
default	$\frac{\frac{1}{4}B c^3x^8+\frac{1}{3}A c^3x^6-\frac{1}{3}x^6Bb c^2-\frac{1}{2}Ab c^2x^4+\frac{1}{2}x^4B b^2c+A b^2c x^2-b^3B x^2}{2c^4}-\frac{b^3(Ac-Bb)\ln(c x^2+b)}{2c^5}$
parallelrisch	$-\frac{-3B x^8c^4-4A x^6c^4+4B x^6b c^3+6A x^4b c^3-6B x^4b^2c^2-12A x^2b^2c^2+12B x^2b^3c+12A\ln(c x^2+b)b^3c-12B\ln(c x^2+b)b^4}{24c^5}$
risch	$\frac{B x^8}{8c}+\frac{A x^6}{6c}-\frac{x^6Bb}{6c^2}-\frac{Ab x^4}{4c^2}+\frac{x^4B b^2}{4c^3}+\frac{A b^2x^2}{2c^3}-\frac{b^3B x^2}{2c^4}-\frac{b^3\ln(c x^2+b)A}{2c^4}+\frac{b^4\ln(c x^2+b)B}{2c^5}$

```
[In] int(x^9*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] (1/8*B*x^9/c+1/6/c^2*(A*c-B*b)*x^7-1/4*b/c^3*(A*c-B*b)*x^5+1/2*b^2*(A*c-B*b
)/c^4*x^3)/x-1/2*b^3*(A*c-B*b)/c^5*ln(c*x^2+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.02

$$\int \frac{x^9(A+Bx^2)}{bx^2+cx^4}\,dx$$
$$=\frac{3\,Bc^4x^8-4\,(Bbc^3-Ac^4)x^6+6\,(Bb^2c^2-Abc^3)x^4-12\,(Bb^3c-Ab^2c^2)x^2+12\,(Bb^4-Ab^3c)\log{(cx^2+b)}}{24\,c^5}$$

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/24*(3*B*c^4*x^8 - 4*(B*b*c^3 - A*c^4)*x^6 + 6*(B*b^2*c^2 - A*b*c^3)*x^4 -
 12*(B*b^3*c - A*b^2*c^2)*x^2 + 12*(B*b^4 - A*b^3*c)*log(c*x^2 + b))/c^5
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98

$$\int \frac{x^9(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{Bx^8}{8c}+\frac{b^3(-Ac+Bb)\log{(b+cx^2)}}{2c^5}+x^6\left(\frac{A}{6c}-\frac{Bb}{6c^2}\right)+x^4\left(-\frac{Ab}{4c^2}+\frac{Bb^2}{4c^3}\right)+x^2\left(\frac{Ab^2}{2c^3}-\frac{Bb^3}{2c^4}\right)$$

```
[In] integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2),x)
[Out] B*x**8/(8*c) + b**3*(-A*c + B*b)*log(b + c*x**2)/(2*c**5) + x**6*(A/(6*c) -
 B*b/(6*c**2)) + x**4*(-A*b/(4*c**2) + B*b**2/(4*c**3)) + x**2*(A*b**2/(2*c
**3) - B*b**3/(2*c**4))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01

$$\int \frac{x^9(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{3\,Bc^3x^8-4\,(Bbc^2-Ac^3)x^6+6\,(Bb^2c-Abc^2)x^4-12\,(Bb^3-Ab^2c)x^2}{24\,c^4} + \frac{(Bb^4-Ab^3c)\log{(cx^2+b)}}{2\,c^5}$$

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/24*(3*B*c^3*x^8 - 4*(B*b*c^2 - A*c^3)*x^6 + 6*(B*b^2*c - A*b*c^2)*x^4 - 1
2*(B*b^3 - A*b^2*c)*x^2)/c^4 + 1/2*(B*b^4 - A*b^3*c)*log(c*x^2 + b)/c^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.05

$$\int \frac{x^9(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{3\,Bc^3x^8-4\,Bbc^2x^6+4\,Ac^3x^6+6\,Bb^2cx^4-6\,Abc^2x^4-12\,Bb^3x^2+12\,Ab^2cx^2}{24\,c^4} + \frac{(Bb^4-Ab^3c)\log{(|cx^2+b|)}}{2\,c^5}$$

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/24*(3*B*c^3*x^8 - 4*B*b*c^2*x^6 + 4*A*c^3*x^6 + 6*B*b^2*c*x^4 - 6*A*b*c^2
*x^4 - 12*B*b^3*x^2 + 12*A*b^2*c*x^2)/c^4 + 1/2*(B*b^4 - A*b^3*c)*log(abs(c
*x^2 + b))/c^5
```

Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.04

$$\int \frac{x^9(A+Bx^2)}{bx^2+cx^4}\,dx = x^6\left(\frac{A}{6\,c}-\frac{B\,b}{6\,c^2}\right)+\frac{B\,x^8}{8\,c}+\frac{\ln{(c\,x^2+b)}\,\left(B\,b^4-A\,b^3\,c\right)}{2\,c^5} + \frac{b^2\,x^2\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{2\,c^2}-\frac{b\,x^4\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{4\,c}$$

```
[In] int((x^9*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^6*(A/(6*c) - (B*b)/(6*c^2)) + (B*x^8)/(8*c) + (log(b + c*x^2)*(B*b^4 - A*
b^3*c))/(2*c^5) + (b^2*x^2*(A/c - (B*b)/c^2))/(2*c^2) - (b*x^4*(A/c - (B*b)
/c^2))/(4*c)
```

3.43 $\int \frac{x^8(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 98

$$\int \frac{x^8(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{b^2(bB-Ac)x}{c^4}+\frac{b(bB-Ac)x^3}{3c^3}-\frac{(bB-Ac)x^5}{5c^2}+\frac{Bx^7}{7c}+\frac{b^{5/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{9/2}}$$

```
[Out] -b^2*(-A*c+B*b)*x/c^4+1/3*b*(-A*c+B*b)*x^3/c^3-1/5*(-A*c+B*b)*x^5/c^2+1/7*B
*x^7/c+b^(5/2)*(-A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/c^(9/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 470, 308, 211}

$$\int \frac{x^8(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{b^{5/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{9/2}}-\frac{b^2x(bB-Ac)}{c^4}+\frac{bx^3(bB-Ac)}{3c^3}-\frac{x^5(bB-Ac)}{5c^2}+\frac{Bx^7}{7c}$$

```
[In] Int[(x^8*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] -((b^2*(b*B - A*c)*x)/c^4) + (b*(b*B - A*c)*x^3)/(3*c^3) - ((b*B - A*c)*x^5
)/(5*c^2) + (B*x^7)/(7*c) + (b^(5/2)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]
])/c^(9/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^6(A+Bx^2)}{b+cx^2}\,dx \\
&= \frac{Bx^7}{7c} - \frac{(7bB-7Ac)\int \frac{x^6}{b+cx^2}\,dx}{7c} \\
&= \frac{Bx^7}{7c} - \frac{(7bB-7Ac)\int \left(\frac{b^2}{c^3}-\frac{bx^2}{c^2}+\frac{x^4}{c}-\frac{b^3}{c^3(b+cx^2)}\right)dx}{7c} \\
&= -\frac{b^2(bB-Ac)x}{c^4}+\frac{b(bB-Ac)x^3}{3c^3}-\frac{(bB-Ac)x^5}{5c^2}+\frac{Bx^7}{7c}+\frac{(b^3(bB-Ac))\int \frac{1}{b+cx^2}\,dx}{c^4} \\
&= -\frac{b^2(bB-Ac)x}{c^4}+\frac{b(bB-Ac)x^3}{3c^3}-\frac{(bB-Ac)x^5}{5c^2}+\frac{Bx^7}{7c}+\frac{b^{5/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{9/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{x^8(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{b^2(bB-Ac)x}{c^4}+\frac{b(bB-Ac)x^3}{3c^3}+\frac{(-bB+Ac)x^5}{5c^2}+\frac{Bx^7}{7c}+\frac{b^{5/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{9/2}}$$

```
[In] Integrate[(x^8*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] -((b^2*(b*B - A*c)*x)/c^4) + (b*(b*B - A*c)*x^3)/(3*c^3) + ((-(b*B) + A*c)*
x^5)/(5*c^2) + (B*x^7)/(7*c) + (b^(5/2)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt
[b]])/c^(9/2)
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

method	result
default	$\frac{\frac{1}{7}Bc^3x^7+\frac{1}{5}Ac^3x^5-\frac{1}{5}Bbc^2x^5-\frac{1}{3}Abc^2x^3+\frac{1}{3}Bb^2cx^3+Ab^2cx-b^3Bx}{c^4}-\frac{b^3(Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{c^4\sqrt{bc}}$
risch	$\frac{Bx^7}{7c}+\frac{Ax^5}{5c}-\frac{Bbx^5}{5c^2}-\frac{Abx^3}{3c^2}+\frac{Bb^2x^3}{3c^3}+\frac{Ab^2x}{c^3}-\frac{b^3Bx}{c^4}+\frac{\sqrt{-bc}\,b^2\ln(-\sqrt{-bc}\,x-b)A}{2c^4}-\frac{\sqrt{-bc}\,b^3\ln(-\sqrt{-bc}\,x-b)B}{2c^5}-$

```
[In] int(x^8*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/7*B*c^3*x^7+1/5*A*c^3*x^5-1/5*B*b*c^2*x^5-1/3*A*b*c^2*x^3+1/3*B*b^
2*c*x^3+A*b^2*c*x-b^3*B*x)-b^3*(A*c-B*b)/c^4/(b*c)^(1/2)*arctan(c*x/(b*c)^(
1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.33

$$\int \frac{x^8(A+Bx^2)}{bx^2+cx^4}\,dx = \left[\frac{30\,Bc^3x^7-42\,(Bbc^2-Ac^3)x^5+70\,(Bb^2c-Abc^2)x^3-105\,(Bb^3-Ab^2c)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)-21}{210\,c^4}\right.$$

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/210*(30*B*c^3*x^7 - 42*(B*b*c^2 - A*c^3)*x^5 + 70*(B*b^2*c - A*b*c^2)*x^
3 - 105*(B*b^3 - A*b^2*c)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*
x^2 + b)) - 210*(B*b^3 - A*b^2*c)*x)/c^4, 1/105*(15*B*c^3*x^7 - 21*(B*b*c^2
 - A*c^3)*x^5 + 35*(B*b^2*c - A*b*c^2)*x^3 + 105*(B*b^3 - A*b^2*c)*sqrt(b/c
)*arctan(c*x*sqrt(b/c)/b) - 105*(B*b^3 - A*b^2*c)*x)/c^4]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(87) = 174$.

Time = 0.22 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.84

$$\int \frac{x^8(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bx^7}{7c} + x^5\left(\frac{A}{5c} - \frac{Bb}{5c^2}\right) + x^3\left(-\frac{Ab}{3c^2} + \frac{Bb^2}{3c^3}\right) + x\left(\frac{Ab^2}{c^3} - \frac{Bb^3}{c^4}\right) - \frac{\sqrt{-\frac{b^5}{c^9}}(-Ac+Bb)\log\left(-\frac{c^4\sqrt{-\frac{b^5}{c^9}}(-Ac+Bb)}{-Ab^2c+Bb^3} + x\right)}{2} + \frac{\sqrt{-\frac{b^5}{c^9}}(-Ac+Bb)\log\left(\frac{c^4\sqrt{-\frac{b^5}{c^9}}(-Ac+Bb)}{-Ab^2c+Bb^3} + x\right)}{2}$$

```
[In] integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**7/(7*c) + x**5*(A/(5*c) - B*b/(5*c**2)) + x**3*(-A*b/(3*c**2) + B*b**2
/(3*c**3)) + x*(A*b**2/c**3 - B*b**3/c**4) - sqrt(-b**5/c**9)*(-A*c + B*b)*
log(-c**4*sqrt(-b**5/c**9)*(-A*c + B*b)/(-A*b**2*c + B*b**3) + x)/2 + sqrt(
-b**5/c**9)*(-A*c + B*b)*log(c**4*sqrt(-b**5/c**9)*(-A*c + B*b)/(-A*b**2*c
+ B*b**3) + x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \frac{x^8(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{(Bb^4 - Ab^3c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^4} + \frac{15\,Bc^3x^7 - 21\,(Bbc^2 - Ac^3)x^5 + 35\,(Bb^2c - Abc^2)x^3 - 105\,(Bb^3 - Ab^2c)x}{105\,c^4}$$

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] (B*b^4 - A*b^3*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/105*(15*B*c^3*x
^7 - 21*(B*b*c^2 - A*c^3)*x^5 + 35*(B*b^2*c - A*b*c^2)*x^3 - 105*(B*b^3 - A
*b^2*c)*x)/c^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10

$$\int \frac{x^8(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{(Bb^4-Ab^3c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^4} + \frac{15\,Bc^6x^7-21\,Bbc^5x^5+21\,Ac^6x^5+35\,Bb^2c^4x^3-35\,Abc^5x^3-105\,Bb^3c^3x+105\,Ab^2c^4x}{105\,c^7}$$

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] (B*b^4 - A*b^3*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/105*(15*B*c^6*x
^7 - 21*B*b*c^5*x^5 + 21*A*c^6*x^5 + 35*B*b^2*c^4*x^3 - 35*A*b*c^5*x^3 - 10
5*B*b^3*c^3*x + 105*A*b^2*c^4*x)/c^7
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.20

$$\int \frac{x^8(A+Bx^2)}{bx^2+cx^4}\,dx = x^5\left(\frac{A}{5\,c}-\frac{B\,b}{5\,c^2}\right)+\frac{B\,x^7}{7\,c}+\frac{b^{5/2}\,\mathrm{atan}\left(\frac{b^{5/2}\,\sqrt{c}\,x\,(A\,c-B\,b)}{B\,b^4-A\,b^3\,c}\right)\,(A\,c-B\,b)}{c^{9/2}} - \frac{b\,x^3\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{3\,c}+\frac{b^2\,x\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{c^2}$$

```
[In] int((x^8*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^5*(A/(5*c) - (B*b)/(5*c^2)) + (B*x^7)/(7*c) + (b^(5/2)*atan((b^(5/2)*c^(1
/2)*x*(A*c - B*b))/(B*b^4 - A*b^3*c))*(A*c - B*b))/c^(9/2) - (b*x^3*(A/c -
(B*b)/c^2))/(3*c) + (b^2*x*(A/c - (B*b)/c^2))/c^2
```

3.44 $\int \frac{x^7(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 75

$$\int \frac{x^7(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{b(bB-Ac)x^2}{2c^3}-\frac{(bB-Ac)x^4}{4c^2}+\frac{Bx^6}{6c}-\frac{b^2(bB-Ac)\log(b+cx^2)}{2c^4}$$

```
[Out] 1/2*b*(-A*c+B*b)*x^2/c^3-1/4*(-A*c+B*b)*x^4/c^2+1/6*B*x^6/c-1/2*b^2*(-A*c+B
*b)*ln(c*x^2+b)/c^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$\int \frac{x^7(A+Bx^2)}{bx^2+cx^4}\,dx=-\frac{b^2(bB-Ac)\log(b+cx^2)}{2c^4}+\frac{bx^2(bB-Ac)}{2c^3}-\frac{x^4(bB-Ac)}{4c^2}+\frac{Bx^6}{6c}$$

```
[In] Int[(x^7*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (b*(b*B - A*c)*x^2)/(2*c^3) - ((b*B - A*c)*x^4)/(4*c^2) + (B*x^6)/(6*c) - (
b^2*(b*B - A*c)*Log[b + c*x^2])/(2*c^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^5(A+Bx^2)}{b+cx^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{b+cx}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b(bB-Ac)}{c^3}+\frac{(-bB+Ac)x}{c^2}+\frac{Bx^2}{c}-\frac{b^2(bB-Ac)}{c^3(b+cx)}\right)\,dx, x, x^2\right) \\
&= \frac{b(bB-Ac)x^2}{2c^3}-\frac{(bB-Ac)x^4}{4c^2}+\frac{Bx^6}{6c}-\frac{b^2(bB-Ac)\log\left(b+cx^2\right)}{2c^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \frac{x^7(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{cx^2(6b^2B-3bc(2A+Bx^2)+c^2x^2(3A+2Bx^2))+6b^2(-bB+Ac)\log\left(b+cx^2\right)}{12c^4}$$

```
[In] Integrate[(x^7*(A + B*x^2))/(b*x^2 + c*x^4),x]
[Out] (c*x^2*(6*b^2*B - 3*b*c*(2*A + B*x^2) + c^2*x^2*(3*A + 2*B*x^2)) + 6*b^2*(-
(b*B) + A*c)*Log[b + c*x^2])/(12*c^4)
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

method	result	size
norman	$\frac{\frac{B x^{7}}{6c}+\frac{(Ac-Bb)x^{5}}{4c^{2}}-\frac{b(Ac-Bb)x^{3}}{2c^{3}}}{x}+\frac{b^{2}(Ac-Bb)\ln(c x^{2}+b)}{2c^{4}}$	73
default	$-\frac{-\frac{1}{3}B c^{2}x^{6}-\frac{1}{2}A c^{2}x^{4}+\frac{1}{2}x^{4}Bbc+Abc x^{2}-b^{2}B x^{2}}{2c^{3}}+\frac{b^{2}(Ac-Bb)\ln(c x^{2}+b)}{2c^{4}}$	74
parallelrisch	$\frac{2B c^{3}x^{6}+3A c^{3}x^{4}-3Bb c^{2}x^{4}-6Ab c^{2}x^{2}+6B b^{2}c x^{2}+6A\ln(c x^{2}+b)b^{2}c-6B\ln(c x^{2}+b)b^{3}}{12c^{4}}$	84
risch	$\frac{B x^{6}}{6c}+\frac{A x^{4}}{4c}-\frac{x^{4}Bb}{4c^{2}}-\frac{Ab x^{2}}{2c^{2}}+\frac{b^{2}B x^{2}}{2c^{3}}+\frac{b^{2}\ln(c x^{2}+b)A}{2c^{3}}-\frac{b^{3}\ln(c x^{2}+b)B}{2c^{4}}$	86

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/6*B*x^7/c+1/4/c^2*(A*c-B*b)*x^5-1/2*b/c^3*(A*c-B*b)*x^3)/x+1/2*b^2*(A*c-
B*b)/c^4*ln(c*x^2+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{x^7(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2\,Bc^3x^6-3\,(Bbc^2-Ac^3)x^4+6\,(Bb^2c-Abc^2)x^2-6\,(Bb^3-Ab^2c)\log\left(cx^2+b\right)}{12\,c^4}$$

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*c^3*x^6 - 3*(B*b*c^2 - A*c^3)*x^4 + 6*(B*b^2*c - A*b*c^2)*x^2 - 6
*(B*b^3 - A*b^2*c)*log(c*x^2 + b))/c^4
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$\int \frac{x^7(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bx^6}{6c}-\frac{b^2(-Ac+Bb)\log\left(b+cx^2\right)}{2c^4}+x^4\left(\frac{A}{4c}-\frac{Bb}{4c^2}\right)+x^2\left(-\frac{Ab}{2c^2}+\frac{Bb^2}{2c^3}\right)$$

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**6/(6*c) - b**2*(-A*c + B*b)*log(b + c*x**2)/(2*c**4) + x**4*(A/(4*c) -
 B*b/(4*c**2)) + x**2*(-A*b/(2*c**2) + B*b**2/(2*c**3))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{x^7(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2\,Bc^2x^6 - 3\,(Bbc - Ac^2)x^4 + 6\,(Bb^2 - Abc)x^2}{12\,c^3} - \frac{(Bb^3 - Ab^2c)\log\left(cx^2+b\right)}{2\,c^4}$$

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/12*(2*B*c^2*x^6 - 3*(B*b*c - A*c^2)*x^4 + 6*(B*b^2 - A*b*c)*x^2)/c^3 - 1/
2*(B*b^3 - A*b^2*c)*log(c*x^2 + b)/c^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int \frac{x^7(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2\,Bc^2x^6 - 3\,Bbcx^4 + 3\,Ac^2x^4 + 6\,Bb^2x^2 - 6\,Abcx^2}{12\,c^3} - \frac{(Bb^3 - Ab^2c)\log\left(|cx^2+b|\right)}{2\,c^4}$$

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/12*(2*B*c^2*x^6 - 3*B*b*c*x^4 + 3*A*c^2*x^4 + 6*B*b^2*x^2 - 6*A*b*c*x^2)/
c^3 - 1/2*(B*b^3 - A*b^2*c)*log(abs(c*x^2 + b))/c^4
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

$$\int \frac{x^7(A+Bx^2)}{bx^2+cx^4}\,dx = x^4\left(\frac{A}{4\,c} - \frac{B\,b}{4\,c^2}\right) + \frac{B\,x^6}{6\,c} - \frac{\ln\left(c\,x^2+b\right)\,\left(B\,b^3 - A\,b^2\,c\right)}{2\,c^4} - \frac{b\,x^2\,\left(\frac{A}{c} - \frac{B\,b}{c^2}\right)}{2\,c}$$

```
[In] int((x^7*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^4*(A/(4*c) - (B*b)/(4*c^2)) + (B*x^6)/(6*c) - (log(b + c*x^2)*(B*b^3 - A*
b^2*c))/(2*c^4) - (b*x^2*(A/c - (B*b)/c^2))/(2*c)
```

3.45 $\int \frac{x^6(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{x^6(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{b(bB-Ac)x}{c^3}-\frac{(bB-Ac)x^3}{3c^2}+\frac{Bx^5}{5c}-\frac{b^{3/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}}$$

```
[Out] b*(-A*c+B*b)*x/c^3-1/3*(-A*c+B*b)*x^3/c^2+1/5*B*x^5/c-b^(3/2)*(-A*c+B*b)*ar
ctan(x*c^(1/2)/b^(1/2))/c^(7/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 470, 308, 211}

$$\int \frac{x^6(A+Bx^2)}{bx^2+cx^4}\,dx=-\frac{b^{3/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}}+\frac{bx(bB-Ac)}{c^3}-\frac{x^3(bB-Ac)}{3c^2}+\frac{Bx^5}{5c}$$

```
[In] Int[(x^6*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (b*(b*B - A*c)*x)/c^3 - ((b*B - A*c)*x^3)/(3*c^2) + (B*x^5)/(5*c) - (b^(3/2
)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(7/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^4(A+Bx^2)}{b+cx^2}\,dx \\
&= \frac{Bx^5}{5c} - \frac{(5bB-5Ac)\int \frac{x^4}{b+cx^2}\,dx}{5c} \\
&= \frac{Bx^5}{5c} - \frac{(5bB-5Ac)\int \left(-\frac{b}{c^2}+\frac{x^2}{c}+\frac{b^2}{c^2(b+cx^2)}\right)dx}{5c} \\
&= \frac{b(bB-Ac)x}{c^3} - \frac{(bB-Ac)x^3}{3c^2} + \frac{Bx^5}{5c} - \frac{(b^2(bB-Ac))\int \frac{1}{b+cx^2}\,dx}{c^3} \\
&= \frac{b(bB-Ac)x}{c^3} - \frac{(bB-Ac)x^3}{3c^2} + \frac{Bx^5}{5c} - \frac{b^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{x^6(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{b(bB-Ac)x}{c^3} + \frac{(-bB+Ac)x^3}{3c^2} + \frac{Bx^5}{5c} - \frac{b^{3/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}}$$

```
[In] Integrate[(x^6*(A + B*x^2))/(b*x^2 + c*x^4),x]
[Out] (b*(b*B - A*c)*x)/c^3 + ((-(b*B) + A*c)*x^3)/(3*c^2) + (B*x^5)/(5*c) - (b^(
3/2)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(7/2)
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

method	result
default	$-\frac{-\frac{1}{5} B c^2 x^5-\frac{1}{3} A c^2 x^3+\frac{1}{3} B b c x^3+A b c x-b^2 B x}{c^3}+\frac{b^2 (A c-B b) \arctan\left(\frac{c x}{\sqrt{b c}}\right)}{c^3 \sqrt{b c}}$
risch	$\frac{B x^5}{5 c}+\frac{A x^3}{3 c}-\frac{B b x^3}{3 c^2}-\frac{A b x}{c^2}+\frac{b^2 B x}{c^3}+\frac{\sqrt{-b c}\, b \ln\left(-\sqrt{-b c}\, x+b\right) A}{2 c^3}-\frac{\sqrt{-b c}\, b^2 \ln\left(-\sqrt{-b c}\, x+b\right) B}{2 c^4}-\frac{\sqrt{-b c}\, b \ln\left(\sqrt{-b c}\, x+b\right)}{2 c^3}$

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^3*(-1/5*B*c^2*x^5-1/3*A*c^2*x^3+1/3*B*b*c*x^3+A*b*c*x-b^2*B*x)+b^2*(A*
c-B*b)/c^3/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.31

$$\int \frac{x^6(A+Bx^2)}{bx^2+cx^4}\,dx = \left[\frac{6\,Bc^2x^5-10\,(Bbc-Ac^2)x^3-15\,(Bb^2-Abc)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)+30\,(Bb^2-Abc)x}{30\,c^3}, \frac{3\,Bc^2x^5-\ldots}{\ldots}\right]$$

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/30*(6*B*c^2*x^5 - 10*(B*b*c - A*c^2)*x^3 - 15*(B*b^2 - A*b*c)*sqrt(-b/c)
*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 30*(B*b^2 - A*b*c)*x)/c^
3, 1/15*(3*B*c^2*x^5 - 5*(B*b*c - A*c^2)*x^3 - 15*(B*b^2 - A*b*c)*sqrt(b/c)
*arctan(c*x*sqrt(b/c)/b) + 15*(B*b^2 - A*b*c)*x)/c^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(68) = 136.

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.99

$$\int \frac{x^6(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bx^5}{5c} + x^3\left(\frac{A}{3c} - \frac{Bb}{3c^2}\right) + x\left(-\frac{Ab}{c^2} + \frac{Bb^2}{c^3}\right) + \frac{\sqrt{-\frac{b^3}{c^7}}(-Ac+Bb)\log\left(-\frac{c^3\sqrt{-\frac{b^3}{c^7}}(-Ac+Bb)}{-Abc+Bb^2} + x\right)}{2} - \frac{\sqrt{-\frac{b^3}{c^7}}(-Ac+Bb)\log\left(\frac{c^3\sqrt{-\frac{b^3}{c^7}}(-Ac+Bb)}{-Abc+Bb^2} + x\right)}{2}$$

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**5/(5*c) + x**3*(A/(3*c) - B*b/(3*c**2)) + x*(-A*b/c**2 + B*b**2/c**3)
+ sqrt(-b**3/c**7)*(-A*c + B*b)*log(-c**3*sqrt(-b**3/c**7)*(-A*c + B*b)/(-A
*b*c + B*b**2) + x)/2 - sqrt(-b**3/c**7)*(-A*c + B*b)*log(c**3*sqrt(-b**3/c
**7)*(-A*c + B*b)/(-A*b*c + B*b**2) + x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{x^6(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{(Bb^3 - Ab^2c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^3} + \frac{3\,Bc^2x^5 - 5\,(Bbc - Ac^2)x^3 + 15\,(Bb^2 - Abc)x}{15\,c^3}$$

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -(B*b^3 - A*b^2*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/15*(3*B*c^2*x^
5 - 5*(B*b*c - A*c^2)*x^3 + 15*(B*b^2 - A*b*c)*x)/c^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \frac{x^6(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{(Bb^3 - Ab^2c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^3} + \frac{3\,Bc^4x^5 - 5\,Bbc^3x^3 + 5\,Ac^4x^3 + 15\,Bb^2c^2x - 15\,Abc^3x}{15\,c^5}$$

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -(B*b^3 - A*b^2*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/15*(3*B*c^4*x^
5 - 5*B*b*c^3*x^3 + 5*A*c^4*x^3 + 15*B*b^2*c^2*x - 15*A*b*c^3*x)/c^5
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.25

$$\int \frac{x^6(A+Bx^2)}{bx^2+cx^4}\,dx = x^3\left(\frac{A}{3\,c}-\frac{B\,b}{3\,c^2}\right)+\frac{B\,x^5}{5\,c} - \frac{b^{3/2}\,\mathrm{atan}\left(\frac{b^{3/2}\,\sqrt{c}\,x\,(A\,c-B\,b)}{B\,b^3-A\,b^2\,c}\right)\,(A\,c-B\,b)}{c^{7/2}} - \frac{b\,x\,\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)}{c}$$

```
[In] int((x^6*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^3*(A/(3*c) - (B*b)/(3*c^2)) + (B*x^5)/(5*c) - (b^(3/2)*atan((b^(3/2)*c^(1
/2)*x*(A*c - B*b))/(B*b^3 - A*b^2*c))*(A*c - B*b))/c^(7/2) - (b*x*(A/c - (B
*b)/c^2))/c
```

3.46 $\int \frac{x^5(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 54

$$\int \frac{x^5(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{b(bB-Ac)\log(b+cx^2)}{2c^3}$$

```
[Out] -1/2*(-A*c+B*b)*x^2/c^2+1/4*B*x^4/c+1/2*b*(-A*c+B*b)*ln(c*x^2+b)/c^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$\int \frac{x^5(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{b(bB-Ac)\log(b+cx^2)}{2c^3}-\frac{x^2(bB-Ac)}{2c^2}+\frac{Bx^4}{4c}$$

```
[In] Int[(x^5*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] -1/2*((b*B - A*c)*x^2)/c^2 + (B*x^4)/(4*c) + (b*(b*B - A*c)*Log[b + c*x^2])
/(2*c^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^3(A+Bx^2)}{b+cx^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{b+cx}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{-bB+Ac}{c^2}+\frac{Bx}{c}+\frac{b(bB-Ac)}{c^2(b+cx)}\right)dx, x, x^2\right) \\
&= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{b(bB-Ac)\log(b+cx^2)}{2c^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{x^5(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{cx^2(-2bB+2Ac+Bcx^2)+2b(bB-Ac)\log(b+cx^2)}{4c^3}$$

```
[In] Integrate[(x^5*(A + B*x^2))/(b*x^2 + c*x^4),x]
[Out] (c*x^2*(-2*b*B + 2*A*c + B*c*x^2) + 2*b*(b*B - A*c)*Log[b + c*x^2])/(4*c^3)
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\frac{1}{2}Bc\,x^4+Ac\,x^2-bB\,x^2}{2c^2}-\frac{b(Ac-Bb)\ln(c\,x^2+b)}{2c^3}$	50
norman	$\frac{\frac{B\,x^5}{4c}+\frac{(Ac-Bb)x^3}{2c^2}}{x}-\frac{b(Ac-Bb)\ln(c\,x^2+b)}{2c^3}$	54
parallelrisch	$-\frac{-B\,c^2x^4-2A\,c^2x^2+2Bbc\,x^2+2A\ln(c\,x^2+b)bc-2B\ln(c\,x^2+b)b^2}{4c^3}$	60
risch	$\frac{B\,x^4}{4c}+\frac{A\,x^2}{2c}-\frac{Bb\,x^2}{2c^2}+\frac{A^2}{4Bc}-\frac{Ab}{2c^2}+\frac{B\,b^2}{4c^3}-\frac{b\ln(c\,x^2+b)A}{2c^2}+\frac{b^2\ln(c\,x^2+b)B}{2c^3}$	89

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] 1/2/c^2*(1/2*B*c*x^4+A*c*x^2-b*B*x^2)-1/2*b/c^3*(A*c-B*b)*ln(c*x^2+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{x^5(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bc^2x^4-2\,(Bbc-Ac^2)x^2+2\,(Bb^2-Abc)\log\left(cx^2+b\right)}{4\,c^3}$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/4*(B*c^2*x^4 - 2*(B*b*c - A*c^2)*x^2 + 2*(B*b^2 - A*b*c)*log(c*x^2 + b))/
c^3
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{x^5(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bx^4}{4c}+\frac{b(-Ac+Bb)\log\left(b+cx^2\right)}{2c^3}+x^2\left(\frac{A}{2c}-\frac{Bb}{2c^2}\right)$$

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2),x)
[Out] B*x**4/(4*c) + b*(-A*c + B*b)*log(b + c*x**2)/(2*c**3) + x**2*(A/(2*c) - B*
b/(2*c**2))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{x^5(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bcx^4-2\,(Bb-Ac)x^2}{4\,c^2}+\frac{(Bb^2-Abc)\log\left(cx^2+b\right)}{2\,c^3}$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 1/4*(B*c*x^4 - 2*(B*b - A*c)*x^2)/c^2 + 1/2*(B*b^2 - A*b*c)*log(c*x^2 + b)/
c^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{x^5(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bcx^4-2\,Bbx^2+2\,Acx^2}{4\,c^2}+\frac{(Bb^2-Abc)\log\left(|cx^2+b|\right)}{2\,c^3}$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
[Out] 1/4*(B*c*x^4 - 2*B*b*x^2 + 2*A*c*x^2)/c^2 + 1/2*(B*b^2 - A*b*c)*log(abs(c*x
^2 + b))/c^3
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{x^5(A+Bx^2)}{bx^2+cx^4}\,dx = x^2\left(\frac{A}{2\,c}-\frac{B\,b}{2\,c^2}\right)+\frac{\ln\left(c\,x^2+b\right)\,\left(B\,b^2-A\,b\,c\right)}{2\,c^3}+\frac{B\,x^4}{4\,c}$$

```
[In] int((x^5*(A + B*x^2))/(b*x^2 + c*x^4),x)
[Out] x^2*(A/(2*c) - (B*b)/(2*c^2)) + (log(b + c*x^2)*(B*b^2 - A*b*c))/(2*c^3) +
(B*x^4)/(4*c)
```

3.47 $\int \frac{x^4(A+Bx^2)}{bx^2+cx^4} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 58

$$\int \frac{x^4(A+Bx^2)}{bx^2+cx^4} \, dx = -\frac{(bB-Ac)x}{c^2}+\frac{Bx^3}{3c}+\frac{\sqrt{b}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}}$$

```
[Out] -(-A*c+B*b)*x/c^2+1/3*B*x^3/c+(-A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))*b^(1/2)/
c^(5/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 470, 327, 211}

$$\int \frac{x^4(A+Bx^2)}{bx^2+cx^4} \, dx = \frac{\sqrt{b}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}}-\frac{x(bB-Ac)}{c^2}+\frac{Bx^3}{3c}$$

```
[In] Int[(x^4*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] -(((b*B - A*c)*x)/c^2) + (B*x^3)/(3*c) + (Sqrt[b]*(b*B - A*c)*ArcTan[(Sqrt[
c]*x)/Sqrt[b]])/c^(5/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^2(A+Bx^2)}{b+cx^2}\,dx\\
&= \frac{Bx^3}{3c} - \frac{(3bB-3Ac)\int \frac{x^2}{b+cx^2}\,dx}{3c}\\
&= -\frac{(bB-Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{(b(bB-Ac))\int \frac{1}{b+cx^2}\,dx}{c^2}\\
&= -\frac{(bB-Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{\sqrt{b}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{x^4(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{(-bB+Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{\sqrt{b}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}}$$

```
[In] Integrate[(x^4*(A + B*x^2))/(b*x^2 + c*x^4),x]
[Out] ((-(b*B) + A*c)*x)/c^2 + (B*x^3)/(3*c) + (Sqrt[b]*(b*B - A*c)*ArcTan[(Sqrt[
c]*x)/Sqrt[b]])/c^(5/2)
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

method	result
default	$\frac{\frac{1}{3}Bc\,x^3+Acx-bBx}{c^2}-\frac{b(Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{c^2\sqrt{bc}}$
risch	$\frac{B\,x^3}{3c}+\frac{Ax}{c}-\frac{bBx}{c^2}+\frac{\sqrt{-bc}\,\ln(-\sqrt{-bc}\,x-b)A}{2c^2}-\frac{\sqrt{-bc}\,\ln(-\sqrt{-bc}\,x-b)Bb}{2c^3}-\frac{\sqrt{-bc}\,\ln(\sqrt{-bc}\,x-b)A}{2c^2}+\frac{\sqrt{-bc}\,\ln(\sqrt{-bc}\,x-b}{2c^3}$

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] 1/c^2*(1/3*B*c*x^3+A*c*x-b*B*x)-b*(A*c-B*b)/c^2/(b*c)^(1/2)*arctan(c*x/(b*c
)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.22

$$\int \frac{x^4(A+Bx^2)}{bx^2+cx^4}\,dx = \left[\frac{2\,Bcx^3-3\,(Bb-Ac)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)-6\,(Bb-Ac)x}{6\,c^2}, \frac{Bcx^3+3\,(Bb-Ac)\sqrt{\frac{b}{c}}\arctan\left(\frac{cx\sqrt{\frac{b}{c}}}{b}\right.}{3\,c^2}\right.$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] [1/6*(2*B*c*x^3 - 3*(B*b - A*c)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) -
b)/(c*x^2 + b)) - 6*(B*b - A*c)*x)/c^2, 1/3*(B*c*x^3 + 3*(B*b - A*c)*sqrt(b
/c)*arctan(c*x*sqrt(b/c)/b) - 3*(B*b - A*c)*x)/c^2]
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.55

$$\int \frac{x^4(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bx^3}{3c}+x\left(\frac{A}{c}-\frac{Bb}{c^2}\right)-\frac{\sqrt{-\frac{b}{c^5}}(-Ac+Bb)\log\left(-c^2\sqrt{-\frac{b}{c^5}}+x\right)}{2}+\frac{\sqrt{-\frac{b}{c^5}}(-Ac+Bb)\log\left(c^2\sqrt{-\frac{b}{c^5}}+x\right)}{2}$$

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x**3/(3*c) + x*(A/c - B*b/c**2) - sqrt(-b/c**5)*(-A*c + B*b)*log(-c**2*sq
rt(-b/c**5) + x)/2 + sqrt(-b/c**5)*(-A*c + B*b)*log(c**2*sqrt(-b/c**5) + x)
/2
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{x^4(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{(Bb^2-Abc)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^2}+\frac{Bcx^3-3\,(Bb-Ac)x}{3\,c^2}$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] (B*b^2 - A*b*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/3*(B*c*x^3 - 3*(B
*b - A*c)*x)/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{x^4(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{(Bb^2-Abc)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^2}+\frac{Bc^2x^3-3\,Bbcx+3\,Ac^2x}{3\,c^3}$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] (B*b^2 - A*b*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/3*(B*c^2*x^3 - 3*
B*b*c*x + 3*A*c^2*x)/c^3
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21

$$\int \frac{x^4(A+Bx^2)}{bx^2+cx^4}\,dx=x\left(\frac{A}{c}-\frac{B\,b}{c^2}\right)+\frac{B\,x^3}{3\,c}+\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c}\,x\,(A\,c-B\,b)}{B\,b^2-A\,b\,c}\right)\,(A\,c-B\,b)}{c^{5/2}}$$

```
[In] int((x^4*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x*(A/c - (B*b)/c^2) + (B*x^3)/(3*c) + (b^(1/2)*atan((b^(1/2)*c^(1/2)*x*(A*c
 - B*b))/(B*b^2 - A*b*c))*(A*c - B*b))/c^(5/2)
```

3.48 $\int \frac{x^3(A+Bx^2)}{bx^2+cx^4} dx$

Optimal result

Integrand size = 24, antiderivative size = 35

$$\int \frac{x^3(A+Bx^2)}{bx^2+cx^4} dx = \frac{Bx^2}{2c} - \frac{(bB-Ac)\log(b+cx^2)}{2c^2}$$

```
[Out] 1/2*B*x^2/c-1/2*(-A*c+B*b)*ln(c*x^2+b)/c^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 455, 45}

$$\int \frac{x^3(A+Bx^2)}{bx^2+cx^4} dx = \frac{Bx^2}{2c} - \frac{(bB-Ac)\log(b+cx^2)}{2c^2}$$

```
[In] Int[(x^3*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (B*x^2)/(2*c) - ((b*B - A*c)*Log[b + c*x^2])/(2*c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x(A+Bx^2)}{b+cx^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{b+cx}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{B}{c}+\frac{-bB+Ac}{c(b+cx)}\right)\,dx, x, x^2\right) \\
&= \frac{Bx^2}{2c}-\frac{(bB-Ac)\log\left(b+cx^2\right)}{2c^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x^3(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bcx^2+(-bB+Ac)\log\left(b+cx^2\right)}{2c^2}$$

```
[In] Integrate[(x^3*(A + B*x^2))/(b*x^2 + c*x^4),x]
[Out] (B*c*x^2 + (-(b*B) + A*c)*Log[b + c*x^2])/(2*c^2)
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{B\,x^2}{2c}+\frac{(Ac-Bb)\ln\left(c\,x^2+b\right)}{2c^2}$	32
norman	$\frac{B\,x^2}{2c}+\frac{(Ac-Bb)\ln\left(c\,x^2+b\right)}{2c^2}$	32
parallelrisch	$\frac{Bc\,x^2+A\ln\left(c\,x^2+b\right)c-B\ln\left(c\,x^2+b\right)b}{2c^2}$	36
risch	$\frac{B\,x^2}{2c}+\frac{\ln\left(c\,x^2+b\right)A}{2c}-\frac{\ln\left(c\,x^2+b\right)Bb}{2c^2}$	40

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] 1/2*B*x^2/c+1/2/c^2*(A*c-B*b)*ln(c*x^2+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{x^3(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bcx^2-(Bb-Ac)\log\left(cx^2+b\right)}{2\,c^2}$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/2*(B*c*x^2 - (B*b - A*c)*log(c*x^2 + b))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x^3(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bx^2}{2c}-\frac{(-Ac+Bb)\log\left(b+cx^2\right)}{2c^2}$$

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2),x)
[Out] B*x**2/(2*c) - (-A*c + B*b)*log(b + c*x**2)/(2*c**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x^3(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bx^2}{2\,c}-\frac{(Bb-Ac)\log\left(cx^2+b\right)}{2\,c^2}$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 1/2*B*x^2/c - 1/2*(B*b - A*c)*log(c*x^2 + b)/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{x^3(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bx^2}{2\,c} - \frac{(Bb-Ac)\log\left(|cx^2+b|\right)}{2\,c^2}$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
[Out] 1/2*B*x^2/c - 1/2*(B*b - A*c)*log(abs(c*x^2 + b))/c^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x^3(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{B\,x^2}{2\,c} + \frac{\ln\left(c\,x^2+b\right)\,\left(A\,c-B\,b\right)}{2\,c^2}$$

```
[In] int((x^3*(A + B*x^2))/(b*x^2 + c*x^4),x)
[Out] (B*x^2)/(2*c) + (log(b + c*x^2)*(A*c - B*b))/(2*c^2)
```

3.49 $\int \frac{x^2(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 40

$$\int \frac{x^2(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{Bx}{c}-\frac{(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}c^{3/2}}$$

```
[Out] B*x/c-(-A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/c^(3/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 396, 211}

$$\int \frac{x^2(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{Bx}{c}-\frac{(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}c^{3/2}}$$

```
[In] Int[(x^2*(A + B*x^2))/(b*x^2 + c*x^4),x]
[Out] (B*x)/c - ((b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(Sqrt[b]*c^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{A+Bx^2}{b+cx^2}\,dx \\ &= \frac{Bx}{c} - \frac{(bB-Ac)\int \frac{1}{b+cx^2}\,dx}{c} \\ &= \frac{Bx}{c} - \frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}c^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^2(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bx}{c} - \frac{(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}c^{3/2}}$$

```
[In] Integrate[(x^2*(A + B*x^2))/(b*x^2 + c*x^4),x]
[Out] (B*x)/c - ((b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(Sqrt[b]*c^(3/2))
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{Bx}{c}+\frac{(Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{c\sqrt{bc}}$	34
risch	$\frac{Bx}{c}-\frac{\ln(cx+\sqrt{-bc})A}{2\sqrt{-bc}}+\frac{\ln(cx+\sqrt{-bc})Bb}{2c\sqrt{-bc}}+\frac{\ln(-cx+\sqrt{-bc})A}{2\sqrt{-bc}}-\frac{\ln(-cx+\sqrt{-bc})Bb}{2c\sqrt{-bc}}$	98

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] B*x/c+(A*c-B*b)/c/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.48

$$\int \frac{x^2(A+Bx^2)}{bx^2+cx^4}\,dx = \left[\frac{2\,Bbcx+(Bb-Ac)\sqrt{-bc}\log\left(\frac{cx^2-2\sqrt{-bc}x-b}{cx^2+b}\right)}{2\,bc^2}, \frac{Bbcx-(Bb-Ac)\sqrt{bc}\arctan\left(\frac{\sqrt{bc}x}{b}\right)}{bc^2}\right]$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*b*c*x + (B*b - A*c)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c)*x - b)/(
c*x^2 + b)))/(b*c^2), (B*b*c*x - (B*b - A*c)*sqrt(b*c)*arctan(sqrt(b*c)*x/b
))/(b*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(34) = 68.

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.05

$$\int \frac{x^2(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bx}{c} + \frac{\sqrt{-\frac{1}{bc^3}}(-Ac+Bb)\log\left(-bc\sqrt{-\frac{1}{bc^3}}+x\right)}{2} - \frac{\sqrt{-\frac{1}{bc^3}}(-Ac+Bb)\log\left(bc\sqrt{-\frac{1}{bc^3}}+x\right)}{2}$$

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] B*x/c + sqrt(-1/(b*c**3))*(-A*c + B*b)*log(-b*c*sqrt(-1/(b*c**3)) + x)/2 -
sqrt(-1/(b*c**3))*(-A*c + B*b)*log(b*c*sqrt(-1/(b*c**3)) + x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{x^2(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bx}{c} - \frac{(Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c}$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] B*x/c - (B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{x^2(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{Bx}{c} - \frac{(Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c}$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
[Out] B*x/c - (B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{x^2(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{B\,x}{c} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(A\,c-B\,b)}{\sqrt{b}\,c^{3/2}}$$

```
[In] int((x^2*(A + B*x^2))/(b*x^2 + c*x^4),x)
[Out] (B*x)/c + (atan((c^(1/2)*x)/b^(1/2))*(A*c - B*b))/(b^(1/2)*c^(3/2))
```

3.50 $\int \frac{x(A+Bx^2)}{bx^2+cx^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 34

$$\int \frac{x(A+Bx^2)}{bx^2+cx^4} dx = \frac{A\log(x)}{b} + \frac{(bB-Ac)\log(b+cx^2)}{2bc}$$

```
[Out] A*ln(x)/b+1/2*(-A*c+B*b)*ln(c*x^2+b)/b/c
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1598, 457, 78}

$$\int \frac{x(A+Bx^2)}{bx^2+cx^4} dx = \frac{(bB-Ac)\log(b+cx^2)}{2bc} + \frac{A\log(x)}{b}$$

```
[In] Int[(x*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (A*Log[x])/b + ((b*B - A*c)*Log[b + c*x^2])/(2*b*c)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{x\,(b+cx^2)}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x(b+cx)}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{bx}+\frac{bB-Ac}{b(b+cx)}\right)\,dx, x, x^2\right) \\
&= \frac{A\log(x)}{b}+\frac{(bB-Ac)\log\left(b+cx^2\right)}{2bc}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{x(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{A\log(x)}{b}+\frac{(bB-Ac)\log\left(b+cx^2\right)}{2bc}$$

```
[In] Integrate[(x*(A + B*x^2))/(b*x^2 + c*x^4),x]
[Out] (A*Log[x])/b + ((b*B - A*c)*Log[b + c*x^2])/(2*b*c)
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{A\ln(x)}{b}-\frac{(Ac-Bb)\ln(c\,x^2+b)}{2bc}$	33
norman	$\frac{A\ln(x)}{b}-\frac{(Ac-Bb)\ln(c\,x^2+b)}{2bc}$	33
risch	$\frac{A\ln(x)}{b}-\frac{\ln(c\,x^2+b)A}{2b}+\frac{\ln(c\,x^2+b)B}{2c}$	37
parallelrisch	$\frac{2A\ln(x)c-A\ln(c\,x^2+b)c+B\ln(c\,x^2+b)b}{2bc}$	39

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] A*ln(x)/b-1/2*(A*c-B*b)/b/c*ln(c*x^2+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{x(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{2\,Ac\log{(x)}+(Bb-Ac)\log{(cx^2+b)}}{2\,bc}$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/2*(2*A*c*log(x) + (B*b - A*c)*log(c*x^2 + b))/(b*c)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{x(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{A\log{(x)}}{b}+\frac{(-Ac+Bb)\log{\left(\frac{b}{c}+x^2\right)}}{2bc}$$

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2),x)
[Out] A*log(x)/b + (-A*c + B*b)*log(b/c + x**2)/(2*b*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{x(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{A\log{(x^2)}}{2\,b}+\frac{(Bb-Ac)\log{(cx^2+b)}}{2\,bc}$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 1/2*A*log(x^2)/b + 1/2*(B*b - A*c)*log(c*x^2 + b)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{x(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{A\log(|x|)}{b} + \frac{(Bb-Ac)\log(|cx^2+b|)}{2\,bc}$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
[Out] A*log(abs(x))/b + 1/2*(B*b - A*c)*log(abs(c*x^2 + b))/(b*c)
```

Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{x(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{A\,\ln(x)}{b} - \frac{\ln(c\,x^2+b)\,(A\,c-B\,b)}{2\,b\,c}$$

```
[In] int((x*(A + B*x^2))/(b*x^2 + c*x^4),x)
[Out] (A*log(x))/b - (log(b + c*x^2)*(A*c - B*b))/(2*b*c)
```

3.51 $\int \frac{A+Bx^2}{bx^2+cx^4} dx$

Optimal result

Integrand size = 21, antiderivative size = 42

$$\int \frac{A+Bx^2}{bx^2+cx^4} dx = -\frac{A}{bx} + \frac{(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}$$

```
[Out] -A/b/x+(-A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/b^(3/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1607, 464, 211}

$$\int \frac{A+Bx^2}{bx^2+cx^4} dx = \frac{(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}} - \frac{A}{bx}$$

```
[In] Int[(A + B*x^2)/(b*x^2 + c*x^4),x]
```

```
[Out] -(A/(b*x)) + ((b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(b^(3/2)*Sqrt[c])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{A+Bx^2}{x^2\left(b+cx^2\right)}\,dx\\ &= -\frac{A}{bx}-\frac{(-bB+Ac)\int \frac{1}{b+cx^2}\,dx}{b}\\ &= -\frac{A}{bx}+\frac{(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{bx^2+cx^4}\,dx=-\frac{A}{bx}+\frac{(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}$$

```
[In] Integrate[(A + B*x^2)/(b*x^2 + c*x^4),x]
[Out] -(A/(b*x)) + ((b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(b^(3/2)*Sqrt[c])
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result	si
default	$-\frac{A}{bx}+\frac{(-Ac+Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{b\sqrt{bc}}$	37
risch	$-\frac{A}{bx}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(b^3c\_Z^2+A^2c^2-2ABbc+B^2b^2\right)}\_R\ln\left(\left(3\_R^2b^3c+2A^2c^2-4ABbc+2B^2b^2\right)x+\left(b^2Ac-B\,b^3\right)\_R\right)\right)}{2}$	99

```
[In] int((B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -A/b/x+(-A*c+B*b)/b/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.50

$$\int \frac{A+Bx^2}{bx^2+cx^4}\,dx = \left[\frac{(Bb-Ac)\sqrt{-bc}x\log\left(\frac{cx^2+2\sqrt{-bc}x-b}{cx^2+b}\right)-2\,Abc}{2\,b^2cx}, \frac{(Bb-Ac)\sqrt{bc}x\arctan\left(\frac{\sqrt{bc}x}{b}\right)-Abc}{b^2cx}\right]$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*((B*b - A*c)*sqrt(-b*c)*x*log((c*x^2 + 2*sqrt(-b*c)*x - b)/(c*x^2 + b)
) - 2*A*b*c)/(b^2*c*x), ((B*b - A*c)*sqrt(b*c)*x*arctan(sqrt(b*c)*x/b) - A*
b*c)/(b^2*c*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(34) = 68$.

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.95

$$\int \frac{A+Bx^2}{bx^2+cx^4}\,dx = -\frac{A}{bx} - \frac{\sqrt{-\frac{1}{b^3c}}(-Ac+Bb)\log\left(-b^2\sqrt{-\frac{1}{b^3c}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{b^3c}}(-Ac+Bb)\log\left(b^2\sqrt{-\frac{1}{b^3c}}+x\right)}{2}$$

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] -A/(b*x) - sqrt(-1/(b**3*c))*(-A*c + B*b)*log(-b**2*sqrt(-1/(b**3*c)) + x)/
2 + sqrt(-1/(b**3*c))*(-A*c + B*b)*log(b**2*sqrt(-1/(b**3*c)) + x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^2}{bx^2+cx^4}\,dx = \frac{(Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b} - \frac{A}{bx}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] (B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) - A/(b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^2}{bx^2+cx^4}\,dx = \frac{(Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b} - \frac{A}{bx}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
[Out] (B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) - A/(b*x)
```

Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^2}{bx^2+cx^4}\,dx = -\frac{A}{b\,x} - \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(A\,c-B\,b)}{b^{3/2}\,\sqrt{c}}$$

```
[In] int((A + B*x^2)/(b*x^2 + c*x^4),x)
[Out] - A/(b*x) - (atan((c^(1/2)*x)/b^(1/2))*(A*c - B*b))/(b^(3/2)*c^(1/2))
```

3.52 $\int \frac{A+Bx^2}{bx^2-cx^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 41

$$\int \frac{A+Bx^2}{bx^2-cx^4} dx = -\frac{A}{bx} + \frac{(bB+Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}$$

```
[Out] -A/b/x+(A*c+B*b)*arctanh(x*c^(1/2)/b^(1/2))/b^(3/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1607, 464, 214}

$$\int \frac{A+Bx^2}{bx^2-cx^4} dx = \frac{(Ac+bB)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}} - \frac{A}{bx}$$

```
[In] Int[(A + B*x^2)/(b*x^2 - c*x^4),x]
```

```
[Out] -(A/(b*x)) + ((b*B + A*c)*ArcTanh[(Sqrt[c]*x)/Sqrt[b]])/(b^(3/2)*Sqrt[c])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{A+Bx^2}{x^2\,(b-cx^2)}\,dx\\ &= -\frac{A}{bx}+\frac{(bB+Ac)\int \frac{1}{b-cx^2}\,dx}{b}\\ &= -\frac{A}{bx}+\frac{(bB+Ac)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{bx^2-cx^4}\,dx = -\frac{A}{bx}+\frac{(bB+Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}$$

```
[In] Integrate[(A + B*x^2)/(b*x^2 - c*x^4),x]
[Out] -(A/(b*x)) + ((b*B + A*c)*ArcTanh[(Sqrt[c]*x)/Sqrt[b]])/(b^(3/2)*Sqrt[c])
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

method	result	si
default	$-\frac{(-Ac-Bb)\,\text{arctanh}\left(\frac{cx}{\sqrt{bc}}\right)}{b\sqrt{bc}}-\frac{A}{bx}$	39
risch	$-\frac{A}{bx}+\frac{\left(\sum_{\_R=\text{RootOf}\left(b^3c\_Z^2-A^2c^2-2ABbc-B^2b^2\right)}\_R\ln\left(\left(3\_R^2b^3c-2A^2c^2-4ABbc-2B^2b^2\right)x+\left(b^2Ac+B\,b^3\right)\_R\right)\right)}{2}$	10

```
[In] int((B*x^2+A)/(-c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-A*c-B*b)/b/(b*c)^(1/2)*arctanh(c*x/(b*c)^(1/2))-A/b/x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.51

$$\int \frac{A+Bx^2}{bx^2-cx^4}\,dx = \left[\frac{(Bb+Ac)\sqrt{bc}x\log\left(\frac{cx^2+2\sqrt{bc}x+b}{cx^2-b}\right)-2\,Abc}{2\,b^2cx},\ -\frac{(Bb+Ac)\sqrt{-bc}x\arctan\left(\frac{\sqrt{-bc}x}{b}\right)+Abc}{b^2cx}\right]$$

```
[In] integrate((B*x^2+A)/(-c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*((B*b + A*c)*sqrt(b*c)*x*log((c*x^2 + 2*sqrt(b*c)*x + b)/(c*x^2 - b))
- 2*A*b*c)/(b^2*c*x), -((B*b + A*c)*sqrt(-b*c)*x*arctan(sqrt(-b*c)*x/b) + A
*b*c)/(b^2*c*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(34) = 68.

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.83

$$\int \frac{A+Bx^2}{bx^2-cx^4}\,dx = -\frac{A}{bx}-\frac{\sqrt{\frac{1}{b^3c}}(Ac+Bb)\log\left(-b^2\sqrt{\frac{1}{b^3c}}+x\right)}{2}+\frac{\sqrt{\frac{1}{b^3c}}(Ac+Bb)\log\left(b^2\sqrt{\frac{1}{b^3c}}+x\right)}{2}$$

```
[In] integrate((B*x**2+A)/(-c*x**4+b*x**2),x)
```

```
[Out] -A/(b*x) - sqrt(1/(b**3*c))*(A*c + B*b)*log(-b**2*sqrt(1/(b**3*c)) + x)/2 +
 sqrt(1/(b**3*c))*(A*c + B*b)*log(b**2*sqrt(1/(b**3*c)) + x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx^2}{bx^2-cx^4}\,dx = -\frac{(Bb+Ac)\log\left(\frac{cx-\sqrt{bc}}{cx+\sqrt{bc}}\right)}{2\sqrt{bc}b} - \frac{A}{bx}$$

[In] integrate((B*x^2+A)/(-c*x^4+b*x^2),x, algorithm="maxima")

[Out] -1/2*(B*b + A*c)*log((c*x - sqrt(b*c))/(c*x + sqrt(b*c)))/(sqrt(b*c)*b) - A
/(b*x)

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^2}{bx^2-cx^4}\,dx = -\frac{(Bb+Ac)\arctan\left(\frac{cx}{\sqrt{-bc}}\right)}{\sqrt{-bc}b} - \frac{A}{bx}$$

[In] integrate((B*x^2+A)/(-c*x^4+b*x^2),x, algorithm="giac")

[Out] -(B*b + A*c)*arctan(c*x/sqrt(-b*c))/(sqrt(-b*c)*b) - A/(b*x)

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx^2}{bx^2-cx^4}\,dx = \frac{\mathrm{atanh}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(A\,c+B\,b)}{b^{3/2}\,\sqrt{c}} - \frac{A}{b\,x}$$

[In] int((A + B*x^2)/(b*x^2 - c*x^4),x)

[Out] (atanh((c^(1/2)*x)/b^(1/2))*(A*c + B*b))/(b^(3/2)*c^(1/2)) - A/(b*x)

3.53 $\int \frac{A+Bx^2}{x(bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 49

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)}\,dx=-\frac{A}{2bx^2}+\frac{(bB-Ac)\log(x)}{b^2}-\frac{(bB-Ac)\log\left(b+cx^2\right)}{2b^2}$$

```
[Out] -1/2*A/b/x^2+(-A*c+B*b)*ln(x)/b^2-1/2*(-A*c+B*b)*ln(c*x^2+b)/b^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)}\,dx=-\frac{(bB-Ac)\log\left(b+cx^2\right)}{2b^2}+\frac{\log(x)(bB-Ac)}{b^2}-\frac{A}{2bx^2}$$

```
[In] Int[(A + B*x^2)/(x*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*A/(b*x^2) + ((b*B - A*c)*Log[x])/b^2 - ((b*B - A*c)*Log[b + c*x^2])/(2
*b^2)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{x^3\left(b+cx^2\right)}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2(b+cx)}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{bx^2}+\frac{bB-Ac}{b^2x}-\frac{c(bB-Ac)}{b^2(b+cx)}\right)\,dx, x, x^2\right) \\
&= -\frac{A}{2bx^2}+\frac{(bB-Ac)\log(x)}{b^2}-\frac{(bB-Ac)\log\left(b+cx^2\right)}{2b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)}\,dx = -\frac{A}{2bx^2}+\frac{(bB-Ac)\log(x)}{b^2}+\frac{(-bB+Ac)\log\left(b+cx^2\right)}{2b^2}$$

```
[In] Integrate[(A + B*x^2)/(x*(b*x^2 + c*x^4)),x]
[Out] -1/2*A/(b*x^2) + ((b*B - A*c)*Log[x])/b^2 + ((-(b*B) + A*c)*Log[b + c*x^2])
/(2*b^2)
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{A}{2b\,x^{2}}+\frac{(-Ac+Bb)\ln(x)}{b^{2}}+\frac{(Ac-Bb)\ln\left(c\,x^{2}+b\right)}{2b^{2}}$	46
norman	$-\frac{A}{2b\,x^{2}}-\frac{(Ac-Bb)\ln(x)}{b^{2}}+\frac{(Ac-Bb)\ln\left(c\,x^{2}+b\right)}{2b^{2}}$	47
parallelrisch	$-\frac{2A\ln(x)x^{2}c-A\ln\left(c\,x^{2}+b\right)x^{2}c-2B\ln(x)x^{2}b+B\ln\left(c\,x^{2}+b\right)x^{2}b+Ab}{2x^{2}b^{2}}$	60
risch	$-\frac{A}{2b\,x^{2}}-\frac{\ln(x)Ac}{b^{2}}+\frac{\ln(x)B}{b}+\frac{\ln\left(-c\,x^{2}-b\right)Ac}{2b^{2}}-\frac{\ln\left(-c\,x^{2}-b\right)B}{2b}$	62

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*A/b/x^2+(-A*c+B*b)*ln(x)/b^2+1/2*(A*c-B*b)/b^2*ln(c*x^2+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)}\,dx=-\frac{(Bb-Ac)x^2\log\left(cx^2+b\right)-2\,(Bb-Ac)x^2\log\left(x\right)+Ab}{2\,b^2x^2}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*((B*b - A*c)*x^2*log(c*x^2 + b) - 2*(B*b - A*c)*x^2*log(x) + A*b)/(b^2
*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)}\,dx=-\frac{A}{2bx^2}+\frac{(-Ac+Bb)\log\left(x\right)}{b^2}-\frac{(-Ac+Bb)\log\left(\frac{b}{c}+x^2\right)}{2b^2}$$

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2),x)
```

```
[Out] -A/(2*b*x**2) + (-A*c + B*b)*log(x)/b**2 - (-A*c + B*b)*log(b/c + x**2)/(2*
b**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2}{x\,(bx^2+cx^4)}\,dx = -\frac{(Bb-Ac)\log{(cx^2+b)}}{2\,b^2}+\frac{(Bb-Ac)\log{(x^2)}}{2\,b^2}-\frac{A}{2\,bx^2}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*(B*b - A*c)*log(c*x^2 + b)/b^2 + 1/2*(B*b - A*c)*log(x^2)/b^2 - 1/2*A/
(b*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.45

$$\int \frac{A+Bx^2}{x\,(bx^2+cx^4)}\,dx = \frac{(Bb-Ac)\log{(x^2)}}{2\,b^2}-\frac{(Bbc-Ac^2)\log{(|cx^2+b|)}}{2\,b^2c}-\frac{Bbx^2-Acx^2+Ab}{2\,b^2x^2}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(B*b - A*c)*log(x^2)/b^2 - 1/2*(B*b*c - A*c^2)*log(abs(c*x^2 + b))/(b^2
*c) - 1/2*(B*b*x^2 - A*c*x^2 + A*b)/(b^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{x\,(bx^2+cx^4)}\,dx = \frac{\ln{(c\,x^2+b)}\,\left(A\,c-B\,b\right)}{2\,b^2}-\frac{A}{2\,b\,x^2}-\frac{\ln{(x)}\,\left(A\,c-B\,b\right)}{b^2}$$

```
[In] int((A + B*x^2)/(x*(b*x^2 + c*x^4)),x)
```

```
[Out] (log(b + c*x^2)*(A*c - B*b))/(2*b^2) - A/(2*b*x^2) - (log(x)*(A*c - B*b))/b
^2
```

3.54 $\int \frac{A+Bx^2}{x^2(bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 61

$$\int \frac{A+Bx^2}{x^2(bx^2+cx^4)}\,dx = -\frac{A}{3bx^3}-\frac{bB-Ac}{b^2x}-\frac{\sqrt{c}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{5/2}}$$

```
[Out] -1/3*A/b/x^3+(A*c-B*b)/b^2/x-(-A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))*c^(1/2)/b
^(5/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 464, 331, 211}

$$\int \frac{A+Bx^2}{x^2(bx^2+cx^4)}\,dx = -\frac{\sqrt{c}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{5/2}}-\frac{bB-Ac}{b^2x}-\frac{A}{3bx^3}$$

```
[In] Int[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/3*A/(b*x^3) - (b*B - A*c)/(b^2*x) - (Sqrt[c]*(b*B - A*c)*ArcTan[(Sqrt[c]
*x)/Sqrt[b]])/b^(5/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{x^4\left(b+cx^2\right)}\,dx \\
&= -\frac{A}{3bx^3}-\frac{(-3bB+3Ac)\int \frac{1}{x^2(b+cx^2)}\,dx}{3b} \\
&= -\frac{A}{3bx^3}-\frac{bB-Ac}{b^2x}-\frac{(c(bB-Ac))\int \frac{1}{b+cx^2}\,dx}{b^2} \\
&= -\frac{A}{3bx^3}-\frac{bB-Ac}{b^2x}-\frac{\sqrt{c}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)}\,dx=-\frac{A}{3bx^3}+\frac{-bB+Ac}{b^2x}-\frac{\sqrt{c}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{5/2}}$$

```
[In] Integrate[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)),x]
[Out] -1/3*A/(b*x^3) + (-(b*B) + A*c)/(b^2*x) - (Sqrt[c]*(b*B - A*c)*ArcTan[(Sqrt
[c]*x)/Sqrt[b]])/b^(5/2)
```

Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

method	result
default	$-\frac{A}{3b\,x^{3}}-\frac{-Ac+Bb}{x\,b^{2}}+\frac{c(Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{b^{2}\sqrt{bc}}$
risch	$\frac{\frac{(Ac-Bb)x^{2}}{b^{2}}-\frac{A}{3b}}{x^{3}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(b^{5}\_Z^{2}+A^{2}c^{3}-2ABb\,c^{2}+B^{2}b^{2}c\right)}\_R\ln\left(\left(3\_R^{2}b^{5}+2A^{2}c^{3}-4ABb\,c^{2}+2B^{2}b^{2}c\right)x+\left(-A\,b^{3}c+B\,b^{4}\right)\_\right.\right.}{2}$

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*A/b/x^3-(-A*c+B*b)/x/b^2+c*(A*c-B*b)/b^2/(b*c)^(1/2)*arctan(c*x/(b*c)^
(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.21

$$\int \frac{A+Bx^2}{x^2\,(bx^2+cx^4)}\,dx=\left[-\frac{3\,(Bb-Ac)x^3\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2+2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)+6\,(Bb-Ac)x^2+2\,Ab}{6\,b^2x^3},\right.$$
$$\left.-\frac{3\,(Bb-Ac)x^3\sqrt{\frac{c}{b}}\arctan\left(x\sqrt{\frac{c}{b}}\right)+3\,(Bb-Ac)x^2+Ab}{3\,b^2x^3}\right]$$

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(B*b - A*c)*x^3*sqrt(-c/b)*log((c*x^2 + 2*b*x*sqrt(-c/b) - b)/(c*x
^2 + b)) + 6*(B*b - A*c)*x^2 + 2*A*b)/(b^2*x^3), -1/3*(3*(B*b - A*c)*x^3*sq
rt(c/b)*arctan(x*sqrt(c/b)) + 3*(B*b - A*c)*x^2 + A*b)/(b^2*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(49) = 98$.

Time = 0.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.11

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)}\,dx = \frac{\sqrt{-\frac{c}{b^5}}(-Ac+Bb)\log\left(-\frac{b^3\sqrt{-\frac{c}{b^5}}(-Ac+Bb)}{-Ac^2+Bbc}+x\right)}{2} - \frac{\sqrt{-\frac{c}{b^5}}(-Ac+Bb)\log\left(\frac{b^3\sqrt{-\frac{c}{b^5}}(-Ac+Bb)}{-Ac^2+Bbc}+x\right)}{2} + \frac{-Ab+x^2\cdot(3Ac-3Bb)}{3b^2x^3}$$

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2),x)
```

```
[Out] sqrt(-c/b**5)*(-A*c + B*b)*log(-b**3*sqrt(-c/b**5)*(-A*c + B*b)/(-A*c**2 +
B*b*c) + x)/2 - sqrt(-c/b**5)*(-A*c + B*b)*log(b**3*sqrt(-c/b**5)*(-A*c + B
*b)/(-A*c**2 + B*b*c) + x)/2 + (-A*b + x**2*(3*A*c - 3*B*b))/(3*b**2*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)}\,dx = -\frac{\left(Bbc-Ac^2\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b^2} - \frac{3\,(Bb-Ac)x^2+Ab}{3\,b^2x^3}$$

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -(B*b*c - A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) - 1/3*(3*(B*b - A*c)
*x^2 + A*b)/(b^2*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)}\,dx = -\frac{\left(Bbc-Ac^2\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b^2} - \frac{3\,Bbx^2-3\,Acx^2+Ab}{3\,b^2x^3}$$

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -(B*b*c - A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) - 1/3*(3*B*b*x^2 - 3
*A*c*x^2 + A*b)/(b^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)}\,dx=\frac{\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,\left(A\,c-B\,b\right)}{b^{5/2}}-\frac{\frac{A}{3\,b}-\frac{x^2\,\left(A\,c-B\,b\right)}{b^2}}{x^3}$$

```
[In] int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)),x)
```

```
[Out] (c^(1/2)*atan((c^(1/2)*x)/b^(1/2))*(A*c - B*b))/b^(5/2) - (A/(3*b) - (x^2*(
A*c - B*b))/b^2)/x^3
```

3.55 $\int \frac{A+Bx^2}{x^3(bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 70

$$\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)}\,dx = -\frac{A}{4bx^4}-\frac{bB-Ac}{2b^2x^2}-\frac{c(bB-Ac)\log(x)}{b^3}+\frac{c(bB-Ac)\log\left(b+cx^2\right)}{2b^3}$$

```
[Out] -1/4*A/b/x^4+1/2*(A*c-B*b)/b^2/x^2-c*(-A*c+B*b)*ln(x)/b^3+1/2*c*(-A*c+B*b)*
ln(c*x^2+b)/b^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)}\,dx = \frac{c(bB-Ac)\log\left(b+cx^2\right)}{2b^3}-\frac{c\log(x)(bB-Ac)}{b^3}-\frac{bB-Ac}{2b^2x^2}-\frac{A}{4bx^4}$$

```
[In] Int[(A + B*x^2)/(x^3*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*A/(b*x^4) - (b*B - A*c)/(2*b^2*x^2) - (c*(b*B - A*c)*Log[x])/b^3 + (c*
(b*B - A*c)*Log[b + c*x^2])/(2*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{x^5\,(b+cx^2)}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^3(b+cx)}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{bx^3}+\frac{bB-Ac}{b^2x^2}-\frac{c(bB-Ac)}{b^3x}+\frac{c^2(bB-Ac)}{b^3(b+cx)}\right)\,dx, x, x^2\right) \\
&= -\frac{A}{4bx^4}-\frac{bB-Ac}{2b^2x^2}-\frac{c(bB-Ac)\log(x)}{b^3}+\frac{c(bB-Ac)\log\left(b+cx^2\right)}{2b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x^3\,(bx^2+cx^4)}\,dx = \frac{-b(Ab+2bBx^2-2Acx^2)+4c(-bB+Ac)x^4\log(x)+2c(bB-Ac)x^4\log\left(b+cx^2\right)}{4b^3x^4}$$

```
[In] Integrate[(A + B*x^2)/(x^3*(b*x^2 + c*x^4)),x]
```

```
[Out] (-(b*(A*b + 2*b*B*x^2 - 2*A*c*x^2)) + 4*c*(-(b*B) + A*c)*x^4*Log[x] + 2*c*(
b*B - A*c)*x^4*Log[b + c*x^2])/(4*b^3*x^4)
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{A}{4b\,x^{4}}-\frac{-Ac+Bb}{2x^{2}b^{2}}+\frac{c(Ac-Bb)\ln(x)}{b^{3}}-\frac{c(Ac-Bb)\ln(c\,x^{2}+b)}{2b^{3}}$	64
norman	$\frac{-\frac{A}{4b}+\frac{(Ac-Bb)x^{2}}{2b^{2}}}{x^{4}}+\frac{c(Ac-Bb)\ln(x)}{b^{3}}-\frac{c(Ac-Bb)\ln(c\,x^{2}+b)}{2b^{3}}$	66
risch	$\frac{-\frac{A}{4b}+\frac{(Ac-Bb)x^{2}}{2b^{2}}}{x^{4}}+\frac{c^{2}\ln(x)A}{b^{3}}-\frac{c\ln(x)B}{b^{2}}-\frac{c^{2}\ln(c\,x^{2}+b)A}{2b^{3}}+\frac{c\ln(c\,x^{2}+b)B}{2b^{2}}$	80
parallelrisch	$\frac{4A\ln(x)x^{4}c^{2}-2A\ln(c\,x^{2}+b)x^{4}c^{2}-4B\ln(x)x^{4}bc+2B\ln(c\,x^{2}+b)x^{4}bc+2Abc\,x^{2}-2b^{2}B\,x^{2}-b^{2}A}{4b^{3}x^{4}}$	87

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*A/b/x^4-1/2*(-A*c+B*b)/x^2/b^2+c*(A*c-B*b)/b^3*ln(x)-1/2*c*(A*c-B*b)/b
^3*ln(c*x^2+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^2}{x^3\,(bx^2+cx^4)}\,dx = \frac{2\,(Bbc-Ac^2)x^4\log\left(cx^2+b\right)-4\,(Bbc-Ac^2)x^4\log\left(x\right)-Ab^2-2\,(Bb^2-Abc)x^2}{4\,b^3x^4}$$

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(B*b*c - A*c^2)*x^4*log(c*x^2 + b) - 4*(B*b*c - A*c^2)*x^4*log(x) -
A*b^2 - 2*(B*b^2 - A*b*c)*x^2)/(b^3*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^2}{x^3\,(bx^2+cx^4)}\,dx = \frac{-Ab+x^2\cdot(2Ac-2Bb)}{4b^2x^4}-\frac{c(-Ac+Bb)\log\left(x\right)}{b^3}+\frac{c(-Ac+Bb)\log\left(\frac{b}{c}+x^2\right)}{2b^3}$$

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2),x)
```

```
[Out] (-A*b + x**2*(2*A*c - 2*B*b))/(4*b**2*x**4) - c*(-A*c + B*b)*log(x)/b**3 +
c*(-A*c + B*b)*log(b/c + x**2)/(2*b**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)}\,dx = \frac{\left(Bbc-Ac^2\right)\log\left(cx^2+b\right)}{2\,b^3} - \frac{\left(Bbc-Ac^2\right)\log\left(x^2\right)}{2\,b^3} - \frac{2\,(Bb-Ac)x^2+Ab}{4\,b^2x^4}$$

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*c - A*c^2)*log(c*x^2 + b)/b^3 - 1/2*(B*b*c - A*c^2)*log(x^2)/b^3 -
 1/4*(2*(B*b - A*c)*x^2 + A*b)/(b^2*x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.43

$$\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)}\,dx = -\frac{\left(Bbc-Ac^2\right)\log\left(x^2\right)}{2\,b^3} + \frac{\left(Bbc^2-Ac^3\right)\log\left(|cx^2+b|\right)}{2\,b^3c} + \frac{3\,Bbcx^4-3\,Ac^2x^4-2\,Bb^2x^2+2\,Abcx^2-Ab^2}{4\,b^3x^4}$$

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*(B*b*c - A*c^2)*log(x^2)/b^3 + 1/2*(B*b*c^2 - A*c^3)*log(abs(c*x^2 + b
))/(b^3*c) + 1/4*(3*B*b*c*x^4 - 3*A*c^2*x^4 - 2*B*b^2*x^2 + 2*A*b*c*x^2 - A
*b^2)/(b^3*x^4)
```

Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)}\,dx = \frac{\ln(x)\,\left(A\,c^2-B\,b\,c\right)}{b^3} - \frac{\ln\left(c\,x^2+b\right)\,\left(A\,c^2-B\,b\,c\right)}{2\,b^3} - \frac{\frac{A}{4\,b}-\frac{x^2\,(A\,c-B\,b)}{2\,b^2}}{x^4}$$

```
[In] int((A + B*x^2)/(x^3*(b*x^2 + c*x^4)),x)
```

```
[Out] (log(x)*(A*c^2 - B*b*c))/b^3 - (log(b + c*x^2)*(A*c^2 - B*b*c))/(2*b^3) - (
A/(4*b) - (x^2*(A*c - B*b))/(2*b^2))/x^4
```

3.56 $\int \frac{A+Bx^2}{x^4(bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 78

$$\int \frac{A+Bx^2}{x^4\left(bx^2+cx^4\right)}\,dx=-\frac{A}{5bx^5}-\frac{bB-Ac}{3b^2x^3}+\frac{c(bB-Ac)}{b^3x}+\frac{c^{3/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{7/2}}$$

```
[Out] -1/5*A/b/x^5+1/3*(A*c-B*b)/b^2/x^3+c*(-A*c+B*b)/b^3/x+c^(3/2)*(-A*c+B*b)*ar
ctan(x*c^(1/2)/b^(1/2))/b^(7/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 464, 331, 211}

$$\int \frac{A+Bx^2}{x^4\left(bx^2+cx^4\right)}\,dx=\frac{c^{3/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{7/2}}+\frac{c(bB-Ac)}{b^3x}-\frac{bB-Ac}{3b^2x^3}-\frac{A}{5bx^5}$$

```
[In] Int[(A + B*x^2)/(x^4*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/5*A/(b*x^5) - (b*B - A*c)/(3*b^2*x^3) + (c*(b*B - A*c))/(b^3*x) + (c^(3/
2)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(7/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{x^6\left(b+cx^2\right)}\,dx \\
&= -\frac{A}{5bx^5}-\frac{(-5bB+5Ac)\int \frac{1}{x^4(b+cx^2)}\,dx}{5b} \\
&= -\frac{A}{5bx^5}-\frac{bB-Ac}{3b^2x^3}-\frac{(c(bB-Ac))\int \frac{1}{x^2(b+cx^2)}\,dx}{b^2} \\
&= -\frac{A}{5bx^5}-\frac{bB-Ac}{3b^2x^3}+\frac{c(bB-Ac)}{b^3x}+\frac{(c^2(bB-Ac))\int \frac{1}{b+cx^2}\,dx}{b^3} \\
&= -\frac{A}{5bx^5}-\frac{bB-Ac}{3b^2x^3}+\frac{c(bB-Ac)}{b^3x}+\frac{c^{3/2}(bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x^4\left(bx^2+cx^4\right)}\,dx = -\frac{A}{5bx^5}+\frac{-bB+Ac}{3b^2x^3}+\frac{c(bB-Ac)}{b^3x}+\frac{c^{3/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{7/2}}$$

```
[In] Integrate[(A + B*x^2)/(x^4*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/5*A/(b*x^5) + (-(b*B) + A*c)/(3*b^2*x^3) + (c*(b*B - A*c))/(b^3*x) + (c^
(3/2)*(b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(7/2)
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

method	result
default	$-\frac{A}{5b\,x^5}-\frac{-Ac+Bb}{3x^3b^2}-\frac{c(Ac-Bb)}{b^3x}-\frac{c^2(Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{b^3\sqrt{bc}}$
risch	$\frac{-\frac{c(Ac-Bb)x^4}{b^3}+\frac{(Ac-Bb)x^2}{3b^2}-\frac{A}{5b}}{x^5}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^7\_Z^2+A^2c^5-2ABb\,c^4+B^2b^2c^3\right)}\_R\ln\left(\left(3\_R^2b^7+2A^2c^5-4ABb\,c^4+2B^2b^2c^3\right)\right.\right.}{2}$

```
[In] int((B*x^2+A)/x^4/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*A/b/x^5-1/3*(-A*c+B*b)/x^3/b^2-c*(A*c-B*b)/b^3/x-c^2*(A*c-B*b)/b^3/(b*
c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.36

$$\int \frac{A+Bx^2}{x^4\left(bx^2+cx^4\right)}\,dx$$
$$=\left[-\frac{15\,(Bbc-Ac^2)x^5\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)-30\,(Bbc-Ac^2)x^4+6\,Ab^2+10\,(Bb^2-Abc)x^2}{30\,b^3x^5},\frac{15\,(Bbc\right.$$

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*(B*b*c - A*c^2)*x^5*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b
)/(c*x^2 + b)) - 30*(B*b*c - A*c^2)*x^4 + 6*A*b^2 + 10*(B*b^2 - A*b*c)*x^2)
```

```
/(b^3*x^5), 1/15*(15*(B*b*c - A*c^2)*x^5*sqrt(c/b)*arctan(x*sqrt(c/b)) + 15
*(B*b*c - A*c^2)*x^4 - 3*A*b^2 - 5*(B*b^2 - A*b*c)*x^2)/(b^3*x^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(68) = 136$.

Time = 0.25 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.09

$$\int \frac{A+Bx^2}{x^4\left(bx^2+cx^4\right)}\,dx = -\frac{\sqrt{-\frac{c^3}{b^7}}(-Ac+Bb)\log\left(-\frac{b^4\sqrt{-\frac{c^3}{b^7}}(-Ac+Bb)}{-Ac^3+Bbc^2}+x\right)}{2} + \frac{\sqrt{-\frac{c^3}{b^7}}(-Ac+Bb)\log\left(\frac{b^4\sqrt{-\frac{c^3}{b^7}}(-Ac+Bb)}{-Ac^3+Bbc^2}+x\right)}{2} + \frac{-3Ab^2+x^4(-15Ac^2+15Bbc)+x^2\cdot(5Abc-5Bb^2)}{15b^3x^5}$$

```
[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2),x)
```

```
[Out] -sqrt(-c**3/b**7)*(-A*c + B*b)*log(-b**4*sqrt(-c**3/b**7)*(-A*c + B*b)/(-A*
c**3 + B*b*c**2) + x)/2 + sqrt(-c**3/b**7)*(-A*c + B*b)*log(b**4*sqrt(-c**3
/b**7)*(-A*c + B*b)/(-A*c**3 + B*b*c**2) + x)/2 + (-3*A*b**2 + x**4*(-15*A*
c**2 + 15*B*b*c) + x**2*(5*A*b*c - 5*B*b**2))/(15*b**3*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2}{x^4\left(bx^2+cx^4\right)}\,dx = \frac{(Bbc^2-Ac^3)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b^3} + \frac{15\,(Bbc-Ac^2)x^4-3\,Ab^2-5\,(Bb^2-Abc)x^2}{15\,b^3x^5}$$

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] (B*b*c^2 - A*c^3)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) + 1/15*(15*(B*b*c -
 A*c^2)*x^4 - 3*A*b^2 - 5*(B*b^2 - A*b*c)*x^2)/(b^3*x^5)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^2}{x^4\,(bx^2+cx^4)}\,dx = \frac{(Bbc^2-Ac^3)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b^3} + \frac{15\,Bbcx^4-15\,Ac^2x^4-5\,Bb^2x^2+5\,Abcx^2-3\,Ab^2}{15\,b^3x^5}$$

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] (B*b*c^2 - A*c^3)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) + 1/15*(15*B*b*c*x^
4 - 15*A*c^2*x^4 - 5*B*b^2*x^2 + 5*A*b*c*x^2 - 3*A*b^2)/(b^3*x^5)
```

Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^2}{x^4\,(bx^2+cx^4)}\,dx = -\frac{\frac{A}{5\,b}-\frac{x^2\,(A\,c-B\,b)}{3\,b^2}+\frac{c\,x^4\,(A\,c-B\,b)}{b^3}}{x^5} - \frac{c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(A\,c-B\,b)}{b^{7/2}}$$

```
[In] int((A + B*x^2)/(x^4*(b*x^2 + c*x^4)),x)
```

```
[Out] - (A/(5*b) - (x^2*(A*c - B*b))/(3*b^2) + (c*x^4*(A*c - B*b))/b^3)/x^5 - (c^
(3/2)*atan((c^(1/2)*x)/b^(1/2))*(A*c - B*b))/b^(7/2)
```

3.57 $\int \frac{A+Bx^2}{x^5(bx^2+cx^4)} dx$

Optimal result

Integrand size = 24, antiderivative size = 92

$$\int \frac{A+Bx^2}{x^5(bx^2+cx^4)} dx = -\frac{A}{6bx^6} - \frac{bB-Ac}{4b^2x^4} + \frac{c(bB-Ac)}{2b^3x^2} + \frac{c^2(bB-Ac)\log(x)}{b^4} - \frac{c^2(bB-Ac)\log(b+cx^2)}{2b^4}$$

```
[Out] -1/6*A/b/x^6+1/4*(A*c-B*b)/b^2/x^4+1/2*c*(-A*c+B*b)/b^3/x^2+c^2*(-A*c+B*b)*
ln(x)/b^4-1/2*c^2*(-A*c+B*b)*ln(c*x^2+b)/b^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$\int \frac{A+Bx^2}{x^5(bx^2+cx^4)} dx = -\frac{c^2(bB-Ac)\log(b+cx^2)}{2b^4} + \frac{c^2\log(x)(bB-Ac)}{b^4} + \frac{c(bB-Ac)}{2b^3x^2} - \frac{bB-Ac}{4b^2x^4} - \frac{A}{6bx^6}$$

```
[In] Int[(A + B*x^2)/(x^5*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/6*A/(b*x^6) - (b*B - A*c)/(4*b^2*x^4) + (c*(b*B - A*c))/(2*b^3*x^2) + (c
^2*(b*B - A*c)*Log[x])/b^4 - (c^2*(b*B - A*c)*Log[b + c*x^2])/(2*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{x^7\left(b+cx^2\right)}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^4(b+cx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{bx^4}+\frac{bB-Ac}{b^2x^3}-\frac{c(bB-Ac)}{b^3x^2}+\frac{c^2(bB-Ac)}{b^4x}-\frac{c^3(bB-Ac)}{b^4(b+cx)}\right)\,dx,x,x^2\right)\\
&= -\frac{A}{6bx^6}-\frac{bB-Ac}{4b^2x^4}+\frac{c(bB-Ac)}{2b^3x^2}+\frac{c^2(bB-Ac)\log(x)}{b^4}-\frac{c^2(bB-Ac)\log\left(b+cx^2\right)}{2b^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)}\,dx = -\frac{A}{6bx^6}+\frac{-bB+Ac}{4b^2x^4}+\frac{c(bB-Ac)}{2b^3x^2}+\frac{\left(bBc^2-Ac^3\right)\log(x)}{b^4}+\frac{\left(-bBc^2+Ac^3\right)\log\left(b+cx^2\right)}{2b^4}$$

```
[In] Integrate[(A + B*x^2)/(x^5*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/6*A/(b*x^6) + (-(b*B) + A*c)/(4*b^2*x^4) + (c*(b*B - A*c))/(2*b^3*x^2) +
 ((b*B*c^2 - A*c^3)*Log[x])/b^4 + ((-(b*B*c^2) + A*c^3)*Log[b + c*x^2])/(2*
b^4)
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.93

method	result
default	$-\frac{A}{6b\,x^6}-\frac{-Ac+Bb}{4x^4b^2}-\frac{c(Ac-Bb)}{2b^3x^2}-\frac{c^2(Ac-Bb)\ln(x)}{b^4}+\frac{c^2(Ac-Bb)\ln(c\,x^2+b)}{2b^4}$
norman	$\frac{-\frac{A}{6b}+\frac{(Ac-Bb)x^2}{4b^2}-\frac{c(Ac-Bb)x^4}{2b^3}}{x^6}-\frac{c^2(Ac-Bb)\ln(x)}{b^4}+\frac{c^2(Ac-Bb)\ln(c\,x^2+b)}{2b^4}$
risch	$\frac{-\frac{A}{6b}+\frac{(Ac-Bb)x^2}{4b^2}-\frac{c(Ac-Bb)x^4}{2b^3}}{x^6}-\frac{c^3\ln(x)A}{b^4}+\frac{c^2\ln(x)B}{b^3}+\frac{c^3\ln(-c\,x^2-b)A}{2b^4}-\frac{c^2\ln(-c\,x^2-b)B}{2b^3}$
parallelrisch	$-\frac{12A\ln(x)x^6c^3-6A\ln(c\,x^2+b)x^6c^3-12B\ln(x)x^6b\,c^2+6B\ln(c\,x^2+b)x^6b\,c^2+6Ab\,c^2x^4-6x^4B\,b^2c-3A\,b^2c\,x^2+3b^3B\,x^2+2b^3A}{12b^4x^6}$

```
[In] int((B*x^2+A)/x^5/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*A/b/x^6-1/4*(-A*c+B*b)/x^4/b^2-1/2*c*(A*c-B*b)/b^3/x^2-c^2*(A*c-B*b)/b
^4*ln(x)+1/2*c^2*(A*c-B*b)/b^4*ln(c*x^2+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^2}{x^5\,(bx^2+cx^4)}\,dx = -\frac{6\,(Bbc^2-Ac^3)x^6\log{(cx^2+b)}-12\,(Bbc^2-Ac^3)x^6\log{(x)}-6\,(Bb^2c-Abc^2)x^4+2\,Ab^3+3\,(Bb^3-Ab^2}{12\,b^4x^6}$$

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/12*(6*(B*b*c^2 - A*c^3)*x^6*log(c*x^2 + b) - 12*(B*b*c^2 - A*c^3)*x^6*lo
g(x) - 6*(B*b^2*c - A*b*c^2)*x^4 + 2*A*b^3 + 3*(B*b^3 - A*b^2*c)*x^2)/(b^4*
x^6)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2}{x^5\,(bx^2+cx^4)}\,dx = \frac{-2Ab^2+x^4(-6Ac^2+6Bbc)+x^2\cdot(3Abc-3Bb^2)}{12b^3x^6} + \frac{c^2(-Ac+Bb)\log{(x)}}{b^4} - \frac{c^2(-Ac+Bb)\log{\left(\frac{b}{c}+x^2\right)}}{2b^4}$$

```
[In] integrate((B*x**2+A)/x**5/(c*x**4+b*x**2),x)
```

```
[Out] (-2*A*b**2 + x**4*(-6*A*c**2 + 6*B*b*c) + x**2*(3*A*b*c - 3*B*b**2))/(12*b*
*3*x**6) + c**2*(-A*c + B*b)*log(x)/b**4 - c**2*(-A*c + B*b)*log(b/c + x**2
)/(2*b**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^2}{x^5\,(bx^2+cx^4)}\,dx = -\frac{(Bbc^2-Ac^3)\log(cx^2+b)}{2\,b^4} + \frac{(Bbc^2-Ac^3)\log(x^2)}{2\,b^4} + \frac{6\,(Bbc-Ac^2)x^4-2\,Ab^2-3\,(Bb^2-Abc)x^2}{12\,b^3x^6}$$

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*(B*b*c^2 - A*c^3)*log(c*x^2 + b)/b^4 + 1/2*(B*b*c^2 - A*c^3)*log(x^2)/
b^4 + 1/12*(6*(B*b*c - A*c^2)*x^4 - 2*A*b^2 - 3*(B*b^2 - A*b*c)*x^2)/(b^3*x
^6)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx^2}{x^5\,(bx^2+cx^4)}\,dx = \frac{(Bbc^2-Ac^3)\log(x^2)}{2\,b^4} - \frac{(Bbc^3-Ac^4)\log(|cx^2+b|)}{2\,b^4c} - \frac{11\,Bbc^2x^6-11\,Ac^3x^6-6\,Bb^2cx^4+6\,Abc^2x^4+3\,Bb^3x^2-3\,Ab^2cx^2+2\,Ab^3}{12\,b^4x^6}$$

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(B*b*c^2 - A*c^3)*log(x^2)/b^4 - 1/2*(B*b*c^3 - A*c^4)*log(abs(c*x^2 +
b))/(b^4*c) - 1/12*(11*B*b*c^2*x^6 - 11*A*c^3*x^6 - 6*B*b^2*c*x^4 + 6*A*b*c
^2*x^4 + 3*B*b^3*x^2 - 3*A*b^2*c*x^2 + 2*A*b^3)/(b^4*x^6)
```

Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x^5\,(bx^2+cx^4)}\,dx = \frac{\ln(c\,x^2+b)\,(A\,c^3-B\,b\,c^2)}{2\,b^4} - \frac{\frac{A}{6\,b}-\frac{x^2\,(A\,c-B\,b)}{4\,b^2}+\frac{c\,x^4\,(A\,c-B\,b)}{2\,b^3}}{x^6} - \frac{\ln(x)\,(A\,c^3-B\,b\,c^2)}{b^4}$$

```
[In] int((A + B*x^2)/(x^5*(b*x^2 + c*x^4)),x)
```

```
[Out] (log(b + c*x^2)*(A*c^3 - B*b*c^2))/(2*b^4) - (A/(6*b) - (x^2*(A*c - B*b))/(
4*b^2) + (c*x^4*(A*c - B*b))/(2*b^3))/x^6 - (log(x)*(A*c^3 - B*b*c^2))/b^4
```

3.58 $\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 133

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^2} dx = -\frac{b^2(4bB-3Ac)x}{c^5} + \frac{b(3bB-2Ac)x^3}{3c^4} - \frac{(2bB-Ac)x^5}{5c^3} + \frac{Bx^7}{7c^2} - \frac{b^3(bB-Ac)x}{2c^5(b+cx^2)} + \frac{b^{5/2}(9bB-7Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{11/2}}$$

```
[Out] -b^2*(-3*A*c+4*B*b)*x/c^5+1/3*b*(-2*A*c+3*B*b)*x^3/c^4-1/5*(-A*c+2*B*b)*x^5
/c^3+1/7*B*x^7/c^2-1/2*b^3*(-A*c+B*b)*x/c^5/(c*x^2+b)+1/2*b^(5/2)*(-7*A*c+9
*B*b)*arctan(x*c^(1/2)/b^(1/2))/c^(11/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 466, 1824, 211}

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^2} dx = \frac{b^{5/2}(9bB-7Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{11/2}} - \frac{b^3x(bB-Ac)}{2c^5(b+cx^2)} - \frac{b^2x(4bB-3Ac)}{c^5} + \frac{bx^3(3bB-2Ac)}{3c^4} - \frac{x^5(2bB-Ac)}{5c^3} + \frac{Bx^7}{7c^2}$$

```
[In] Int[(x^12*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b^2*(4*b*B - 3*A*c)*x)/c^5) + (b*(3*b*B - 2*A*c)*x^3)/(3*c^4) - ((2*b*B
- A*c)*x^5)/(5*c^3) + (B*x^7)/(7*c^2) - (b^3*(b*B - A*c)*x)/(2*c^5*(b + c*x
^2)) + (b^(5/2)*(9*b*B - 7*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \frac{x^8(A+Bx^2)}{(b+cx^2)^2}\,dx$$

$$= -\frac{b^3(bB-Ac)x}{2c^5(b+cx^2)} - \frac{\int \frac{-b^3(bB-Ac)+2b^2c(bB-Ac)x^2-2bc^2(bB-Ac)x^4+2c^3(bB-Ac)x^6-2Bc^4x^8}{b+cx^2}\,dx}{2c^5}$$

$$= -\frac{b^3(bB-Ac)x}{2c^5(b+cx^2)} - \frac{\int \left(2b^2(4bB-3Ac)-2bc(3bB-2Ac)x^2+2c^2(2bB-Ac)x^4-2Bc^3x^6+\frac{-9b^4B+7Ab^3c}{b+cx^2}\right)dx}{2c^5}$$

$$= -\frac{b^2(4bB-3Ac)x}{c^5} + \frac{b(3bB-2Ac)x^3}{3c^4} - \frac{(2bB-Ac)x^5}{5c^3} + \frac{Bx^7}{7c^2} - \frac{b^3(bB-Ac)x}{2c^5(b+cx^2)} + \frac{(b^3(9bB-7Ac))\int \frac{1}{b+cx^2}\,dx}{2c^5}$$

$$= -\frac{b^2(4bB - 3Ac)x}{c^5} + \frac{b(3bB - 2Ac)x^3}{3c^4} - \frac{(2bB - Ac)x^5}{5c^3} + \frac{Bx^7}{7c^2} - \frac{b^3(bB - Ac)x}{2c^5\,(b + cx^2)} + \frac{b^{5/2}(9bB - 7Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{11/2}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.01

$$\int \frac{x^{12}(A + Bx^2)}{(bx^2 + cx^4)^2}\,dx = -\frac{b^2(4bB - 3Ac)x}{c^5} + \frac{b(3bB - 2Ac)x^3}{3c^4} + \frac{(-2bB + Ac)x^5}{5c^3} + \frac{Bx^7}{7c^2} + \frac{(-b^4B + Ab^3c)\,x}{2c^5\,(b + cx^2)} + \frac{b^{5/2}(9bB - 7Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{11/2}}$$

```
[In] Integrate[(x^12*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b^2*(4*b*B - 3*A*c)*x)/c^5) + (b*(3*b*B - 2*A*c)*x^3)/(3*c^4) + ((-2*b*B
 + A*c)*x^5)/(5*c^3) + (B*x^7)/(7*c^2) + ((-(b^4*B) + A*b^3*c)*x)/(2*c^5*(b
 + c*x^2)) + (b^(5/2)*(9*b*B - 7*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(11
/2))
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92

method	result
default	$\frac{\frac{1}{7}B\,c^3x^7+\frac{1}{5}A\,c^3x^5-\frac{2}{5}Bb\,c^2x^5-\frac{2}{3}Ab\,c^2x^3+B\,b^2c\,x^3+3A\,b^2cx-4b^3\,Bx}{c^5}-\frac{b^3\left(\frac{\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)x}{c\,x^2+b}+\frac{(7Ac-9Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{c^5}$
risch	$\frac{B\,x^7}{7c^2}+\frac{A\,x^5}{5c^2}-\frac{2Bb\,x^5}{5c^3}-\frac{2Ab\,x^3}{3c^3}+\frac{B\,b^2x^3}{c^4}+\frac{3A\,b^2x}{c^4}-\frac{4b^3Bx}{c^5}+\frac{\left(\frac{1}{2}A\,b^3c-\frac{1}{2}B\,b^4\right)x}{c^5(c\,x^2+b)}+\frac{7\sqrt{-bc}\,b^2\ln\left(-\sqrt{-bc}\,x-b\right)A}{4c^5}-9$

```
[In] int(x^12*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(1/7*B*c^3*x^7+1/5*A*c^3*x^5-2/5*B*b*c^2*x^5-2/3*A*b*c^2*x^3+B*b^2*c*
x^3+3*A*b^2*c*x-4*b^3*B*x)-b^3/c^5*((-1/2*A*c+1/2*B*b)*x/(c*x^2+b)+1/2*(7*A
*c-9*B*b)/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.63

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$
$$= \left[\frac{60\,Bc^4x^9-12\,(9\,Bbc^3-7\,Ac^4)x^7+28\,(9\,Bb^2c^2-7\,Abc^3)x^5-140\,(9\,Bb^3c-7\,Ab^2c^2)x^3-105\,(9\,Bb^4-}{420\,(c^6x^2+bc^5)}\right.$$

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
[Out] [1/420*(60*B*c^4*x^9 - 12*(9*B*b*c^3 - 7*A*c^4)*x^7 + 28*(9*B*b^2*c^2 - 7*A
*b*c^3)*x^5 - 140*(9*B*b^3*c - 7*A*b^2*c^2)*x^3 - 105*(9*B*b^4 - 7*A*b^3*c
+ (9*B*b^3*c - 7*A*b^2*c^2)*x^2)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) -
 b)/(c*x^2 + b)) - 210*(9*B*b^4 - 7*A*b^3*c)*x)/(c^6*x^2 + b*c^5), 1/210*(3
0*B*c^4*x^9 - 6*(9*B*b*c^3 - 7*A*c^4)*x^7 + 14*(9*B*b^2*c^2 - 7*A*b*c^3)*x^
5 - 70*(9*B*b^3*c - 7*A*b^2*c^2)*x^3 + 105*(9*B*b^4 - 7*A*b^3*c + (9*B*b^3*
c - 7*A*b^2*c^2)*x^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) - 105*(9*B*b^4 - 7*
A*b^3*c)*x)/(c^6*x^2 + b*c^5)]
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.79

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bx^7}{7c^2}+x^5\left(\frac{A}{5c^2}-\frac{2Bb}{5c^3}\right)+x^3\left(-\frac{2Ab}{3c^3}+\frac{Bb^2}{c^4}\right)+x\left(\frac{3Ab^2}{c^4}-\frac{4Bb^3}{c^5}\right)+\frac{x(Ab^3c-Bb^4)}{2bc^5+2c^6x^2}-\frac{\sqrt{-\frac{b^5}{c^{11}}}(-7Ac+9Bb)\log\left(-\frac{c^5\sqrt{-\frac{b^5}{c^{11}}}(-7Ac+9Bb)}{-7Ab^2c+9Bb^3}+x\right)}{4}+\frac{\sqrt{-\frac{b^5}{c^{11}}}(-7Ac+9Bb)\log\left(\frac{c^5\sqrt{-\frac{b^5}{c^{11}}}(-7Ac+9Bb)}{-7Ab^2c+9Bb^3}+x\right)}{4}$$

```
[In] integrate(x**12*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
[Out] B*x**7/(7*c**2) + x**5*(A/(5*c**2) - 2*B*b/(5*c**3)) + x**3*(-2*A*b/(3*c**3
) + B*b**2/c**4) + x*(3*A*b**2/c**4 - 4*B*b**3/c**5) + x*(A*b**3*c - B*b**4
)/(2*b*c**5 + 2*c**6*x**2) - sqrt(-b**5/c**11)*(-7*A*c + 9*B*b)*log(-c**5*s
qrt(-b**5/c**11)*(-7*A*c + 9*B*b)/(-7*A*b**2*c + 9*B*b**3) + x)/4 + sqrt(-b
**5/c**11)*(-7*A*c + 9*B*b)*log(c**5*sqrt(-b**5/c**11)*(-7*A*c + 9*B*b)/(-7
*A*b**2*c + 9*B*b**3) + x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.02

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(Bb^4-Ab^3c)x}{2\,(c^6x^2+bc^5)} + \frac{(9\,Bb^4-7\,Ab^3c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^5} + \frac{15\,Bc^3x^7-21\,(2\,Bbc^2-Ac^3)x^5+35\,(3\,Bb^2c-2\,Abc^2)x^3-105\,(4\,Bb^3-3\,Ab^2c)x}{105\,c^5}$$

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b^4 - A*b^3*c)*x/(c^6*x^2 + b*c^5) + 1/2*(9*B*b^4 - 7*A*b^3*c)*arct
an(c*x/sqrt(b*c))/(sqrt(b*c)*c^5) + 1/105*(15*B*c^3*x^7 - 21*(2*B*b*c^2 - A
*c^3)*x^5 + 35*(3*B*b^2*c - 2*A*b*c^2)*x^3 - 105*(4*B*b^3 - 3*A*b^2*c)*x)/c
^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(9\,Bb^4-7\,Ab^3c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^5} - \frac{Bb^4x-Ab^3cx}{2\,(cx^2+b)c^5} + \frac{15\,Bc^{12}x^7-42\,Bbc^{11}x^5+21\,Ac^{12}x^5+105\,Bb^2c^{10}x^3-70\,Abc^{11}x^3-420\,Bb^3c^9x+315\,Ab^2c^{10}x}{105\,c^{14}}$$

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(9*B*b^4 - 7*A*b^3*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^5) - 1/2*(B*b^
4*x - A*b^3*c*x)/((c*x^2 + b)*c^5) + 1/105*(15*B*c^12*x^7 - 42*B*b*c^11*x^5
 + 21*A*c^12*x^5 + 105*B*b^2*c^10*x^3 - 70*A*b*c^11*x^3 - 420*B*b^3*c^9*x +
 315*A*b^2*c^10*x)/c^14
```

Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.53

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = x\left(\frac{2\,b\left(\frac{2\,b\left(\frac{A}{c^2}-\frac{2\,B\,b}{c^3}\right)}{c}+\frac{B\,b^2}{c^4}\right)}{c}-\frac{b^2\left(\frac{A}{c^2}-\frac{2\,B\,b}{c^3}\right)}{c^2}\right) + x^5\left(\frac{A}{5\,c^2}-\frac{2\,B\,b}{5\,c^3}\right)-x^3\left(\frac{2\,b\left(\frac{A}{c^2}-\frac{2\,B\,b}{c^3}\right)}{3\,c}+\frac{B\,b^2}{3\,c^4}\right)+\frac{B\,x^7}{7\,c^2} - \frac{x\left(\frac{B\,b^4}{2}-\frac{A\,b^3\,c}{2}\right)}{c^6\,x^2+b\,c^5}+\frac{b^{5/2}\,\mathrm{atan}\left(\frac{b^{5/2}\,\sqrt{c}\,x\,(7\,A\,c-9\,B\,b)}{9\,B\,b^4-7\,A\,b^3\,c}\right)\,(7\,A\,c-9\,B\,b)}{2\,c^{11/2}}$$

```
[In] int((x^12*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x*((2*b*((2*b*(A/c^2 - (2*B*b)/c^3))/c + (B*b^2)/c^4))/c - (b^2*(A/c^2 - (2
*B*b)/c^3))/c^2) + x^5*(A/(5*c^2) - (2*B*b)/(5*c^3)) - x^3*((2*b*(A/c^2 - (
2*B*b)/c^3))/(3*c) + (B*b^2)/(3*c^4)) + (B*x^7)/(7*c^2) - (x*((B*b^4)/2 - (
A*b^3*c)/2))/(b*c^5 + c^6*x^2) + (b^(5/2)*atan((b^(5/2)*c^(1/2)*x*(7*A*c -
9*B*b))/(9*B*b^4 - 7*A*b^3*c))*(7*A*c - 9*B*b))/(2*c^(11/2))
```

3.59 $\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 105

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{b(3bB-2Ac)x^2}{2c^4} - \frac{(2bB-Ac)x^4}{4c^3} + \frac{Bx^6}{6c^2} - \frac{b^3(bB-Ac)}{2c^5(b+cx^2)} - \frac{b^2(4bB-3Ac)\log(b+cx^2)}{2c^5}$$

```
[Out] 1/2*b*(-2*A*c+3*B*b)*x^2/c^4-1/4*(-A*c+2*B*b)*x^4/c^3+1/6*B*x^6/c^2-1/2*b^3
*(-A*c+B*b)/c^5/(c*x^2+b)-1/2*b^2*(-3*A*c+4*B*b)*ln(c*x^2+b)/c^5
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 457, 78}

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{b^3(bB-Ac)}{2c^5(b+cx^2)} - \frac{b^2(4bB-3Ac)\log(b+cx^2)}{2c^5} + \frac{bx^2(3bB-2Ac)}{2c^4} - \frac{x^4(2bB-Ac)}{4c^3} + \frac{Bx^6}{6c^2}$$

```
[In] Int[(x^11*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*(3*b*B - 2*A*c)*x^2)/(2*c^4) - ((2*b*B - A*c)*x^4)/(4*c^3) + (B*x^6)/(6*
c^2) - (b^3*(b*B - A*c))/(2*c^5*(b + c*x^2)) - (b^2*(4*b*B - 3*A*c)*Log[b +
 c*x^2])/(2*c^5)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^7(A+Bx^2)}{(b+cx^2)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(A+Bx)}{(b+cx)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b(3bB-2Ac)}{c^4}+\frac{(-2bB+Ac)x}{c^3}+\frac{Bx^2}{c^2}+\frac{b^3(bB-Ac)}{c^4(b+cx)^2} - \frac{b^2(4bB-3Ac)}{c^4(b+cx)}\right)dx, x, x^2\right) \\
&= \frac{b(3bB-2Ac)x^2}{2c^4}-\frac{(2bB-Ac)x^4}{4c^3}+\frac{Bx^6}{6c^2}-\frac{b^3(bB-Ac)}{2c^5\,(b+cx^2)}-\frac{b^2(4bB-3Ac)\log{(b+cx^2)}}{2c^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{6bc(3bB-2Ac)x^2+3c^2(-2bB+Ac)x^4+2Bc^3x^6+\frac{6b^3(-bB+Ac)}{b+cx^2}+6b^2(-4bB+3Ac)\log{(b+cx^2)}}{12c^5}$$

```
[In] Integrate[(x^11*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (6*b*c*(3*b*B - 2*A*c)*x^2 + 3*c^2*(-2*b*B + A*c)*x^4 + 2*B*c^3*x^6 + (6*b^
3*(-(b*B) + A*c))/(b + c*x^2) + 6*b^2*(-4*b*B + 3*A*c)*Log[b + c*x^2])/(12*
c^5)
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.98

method	result
default	$-\frac{-\frac{B\,c^{2} x^{6}}{6}+\frac{\left(-A\,c^{2}+2Bbc\right) x^{4}}{4}+\frac{\left(2Abc-3B\,b^{2}\right) x^{2}}{2}}{c^{4}}+\frac{b^{2} \left(\frac{b \left(Ac-Bb\right)}{c \left(c\,x^{2}+b\right)}+\frac{\left(3Ac-4Bb\right) \ln \left(c\,x^{2}+b\right)}{c}\right)}{2c^{4}}$
norman	$\frac{\frac{B\,x^{11}}{6c}+\frac{\left(3Ac-4Bb\right) x^{9}}{12c^{2}}-\frac{b \left(3Ac-4Bb\right) x^{7}}{4c^{3}}+\frac{b \left(3b^{2} Ac-4B\,b^{3}\right) x^{3}}{2c^{5}}}{x^{3} \left(c\,x^{2}+b\right)}+\frac{b^{2} \left(3Ac-4Bb\right) \ln \left(c\,x^{2}+b\right)}{2c^{5}}$
risch	$\frac{B\,x^{6}}{6c^{2}}+\frac{A\,x^{4}}{4c^{2}}-\frac{x^{4} Bb}{2c^{3}}-\frac{Ab\,x^{2}}{c^{3}}+\frac{3b^{2} B\,x^{2}}{2c^{4}}+\frac{b^{3} A}{2c^{4} \left(c\,x^{2}+b\right)}-\frac{b^{4} B}{2c^{5} \left(c\,x^{2}+b\right)}+\frac{3b^{2} \ln \left(c\,x^{2}+b\right) A}{2c^{4}}-\frac{2b^{3} \ln \left(c\,x^{2}+b\right) B}{c^{5}}$
parallelrisch	$\frac{2B\,x^{8} c^{4}+3A\,x^{6} c^{4}-4B\,x^{6} b\,c^{3}-9A\,x^{4} b\,c^{3}+12B\,x^{4} b^{2} c^{2}+18A \ln \left(c\,x^{2}+b\right) x^{2} b^{2} c^{2}-24B \ln \left(c\,x^{2}+b\right) x^{2} b^{3} c+18A \ln \left(c\,x^{2}+b\right) b^{3} c-24B}{12c^{5} \left(c\,x^{2}+b\right)}$

```
[In] int(x^11*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^4*(-1/6*B*c^2*x^6+1/4*(-A*c^2+2*B*b*c)*x^4+1/2*(2*A*b*c-3*B*b^2)*x^2)+
1/2*b^2/c^4*(b*(A*c-B*b)/c/(c*x^2+b)+(3*A*c-4*B*b)/c*ln(c*x^2+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.41

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$
$$=\frac{2\,Bc^4x^8-(4\,Bbc^3-3\,Ac^4)x^6-6\,Bb^4+6\,Ab^3c+3\,(4\,Bb^2c^2-3\,Abc^3)x^4+6\,(3\,Bb^3c-2\,Ab^2c^2)x^2-6\,(4\,B}{12\,(c^6x^2+bc^5)}$$

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*c^4*x^8 - (4*B*b*c^3 - 3*A*c^4)*x^6 - 6*B*b^4 + 6*A*b^3*c + 3*(4*
B*b^2*c^2 - 3*A*b*c^3)*x^4 + 6*(3*B*b^3*c - 2*A*b^2*c^2)*x^2 - 6*(4*B*b^4 -
 3*A*b^3*c + (4*B*b^3*c - 3*A*b^2*c^2)*x^2)*log(c*x^2 + b))/(c^6*x^2 + b*c^
5)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bx^6}{6c^2} - \frac{b^2(-3Ac+4Bb)\log(b+cx^2)}{2c^5} + x^4\left(\frac{A}{4c^2}-\frac{Bb}{2c^3}\right) + x^2\left(-\frac{Ab}{c^3}+\frac{3Bb^2}{2c^4}\right) + \frac{Ab^3c-Bb^4}{2bc^5+2c^6x^2}$$

```
[In] integrate(x**11*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x**6/(6*c**2) - b**2*(-3*A*c + 4*B*b)*log(b + c*x**2)/(2*c**5) + x**4*(A/
(4*c**2) - B*b/(2*c**3)) + x**2*(-A*b/c**3 + 3*B*b**2/(2*c**4)) + (A*b**3*c
 - B*b**4)/(2*b*c**5 + 2*c**6*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.02

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{Bb^4-Ab^3c}{2\,(c^6x^2+bc^5)} + \frac{2\,Bc^2x^6-3\,(2\,Bbc-Ac^2)x^4+6\,(3\,Bb^2-2\,Abc)x^2}{12\,c^4} - \frac{(4\,Bb^3-3\,Ab^2c)\log(cx^2+b)}{2\,c^5}$$

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b^4 - A*b^3*c)/(c^6*x^2 + b*c^5) + 1/12*(2*B*c^2*x^6 - 3*(2*B*b*c -
 A*c^2)*x^4 + 6*(3*B*b^2 - 2*A*b*c)*x^2)/c^4 - 1/2*(4*B*b^3 - 3*A*b^2*c)*lo
g(c*x^2 + b)/c^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.29

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(4\,Bb^3-3\,Ab^2c)\log(|cx^2+b|)}{2\,c^5} + \frac{2\,Bc^4x^6-6\,Bbc^3x^4+3\,Ac^4x^4+18\,Bb^2c^2x^2-12\,Abc^3x^2}{12\,c^6} + \frac{4\,Bb^3cx^2-3\,Ab^2c^2x^2+3\,Bb^4-2\,Ab^3c}{2\,(cx^2+b)c^5}$$

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*B*b^3 - 3*A*b^2*c)*log(abs(c*x^2 + b))/c^5 + 1/12*(2*B*c^4*x^6 - 6*
B*b*c^3*x^4 + 3*A*c^4*x^4 + 18*B*b^2*c^2*x^2 - 12*A*b*c^3*x^2)/c^6 + 1/2*(4
*B*b^3*c*x^2 - 3*A*b^2*c^2*x^2 + 3*B*b^4 - 2*A*b^3*c)/((c*x^2 + b)*c^5)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.15

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = x^4\left(\frac{A}{4\,c^2}-\frac{B\,b}{2\,c^3}\right)-x^2\left(\frac{b\,\left(\frac{A}{c^2}-\frac{2\,B\,b}{c^3}\right)}{c}+\frac{B\,b^2}{2\,c^4}\right)+\frac{B\,x^6}{6\,c^2}-\frac{\ln\left(c\,x^2+b\right)\,\left(4\,B\,b^3-3\,A\,b^2\,c\right)}{2\,c^5}-\frac{B\,b^4-A\,b^3\,c}{2\,c\,\left(c^5\,x^2+b\,c^4\right)}$$

```
[In] int((x^11*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^4*(A/(4*c^2) - (B*b)/(2*c^3)) - x^2*((b*(A/c^2 - (2*B*b)/c^3))/c + (B*b^2
)/(2*c^4)) + (B*x^6)/(6*c^2) - (log(b + c*x^2)*(4*B*b^3 - 3*A*b^2*c))/(2*c^
5) - (B*b^4 - A*b^3*c)/(2*c*(b*c^4 + c^5*x^2))
```

3.60 $\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 110

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2} dx = \frac{b(3bB-2Ac)x}{c^4} - \frac{(2bB-Ac)x^3}{3c^3} + \frac{Bx^5}{5c^2} + \frac{b^2(bB-Ac)x}{2c^4(b+cx^2)} - \frac{b^{3/2}(7bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{9/2}}$$

```
[Out] b*(-2*A*c+3*B*b)*x/c^4-1/3*(-A*c+2*B*b)*x^3/c^3+1/5*B*x^5/c^2+1/2*b^2*(-A*c
+B*b)*x/c^4/(c*x^2+b)-1/2*b^(3/2)*(-5*A*c+7*B*b)*arctan(x*c^(1/2)/b^(1/2))/
c^(9/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 466, 1824, 211}

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2} dx = -\frac{b^{3/2}(7bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{9/2}} + \frac{b^2x(bB-Ac)}{2c^4(b+cx^2)} + \frac{bx(3bB-2Ac)}{c^4} - \frac{x^3(2bB-Ac)}{3c^3} + \frac{Bx^5}{5c^2}$$

```
[In] Int[(x^10*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*(3*b*B - 2*A*c)*x)/c^4 - ((2*b*B - A*c)*x^3)/(3*c^3) + (B*x^5)/(5*c^2) +
 (b^2*(b*B - A*c)*x)/(2*c^4*(b + c*x^2)) - (b^(3/2)*(7*b*B - 5*A*c)*ArcTan[
(Sqrt[c]*x)/Sqrt[b]])/(2*c^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^6(A+Bx^2)}{(b+cx^2)^2}\,dx\\
&= \frac{b^2(bB-Ac)x}{2c^4(b+cx^2)} - \frac{\int \frac{b^2(bB-Ac)-2bc(bB-Ac)x^2+2c^2(bB-Ac)x^4-2Bc^3x^6}{b+cx^2}\,dx}{2c^4}\\
&= \frac{b^2(bB-Ac)x}{2c^4(b+cx^2)} - \frac{\int \left(-2b(3bB-2Ac)+2c(2bB-Ac)x^2-2Bc^2x^4+\frac{7b^3B-5Ab^2c}{b+cx^2}\right)dx}{2c^4}\\
&= \frac{b(3bB-2Ac)x}{c^4} - \frac{(2bB-Ac)x^3}{3c^3} + \frac{Bx^5}{5c^2} + \frac{b^2(bB-Ac)x}{2c^4(b+cx^2)} - \frac{(b^2(7bB-5Ac))\int \frac{1}{b+cx^2}\,dx}{2c^4}\\
&= \frac{b(3bB-2Ac)x}{c^4} - \frac{(2bB-Ac)x^3}{3c^3} + \frac{Bx^5}{5c^2} + \frac{b^2(bB-Ac)x}{2c^4(b+cx^2)} - \frac{b^{3/2}(7bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{9/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{b(3bB-2Ac)x}{c^4}+\frac{(-2bB+Ac)x^3}{3c^3}+\frac{Bx^5}{5c^2} - \frac{(-b^3B+Ab^2c)\,x}{2c^4\,(b+cx^2)} - \frac{b^{3/2}(7bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{9/2}}$$

```
[In] Integrate[(x^10*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*(3*b*B - 2*A*c)*x)/c^4 + ((-2*b*B + A*c)*x^3)/(3*c^3) + (B*x^5)/(5*c^2)
- ((-(b^3*B) + A*b^2*c)*x)/(2*c^4*(b + c*x^2)) - (b^(3/2)*(7*b*B - 5*A*c)*A
rcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(9/2))
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

method	result
default	$-\frac{-\frac{1}{5}B\,c^2x^5-\frac{1}{3}A\,c^2x^3+\frac{2}{3}Bbc\,x^3+2Abcx-3b^2Bx}{c^4}+\frac{b^2\left(\frac{\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)x}{c\,x^2+b}+\frac{(5Ac-7Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{c^4}$
risch	$\frac{B\,x^5}{5c^2}+\frac{A\,x^3}{3c^2}-\frac{2Bb\,x^3}{3c^3}-\frac{2Abx}{c^3}+\frac{3b^2Bx}{c^4}+\frac{\left(-\frac{1}{2}b^2Ac+\frac{1}{2}B\,b^3\right)x}{c^4(c\,x^2+b)}+\frac{5\sqrt{-bc}\,b\ln(-\sqrt{-bc}\,x+b)A}{4c^4}-\frac{7\sqrt{-bc}\,b^2\ln(-\sqrt{-bc}\,x+}{4c^5}$

```
[In] int(x^10*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^4*(-1/5*B*c^2*x^5-1/3*A*c^2*x^3+2/3*B*b*c*x^3+2*A*b*c*x-3*b^2*B*x)+b^2
/c^4*((-1/2*A*c+1/2*B*b)*x/(c*x^2+b)+1/2*(5*A*c-7*B*b)/(b*c)^(1/2)*arctan(c
*x/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.71

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$

$$= \left[\frac{12\,Bc^3x^7-4\,(7\,Bbc^2-5\,Ac^3)x^5+20\,(7\,Bb^2c-5\,Abc^2)x^3-15\,(7\,Bb^3-5\,Ab^2c+(7\,Bb^2c-5\,Abc^2)x^2)}{60\,(c^5x^2+bc^4)}\right.$$

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*B*c^3*x^7 - 4*(7*B*b*c^2 - 5*A*c^3)*x^5 + 20*(7*B*b^2*c - 5*A*b*c
^2)*x^3 - 15*(7*B*b^3 - 5*A*b^2*c + (7*B*b^2*c - 5*A*b*c^2)*x^2)*sqrt(-b/c)
*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 30*(7*B*b^3 - 5*A*b^2*c)
*x)/(c^5*x^2 + b*c^4), 1/30*(6*B*c^3*x^7 - 2*(7*B*b*c^2 - 5*A*c^3)*x^5 + 10
*(7*B*b^2*c - 5*A*b*c^2)*x^3 - 15*(7*B*b^3 - 5*A*b^2*c + (7*B*b^2*c - 5*A*b
*c^2)*x^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) + 15*(7*B*b^3 - 5*A*b^2*c)*x)/
(c^5*x^2 + b*c^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(104) = 208$.

Time = 0.39 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.92

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bx^5}{5c^2} + x^3\left(\frac{A}{3c^2} - \frac{2Bb}{3c^3}\right) + x\left(-\frac{2Ab}{c^3} + \frac{3Bb^2}{c^4}\right) + \frac{x(-Ab^2c+Bb^3)}{2bc^4+2c^5x^2} + \frac{\sqrt{-\frac{b^3}{c^9}}(-5Ac+7Bb)\log\left(-\frac{c^4\sqrt{-\frac{b^3}{c^9}}(-5Ac+7Bb)}{-5Abc+7Bb^2}+x\right)}{4} - \frac{\sqrt{-\frac{b^3}{c^9}}(-5Ac+7Bb)\log\left(\frac{c^4\sqrt{-\frac{b^3}{c^9}}(-5Ac+7Bb)}{-5Abc+7Bb^2}+x\right)}{4}$$

```
[In] integrate(x**10*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x**5/(5*c**2) + x**3*(A/(3*c**2) - 2*B*b/(3*c**3)) + x*(-2*A*b/c**3 + 3*B
*b**2/c**4) + x*(-A*b**2*c + B*b**3)/(2*b*c**4 + 2*c**5*x**2) + sqrt(-b**3/
c**9)*(-5*A*c + 7*B*b)*log(-c**4*sqrt(-b**3/c**9)*(-5*A*c + 7*B*b)/(-5*A*b*
c + 7*B*b**2) + x)/4 - sqrt(-b**3/c**9)*(-5*A*c + 7*B*b)*log(c**4*sqrt(-b**
3/c**9)*(-5*A*c + 7*B*b)/(-5*A*b*c + 7*B*b**2) + x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.02

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(Bb^3-Ab^2c)x}{2\,(c^5x^2+bc^4)} - \frac{(7\,Bb^3-5\,Ab^2c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^4} + \frac{3\,Bc^2x^5 - 5\,(2\,Bbc-Ac^2)x^3 + 15\,(3\,Bb^2-2\,Abc)x}{15\,c^4}$$

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b^3 - A*b^2*c)*x/(c^5*x^2 + b*c^4) - 1/2*(7*B*b^3 - 5*A*b^2*c)*arcta
n(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/15*(3*B*c^2*x^5 - 5*(2*B*b*c - A*c^2)*
x^3 + 15*(3*B*b^2 - 2*A*b*c)*x)/c^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(7\,Bb^3-5\,Ab^2c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^4} + \frac{Bb^3x-Ab^2cx}{2\,(cx^2+b)c^4} + \frac{3\,Bc^8x^5-10\,Bbc^7x^3+5\,Ac^8x^3+45\,Bb^2c^6x-30\,Abc^7x}{15\,c^{10}}$$

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*B*b^3 - 5*A*b^2*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/2*(B*b
^3*x - A*b^2*c*x)/((c*x^2 + b)*c^4) + 1/15*(3*B*c^8*x^5 - 10*B*b*c^7*x^3 +
5*A*c^8*x^3 + 45*B*b^2*c^6*x - 30*A*b*c^7*x)/c^10
```

Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.28

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = x^3\left(\frac{A}{3\,c^2}-\frac{2\,B\,b}{3\,c^3}\right) - x\left(\frac{2\,b\,\left(\frac{A}{c^2}-\frac{2\,B\,b}{c^3}\right)}{c}+\frac{B\,b^2}{c^4}\right) + \frac{B\,x^5}{5\,c^2} + \frac{x\,\left(\frac{B\,b^3}{2}-\frac{A\,b^2\,c}{2}\right)}{c^5\,x^2+b\,c^4} - \frac{b^{3/2}\,\mathrm{atan}\left(\frac{b^{3/2}\,\sqrt{c}\,x\,(5\,A\,c-7\,B\,b)}{7\,B\,b^3-5\,A\,b^2\,c}\right)\,(5\,A\,c-7\,B\,b)}{2\,c^{9/2}}$$

```
[In] int((x^10*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^3*(A/(3*c^2) - (2*B*b)/(3*c^3)) - x*((2*b*(A/c^2 - (2*B*b)/c^3))/c + (B*b
^2)/c^4) + (B*x^5)/(5*c^2) + (x*((B*b^3)/2 - (A*b^2*c)/2))/(b*c^4 + c^5*x^2
) - (b^(3/2)*atan((b^(3/2)*c^(1/2)*x*(5*A*c - 7*B*b))/(7*B*b^3 - 5*A*b^2*c)
)*(5*A*c - 7*B*b))/(2*c^(9/2))
```

3.61 $\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2} dx = -\frac{(2bB-Ac)x^2}{2c^3} + \frac{Bx^4}{4c^2} + \frac{b^2(bB-Ac)}{2c^4(b+cx^2)} + \frac{b(3bB-2Ac)\log(b+cx^2)}{2c^4}$$

```
[Out] -1/2*(-A*c+2*B*b)*x^2/c^3+1/4*B*x^4/c^2+1/2*b^2*(-A*c+B*b)/c^4/(c*x^2+b)+1/
2*b*(-2*A*c+3*B*b)*ln(c*x^2+b)/c^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 457, 78}

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2} dx = \frac{b^2(bB-Ac)}{2c^4(b+cx^2)} + \frac{b(3bB-2Ac)\log(b+cx^2)}{2c^4} - \frac{x^2(2bB-Ac)}{2c^3} + \frac{Bx^4}{4c^2}$$

```
[In] Int[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*((2*b*B - A*c)*x^2)/c^3 + (B*x^4)/(4*c^2) + (b^2*(b*B - A*c))/(2*c^4*(
b + c*x^2)) + (b*(3*b*B - 2*A*c)*Log[b + c*x^2])/(2*c^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^5(A+Bx^2)}{(b+cx^2)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(b+cx)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{-2bB+Ac}{c^3}+\frac{Bx}{c^2}-\frac{b^2(bB-Ac)}{c^3(b+cx)^2}+\frac{b(3bB-2Ac)}{c^3(b+cx)}\right)\,dx, x, x^2\right) \\
&= -\frac{(2bB-Ac)x^2}{2c^3}+\frac{Bx^4}{4c^2}+\frac{b^2(bB-Ac)}{2c^4\,(b+cx^2)}+\frac{b(3bB-2Ac)\log\left(b+cx^2\right)}{2c^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{2c(-2bB+Ac)x^2+Bc^2x^4+\frac{2b^2(bB-Ac)}{b+cx^2}+2b(3bB-2Ac)\log\left(b+cx^2\right)}{4c^4}$$

```
[In] Integrate[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*c*(-2*b*B + A*c)*x^2 + B*c^2*x^4 + (2*b^2*(b*B - A*c))/(b + c*x^2) + 2*b
*(3*b*B - 2*A*c)*Log[b + c*x^2])/(4*c^4)
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92

method	result
default	$\frac{\left(B c \,x^{2}+A c -2 B b \right)^{2}}{4 c^{4} B}-\frac{b \left(\frac{b \left(A c -B b \right)}{c \left(c \,x^{2}+b \right)}+\frac{\left(2 A c -3 B b \right) \ln \left(c \,x^{2}+b \right)}{c}\right)}{2 c^{3}}$
norman	$\frac{\frac{B \,x^{9}}{4 c}+\frac{\left(2 A c -3 B b \right) x^{7}}{4 c^{2}}-\frac{b \left(2 A b c -3 B \,b^{2}\right) x^{3}}{2 c^{4}}}{x^{3} \left(c \,x^{2}+b \right)}-\frac{b \left(2 A c -3 B b \right) \ln \left(c \,x^{2}+b \right)}{2 c^{4}}$
parallelrisch	$-\frac{-B \,c^{3} x^{6}-2 A \,c^{3} x^{4}+3 B b \,c^{2} x^{4}+4 A \ln \left(c \,x^{2}+b \right) x^{2} b \,c^{2}-6 B \ln \left(c \,x^{2}+b \right) x^{2} b^{2} c +4 A \ln \left(c \,x^{2}+b \right) b^{2} c -6 B \ln \left(c \,x^{2}+b \right) b^{3}+4 b^{2} A c -6 B b}{4 c^{4} \left(c \,x^{2}+b \right)}$
risch	$\frac{B \,x^{4}}{4 c^{2}}+\frac{A \,x^{2}}{2 c^{2}}-\frac{B b \,x^{2}}{c^{3}}+\frac{A^{2}}{4 c^{2} B}-\frac{A b}{c^{3}}+\frac{B \,b^{2}}{c^{4}}-\frac{b^{2} A}{2 c^{3} \left(c \,x^{2}+b \right)}+\frac{b^{3} B}{2 c^{4} \left(c \,x^{2}+b \right)}-\frac{b \ln \left(c \,x^{2}+b \right) A}{c^{3}}+\frac{3 b^{2} \ln \left(c \,x^{2}+b \right) B}{2 c^{4}}$

```
[In] int(x^9*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(B*c*x^2+A*c-2*B*b)^2/c^4/B-1/2*b/c^3*(b*(A*c-B*b)/c/(c*x^2+b)+(2*A*c-3
*B*b)/c*ln(c*x^2+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.46

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bc^3x^6 - (3\,Bbc^2 - 2\,Ac^3)x^4 + 2\,Bb^3 - 2\,Ab^2c - 2\,(2\,Bb^2c - Abc^2)x^2 + 2\,(3\,Bb^3 - 2\,Ab^2c + (3\,Bb^2c - 2\,Ab}{4\,(c^5x^2 + bc^4)}$$

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(B*c^3*x^6 - (3*B*b*c^2 - 2*A*c^3)*x^4 + 2*B*b^3 - 2*A*b^2*c - 2*(2*B*b
^2*c - A*b*c^2)*x^2 + 2*(3*B*b^3 - 2*A*b^2*c + (3*B*b^2*c - 2*A*b*c^2)*x^2)
*log(c*x^2 + b))/(c^5*x^2 + b*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bx^4}{4c^2} + \frac{b(-2Ac+3Bb)\log{(b+cx^2)}}{2c^4} + x^2\left(\frac{A}{2c^2} - \frac{Bb}{c^3}\right) + \frac{-Ab^2c+Bb^3}{2bc^4+2c^5x^2}$$

```
[In] integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x**4/(4*c**2) + b*(-2*A*c + 3*B*b)*log(b + c*x**2)/(2*c**4) + x**2*(A/(2*
c**2) - B*b/c**3) + (-A*b**2*c + B*b**3)/(2*b*c**4 + 2*c**5*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bb^3-Ab^2c}{2\,(c^5x^2+bc^4)} + \frac{Bcx^4-2\,(2\,Bb-Ac)x^2}{4\,c^3} + \frac{(3\,Bb^2-2\,Abc)\log{(cx^2+b)}}{2\,c^4}$$

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b^3 - A*b^2*c)/(c^5*x^2 + b*c^4) + 1/4*(B*c*x^4 - 2*(2*B*b - A*c)*x^
2)/c^3 + 1/2*(3*B*b^2 - 2*A*b*c)*log(c*x^2 + b)/c^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.28

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(3\,Bb^2-2\,Abc)\log{(|cx^2+b|)}}{2\,c^4} + \frac{Bc^2x^4-4\,Bbcx^2+2\,Ac^2x^2}{4\,c^4} - \frac{3\,Bb^2cx^2-2\,Abc^2x^2+2\,Bb^3-Ab^2c}{2\,(cx^2+b)c^4}$$

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*B*b^2 - 2*A*b*c)*log(abs(c*x^2 + b))/c^4 + 1/4*(B*c^2*x^4 - 4*B*b*c*
x^2 + 2*A*c^2*x^2)/c^4 - 1/2*(3*B*b^2*c*x^2 - 2*A*b*c^2*x^2 + 2*B*b^3 - A*b
^2*c)/((c*x^2 + b)*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = x^2\left(\frac{A}{2\,c^2}-\frac{B\,b}{c^3}\right) + \frac{\ln{(c\,x^2+b)}\,\left(3\,B\,b^2-2\,A\,b\,c\right)}{2\,c^4} + \frac{B\,x^4}{4\,c^2} + \frac{B\,b^3-A\,b^2\,c}{2\,c\,\left(c^4\,x^2+b\,c^3\right)}$$

```
[In] int((x^9*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^2*(A/(2*c^2) - (B*b)/c^3) + (log(b + c*x^2)*(3*B*b^2 - 2*A*b*c))/(2*c^4)
+ (B*x^4)/(4*c^2) + (B*b^3 - A*b^2*c)/(2*c*(b*c^3 + c^4*x^2))
```

3.62 $\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 89

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^2}\,dx=-\frac{(2bB-Ac)x}{c^3}+\frac{Bx^3}{3c^2}-\frac{b(bB-Ac)x}{2c^3(b+cx^2)}+\frac{\sqrt{b}(5bB-3Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{7/2}}$$

```
[Out] -(-A*c+2*B*b)*x/c^3+1/3*B*x^3/c^2-1/2*b*(-A*c+B*b)*x/c^3/(c*x^2+b)+1/2*(-3*
A*c+5*B*b)*arctan(x*c^(1/2)/b^(1/2))*b^(1/2)/c^(7/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 466, 1167, 211}

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^2}\,dx=\frac{\sqrt{b}(5bB-3Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{7/2}}-\frac{bx(bB-Ac)}{2c^3(b+cx^2)}-\frac{x(2bB-Ac)}{c^3}+\frac{Bx^3}{3c^2}$$

```
[In] Int[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -(((2*b*B - A*c)*x)/c^3) + (B*x^3)/(3*c^2) - (b*(b*B - A*c)*x)/(2*c^3*(b +
c*x^2)) + (Sqrt[b]*(5*b*B - 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^4(A+Bx^2)}{(b+cx^2)^2}\,dx \\
&= -\frac{b(bB-Ac)x}{2c^3(b+cx^2)}-\frac{\int \frac{-b(bB-Ac)+2c(bB-Ac)x^2-2Bc^2x^4}{b+cx^2}\,dx}{2c^3} \\
&= -\frac{b(bB-Ac)x}{2c^3(b+cx^2)}-\frac{\int \left(2(2bB-Ac)-2Bcx^2+\frac{-5b^2B+3Abc}{b+cx^2}\right)dx}{2c^3} \\
&= -\frac{(2bB-Ac)x}{c^3}+\frac{Bx^3}{3c^2}-\frac{b(bB-Ac)x}{2c^3(b+cx^2)}+\frac{(b(5bB-3Ac))\int \frac{1}{b+cx^2}\,dx}{2c^3} \\
&= -\frac{(2bB-Ac)x}{c^3}+\frac{Bx^3}{3c^2}-\frac{b(bB-Ac)x}{2c^3(b+cx^2)}+\frac{\sqrt{b}(5bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(-2bB+Ac)x}{c^3}+\frac{Bx^3}{3c^2}+\frac{(-b^2B+Abc)\,x}{2c^3\,(b+cx^2)} + \frac{\sqrt{b}(5bB-3Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{7/2}}$$

```
[In] Integrate[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*b*B + A*c)*x)/c^3 + (B*x^3)/(3*c^2) + ((-(b^2*B) + A*b*c)*x)/(2*c^3*(b
 + c*x^2)) + (Sqrt[b]*(5*b*B - 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(7/
2))
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.84

method	result
default	$\frac{\frac{1}{3}Bc\,x^3+Acx-2bBx}{c^3}-\frac{b\left(\frac{\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)x}{c\,x^2+b}+\frac{(3Ac-5Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{c^3}$
risch	$\frac{B\,x^3}{3c^2}+\frac{Ax}{c^2}-\frac{2bBx}{c^3}+\frac{\left(\frac{1}{2}Abc-\frac{1}{2}B\,b^2\right)x}{c^3(c\,x^2+b)}+\frac{3\sqrt{-bc}\,\ln(-\sqrt{-bc}\,x-b)A}{4c^3}-\frac{5\sqrt{-bc}\,\ln(-\sqrt{-bc}\,x-b)Bb}{4c^4}-\frac{3\sqrt{-bc}\,\ln(\sqrt{-bc}\,x-b)}{4c^3}$

```
[In] int(x^8*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/3*B*c*x^3+A*c*x-2*b*B*x)-b/c^3*((-1/2*A*c+1/2*B*b)*x/(c*x^2+b)+1/2
*(3*A*c-5*B*b)/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.70

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \left[\frac{4\,Bc^2x^5-4\,(5\,Bbc-3\,Ac^2)x^3-3\,(5\,Bb^2-3\,Abc+(5\,Bbc-3\,Ac^2)x^2)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)-6\,(5}{12\,(c^4x^2+bc^3)}\right.$$

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*B*c^2*x^5 - 4*(5*B*b*c - 3*A*c^2)*x^3 - 3*(5*B*b^2 - 3*A*b*c + (5*
B*b*c - 3*A*c^2)*x^2)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2
+ b)) - 6*(5*B*b^2 - 3*A*b*c)*x)/(c^4*x^2 + b*c^3), 1/6*(2*B*c^2*x^5 - 2*(5
*B*b*c - 3*A*c^2)*x^3 + 3*(5*B*b^2 - 3*A*b*c + (5*B*b*c - 3*A*c^2)*x^2)*sqr
t(b/c)*arctan(c*x*sqrt(b/c)/b) - 3*(5*B*b^2 - 3*A*b*c)*x)/(c^4*x^2 + b*c^3)
]
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.45

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bx^3}{3c^2} + x\left(\frac{A}{c^2} - \frac{2Bb}{c^3}\right) + \frac{x(Abc-Bb^2)}{2bc^3+2c^4x^2} - \frac{\sqrt{-\frac{b}{c^7}}(-3Ac+5Bb)\log\left(-c^3\sqrt{-\frac{b}{c^7}}+x\right)}{4} + \frac{\sqrt{-\frac{b}{c^7}}(-3Ac+5Bb)\log\left(c^3\sqrt{-\frac{b}{c^7}}+x\right)}{4}$$

```
[In] integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x**3/(3*c**2) + x*(A/c**2 - 2*B*b/c**3) + x*(A*b*c - B*b**2)/(2*b*c**3 +
2*c**4*x**2) - sqrt(-b/c**7)*(-3*A*c + 5*B*b)*log(-c**3*sqrt(-b/c**7) + x)/
4 + sqrt(-b/c**7)*(-3*A*c + 5*B*b)*log(c**3*sqrt(-b/c**7) + x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(Bb^2-Abc)x}{2\,(c^4x^2+bc^3)} + \frac{(5\,Bb^2-3\,Abc)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^3} + \frac{Bcx^3-3\,(2\,Bb-Ac)x}{3\,c^3}$$

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b^2 - A*b*c)*x/(c^4*x^2 + b*c^3) + 1/2*(5*B*b^2 - 3*A*b*c)*arctan(c
*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/3*(B*c*x^3 - 3*(2*B*b - A*c)*x)/c^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(5\,Bb^2-3\,Abc)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^3} - \frac{Bb^2x-Abcx}{2\,(cx^2+b)c^3} + \frac{Bc^4x^3-6\,Bbc^3x+3\,Ac^4x}{3\,c^6}$$

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*B*b^2 - 3*A*b*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) - 1/2*(B*b^2*
x - A*b*c*x)/((c*x^2 + b)*c^3) + 1/3*(B*c^4*x^3 - 6*B*b*c^3*x + 3*A*c^4*x)/
c^6
```

Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.17

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = x\left(\frac{A}{c^2}-\frac{2\,B\,b}{c^3}\right) - \frac{x\left(\frac{B\,b^2}{2}-\frac{A\,b\,c}{2}\right)}{c^4\,x^2+b\,c^3} + \frac{B\,x^3}{3\,c^2} + \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c}\,x\,(3\,A\,c-5\,B\,b)}{5\,B\,b^2-3\,A\,b\,c}\right)\,(3\,A\,c-5\,B\,b)}{2\,c^{7/2}}$$

```
[In] int((x^8*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x*(A/c^2 - (2*B*b)/c^3) - (x*((B*b^2)/2 - (A*b*c)/2))/(b*c^3 + c^4*x^2) + (
B*x^3)/(3*c^2) + (b^(1/2)*atan((b^(1/2)*c^(1/2)*x*(3*A*c - 5*B*b))/(5*B*b^2
 - 3*A*b*c))*(3*A*c - 5*B*b))/(2*c^(7/2))
```

3.63 $\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 61

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^2} dx = \frac{Bx^2}{2c^2} - \frac{b(bB-Ac)}{2c^3(b+cx^2)} - \frac{(2bB-Ac)\log(b+cx^2)}{2c^3}$$

```
[Out] 1/2*B*x^2/c^2-1/2*b*(-A*c+B*b)/c^3/(c*x^2+b)-1/2*(-A*c+2*B*b)*ln(c*x^2+b)/c
^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 457, 78}

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^2} dx = -\frac{b(bB-Ac)}{2c^3(b+cx^2)} - \frac{(2bB-Ac)\log(b+cx^2)}{2c^3} + \frac{Bx^2}{2c^2}$$

```
[In] Int[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*x^2)/(2*c^2) - (b*(b*B - A*c))/(2*c^3*(b + c*x^2)) - ((2*b*B - A*c)*Log[
b + c*x^2])/(2*c^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^3(A+Bx^2)}{(b+cx^2)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{(b+cx)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{B}{c^2}+\frac{b(bB-Ac)}{c^2(b+cx)^2}+\frac{-2bB+Ac}{c^2(b+cx)}\right)\,dx, x, x^2\right) \\
&= \frac{Bx^2}{2c^2}-\frac{b(bB-Ac)}{2c^3\,(b+cx^2)}-\frac{(2bB-Ac)\log\left(b+cx^2\right)}{2c^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bcx^2+\frac{b(-bB+Ac)}{b+cx^2}+(-2bB+Ac)\log\left(b+cx^2\right)}{2c^3}$$

```
[In] Integrate[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*c*x^2 + (b*(-(b*B) + A*c))/(b + c*x^2) + (-2*b*B + A*c)*Log[b + c*x^2])/
(2*c^3)
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{B\,x^{2}}{2c^{2}}+\frac{\frac{b(Ac-Bb)}{c\left(c\,x^{2}+b\right)}+\frac{(Ac-2Bb)\ln\left(c\,x^{2}+b\right)}{c}}{2c^{2}}$	59
norman	$\frac{\frac{B\,x^{7}}{2c}+\frac{b(Ac-2Bb)x^{3}}{2c^{3}}}{x^{3}(c\,x^{2}+b)}+\frac{(Ac-2Bb)\ln(c\,x^{2}+b)}{2c^{3}}$	63
risch	$\frac{B\,x^{2}}{2c^{2}}+\frac{bA}{2c^{2}(c\,x^{2}+b)}-\frac{b^{2}B}{2c^{3}(c\,x^{2}+b)}+\frac{\ln(c\,x^{2}+b)A}{2c^{2}}-\frac{\ln(c\,x^{2}+b)Bb}{c^{3}}$	74
parallelrisch	$\frac{B\,c^{2}x^{4}+A\ln(c\,x^{2}+b)x^{2}c^{2}-2B\ln(c\,x^{2}+b)x^{2}bc+A\ln(c\,x^{2}+b)bc-2B\ln(c\,x^{2}+b)b^{2}+Abc-2B\,b^{2}}{2c^{3}(c\,x^{2}+b)}$	92

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*x^2/c^2+1/2/c^2*(b*(A*c-B*b)/c/(c*x^2+b)+1/c*(A*c-2*B*b)*ln(c*x^2+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.33

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bc^2x^4+Bbcx^2-Bb^2+Abc-(2\,Bb^2-Abc+(2\,Bbc-Ac^2)x^2)\log(cx^2+b)}{2\,(c^4x^2+bc^3)}$$

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(B*c^2*x^4 + B*b*c*x^2 - B*b^2 + A*b*c - (2*B*b^2 - A*b*c + (2*B*b*c -
A*c^2)*x^2)*log(c*x^2 + b))/(c^4*x^2 + b*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bx^2}{2c^2}+\frac{Abc-Bb^2}{2bc^3+2c^4x^2}-\frac{(-Ac+2Bb)\log(b+cx^2)}{2c^3}$$

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x**2/(2*c**2) + (A*b*c - B*b**2)/(2*b*c**3 + 2*c**4*x**2) - (-A*c + 2*B*b
)*log(b + c*x**2)/(2*c**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \frac{x^7(A + Bx^2)}{(bx^2 + cx^4)^2}\,dx = \frac{Bx^2}{2\,c^2} - \frac{Bb^2 - Abc}{2\,(c^4x^2 + bc^3)} - \frac{(2\,Bb - Ac)\log\left(cx^2 + b\right)}{2\,c^3}$$

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*B*x^2/c^2 - 1/2*(B*b^2 - A*b*c)/(c^4*x^2 + b*c^3) - 1/2*(2*B*b - A*c)*l
og(c*x^2 + b)/c^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

$$\int \frac{x^7(A + Bx^2)}{(bx^2 + cx^4)^2}\,dx = \frac{Bx^2}{2\,c^2} - \frac{(2\,Bb - Ac)\log\left(|cx^2 + b|\right)}{2\,c^3} + \frac{2\,Bbcx^2 - Ac^2x^2 + Bb^2}{2\,(cx^2 + b)c^3}$$

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/c^2 - 1/2*(2*B*b - A*c)*log(abs(c*x^2 + b))/c^3 + 1/2*(2*B*b*c*x^
2 - A*c^2*x^2 + B*b^2)/((c*x^2 + b)*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02

$$\int \frac{x^7(A + Bx^2)}{(bx^2 + cx^4)^2}\,dx = \frac{B\,x^2}{2\,c^2} + \frac{\ln\left(c\,x^2 + b\right)\,\left(A\,c - 2\,B\,b\right)}{2\,c^3} - \frac{B\,b^2 - A\,b\,c}{2\,c\,\left(c^3\,x^2 + b\,c^2\right)}$$

```
[In] int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (B*x^2)/(2*c^2) + (log(b + c*x^2)*(A*c - 2*B*b))/(2*c^3) - (B*b^2 - A*b*c)/
(2*c*(b*c^2 + c^3*x^2))
```

3.64 $\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^2}\,dx=\frac{Bx}{c^2}+\frac{(bB-Ac)x}{2c^2(b+cx^2)}-\frac{(3bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2\sqrt{b}c^{5/2}}$$

```
[Out] B*x/c^2+1/2*(-A*c+B*b)*x/c^2/(c*x^2+b)-1/2*(-A*c+3*B*b)*arctan(x*c^(1/2)/b^
(1/2))/c^(5/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 466, 396, 211}

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^2}\,dx=-\frac{(3bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2\sqrt{b}c^{5/2}}+\frac{x(bB-Ac)}{2c^2(b+cx^2)}+\frac{Bx}{c^2}$$

```
[In] Int[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*x)/c^2 + ((b*B - A*c)*x)/(2*c^2*(b + c*x^2)) - ((3*b*B - A*c)*ArcTan[(Sq
rt[c]*x)/Sqrt[b]])/(2*Sqrt[b]*c^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^2(A+Bx^2)}{(b+cx^2)^2}\,dx \\
&= \frac{(bB-Ac)x}{2c^2\,(b+cx^2)} - \frac{\int \frac{bB-Ac-2Bcx^2}{b+cx^2}\,dx}{2c^2} \\
&= \frac{Bx}{c^2} + \frac{(bB-Ac)x}{2c^2\,(b+cx^2)} - \frac{(3bB-Ac)\int \frac{1}{b+cx^2}\,dx}{2c^2} \\
&= \frac{Bx}{c^2} + \frac{(bB-Ac)x}{2c^2\,(b+cx^2)} - \frac{(3bB-Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2\sqrt{b}c^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bx}{c^2} - \frac{(-bB+Ac)x}{2c^2\,(b+cx^2)} - \frac{(3bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2\sqrt{b}c^{5/2}}$$

```
[In] Integrate[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
[Out] (B*x)/c^2 - ((-(b*B) + A*c)*x)/(2*c^2*(b + c*x^2)) - ((3*b*B - A*c)*ArcTan[
(Sqrt[c]*x)/Sqrt[b]])/(2*Sqrt[b]*c^(5/2))
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{Bx}{c^2}+\frac{\frac{\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)x}{c\,x^2+b}+\frac{(Ac-3Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}}{c^2}$	57
risch	$\frac{Bx}{c^2}+\frac{\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)x}{c^2(c\,x^2+b)}-\frac{\ln\left(cx+\sqrt{-bc}\right)A}{4c\sqrt{-bc}}+\frac{3\ln\left(cx+\sqrt{-bc}\right)Bb}{4c^2\sqrt{-bc}}+\frac{\ln\left(-cx+\sqrt{-bc}\right)A}{4c\sqrt{-bc}}-\frac{3\ln\left(-cx+\sqrt{-bc}\right)Bb}{4c^2\sqrt{-bc}}$	127

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] B*x/c^2+1/c^2*((-1/2*A*c+1/2*B*b)*x/(c*x^2+b)+1/2*(A*c-3*B*b)/(b*c)^(1/2)*a
rctan(c*x/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 208, normalized size of antiderivative = 3.06

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \left[\frac{4\,Bbc^2x^3+(3\,Bb^2-Abc+(3\,Bbc-Ac^2)x^2)\sqrt{-bc}\log\left(\frac{cx^2-2\,\sqrt{-bc}x-b}{cx^2+b}\right)+2\,(3\,Bb^2c-Abc^2)x}{4\,(bc^4x^2+b^2c^3)},\ \frac{2\,Bbc^2x^3 +}{\ }\right.$$

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*B*b*c^2*x^3 + (3*B*b^2 - A*b*c + (3*B*b*c - A*c^2)*x^2)*sqrt(-b*c)*
log((c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)) + 2*(3*B*b^2*c - A*b*c^2)*x)/
(b*c^4*x^2 + b^2*c^3), 1/2*(2*B*b*c^2*x^3 - (3*B*b^2 - A*b*c + (3*B*b*c - A
*c^2)*x^2)*sqrt(b*c)*arctan(sqrt(b*c)*x/b) + (3*B*b^2*c - A*b*c^2)*x)/(b*c^
4*x^2 + b^2*c^3)]
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.68

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bx}{c^2}+\frac{x(-Ac+Bb)}{2bc^2+2c^3x^2}+\frac{\sqrt{-\frac{1}{bc^5}}(-Ac+3Bb)\log\left(-bc^2\sqrt{-\frac{1}{bc^5}}+x\right)}{4} - \frac{\sqrt{-\frac{1}{bc^5}}(-Ac+3Bb)\log\left(bc^2\sqrt{-\frac{1}{bc^5}}+x\right)}{4}$$

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] B*x/c**2 + x*(-A*c + B*b)/(2*b*c**2 + 2*c**3*x**2) + sqrt(-1/(b*c**5))*(-A*
c + 3*B*b)*log(-b*c**2*sqrt(-1/(b*c**5)) + x)/4 - sqrt(-1/(b*c**5))*(-A*c +
 3*B*b)*log(b*c**2*sqrt(-1/(b*c**5)) + x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(Bb-Ac)x}{2\,(c^3x^2+bc^2)} + \frac{Bx}{c^2} - \frac{(3\,Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^2}$$

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b - A*c)*x/(c^3*x^2 + b*c^2) + B*x/c^2 - 1/2*(3*B*b - A*c)*arctan(c*
x/sqrt(b*c))/(sqrt(b*c)*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bx}{c^2} - \frac{(3\,Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^2} + \frac{Bbx-Acx}{2\,(cx^2+b)c^2}$$

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] B*x/c^2 - 1/2*(3*B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/2*(B*
b*x - A*c*x)/((c*x^2 + b)*c^2)
```

Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{B\,x}{c^2} - \frac{x\,\left(\frac{A\,c}{2}-\frac{B\,b}{2}\right)}{c^3\,x^2+b\,c^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,\left(A\,c-3\,B\,b\right)}{2\,\sqrt{b}\,c^{5/2}}$$

```
[In] int((x^6*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (B*x)/c^2 - (x*((A*c)/2 - (B*b)/2))/(b*c^2 + c^3*x^2) + (atan((c^(1/2)*x)/b
^(1/2))*(A*c - 3*B*b))/(2*b^(1/2)*c^(5/2))
```

3.65 $\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 41

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{bB-Ac}{2c^2(b+cx^2)}+\frac{B\log(b+cx^2)}{2c^2}$$

```
[Out] 1/2*(-A*c+B*b)/c^2/(c*x^2+b)+1/2*B*ln(c*x^2+b)/c^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 455, 45}

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{bB-Ac}{2c^2(b+cx^2)}+\frac{B\log(b+cx^2)}{2c^2}$$

```
[In] Int[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*B - A*c)/(2*c^2*(b + c*x^2)) + (B*Log[b + c*x^2])/(2*c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x(A+Bx^2)}{(b+cx^2)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{(b+cx)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{-bB+Ac}{c(b+cx)^2}+\frac{B}{c(b+cx)}\right)\,dx, x, x^2\right) \\
&= \frac{bB-Ac}{2c^2(b+cx^2)}+\frac{B\log(b+cx^2)}{2c^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{bB-Ac}{2c^2(b+cx^2)}+\frac{B\log(b+cx^2)}{2c^2}$$

```
[In] Integrate[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
[Out] (b*B - A*c)/(2*c^2*(b + c*x^2)) + (B*Log[b + c*x^2])/(2*c^2)
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{Ac-Bb}{2c^2(c\,x^2+b)}+\frac{B\ln(c\,x^2+b)}{2c^2}$	38
norman	$-\frac{Ac-Bb}{2c^2(c\,x^2+b)}+\frac{B\ln(c\,x^2+b)}{2c^2}$	38
risch	$-\frac{A}{2c(c\,x^2+b)}+\frac{Bb}{2c^2(c\,x^2+b)}+\frac{B\ln(c\,x^2+b)}{2c^2}$	47
parallelrisch	$-\frac{-B\ln(c\,x^2+b)x^2c-B\ln(c\,x^2+b)b+Ac-Bb}{2c^2(c\,x^2+b)}$	50

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
[Out] -1/2/c^2*(A*c-B*b)/(c*x^2+b)+1/2*B*ln(c*x^2+b)/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2}\,dx=\frac{Bb-Ac+(Bcx^2+Bb)\log(cx^2+b)}{2(c^3x^2+bc^2)}$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
[Out] 1/2*(B*b - A*c + (B*c*x^2 + B*b)*log(c*x^2 + b))/(c^3*x^2 + b*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2}\,dx=\frac{B\log(b+cx^2)}{2c^2}+\frac{-Ac+Bb}{2bc^2+2c^3x^2}$$

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
[Out] B*log(b + c*x**2)/(2*c**2) + (-A*c + B*b)/(2*b*c**2 + 2*c**3*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bb-Ac}{2\,(c^3x^2+bc^2)} + \frac{B\log\left(cx^2+b\right)}{2\,c^2}$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
[Out] 1/2*(B*b - A*c)/(c^3*x^2 + b*c^2) + 1/2*B*log(c*x^2 + b)/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{B\log\left(|cx^2+b|\right)}{2\,c^2} - \frac{Bx^2+A}{2\,(cx^2+b)c}$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
[Out] 1/2*B*log(abs(c*x^2 + b))/c^2 - 1/2*(B*x^2 + A)/((c*x^2 + b)*c)
```

Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{B\,\ln\left(c\,x^2+b\right)}{2\,c^2} - \frac{A\,c-B\,b}{2\,c^2\,\left(c\,x^2+b\right)}$$

```
[In] int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
[Out] (B*log(b + c*x^2))/(2*c^2) - (A*c - B*b)/(2*c^2*(b + c*x^2))
```

3.66 $\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^2}\,dx=-\frac{(bB-Ac)x}{2bc\,(b+cx^2)}+\frac{(bB+Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}c^{3/2}}$$

```
[Out] -1/2*(-A*c+B*b)*x/b/c/(c*x^2+b)+1/2*(A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/b^(
3/2)/c^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 393, 211}

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^2}\,dx=\frac{(Ac+bB)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}c^{3/2}}-\frac{x(bB-Ac)}{2bc\,(b+cx^2)}$$

```
[In] Int[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*((b*B - A*c)*x)/(b*c*(b + c*x^2)) + ((b*B + A*c)*ArcTan[(Sqrt[c]*x)/Sq
rt[b]])/(2*b^(3/2)*c^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{A+Bx^2}{(b+cx^2)^2}\,dx\\ &= -\frac{(bB-Ac)x}{2bc\,(b+cx^2)}+\frac{(bB+Ac)\int \frac{1}{b+cx^2}\,dx}{2bc}\\ &= -\frac{(bB-Ac)x}{2bc\,(b+cx^2)}+\frac{(bB+Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}c^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(bB-Ac)x}{2bc\,(b+cx^2)}+\frac{(bB+Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}c^{3/2}}$$

```
[In] Integrate[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
[Out] -1/2*((b*B - A*c)*x)/(b*c*(b + c*x^2)) + ((b*B + A*c)*ArcTan[(Sqrt[c]*x)/Sq
rt[b]])/(2*b^(3/2)*c^(3/2))
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{(Ac-Bb)x}{2bc(c\,x^2+b)}+\frac{(Ac+Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2bc\sqrt{bc}}$	57
risch	$\frac{(Ac-Bb)x}{2bc(c\,x^2+b)}-\frac{\ln\left(cx+\sqrt{-bc}\right)A}{4\sqrt{-bc}\,b}-\frac{\ln\left(cx+\sqrt{-bc}\right)B}{4\sqrt{-bc}\,c}+\frac{\ln\left(-cx+\sqrt{-bc}\right)A}{4\sqrt{-bc}\,b}+\frac{\ln\left(-cx+\sqrt{-bc}\right)B}{4\sqrt{-bc}\,c}$	122

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(A*c-B*b)/b/c*x/(c*x^2+b)+1/2*(A*c+B*b)/b/c/(b*c)^(1/2)*arctan(c*x/(b*c
)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.89

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$
$$=\left[-\frac{(Bb^2+Abc+(Bbc+Ac^2)x^2)\sqrt{-bc}\log\left(\frac{cx^2-2\sqrt{-bc}x-b}{cx^2+b}\right)+2\,(Bb^2c-Abc^2)x}{4\,(b^2c^3x^2+b^3c^2)},\frac{(Bb^2+Abc+(Bbc+Ac\right.$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((B*b^2 + A*b*c + (B*b*c + A*c^2)*x^2)*sqrt(-b*c)*log((c*x^2 - 2*sqrt
(-b*c)*x - b)/(c*x^2 + b)) + 2*(B*b^2*c - A*b*c^2)*x)/(b^2*c^3*x^2 + b^3*c^
2), 1/2*((B*b^2 + A*b*c + (B*b*c + A*c^2)*x^2)*sqrt(b*c)*arctan(sqrt(b*c)*x
/b) - (B*b^2*c - A*b*c^2)*x)/(b^2*c^3*x^2 + b^3*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(54) = 108.

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.78

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^2}\,dx=\frac{x(Ac-Bb)}{2b^2c+2bc^2x^2}-\frac{\sqrt{-\frac{1}{b^3c^3}}(Ac+Bb)\log\left(-b^2c\sqrt{-\frac{1}{b^3c^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{b^3c^3}}(Ac+Bb)\log\left(b^2c\sqrt{-\frac{1}{b^3c^3}}+x\right)}{4}$$

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] x*(A*c - B*b)/(2*b**2*c + 2*b*c**2*x**2) - sqrt(-1/(b**3*c**3))*(A*c + B*b)
*log(-b**2*c*sqrt(-1/(b**3*c**3)) + x)/4 + sqrt(-1/(b**3*c**3))*(A*c + B*b)
*log(b**2*c*sqrt(-1/(b**3*c**3)) + x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{x^4(A+Bx^2)}{\left(bx^2+cx^4\right)^2}\,dx = -\frac{(Bb-Ac)x}{2\,(bc^2x^2+b^2c)} + \frac{(Bb+Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}bc}$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b - A*c)*x/(b*c^2*x^2 + b^2*c) + 1/2*(B*b + A*c)*arctan(c*x/sqrt(b*
c))/(sqrt(b*c)*b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{x^4(A+Bx^2)}{\left(bx^2+cx^4\right)^2}\,dx = \frac{(Bb+Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}bc} - \frac{Bbx-Acx}{2\,(cx^2+b)bc}$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*b + A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b*c) - 1/2*(B*b*x - A*c*x)
/((c*x^2 + b)*b*c)
```

Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{x^4(A+Bx^2)}{\left(bx^2+cx^4\right)^2}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,\left(A\,c+B\,b\right)}{2\,b^{3/2}\,c^{3/2}} + \frac{x\,\left(A\,c-B\,b\right)}{2\,b\,c\,\left(c\,x^2+b\right)}$$

```
[In] int((x^4*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (atan((c^(1/2)*x)/b^(1/2))*(A*c + B*b))/(2*b^(3/2)*c^(3/2)) + (x*(A*c - B*b
))/(2*b*c*(b + c*x^2))
```

3.67 $\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{bB-Ac}{2bc(b+cx^2)}+\frac{A\log(x)}{b^2}-\frac{A\log(b+cx^2)}{2b^2}$$

```
[Out] 1/2*(A*c-B*b)/b/c/(c*x^2+b)+A*ln(x)/b^2-1/2*A*ln(c*x^2+b)/b^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{A\log(b+cx^2)}{2b^2}+\frac{A\log(x)}{b^2}-\frac{bB-Ac}{2bc(b+cx^2)}$$

```
[In] Int[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(b*B - A*c)/(b*c*(b + c*x^2)) + (A*Log[x])/b^2 - (A*Log[b + c*x^2])/(2
*b^2)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{A+Bx^2}{x\left(b+cx^2\right)^2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x(b+cx)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{b^2x}+\frac{bB-Ac}{b(b+cx)^2}-\frac{Ac}{b^2(b+cx)}\right)\,dx, x, x^2\right)\\ &= -\frac{bB-Ac}{2bc\left(b+cx^2\right)}+\frac{A\log(x)}{b^2}-\frac{A\log\left(b+cx^2\right)}{2b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{x^3(A+Bx^2)}{\left(bx^2+cx^4\right)^2}\,dx = \frac{\frac{b(-bB+Ac)}{c(b+cx^2)}+2A\log(x)-A\log\left(b+cx^2\right)}{2b^2}$$

```
[In] Integrate[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
[Out] ((b*(-(b*B) + A*c))/(c*(b + c*x^2)) + 2*A*Log[x] - A*Log[b + c*x^2])/(2*b^2
)
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{A\ln(x)}{b^2}-\frac{-\frac{b(Ac-Bb)}{c(c\,x^2+b)}+A\ln(c\,x^2+b)}{2b^2}$	48
norman	$-\frac{(Ac-Bb)x^2}{2b^2(c\,x^2+b)}+\frac{A\ln(x)}{b^2}-\frac{A\ln(c\,x^2+b)}{2b^2}$	48
risch	$\frac{A}{2b(c\,x^2+b)}-\frac{B}{2c(c\,x^2+b)}+\frac{A\ln(x)}{b^2}-\frac{A\ln(c\,x^2+b)}{2b^2}$	53
parallelrisch	$\frac{2A\ln(x)x^2c-A\ln(c\,x^2+b)x^2c-Ac\,x^2+bB\,x^2+2Ab\ln(x)-A\ln(c\,x^2+b)b}{2b^2(c\,x^2+b)}$	71

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
[Out] A*ln(x)/b^2-1/2/b^2*(-b*(A*c-B*b)/c/(c*x^2+b)+A*ln(c*x^2+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^2}\,dx=-\frac{Bb^2-Abc+(Ac^2x^2+Abc)\log(cx^2+b)-2\,(Ac^2x^2+Abc)\log(x)}{2\,(b^2c^2x^2+b^3c)}$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
[Out] -1/2*(B*b^2 - A*b*c + (A*c^2*x^2 + A*b*c)*log(c*x^2 + b) - 2*(A*c^2*x^2 + A
*b*c)*log(x))/(b^2*c^2*x^2 + b^3*c)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^2}\,dx=\frac{A\log(x)}{b^2}-\frac{A\log\left(\frac{b}{c}+x^2\right)}{2b^2}+\frac{Ac-Bb}{2b^2c+2bc^2x^2}$$

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
[Out] A*log(x)/b**2 - A*log(b/c + x**2)/(2*b**2) + (A*c - B*b)/(2*b**2*c + 2*b*c*
*2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{Bb-Ac}{2\,(bc^2x^2+b^2c)} - \frac{A\log{(cx^2+b)}}{2\,b^2} + \frac{A\log{(x^2)}}{2\,b^2}$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
[Out] -1/2*(B*b - A*c)/(b*c^2*x^2 + b^2*c) - 1/2*A*log(c*x^2 + b)/b^2 + 1/2*A*log
(x^2)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{A\log{(|cx^2+b|)}}{2\,b^2} + \frac{A\log{(|x|)}}{b^2} - \frac{Bb^2-Abc}{2\,(cx^2+b)b^2c}$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
[Out] -1/2*A*log(abs(c*x^2 + b))/b^2 + A*log(abs(x))/b^2 - 1/2*(B*b^2 - A*b*c)/((
c*x^2 + b)*b^2*c)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{A\,\ln{(x)}}{b^2} - \frac{A\,\ln{(c\,x^2+b)}}{2\,b^2} + \frac{A\,c-B\,b}{2\,b\,c\,\left(c\,x^2+b\right)}$$

```
[In] int((x^3*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
[Out] (A*log(x))/b^2 - (A*log(b + c*x^2))/(2*b^2) + (A*c - B*b)/(2*b*c*(b + c*x^2
))
```

3.68 $\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 70

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^2} dx = -\frac{A}{b^2x} + \frac{(bB-Ac)x}{2b^2(b+cx^2)} + \frac{(bB-3Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{5/2}\sqrt{c}}$$

```
[Out] -A/b^2/x+1/2*(-A*c+B*b)*x/b^2/(c*x^2+b)+1/2*(-3*A*c+B*b)*arctan(x*c^(1/2)/b
^(1/2))/b^(5/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 467, 464, 211}

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^2} dx = \frac{(bB-3Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{5/2}\sqrt{c}} + \frac{x(bB-Ac)}{2b^2(b+cx^2)} - \frac{A}{b^2x}$$

```
[In] Int[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -(A/(b^2*x)) + ((b*B - A*c)*x)/(2*b^2*(b + c*x^2)) + ((b*B - 3*A*c)*ArcTan[
(Sqrt[c]*x)/Sqrt[b]])/(2*b^(5/2)*Sqrt[c])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{x^2\left(b+cx^2\right)^2}\,dx \\
&= \frac{(bB-Ac)x}{2b^2\left(b+cx^2\right)}-\frac{1}{2}\int \frac{-\frac{2A}{b}-\frac{(bB-Ac)x^2}{b^2}}{x^2\left(b+cx^2\right)}\,dx \\
&= -\frac{A}{b^2x}+\frac{(bB-Ac)x}{2b^2\left(b+cx^2\right)}+\frac{(bB-3Ac)\int \frac{1}{b+cx^2}\,dx}{2b^2} \\
&= -\frac{A}{b^2x}+\frac{(bB-Ac)x}{2b^2\left(b+cx^2\right)}+\frac{(bB-3Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{5/2}\sqrt{c}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{A}{b^2x}+\frac{(bB-Ac)x}{2b^2(b+cx^2)}+\frac{(bB-3Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{5/2}\sqrt{c}}$$

```
[In] Integrate[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -(A/(b^2*x)) + ((b*B - A*c)*x)/(2*b^2*(b + c*x^2)) + ((b*B - 3*A*c)*ArcTan[
(Sqrt[c]*x)/Sqrt[b]])/(2*b^(5/2)*Sqrt[c])
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{A}{b^2x}-\frac{\frac{\left(\frac{Ac}{2}-\frac{Bb}{2}\right)x}{c\,x^2+b}+\frac{(3Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}}{b^2}$	62
risch	$\frac{-\frac{(3Ac-Bb)x^2}{2b^2}-\frac{A}{b}}{(c\,x^2+b)x}-\frac{3\ln\left(-\sqrt{-bc}\,x-b\right)Ac}{4\sqrt{-bc}\,b^2}+\frac{\ln\left(-\sqrt{-bc}\,x-b\right)B}{4\sqrt{-bc}\,b}+\frac{3\ln\left(-\sqrt{-bc}\,x+b\right)Ac}{4\sqrt{-bc}\,b^2}-\frac{\ln\left(-\sqrt{-bc}\,x+b\right)B}{4\sqrt{-bc}\,b}$	141

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -A/b^2/x-1/b^2*((1/2*A*c-1/2*B*b)*x/(c*x^2+b)+1/2*(3*A*c-B*b)/(b*c)^(1/2)*a
rctan(c*x/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 210, normalized size of antiderivative = 3.00

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$

$$= \left[-\frac{4\,Ab^2c-2\,(Bb^2c-3\,Abc^2)x^2-((Bbc-3\,Ac^2)x^3+(Bb^2-3\,Abc)x)\sqrt{-bc}\log\left(\frac{cx^2+2\,\sqrt{-bc}x-b}{cx^2+b}\right)}{4\,(b^3c^2x^3+b^4cx)},\right.$$
$$\left.-\frac{2\,Ab^2c-(Bb^2c-3\,Abc^2)x^2-((Bbc-3\,Ac^2)x^3+(Bb^2-3\,Abc)x)\sqrt{bc}\arctan\left(\frac{\sqrt{bc}x}{b}\right)}{2\,(b^3c^2x^3+b^4cx)}\right]$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*A*b^2*c - 2*(B*b^2*c - 3*A*b*c^2)*x^2 - ((B*b*c - 3*A*c^2)*x^3 + (
B*b^2 - 3*A*b*c)*x)*sqrt(-b*c)*log((c*x^2 + 2*sqrt(-b*c)*x - b)/(c*x^2 + b)
))/(b^3*c^2*x^3 + b^4*c*x), -1/2*(2*A*b^2*c - (B*b^2*c - 3*A*b*c^2)*x^2 - (
(B*b*c - 3*A*c^2)*x^3 + (B*b^2 - 3*A*b*c)*x)*sqrt(b*c)*arctan(sqrt(b*c)*x/b
))/(b^3*c^2*x^3 + b^4*c*x)]
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.63

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{\sqrt{-\frac{1}{b^5c}}(-3Ac+Bb)\log\left(-b^3\sqrt{-\frac{1}{b^5c}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{b^5c}}(-3Ac+Bb)\log\left(b^3\sqrt{-\frac{1}{b^5c}}+x\right)}{4} + \frac{-2Ab+x^2(-3Ac+Bb)}{2b^3x+2b^2cx^3}$$

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] -sqrt(-1/(b**5*c))*(-3*A*c + B*b)*log(-b**3*sqrt(-1/(b**5*c)) + x)/4 + sqrt
(-1/(b**5*c))*(-3*A*c + B*b)*log(b**3*sqrt(-1/(b**5*c)) + x)/4 + (-2*A*b +
x**2*(-3*A*c + B*b))/(2*b**3*x + 2*b**2*c*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(Bb-3\,Ac)x^2-2\,Ab}{2\,(b^2cx^3+b^3x)} + \frac{(Bb-3\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^2}$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*b - 3*A*c)*x^2 - 2*A*b)/(b^2*c*x^3 + b^3*x) + 1/2*(B*b - 3*A*c)*arc
tan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(Bb-3\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^2} + \frac{Bbx^2-3\,Acx^2-2\,Ab}{2\,(cx^3+bx)b^2}$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*b - 3*A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) + 1/2*(B*b*x^2 - 3*
A*c*x^2 - 2*A*b)/((c*x^3 + b*x)*b^2)
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{\frac{A}{b}+\frac{x^2\,(3\,A\,c-B\,b)}{2\,b^2}}{c\,x^3+b\,x} - \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(3\,A\,c-B\,b)}{2\,b^{5/2}\,\sqrt{c}}$$

```
[In] int((x^2*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] - (A/b + (x^2*(3*A*c - B*b))/(2*b^2))/(b*x + c*x^3) - (atan((c^(1/2)*x)/b^(
1/2))*(3*A*c - B*b))/(2*b^(5/2)*c^(1/2))
```

3.69 $\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^2}\,dx=-\frac{A}{2b^2x^2}+\frac{bB-Ac}{2b^2(b+cx^2)}+\frac{(bB-2Ac)\log(x)}{b^3}-\frac{(bB-2Ac)\log(b+cx^2)}{2b^3}$$

```
[Out] -1/2*A/b^2/x^2+1/2*(-A*c+B*b)/b^2/(c*x^2+b)+(-2*A*c+B*b)*ln(x)/b^3-1/2*(-2*
A*c+B*b)*ln(c*x^2+b)/b^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1598, 457, 78}

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^2}\,dx=-\frac{(bB-2Ac)\log(b+cx^2)}{2b^3}+\frac{\log(x)(bB-2Ac)}{b^3}+\frac{bB-Ac}{2b^2(b+cx^2)}-\frac{A}{2b^2x^2}$$

```
[In] Int[(x*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*A/(b^2*x^2) + (b*B - A*c)/(2*b^2*(b + c*x^2)) + ((b*B - 2*A*c)*Log[x])
/b^3 - ((b*B - 2*A*c)*Log[b + c*x^2])/(2*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{x^3\left(b+cx^2\right)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2(b+cx)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{b^2x^2}+\frac{bB-2Ac}{b^3x}-\frac{c(bB-Ac)}{b^2(b+cx)^2}-\frac{c(bB-2Ac)}{b^3(b+cx)}\right) dx, x, x^2\right) \\
&= -\frac{A}{2b^2x^2}+\frac{bB-Ac}{2b^2\left(b+cx^2\right)}+\frac{(bB-2Ac)\log(x)}{b^3}-\frac{(bB-2Ac)\log\left(b+cx^2\right)}{2b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \frac{x(A+Bx^2)}{\left(bx^2+cx^4\right)^2}\,dx = \frac{-\frac{Ab}{x^2}+\frac{b(bB-Ac)}{b+cx^2}+2(bB-2Ac)\log(x)+(-bB+2Ac)\log\left(b+cx^2\right)}{2b^3}$$

```
[In] Integrate[(x*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
[Out] (-((A*b)/x^2) + (b*(b*B - A*c))/(b + c*x^2) + 2*(b*B - 2*A*c)*Log[x] + (-(b
*B) + 2*A*c)*Log[b + c*x^2])/(2*b^3)
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

method	result
default	$-\frac{A}{2b^2x^2}+\frac{(-2Ac+Bb)\ln(x)}{b^3}+\frac{c\left(\frac{(2Ac-Bb)\ln\left(c\,x^2+b\right)}{c}-\frac{b(Ac-Bb)}{c\left(c\,x^2+b\right)}\right)}{2b^3}$
norman	$\frac{-\frac{Ax}{2b}+\frac{c(2Ac-Bb)x^5}{2b^3}}{x^3(c\,x^2+b)}-\frac{(2Ac-Bb)\ln(x)}{b^3}+\frac{(2Ac-Bb)\ln\left(c\,x^2+b\right)}{2b^3}$
risch	$\frac{-\frac{(2Ac-Bb)x^2}{2b^2}-\frac{A}{2b}}{x^2(c\,x^2+b)}-\frac{2\ln(x)Ac}{b^3}+\frac{\ln(x)B}{b^2}+\frac{\ln(-c\,x^2-b)Ac}{b^3}-\frac{\ln(-c\,x^2-b)B}{2b^2}$
parallelrisch	$-\frac{4A\ln(x)x^4c^2-2A\ln\left(c\,x^2+b\right)x^4c^2-2B\ln(x)x^4bc+B\ln\left(c\,x^2+b\right)x^4bc-2A\,c^2x^4+x^4Bbc+4A\ln(x)x^2bc-2A\ln\left(c\,x^2+b\right)x^2bc-2}{2b^3x^2(c\,x^2+b)}$

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*A/b^2/x^2+(-2*A*c+B*b)*ln(x)/b^3+1/2/b^3*c*((2*A*c-B*b)/c*ln(c*x^2+b)-
b*(A*c-B*b)/c/(c*x^2+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.60

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{Ab^2-(Bb^2-2\,Abc)x^2+((Bbc-2\,Ac^2)x^4+(Bb^2-2\,Abc)x^2)\log(cx^2+b)-2\,((Bbc-2\,Ac^2)x^4+(B}{2\,(b^3cx^4+b^4x^2)}$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(A*b^2 - (B*b^2 - 2*A*b*c)*x^2 + ((B*b*c - 2*A*c^2)*x^4 + (B*b^2 - 2*A
*b*c)*x^2)*log(c*x^2 + b) - 2*((B*b*c - 2*A*c^2)*x^4 + (B*b^2 - 2*A*b*c)*x^
2)*log(x))/(b^3*c*x^4 + b^4*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{-Ab+x^2(-2Ac+Bb)}{2b^3x^2+2b^2cx^4}+\frac{(-2Ac+Bb)\log(x)}{b^3} - \frac{(-2Ac+Bb)\log\left(\frac{b}{c}+x^2\right)}{2b^3}$$

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] (-A*b + x**2*(-2*A*c + B*b))/(2*b**3*x**2 + 2*b**2*c*x**4) + (-2*A*c + B*b)
*log(x)/b**3 - (-2*A*c + B*b)*log(b/c + x**2)/(2*b**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(Bb-2\,Ac)x^2-Ab}{2\,(b^2cx^4+b^3x^2)} - \frac{(Bb-2\,Ac)\log(cx^2+b)}{2\,b^3} + \frac{(Bb-2\,Ac)\log(x^2)}{2\,b^3}$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*b - 2*A*c)*x^2 - A*b)/(b^2*c*x^4 + b^3*x^2) - 1/2*(B*b - 2*A*c)*log
(c*x^2 + b)/b^3 + 1/2*(B*b - 2*A*c)*log(x^2)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(Bb-2\,Ac)\log(|x|)}{b^3} + \frac{Bbx^2-2\,Acx^2-Ab}{2\,(cx^4+bx^2)b^2} - \frac{(Bbc-2\,Ac^2)\log(|cx^2+b|)}{2\,b^3c}$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] (B*b - 2*A*c)*log(abs(x))/b^3 + 1/2*(B*b*x^2 - 2*A*c*x^2 - A*b)/((c*x^4 + b
*x^2)*b^2) - 1/2*(B*b*c - 2*A*c^2)*log(abs(c*x^2 + b))/(b^3*c)
```

Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\ln(c\,x^2+b)\,\left(2\,A\,c-B\,b\right)}{2\,b^3} - \frac{\frac{A}{2\,b}+\frac{x^2\,(2\,A\,c-B\,b)}{2\,b^2}}{c\,x^4+b\,x^2} - \frac{\ln(x)\,\left(2\,A\,c-B\,b\right)}{b^3}$$

```
[In] int((x*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (log(b + c*x^2)*(2*A*c - B*b))/(2*b^3) - (A/(2*b) + (x^2*(2*A*c - B*b))/(2*
b^2))/(b*x^2 + c*x^4) - (log(x)*(2*A*c - B*b))/b^3
```

3.70 $\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 90

$$\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^2} \, dx = -\frac{A}{3b^2x^3}-\frac{bB-2Ac}{b^3x}-\frac{c(bB-Ac)x}{2b^3\left(b+cx^2\right)}-\frac{\sqrt{c}(3bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{7/2}}$$

```
[Out] -1/3*A/b^2/x^3+(2*A*c-B*b)/b^3/x-1/2*c*(-A*c+B*b)*x/b^3/(c*x^2+b)-1/2*(-5*A
*c+3*B*b)*arctan(x*c^(1/2)/b^(1/2))*c^(1/2)/b^(7/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1607, 467, 1275, 211}

$$\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^2} \, dx = -\frac{\sqrt{c}(3bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{7/2}}-\frac{cx(bB-Ac)}{2b^3\left(b+cx^2\right)}-\frac{bB-2Ac}{b^3x}-\frac{A}{3b^2x^3}$$

```
[In] Int[(A + B*x^2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/3*A/(b^2*x^3) - (b*B - 2*A*c)/(b^3*x) - (c*(b*B - A*c)*x)/(2*b^3*(b + c*
x^2)) - (Sqrt[c]*(3*b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{x^4\left(b+cx^2\right)^2}\,dx\\
&= -\frac{c(bB-Ac)x}{2b^3\left(b+cx^2\right)} - \frac{1}{2}c\int \frac{-\frac{2A}{bc} - \frac{2(bB-Ac)x^2}{b^2c} + \frac{(bB-Ac)x^4}{b^3}}{x^4\left(b+cx^2\right)}\,dx\\
&= -\frac{c(bB-Ac)x}{2b^3\left(b+cx^2\right)} - \frac{1}{2}c\int \left(-\frac{2A}{b^2cx^4} - \frac{2(bB-2Ac)}{b^3cx^2} + \frac{3bB-5Ac}{b^3\left(b+cx^2\right)}\right)dx\\
&= -\frac{A}{3b^2x^3} - \frac{bB-2Ac}{b^3x} - \frac{c(bB-Ac)x}{2b^3\left(b+cx^2\right)} - \frac{(c(3bB-5Ac))\int \frac{1}{b+cx^2}\,dx}{2b^3}\\
&= -\frac{A}{3b^2x^3} - \frac{bB-2Ac}{b^3x} - \frac{c(bB-Ac)x}{2b^3\left(b+cx^2\right)} - \frac{\sqrt{c}(3bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^2}\,dx = -\frac{A}{3b^2x^3}+\frac{-bB+2Ac}{b^3x}-\frac{c(bB-Ac)x}{2b^3(b+cx^2)}-\frac{\sqrt{c}(3bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{7/2}}$$

```
[In] Integrate[(A + B*x^2)/(b*x^2 + c*x^4)^2,x]
[Out] -1/3*A/(b^2*x^3) + (-(b*B) + 2*A*c)/(b^3*x) - (c*(b*B - A*c)*x)/(2*b^3*(b +
 c*x^2)) - (Sqrt[c]*(3*b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(7/2)
)
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

method	result
default	$-\frac{A}{3b^2x^3}-\frac{-2Ac+Bb}{x\,b^3}+\frac{c\left(\frac{\left(\frac{Ac}{2}-\frac{Bb}{2}\right)x}{c\,x^2+b}+\frac{(5Ac-3Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{b^3}$
risch	$\frac{\frac{c(5Ac-3Bb)x^4}{2b^3}+\frac{(5Ac-3Bb)x^2}{3b^2}-\frac{A}{3b}}{x^3(c\,x^2+b)}+\frac{5\sqrt{-bc}\,\ln(-cx-\sqrt{-bc})Ac}{4b^4}-\frac{3\sqrt{-bc}\,\ln(-cx-\sqrt{-bc})B}{4b^3}-\frac{5\sqrt{-bc}\,\ln(-cx+\sqrt{-bc})Ac}{4b^4}+3\sqrt{}$

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
[Out] -1/3*A/b^2/x^3-(-2*A*c+B*b)/x/b^3+1/b^3*c*((1/2*A*c-1/2*B*b)*x/(c*x^2+b)+1/
2*(5*A*c-3*B*b)/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.78

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^2}\,dx$$

$$= \left[-\frac{6\,(3\,Bbc-5\,Ac^2)x^4+4\,Ab^2+4\,(3\,Bb^2-5\,Abc)x^2+3\,((3\,Bbc-5\,Ac^2)x^5+(3\,Bb^2-5\,Abc)x^3)\sqrt{-\frac{c}{b}}\,\mathrm{lo}}{12\,(b^3cx^5+b^4x^3)}\right.$$

$$-\frac{3\,(3\,Bbc-5\,Ac^2)x^4+2\,Ab^2+2\,(3\,Bb^2-5\,Abc)x^2+3\,((3\,Bbc-5\,Ac^2)x^5+(3\,Bb^2-5\,Abc)x^3)\sqrt{\frac{c}{b}}\,\mathrm{arcta}}{6\,(b^3cx^5+b^4x^3)}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(6*(3*B*b*c - 5*A*c^2)*x^4 + 4*A*b^2 + 4*(3*B*b^2 - 5*A*b*c)*x^2 + 3
*((3*B*b*c - 5*A*c^2)*x^5 + (3*B*b^2 - 5*A*b*c)*x^3)*sqrt(-c/b)*log((c*x^2
+ 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)))/(b^3*c*x^5 + b^4*x^3), -1/6*(3*(3*B*b
*c - 5*A*c^2)*x^4 + 2*A*b^2 + 2*(3*B*b^2 - 5*A*b*c)*x^2 + 3*((3*B*b*c - 5*A
*c^2)*x^5 + (3*B*b^2 - 5*A*b*c)*x^3)*sqrt(c/b)*arctan(x*sqrt(c/b)))/(b^3*c*
x^5 + b^4*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(82) = 164$.

Time = 0.31 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.04

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^2}\,dx = \frac{\sqrt{-\frac{c}{b^7}}(-5Ac+3Bb)\log\left(-\frac{b^4\sqrt{-\frac{c}{b^7}}(-5Ac+3Bb)}{-5Ac^2+3Bbc}+x\right)}{4} - \frac{\sqrt{-\frac{c}{b^7}}(-5Ac+3Bb)\log\left(\frac{b^4\sqrt{-\frac{c}{b^7}}(-5Ac+3Bb)}{-5Ac^2+3Bbc}+x\right)}{4} + \frac{-2Ab^2+x^4\cdot(15Ac^2-9Bbc)+x^2\cdot(10Abc-6Bb^2)}{6b^4x^3+6b^3cx^5}$$

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] sqrt(-c/b**7)*(-5*A*c + 3*B*b)*log(-b**4*sqrt(-c/b**7)*(-5*A*c + 3*B*b)/(-5
*A*c**2 + 3*B*b*c) + x)/4 - sqrt(-c/b**7)*(-5*A*c + 3*B*b)*log(b**4*sqrt(-c
/b**7)*(-5*A*c + 3*B*b)/(-5*A*c**2 + 3*B*b*c) + x)/4 + (-2*A*b**2 + x**4*(1
5*A*c**2 - 9*B*b*c) + x**2*(10*A*b*c - 6*B*b**2))/(6*b**4*x**3 + 6*b**3*c*x
**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^2}\,dx = -\frac{3\,(3\,Bbc-5\,Ac^2)x^4+2\,Ab^2+2\,(3\,Bb^2-5\,Abc)x^2}{6\,(b^3cx^5+b^4x^3)} - \frac{(3\,Bbc-5\,Ac^2)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^3}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(3*B*b*c - 5*A*c^2)*x^4 + 2*A*b^2 + 2*(3*B*b^2 - 5*A*b*c)*x^2)/(b^3
*c*x^5 + b^4*x^3) - 1/2*(3*B*b*c - 5*A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c
)*b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^2}\,dx = -\frac{(3\,Bbc-5\,Ac^2)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^3} - \frac{Bbcx-Ac^2x}{2\,(cx^2+b)b^3} - \frac{3\,Bbx^2-6\,Acx^2+Ab}{3\,b^3x^3}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*B*b*c - 5*A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - 1/2*(B*b*c
*x - A*c^2*x)/((c*x^2 + b)*b^3) - 1/3*(3*B*b*x^2 - 6*A*c*x^2 + A*b)/(b^3*x^
3)
```

Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^2}\,dx = \frac{\frac{x^2\,(5\,A\,c-3\,B\,b)}{3\,b^2}-\frac{A}{3\,b}+\frac{c\,x^4\,(5\,A\,c-3\,B\,b)}{2\,b^3}}{c\,x^5+b\,x^3} + \frac{\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(5\,A\,c-3\,B\,b)}{2\,b^{7/2}}$$

```
[In] int((A + B*x^2)/(b*x^2 + c*x^4)^2,x)
```

```
[Out] ((x^2*(5*A*c - 3*B*b))/(3*b^2) - A/(3*b) + (c*x^4*(5*A*c - 3*B*b))/(2*b^3))
/(b*x^3 + c*x^5) + (c^(1/2)*atan((c^(1/2)*x)/b^(1/2))*(5*A*c - 3*B*b))/(2*b
^(7/2))
```

3.71 $\int \frac{A+Bx^2}{x(bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 97

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^2} dx = -\frac{A}{4b^2x^4}-\frac{bB-2Ac}{2b^3x^2}-\frac{c(bB-Ac)}{2b^3\left(b+cx^2\right)} -\frac{c(2bB-3Ac)\log(x)}{b^4}+\frac{c(2bB-3Ac)\log\left(b+cx^2\right)}{2b^4}$$

```
[Out] -1/4*A/b^2/x^4+1/2*(2*A*c-B*b)/b^3/x^2-1/2*c*(-A*c+B*b)/b^3/(c*x^2+b)-c*(-3
*A*c+2*B*b)*ln(x)/b^4+1/2*c*(-3*A*c+2*B*b)*ln(c*x^2+b)/b^4
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^2} dx = \frac{c(2bB-3Ac)\log\left(b+cx^2\right)}{2b^4}-\frac{c\log(x)(2bB-3Ac)}{b^4} -\frac{c(bB-Ac)}{2b^3\left(b+cx^2\right)}-\frac{bB-2Ac}{2b^3x^2}-\frac{A}{4b^2x^4}$$

```
[In] Int[(A + B*x^2)/(x*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/4*A/(b^2*x^4) - (b*B - 2*A*c)/(2*b^3*x^2) - (c*(b*B - A*c))/(2*b^3*(b +
c*x^2)) - (c*(2*b*B - 3*A*c)*Log[x])/b^4 + (c*(2*b*B - 3*A*c)*Log[b + c*x^2
])/(2*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{x^5\left(b+cx^2\right)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^3(b+cx)^2}\,dx,x,x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{b^2x^3}+\frac{bB-2Ac}{b^3x^2}-\frac{c(2bB-3Ac)}{b^4x}+\frac{c^2(bB-Ac)}{b^3(b+cx)^2}+\frac{c^2(2bB-3Ac)}{b^4(b+cx)}\right)dx,x,x^2\right) \\
&= -\frac{A}{4b^2x^4}-\frac{bB-2Ac}{2b^3x^2}-\frac{c(bB-Ac)}{2b^3\left(b+cx^2\right)}-\frac{c(2bB-3Ac)\log(x)}{b^4}+\frac{c(2bB-3Ac)\log\left(b+cx^2\right)}{2b^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$
\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^2}\,dx = -\frac{\frac{Ab^2}{x^4}+\frac{2b(bB-2Ac)}{x^2}+\frac{2bc(bB-Ac)}{b+cx^2}-4c(-2bB+3Ac)\log(x)+2c(-2bB+3Ac)\log\left(b+cx^2\right)}{4b^4}
$$

```
[In] Integrate[(A + B*x^2)/(x*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/4*((A*b^2)/x^4 + (2*b*(b*B - 2*A*c))/x^2 + (2*b*c*(b*B - A*c))/(b + c*x^
2) - 4*c*(-2*b*B + 3*A*c)*Log[x] + 2*c*(-2*b*B + 3*A*c)*Log[b + c*x^2])/b^4
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

method	result
default	$-\frac{A}{4b^2x^4}-\frac{-2Ac+Bb}{2x^2b^3}+\frac{c(3Ac-2Bb)\ln(x)}{b^4}-\frac{c^2\left(\frac{(3Ac-2Bb)\ln\left(cx^2+b\right)}{c}-\frac{b(Ac-Bb)}{c\left(cx^2+b\right)}\right)}{2b^4}$
norman	$\frac{-\frac{A}{4b}+\frac{(3Ac-2Bb)x^2}{4b^2}-\frac{c\left(3Ac^2-2Bbc\right)x^6}{2b^4}}{x^4(cx^2+b)}+\frac{c(3Ac-2Bb)\ln(x)}{b^4}-\frac{c(3Ac-2Bb)\ln\left(cx^2+b\right)}{2b^4}$
risch	$\frac{\frac{c(3Ac-2Bb)x^4}{2b^3}+\frac{(3Ac-2Bb)x^2}{4b^2}-\frac{A}{4b}}{x^4(cx^2+b)}+\frac{3c^2\ln(x)A}{b^4}-\frac{2c\ln(x)B}{b^3}-\frac{3c^2\ln\left(cx^2+b\right)A}{2b^4}+\frac{c\ln(cx^2+b)B}{b^3}$
parallelrisch	$\frac{12A\ln(x)x^6c^3-6A\ln\left(cx^2+b\right)x^6c^3-8B\ln(x)x^6bc^2+4B\ln\left(cx^2+b\right)x^6bc^2-6Ac^3x^6+4x^6Bbc^2+12A\ln(x)x^4bc^2-6A\ln(cx^2+}{4b^4x^4(cx^2+b)}$

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*A/b^2/x^4-1/2*(-2*A*c+B*b)/x^2/b^3+c*(3*A*c-2*B*b)/b^4*ln(x)-1/2/b^4*c
^2*((3*A*c-2*B*b)/c*ln(c*x^2+b)-b*(A*c-B*b)/c/(c*x^2+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.59

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^2}\,dx = -\frac{2\left(2\,Bb^2c-3\,Abc^2\right)x^4+Ab^3+\left(2\,Bb^3-3\,Ab^2c\right)x^2-2\left(\left(2\,Bbc^2-3\,Ac^3\right)x^6+\left(2\,Bb^2c-3\,Abc^2\right)x^4\right)\log(}{4\left(b^4cx^6+b^5x^4\right)}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*B*b^2*c - 3*A*b*c^2)*x^4 + A*b^3 + (2*B*b^3 - 3*A*b^2*c)*x^2 - 2
*((2*B*b*c^2 - 3*A*c^3)*x^6 + (2*B*b^2*c - 3*A*b*c^2)*x^4)*log(c*x^2 + b) +
 4*((2*B*b*c^2 - 3*A*c^3)*x^6 + (2*B*b^2*c - 3*A*b*c^2)*x^4)*log(x))/(b^4*c
*x^6 + b^5*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^2}\,dx=\frac{-Ab^2+x^4\cdot(6Ac^2-4Bbc)+x^2\cdot(3Abc-2Bb^2)}{4b^4x^4+4b^3cx^6}-\frac{c(-3Ac+2Bb)\log{(x)}}{b^4}+\frac{c(-3Ac+2Bb)\log{\left(\frac{b}{c}+x^2\right)}}{2b^4}$$

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2)**2,x)
```

```
[Out] (-A*b**2 + x**4*(6*A*c**2 - 4*B*b*c) + x**2*(3*A*b*c - 2*B*b**2))/(4*b**4*x
**4 + 4*b**3*c*x**6) - c*(-3*A*c + 2*B*b)*log(x)/b**4 + c*(-3*A*c + 2*B*b)*
log(b/c + x**2)/(2*b**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^2}\,dx=-\frac{2\,(2\,Bbc-3\,Ac^2)x^4+Ab^2+(2\,Bb^2-3\,Abc)x^2}{4\,(b^3cx^6+b^4x^4)}+\frac{(2\,Bbc-3\,Ac^2)\log{(cx^2+b)}}{2\,b^4}-\frac{(2\,Bbc-3\,Ac^2)\log{(x^2)}}{2\,b^4}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(2*B*b*c - 3*A*c^2)*x^4 + A*b^2 + (2*B*b^2 - 3*A*b*c)*x^2)/(b^3*c*x
^6 + b^4*x^4) + 1/2*(2*B*b*c - 3*A*c^2)*log(c*x^2 + b)/b^4 - 1/2*(2*B*b*c -
 3*A*c^2)*log(x^2)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.55

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^2}\,dx=-\frac{(2\,Bbc-3\,Ac^2)\log{(x^2)}}{2\,b^4}+\frac{(2\,Bbc^2-3\,Ac^3)\log{(|cx^2+b|)}}{2\,b^4c}-\frac{2\,Bbc^2x^2-3\,Ac^3x^2+3\,Bb^2c-4\,Abc^2}{2\,(cx^2+b)b^4}+\frac{6\,Bbcx^4-9\,Ac^2x^4-2\,Bb^2x^2+4\,Abcx^2-Ab^2}{4\,b^4x^4}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*B*b*c - 3*A*c^2)*log(x^2)/b^4 + 1/2*(2*B*b*c^2 - 3*A*c^3)*log(abs(c
*x^2 + b))/(b^4*c) - 1/2*(2*B*b*c^2*x^2 - 3*A*c^3*x^2 + 3*B*b^2*c - 4*A*b*c
^2)/((c*x^2 + b)*b^4) + 1/4*(6*B*b*c*x^4 - 9*A*c^2*x^4 - 2*B*b^2*x^2 + 4*A*
b*c*x^2 - A*b^2)/(b^4*x^4)
```

Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^2}\,dx = \frac{\frac{x^2\,(3\,A\,c-2\,B\,b)}{4\,b^2}-\frac{A}{4\,b}+\frac{c\,x^4\,(3\,A\,c-2\,B\,b)}{2\,b^3}}{c\,x^6+b\,x^4} - \frac{\ln\left(c\,x^2+b\right)\,\left(3\,A\,c^2-2\,B\,b\,c\right)}{2\,b^4} + \frac{\ln\left(x\right)\,\left(3\,A\,c^2-2\,B\,b\,c\right)}{b^4}$$

```
[In] int((A + B*x^2)/(x*(b*x^2 + c*x^4)^2),x)
```

```
[Out] ((x^2*(3*A*c - 2*B*b))/(4*b^2) - A/(4*b) + (c*x^4*(3*A*c - 2*B*b))/(2*b^3))
/(b*x^4 + c*x^6) - (log(b + c*x^2)*(3*A*c^2 - 2*B*b*c))/(2*b^4) + (log(x)*(
3*A*c^2 - 2*B*b*c))/b^4
```

3.72 $\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^2} dx = -\frac{A}{5b^2x^5} - \frac{bB-2Ac}{3b^3x^3} + \frac{c(2bB-3Ac)}{b^4x} + \frac{c^2(bB-Ac)x}{2b^4\left(b+cx^2\right)} + \frac{c^{3/2}(5bB-7Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{9/2}}$$

```
[Out] -1/5*A/b^2/x^5+1/3*(2*A*c-B*b)/b^3/x^3+c*(-3*A*c+2*B*b)/b^4/x+1/2*c^2*(-A*c
+B*b)*x/b^4/(c*x^2+b)+1/2*c^(3/2)*(-7*A*c+5*B*b)*arctan(x*c^(1/2)/b^(1/2))/
b^(9/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 467, 1816, 211}

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^2} dx = \frac{c^{3/2}(5bB-7Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{9/2}} + \frac{c^2x(bB-Ac)}{2b^4\left(b+cx^2\right)} + \frac{c(2bB-3Ac)}{b^4x} - \frac{bB-2Ac}{3b^3x^3} - \frac{A}{5b^2x^5}$$

```
[In] Int[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/5*A/(b^2*x^5) - (b*B - 2*A*c)/(3*b^3*x^3) + (c*(2*b*B - 3*A*c))/(b^4*x)
+ (c^2*(b*B - A*c)*x)/(2*b^4*(b + c*x^2)) + (c^(3/2)*(5*b*B - 7*A*c)*ArcTan
[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{x^6\left(b+cx^2\right)^2}\,dx \\
&= \frac{c^2(bB-Ac)x}{2b^4\left(b+cx^2\right)}-\frac{1}{2}c^2\int \frac{-\frac{2A}{bc^2}-\frac{2(bB-Ac)x^2}{b^2c^2}+\frac{2(bB-Ac)x^4}{b^3c}-\frac{(bB-Ac)x^6}{b^4}}{x^6\left(b+cx^2\right)}\,dx \\
&= \frac{c^2(bB-Ac)x}{2b^4\left(b+cx^2\right)}-\frac{1}{2}c^2\int \left(-\frac{2A}{b^2c^2x^6}-\frac{2(bB-2Ac)}{b^3c^2x^4}+\frac{2(2bB-3Ac)}{b^4cx^2}+\frac{-5bB+7Ac}{b^4\left(b+cx^2\right)}\right)dx \\
&= -\frac{A}{5b^2x^5}-\frac{bB-2Ac}{3b^3x^3}+\frac{c(2bB-3Ac)}{b^4x}+\frac{c^2(bB-Ac)x}{2b^4\left(b+cx^2\right)}+\frac{\left(c^2(5bB-7Ac)\right)\int \frac{1}{b+cx^2}\,dx}{2b^4} \\
&= -\frac{A}{5b^2x^5}-\frac{bB-2Ac}{3b^3x^3}+\frac{c(2bB-3Ac)}{b^4x}+\frac{c^2(bB-Ac)x}{2b^4\left(b+cx^2\right)}+\frac{c^{3/2}(5bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{9/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^2}\,dx = -\frac{A}{5b^2x^5}+\frac{-bB+2Ac}{3b^3x^3}+\frac{c(2bB-3Ac)}{b^4x}+\frac{c^2(bB-Ac)x}{2b^4\left(b+cx^2\right)}+\frac{c^{3/2}(5bB-7Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{9/2}}$$

```
[In] Integrate[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/5*A/(b^2*x^5) + (-(b*B) + 2*A*c)/(3*b^3*x^3) + (c*(2*b*B - 3*A*c))/(b^4*
x) + (c^2*(b*B - A*c)*x)/(2*b^4*(b + c*x^2)) + (c^(3/2)*(5*b*B - 7*A*c)*Arc
Tan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(9/2))
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.89

method	result
default	$-\frac{A}{5b^2x^5}-\frac{-2Ac+Bb}{3x^3b^3}-\frac{c(3Ac-2Bb)}{b^4x}-\frac{c^2\left(\frac{\left(\frac{Ac}{2}-\frac{Bb}{2}\right)x}{c\,x^2+b}+\frac{(7Ac-5Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{b^4}$
risch	$\frac{-\frac{c^2(7Ac-5Bb)x^6}{2b^4}-\frac{c(7Ac-5Bb)x^4}{3b^3}+\frac{(7Ac-5Bb)x^2}{15b^2}-\frac{A}{5b}}{x^5(c\,x^2+b)}+\frac{\left(\sum_{\_R=\text{RootOf}\left(b^9\_Z^2+49A^2c^5-70ABb\,c^4+25B^2b^2c^3\right)}\_R\ln\left(\left(3\_R^2b^9+98A\right.\right.\right.}{4}$

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*A/b^2/x^5-1/3*(-2*A*c+B*b)/x^3/b^3-c*(3*A*c-2*B*b)/b^4/x-1/b^4*c^2*((1
/2*A*c-1/2*B*b)*x/(c*x^2+b)+1/2*(7*A*c-5*B*b)/(b*c)^(1/2)*arctan(c*x/(b*c)^
(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.77

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^2}\,dx$$

$$= \left[\frac{30\,(5\,Bbc^2-7\,Ac^3)x^6+20\,(5\,Bb^2c-7\,Abc^2)x^4-12\,Ab^3-4\,(5\,Bb^3-7\,Ab^2c)x^2-15\,((5\,Bbc^2-7\,Ac^3)x^7}{60\,(b^4cx^7+b^5x^5)}\right.$$

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(30*(5*B*b*c^2 - 7*A*c^3)*x^6 + 20*(5*B*b^2*c - 7*A*b*c^2)*x^4 - 12*A
*b^3 - 4*(5*B*b^3 - 7*A*b^2*c)*x^2 - 15*((5*B*b*c^2 - 7*A*c^3)*x^7 + (5*B*b
^2*c - 7*A*b*c^2)*x^5)*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2
 + b)))/(b^4*c*x^7 + b^5*x^5), 1/30*(15*(5*B*b*c^2 - 7*A*c^3)*x^6 + 10*(5*B
*b^2*c - 7*A*b*c^2)*x^4 - 6*A*b^3 - 2*(5*B*b^3 - 7*A*b^2*c)*x^2 + 15*((5*B*
b*c^2 - 7*A*c^3)*x^7 + (5*B*b^2*c - 7*A*b*c^2)*x^5)*sqrt(c/b)*arctan(x*sqrt
(c/b)))/(b^4*c*x^7 + b^5*x^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(104) = 208$.

Time = 0.36 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.96

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^2}\,dx = -\frac{\sqrt{-\frac{c^3}{b^9}}(-7Ac+5Bb)\log\left(-\frac{b^5\sqrt{-\frac{c^3}{b^9}}(-7Ac+5Bb)}{-7Ac^3+5Bbc^2}+x\right)}{4} + \frac{\sqrt{-\frac{c^3}{b^9}}(-7Ac+5Bb)\log\left(\frac{b^5\sqrt{-\frac{c^3}{b^9}}(-7Ac+5Bb)}{-7Ac^3+5Bbc^2}+x\right)}{4} + \frac{-6Ab^3+x^6(-105Ac^3+75Bbc^2)+x^4(-70Abc^2+50Bb^2c)+x^2\cdot(14Ab^2c-10Bb^3)}{30b^5x^5+30b^4cx^7}$$

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2)**2,x)
```

```
[Out] -sqrt(-c**3/b**9)*(-7*A*c + 5*B*b)*log(-b**5*sqrt(-c**3/b**9)*(-7*A*c + 5*B
*b)/(-7*A*c**3 + 5*B*b*c**2) + x)/4 + sqrt(-c**3/b**9)*(-7*A*c + 5*B*b)*log
(b**5*sqrt(-c**3/b**9)*(-7*A*c + 5*B*b)/(-7*A*c**3 + 5*B*b*c**2) + x)/4 + (
-6*A*b**3 + x**6*(-105*A*c**3 + 75*B*b*c**2) + x**4*(-70*A*b*c**2 + 50*B*b*
*2*c) + x**2*(14*A*b**2*c - 10*B*b**3))/(30*b**5*x**5 + 30*b**4*c*x**7)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^2}\,dx = \frac{15\,(5\,Bbc^2-7\,Ac^3)x^6+10\,(5\,Bb^2c-7\,Abc^2)x^4-6\,Ab^3-2\,(5\,Bb^3-7\,Ab^2c)x^2}{30\,(b^4cx^7+b^5x^5)} + \frac{(5\,Bbc^2-7\,Ac^3)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^4}$$

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/30*(15*(5*B*b*c^2 - 7*A*c^3)*x^6 + 10*(5*B*b^2*c - 7*A*b*c^2)*x^4 - 6*A*b
^3 - 2*(5*B*b^3 - 7*A*b^2*c)*x^2)/(b^4*c*x^7 + b^5*x^5) + 1/2*(5*B*b*c^2 -
7*A*c^3)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^2}\,dx = \frac{\left(5\,Bbc^2-7\,Ac^3\right)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^4}+\frac{Bbc^2x-Ac^3x}{2\,(cx^2+b)b^4} + \frac{30\,Bbcx^4-45\,Ac^2x^4-5\,Bb^2x^2+10\,Abcx^2-3\,Ab^2}{15\,b^4x^5}$$

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*B*b*c^2 - 7*A*c^3)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4) + 1/2*(B*b*
c^2*x - A*c^3*x)/((c*x^2 + b)*b^4) + 1/15*(30*B*b*c*x^4 - 45*A*c^2*x^4 - 5*
B*b^2*x^2 + 10*A*b*c*x^2 - 3*A*b^2)/(b^4*x^5)
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^2}\,dx = -\frac{\frac{A}{5\,b}-\frac{x^2\,\left(7\,A\,c-5\,B\,b\right)}{15\,b^2}+\frac{c^2\,x^6\,\left(7\,A\,c-5\,B\,b\right)}{2\,b^4}+\frac{c\,x^4\,\left(7\,A\,c-5\,B\,b\right)}{3\,b^3}}{c\,x^7+b\,x^5} - \frac{c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,\left(7\,A\,c-5\,B\,b\right)}{2\,b^{9/2}}$$

```
[In] int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)^2),x)
```

```
[Out] - (A/(5*b) - (x^2*(7*A*c - 5*B*b))/(15*b^2) + (c^2*x^6*(7*A*c - 5*B*b))/(2*
b^4) + (c*x^4*(7*A*c - 5*B*b))/(3*b^3))/(b*x^5 + c*x^7) - (c^(3/2)*atan((c^
(1/2)*x)/b^(1/2))*(7*A*c - 5*B*b))/(2*b^(9/2))
```

3.73 $\int \frac{x^{14}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 140

$$\int \frac{x^{14}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3b(2bB-Ac)x}{c^5} - \frac{(3bB-Ac)x^3}{3c^4} + \frac{Bx^5}{5c^3} - \frac{b^3(bB-Ac)x}{4c^5(b+cx^2)^2} + \frac{b^2(17bB-13Ac)x}{8c^5(b+cx^2)} - \frac{7b^{3/2}(9bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{11/2}}$$

```
[Out] 3*b*(-A*c+2*B*b)*x/c^5-1/3*(-A*c+3*B*b)*x^3/c^4+1/5*B*x^5/c^3-1/4*b^3*(-A*c
+B*b)*x/c^5/(c*x^2+b)^2+1/8*b^2*(-13*A*c+17*B*b)*x/c^5/(c*x^2+b)-7/8*b^(3/2
)*(-5*A*c+9*B*b)*arctan(x*c^(1/2)/b^(1/2))/c^(11/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1598, 466, 1828, 1824, 211}

$$\int \frac{x^{14}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{7b^{3/2}(9bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{11/2}} - \frac{b^3x(bB-Ac)}{4c^5(b+cx^2)^2} + \frac{b^2x(17bB-13Ac)}{8c^5(b+cx^2)} + \frac{3bx(2bB-Ac)}{c^5} - \frac{x^3(3bB-Ac)}{3c^4} + \frac{Bx^5}{5c^3}$$

```
[In] Int[(x^14*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (3*b*(2*b*B - A*c)*x)/c^5 - ((3*b*B - A*c)*x^3)/(3*c^4) + (B*x^5)/(5*c^3) -
 (b^3*(b*B - A*c)*x)/(4*c^5*(b + c*x^2)^2) + (b^2*(17*b*B - 13*A*c)*x)/(8*c
```

```
^5*(b + c*x^2)) - (7*b^(3/2)*(9*b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(
8*c^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
 + 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \int \frac{x^8(A+Bx^2)}{(b+cx^2)^3}\,dx$$

$$= -\frac{b^3(bB-Ac)x}{4c^5(b+cx^2)^2} - \frac{\int \frac{-b^3(bB-Ac)+4b^2c(bB-Ac)x^2-4bc^2(bB-Ac)x^4+4c^3(bB-Ac)x^6-4Bc^4x^8}{(b+cx^2)^2}\,dx}{4c^5}$$

$$= -\frac{b^3(bB-Ac)x}{4c^5(b+cx^2)^2} + \frac{b^2(17bB-13Ac)x}{8c^5(b+cx^2)} + \frac{\int \frac{-b^3(15bB-11Ac)+8b^2c(3bB-2Ac)x^2-8bc^2(2bB-Ac)x^4+8bBc^3x^6}{b+cx^2}dx}{8bc^5}$$

$$= -\frac{b^3(bB-Ac)x}{4c^5(b+cx^2)^2} + \frac{b^2(17bB-13Ac)x}{8c^5(b+cx^2)} + \frac{\int \left(24b^2(2bB-Ac)-8bc(3bB-Ac)x^2+8bBc^2x^4-\frac{7(9b^4B-5Ab^3c)}{b+cx^2}\right)dx}{8bc^5}$$

$$= \frac{3b(2bB-Ac)x}{c^5} - \frac{(3bB-Ac)x^3}{3c^4} + \frac{Bx^5}{5c^3} - \frac{b^3(bB-Ac)x}{4c^5(b+cx^2)^2} + \frac{b^2(17bB-13Ac)x}{8c^5(b+cx^2)} - \frac{(7b^2(9bB-5Ac))\int \frac{1}{b+cx^2}dx}{8c^5}$$

$$= \frac{3b(2bB-Ac)x}{c^5} - \frac{(3bB-Ac)x^3}{3c^4} + \frac{Bx^5}{5c^3} - \frac{b^3(bB-Ac)x}{4c^5(b+cx^2)^2} + \frac{b^2(17bB-13Ac)x}{8c^5(b+cx^2)} - \frac{7b^{3/2}(9bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{11/2}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.95

$$\int \frac{x^{14}(A+Bx^2)}{(bx^2+cx^4)^3}dx = \frac{x(945b^4B-525b^3c(A-3Bx^2)+8c^4x^6(5A+3Bx^2)-8bc^3x^4(35A+9Bx^2)+7b^2c^2x^2(-125A+72Bx^2))}{120c^5(b+cx^2)^2} - \frac{7b^{3/2}(9bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{11/2}}$$

```
[In] Integrate[(x^14*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
[Out] (x*(945*b^4*B - 525*b^3*c*(A - 3*B*x^2) + 8*c^4*x^6*(5*A + 3*B*x^2) - 8*b*c
^3*x^4*(35*A + 9*B*x^2) + 7*b^2*c^2*x^2*(-125*A + 72*B*x^2)))/(120*c^5*(b +
 c*x^2)^2) - (7*b^(3/2)*(9*b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(
11/2))
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.85

method	result
default	$-\frac{-\frac{1}{5} B\,c^{2} x^{5}-\frac{1}{3} A\,c^{2} x^{3}+B b c\,x^{3}+3 A b c x-6 b^{2} B x}{c^{5}}+\frac{b^{2} \left(\frac{\left(-\frac{13}{8} A\,c^{2}+\frac{17}{8} B b c\right) x^{3}-\frac{b \left(11 A c-15 B b\right) x}{8}}{\left(c\,x^{2}+b\right)^{2}}+\frac{7 \left(5 A c-9 B b\right) \arctan\left(\frac{c x}{\sqrt{b c}}\right)}{8 \sqrt{b c}}\right)}{c^{5}}$
risch	$\frac{B\,x^{5}}{5 c^{3}}+\frac{A\,x^{3}}{3 c^{3}}-\frac{B b\,x^{3}}{c^{4}}-\frac{3 A b x}{c^{4}}+\frac{6 b^{2} B x}{c^{5}}+\frac{\left(-\frac{13}{8} b^{2} A\,c^{2}+\frac{17}{8} B\,b^{3} c\right) x^{3}-\frac{b^{3} \left(11 A c-15 B b\right) x}{8}}{c^{5} \left(c\,x^{2}+b\right)^{2}}+\frac{35 \sqrt{-b c}\,b \ln\left(-\sqrt{-b c}\,x+b\right) A}{16 c^{5}}-\frac{63}{}$

[In] int(x^14*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)

[Out] -1/c^5*(-1/5*B*c^2*x^5-1/3*A*c^2*x^3+B*b*c*x^3+3*A*b*c*x-6*b^2*B*x)+b^2/c^5
*(((-13/8*A*c^2+17/8*B*b*c)*x^3-1/8*b*(11*A*c-15*B*b)*x)/(c*x^2+b)^2+7/8*(5
*A*c-9*B*b)/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.97

$$\int \frac{x^{14}(A+Bx^2)}{\left(bx^2+cx^4\right)^3}\,dx$$
$$= \left[\frac{48\,Bc^4x^9 - 16\,(9\,Bbc^3 - 5\,Ac^4)x^7 + 112\,(9\,Bb^2c^2 - 5\,Abc^3)x^5 + 350\,(9\,Bb^3c - 5\,Ab^2c^2)x^3 - 105\,(9\,Bb^4 -}{240\,(c^7x^4 +}\right.$$

[In] integrate(x^14*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")

[Out] [1/240*(48*B*c^4*x^9 - 16*(9*B*b*c^3 - 5*A*c^4)*x^7 + 112*(9*B*b^2*c^2 - 5*
A*b*c^3)*x^5 + 350*(9*B*b^3*c - 5*A*b^2*c^2)*x^3 - 105*(9*B*b^4 - 5*A*b^3*c
 + (9*B*b^2*c^2 - 5*A*b*c^3)*x^4 + 2*(9*B*b^3*c - 5*A*b^2*c^2)*x^2)*sqrt(-b
/c)*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 210*(9*B*b^4 - 5*A*b^
3*c)*x)/(c^7*x^4 + 2*b*c^6*x^2 + b^2*c^5), 1/120*(24*B*c^4*x^9 - 8*(9*B*b*c
^3 - 5*A*c^4)*x^7 + 56*(9*B*b^2*c^2 - 5*A*b*c^3)*x^5 + 175*(9*B*b^3*c - 5*A
*b^2*c^2)*x^3 - 105*(9*B*b^4 - 5*A*b^3*c + (9*B*b^2*c^2 - 5*A*b*c^3)*x^4 +
2*(9*B*b^3*c - 5*A*b^2*c^2)*x^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) + 105*(9
*B*b^4 - 5*A*b^3*c)*x)/(c^7*x^4 + 2*b*c^6*x^2 + b^2*c^5)]

Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.80

$$\int \frac{x^{14}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{Bx^5}{5c^3} + x^3\left(\frac{A}{3c^3} - \frac{Bb}{c^4}\right) + x\left(-\frac{3Ab}{c^4} + \frac{6Bb^2}{c^5}\right) + \frac{7\sqrt{-\frac{b^3}{c^{11}}}(-5Ac+9Bb)\log\left(-\frac{7c^5\sqrt{-\frac{b^3}{c^{11}}}(-5Ac+9Bb)}{-35Abc+63Bb^2}+x\right)}{16} - \frac{7\sqrt{-\frac{b^3}{c^{11}}}(-5Ac+9Bb)\log\left(\frac{7c^5\sqrt{-\frac{b^3}{c^{11}}}(-5Ac+9Bb)}{-35Abc+63Bb^2}+x\right)}{16} + \frac{x^3(-13Ab^2c^2+17Bb^3c)+x(-11Ab^3c+15Bb^4)}{8b^2c^5+16bc^6x^2+8c^7x^4}$$

```
[In] integrate(x**14*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*x**5/(5*c**3) + x**3*(A/(3*c**3) - B*b/c**4) + x*(-3*A*b/c**4 + 6*B*b**2/
c**5) + 7*sqrt(-b**3/c**11)*(-5*A*c + 9*B*b)*log(-7*c**5*sqrt(-b**3/c**11)*
(-5*A*c + 9*B*b)/(-35*A*b*c + 63*B*b**2) + x)/16 - 7*sqrt(-b**3/c**11)*(-5*
A*c + 9*B*b)*log(7*c**5*sqrt(-b**3/c**11)*(-5*A*c + 9*B*b)/(-35*A*b*c + 63*
B*b**2) + x)/16 + (x**3*(-13*A*b**2*c**2 + 17*B*b**3*c) + x*(-11*A*b**3*c +
 15*B*b**4))/(8*b**2*c**5 + 16*b*c**6*x**2 + 8*c**7*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.05

$$\int \frac{x^{14}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(17\,Bb^3c-13\,Ab^2c^2)x^3+(15\,Bb^4-11\,Ab^3c)x}{8\,(c^7x^4+2\,bc^6x^2+b^2c^5)} - \frac{7\,(9\,Bb^3-5\,Ab^2c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^5} + \frac{3\,Bc^2x^5-5\,(3\,Bbc-Ac^2)x^3+45\,(2\,Bb^2-Abc)x}{15\,c^5}$$

```
[In] integrate(x^14*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((17*B*b^3*c - 13*A*b^2*c^2)*x^3 + (15*B*b^4 - 11*A*b^3*c)*x)/(c^7*x^4
+ 2*b*c^6*x^2 + b^2*c^5) - 7/8*(9*B*b^3 - 5*A*b^2*c)*arctan(c*x/sqrt(b*c))/
(sqrt(b*c)*c^5) + 1/15*(3*B*c^2*x^5 - 5*(3*B*b*c - A*c^2)*x^3 + 45*(2*B*b^2
 - A*b*c)*x)/c^5
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99

$$\int \frac{x^{14}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{7\,(9\,Bb^3-5\,Ab^2c)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^5} + \frac{17\,Bb^3cx^3-13\,Ab^2c^2x^3+15\,Bb^4x-11\,Ab^3cx}{8\,(cx^2+b)^2c^5} + \frac{3\,Bc^{12}x^5-15\,Bbc^{11}x^3+5\,Ac^{12}x^3+90\,Bb^2c^{10}x-45\,Abc^{11}x}{15\,c^{15}}$$

```
[In] integrate(x^14*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -7/8*(9*B*b^3 - 5*A*b^2*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^5) + 1/8*(17*
B*b^3*c*x^3 - 13*A*b^2*c^2*x^3 + 15*B*b^4*x - 11*A*b^3*c*x)/((c*x^2 + b)^2*
c^5) + 1/15*(3*B*c^12*x^5 - 15*B*b*c^11*x^3 + 5*A*c^12*x^3 + 90*B*b^2*c^10*
x - 45*A*b*c^11*x)/c^15
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.26

$$\int \frac{x^{14}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{x\left(\frac{15\,B\,b^4}{8}-\frac{11\,A\,b^3\,c}{8}\right)-x^3\left(\frac{13\,A\,b^2\,c^2}{8}-\frac{17\,B\,b^3\,c}{8}\right)}{b^2\,c^5+2\,b\,c^6\,x^2+c^7\,x^4} - x\left(\frac{3\,b\left(\frac{A}{c^3}-\frac{3\,B\,b}{c^4}\right)}{c}+\frac{3\,B\,b^2}{c^5}\right)+x^3\left(\frac{A}{3\,c^3}-\frac{B\,b}{c^4}\right) + \frac{B\,x^5}{5\,c^3}-\frac{7\,b^{3/2}\,\mathrm{atan}\left(\frac{b^{3/2}\,\sqrt{c}\,x\,(5\,A\,c-9\,B\,b)}{9\,B\,b^3-5\,A\,b^2\,c}\right)\,(5\,A\,c-9\,B\,b)}{8\,c^{11/2}}$$

```
[In] int((x^14*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (x*((15*B*b^4)/8 - (11*A*b^3*c)/8) - x^3*((13*A*b^2*c^2)/8 - (17*B*b^3*c)/8
))/(b^2*c^5 + c^7*x^4 + 2*b*c^6*x^2) - x*((3*b*(A/c^3 - (3*B*b)/c^4))/c + (
3*B*b^2)/c^5) + x^3*(A/(3*c^3) - (B*b)/c^4) + (B*x^5)/(5*c^3) - (7*b^(3/2)*
atan((b^(3/2)*c^(1/2)*x*(5*A*c - 9*B*b))/(9*B*b^3 - 5*A*b^2*c))*(5*A*c - 9*
B*b))/(8*c^(11/2))
```

3.74 $\int \frac{x^{13}(A+Bx^2)}{(bx^2+cx^4)^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{x^{13}(A+Bx^2)}{(bx^2+cx^4)^3} dx = -\frac{(3bB-Ac)x^2}{2c^4}+\frac{Bx^4}{4c^3}-\frac{b^3(bB-Ac)}{4c^5(b+cx^2)^2}+\frac{b^2(4bB-3Ac)}{2c^5(b+cx^2)}+\frac{3b(2bB-Ac)\log(b+cx^2)}{2c^5}$$

```
[Out] -1/2*(-A*c+3*B*b)*x^2/c^4+1/4*B*x^4/c^3-1/4*b^3*(-A*c+B*b)/c^5/(c*x^2+b)^2+
1/2*b^2*(-3*A*c+4*B*b)/c^5/(c*x^2+b)+3/2*b*(-A*c+2*B*b)*ln(c*x^2+b)/c^5
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$\int \frac{x^{13}(A+Bx^2)}{(bx^2+cx^4)^3} dx = -\frac{b^3(bB-Ac)}{4c^5(b+cx^2)^2}+\frac{b^2(4bB-3Ac)}{2c^5(b+cx^2)}+\frac{3b(2bB-Ac)\log(b+cx^2)}{2c^5}-\frac{x^2(3bB-Ac)}{2c^4}+\frac{Bx^4}{4c^3}$$

```
[In] Int[(x^13*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/2*((3*b*B - A*c)*x^2)/c^4 + (B*x^4)/(4*c^3) - (b^3*(b*B - A*c))/(4*c^5*(
b + c*x^2)^2) + (b^2*(4*b*B - 3*A*c))/(2*c^5*(b + c*x^2)) + (3*b*(2*b*B - A
*c)*Log[b + c*x^2])/(2*c^5)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^7(A+Bx^2)}{(b+cx^2)^3}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(A+Bx)}{(b+cx)^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{-3bB+Ac}{c^4}+\frac{Bx}{c^3}+\frac{b^3(bB-Ac)}{c^4(b+cx)^3}-\frac{b^2(4bB-3Ac)}{c^4(b+cx)^2}\right.\right. \\
&\qquad \left.\left.+\frac{3b(2bB-Ac)}{c^4(b+cx)}\right)dx, x, x^2\right) \\
&= -\frac{(3bB-Ac)x^2}{2c^4}+\frac{Bx^4}{4c^3}-\frac{b^3(bB-Ac)}{4c^5(b+cx^2)^2}+\frac{b^2(4bB-3Ac)}{2c^5(b+cx^2)}+\frac{3b(2bB-Ac)\log(b+cx^2)}{2c^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \frac{x^{13}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{2c(-3bB+Ac)x^2+Bc^2x^4+\frac{b^3(-bB+Ac)}{(b+cx^2)^2}+\frac{2b^2(4bB-3Ac)}{b+cx^2}+6b(2bB-Ac)\log(b+cx^2)}{4c^5}$$

```
[In] Integrate[(x^13*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*c*(-3*b*B + A*c)*x^2 + B*c^2*x^4 + (b^3*(-(b*B) + A*c))/(b + c*x^2)^2 +
(2*b^2*(4*b*B - 3*A*c))/(b + c*x^2) + 6*b*(2*b*B - A*c)*Log[b + c*x^2])/(4*
c^5)
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.92

method	result
default	$\frac{\left(B c\,x^{2}+A c-3 B b\right)^{2}}{4 c^{5} B}-\frac{b\left(\frac{\left(3 A c-6 B b\right) \ln\left(c\,x^{2}+b\right)}{c}-\frac{b^{2}\left(A c-B b\right)}{2 c\left(c\,x^{2}+b\right)^{2}}+\frac{b\left(3 A c-4 B b\right)}{c\left(c\,x^{2}+b\right)}\right)}{2 c^{4}}$
norman	$\frac{\frac{B\,x^{13}}{4 c}+\frac{\left(A c-2 B b\right) x^{11}}{2 c^{2}}-\frac{b\left(3 A b c-6 B\,b^{2}\right) x^{7}}{c^{4}}-\frac{b^{2}\left(9 A b c-18 B\,b^{2}\right) x^{5}}{4 c^{5}}}{x^{5}\left(c\,x^{2}+b\right)^{2}}-\frac{3 b\left(A c-2 B b\right) \ln\left(c\,x^{2}+b\right)}{2 c^{5}}$
risch	$\frac{B\,x^{4}}{4 c^{3}}+\frac{A\,x^{2}}{2 c^{3}}-\frac{3 B b\,x^{2}}{2 c^{4}}+\frac{A^{2}}{4 c^{3} B}-\frac{3 A b}{2 c^{4}}+\frac{9 B\,b^{2}}{4 c^{5}}+\frac{\left(-\frac{3}{2} b^{2} A c+2 B\,b^{3}\right) x^{2}-\frac{b^{3}\left(5 A c-7 B b\right)}{4 c}}{c^{4}\left(c\,x^{2}+b\right)^{2}}-\frac{3 b \ln\left(c\,x^{2}+b\right) A}{2 c^{4}}+\frac{3 b^{2} \ln(}{}$
parallelrisch	$-\frac{-B\,x^{8} c^{4}-2 A\,x^{6} c^{4}+4 B\,x^{6} b\,c^{3}+6 A \ln\left(c\,x^{2}+b\right) x^{4} b\,c^{3}-12 B \ln\left(c\,x^{2}+b\right) x^{4} b^{2} c^{2}+12 A \ln\left(c\,x^{2}+b\right) x^{2} b^{2} c^{2}-24 B \ln\left(c\,x^{2}+b\right) x^{2} b^{3} c}{4 c^{5}\left(c\,x^{2}+b\right)^{2}}$

```
[In] int(x^13*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(B*c*x^2+A*c-3*B*b)^2/c^5/B-1/2*b/c^4*((3*A*c-6*B*b)/c*ln(c*x^2+b)-1/2*
b^2*(A*c-B*b)/c/(c*x^2+b)^2+b*(3*A*c-4*B*b)/c/(c*x^2+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.61

$$\int \frac{x^{13}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{Bc^4x^8-2\,(2\,Bbc^3-Ac^4)x^6+7\,Bb^4-5\,Ab^3c-(11\,Bb^2c^2-4\,Abc^3)x^4+2\,(Bb^3c-2\,Ab^2c^2)x^2+6\,(2\,Bb^4}{4\,(c^7x^4+2\,bc^6x^2+b^2c^5)}$$

```
[In] integrate(x^13*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(B*c^4*x^8 - 2*(2*B*b*c^3 - A*c^4)*x^6 + 7*B*b^4 - 5*A*b^3*c - (11*B*b^
2*c^2 - 4*A*b*c^3)*x^4 + 2*(B*b^3*c - 2*A*b^2*c^2)*x^2 + 6*(2*B*b^4 - A*b^3
*c + (2*B*b^2*c^2 - A*b*c^3)*x^4 + 2*(2*B*b^3*c - A*b^2*c^2)*x^2)*log(c*x^2
 + b))/(c^7*x^4 + 2*b*c^6*x^2 + b^2*c^5)
```

Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.07

$$\int \frac{x^{13}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{Bx^4}{4c^3}+\frac{3b(-Ac+2Bb)\log(b+cx^2)}{2c^5}+x^2\left(\frac{A}{2c^3}-\frac{3Bb}{2c^4}\right)+\frac{-5Ab^3c+7Bb^4+x^2(-6Ab^2c^2+8Bb^3c)}{4b^2c^5+8bc^6x^2+4c^7x^4}$$

```
[In] integrate(x**13*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*x**4/(4*c**3) + 3*b*(-A*c + 2*B*b)*log(b + c*x**2)/(2*c**5) + x**2*(A/(2*
c**3) - 3*B*b/(2*c**4)) + (-5*A*b**3*c + 7*B*b**4 + x**2*(-6*A*b**2*c**2 +
8*B*b**3*c))/(4*b**2*c**5 + 8*b*c**6*x**2 + 4*c**7*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int \frac{x^{13}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{7\,Bb^4-5\,Ab^3c+2\,(4\,Bb^3c-3\,Ab^2c^2)x^2}{4\,(c^7x^4+2\,bc^6x^2+b^2c^5)}+\frac{Bcx^4-2\,(3\,Bb-Ac)x^2}{4\,c^4}+\frac{3\,(2\,Bb^2-Abc)\log(cx^2+b)}{2\,c^5}$$

```
[In] integrate(x^13*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(7*B*b^4 - 5*A*b^3*c + 2*(4*B*b^3*c - 3*A*b^2*c^2)*x^2)/(c^7*x^4 + 2*b*
c^6*x^2 + b^2*c^5) + 1/4*(B*c*x^4 - 2*(3*B*b - A*c)*x^2)/c^4 + 3/2*(2*B*b^2
 - A*b*c)*log(c*x^2 + b)/c^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.19

$$\int \frac{x^{13}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3\,(2\,Bb^2-Abc)\log(|cx^2+b|)}{2\,c^5}+\frac{Bc^3x^4-6\,Bbc^2x^2+2\,Ac^3x^2}{4\,c^6}-\frac{18\,Bb^2c^2x^4-9\,Abc^3x^4+28\,Bb^3cx^2-12\,Ab^2c^2x^2+11\,Bb^4-4\,Ab^3c}{4\,(cx^2+b)^2c^5}$$

```
[In] integrate(x^13*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/2*(2*B*b^2 - A*b*c)*log(abs(c*x^2 + b))/c^5 + 1/4*(B*c^3*x^4 - 6*B*b*c^2*
x^2 + 2*A*c^3*x^2)/c^6 - 1/4*(18*B*b^2*c^2*x^4 - 9*A*b*c^3*x^4 + 28*B*b^3*c
*x^2 - 12*A*b^2*c^2*x^2 + 11*B*b^4 - 4*A*b^3*c)/((c*x^2 + b)^2*c^5)
```

Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.06

$$\int \frac{x^{13}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{7\,B\,b^4-5\,A\,b^3\,c}{4\,c}+x^2\,\left(2\,B\,b^3-\frac{3\,A\,b^2\,c}{2}\right)}{b^2\,c^4+2\,b\,c^5\,x^2+c^6\,x^4}+x^2\,\left(\frac{A}{2\,c^3}-\frac{3\,B\,b}{2\,c^4}\right) + \frac{\ln\left(c\,x^2+b\right)\,\left(6\,B\,b^2-3\,A\,b\,c\right)}{2\,c^5}+\frac{B\,x^4}{4\,c^3}$$

```
[In] int((x^13*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((7*B*b^4 - 5*A*b^3*c)/(4*c) + x^2*(2*B*b^3 - (3*A*b^2*c)/2))/(b^2*c^4 + c^
6*x^4 + 2*b*c^5*x^2) + x^2*(A/(2*c^3) - (3*B*b)/(2*c^4)) + (log(b + c*x^2)*
(6*B*b^2 - 3*A*b*c))/(2*c^5) + (B*x^4)/(4*c^3)
```

3.75 $\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 118

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{(3bB-Ac)x}{c^4}+\frac{Bx^3}{3c^3}+\frac{b^2(bB-Ac)x}{4c^4(b+cx^2)^2} - \frac{b(13bB-9Ac)x}{8c^4(b+cx^2)}+\frac{5\sqrt{b}(7bB-3Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{9/2}}$$

```
[Out] -(-A*c+3*B*b)*x/c^4+1/3*B*x^3/c^3+1/4*b^2*(-A*c+B*b)*x/c^4/(c*x^2+b)^2-1/8*
b*(-9*A*c+13*B*b)*x/c^4/(c*x^2+b)+5/8*(-3*A*c+7*B*b)*arctan(x*c^(1/2)/b^(1/
2))*b^(1/2)/c^(9/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1598, 466, 1828, 1167, 211}

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{5\sqrt{b}(7bB-3Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{9/2}}+\frac{b^2x(bB-Ac)}{4c^4(b+cx^2)^2} - \frac{bx(13bB-9Ac)}{8c^4(b+cx^2)}-\frac{x(3bB-Ac)}{c^4}+\frac{Bx^3}{3c^3}$$

```
[In] Int[(x^12*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(((3*b*B - A*c)*x)/c^4) + (B*x^3)/(3*c^3) + (b^2*(b*B - A*c)*x)/(4*c^4*(b
+ c*x^2)^2) - (b*(13*b*B - 9*A*c)*x)/(8*c^4*(b + c*x^2)) + (5*Sqrt[b]*(7*b*
B - 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^6(A+Bx^2)}{(b+cx^2)^3}\,dx \\
&= \frac{b^2(bB-Ac)x}{4c^4(b+cx^2)^2} - \frac{\int \frac{b^2(bB-Ac)-4bc(bB-Ac)x^2+4c^2(bB-Ac)x^4-4Bc^3x^6}{(b+cx^2)^2}\,dx}{4c^4} \\
&= \frac{b^2(bB-Ac)x}{4c^4(b+cx^2)^2} - \frac{b(13bB-9Ac)x}{8c^4(b+cx^2)} + \frac{\int \frac{b^2(11bB-7Ac)-8bc(2bB-Ac)x^2+8bBc^2x^4}{b+cx^2}\,dx}{8bc^4}
\end{aligned}
$$

$$= \frac{b^2(bB - Ac)x}{4c^4 (b+cx^2)^2} - \frac{b(13bB - 9Ac)x}{8c^4 (b+cx^2)} + \frac{\int \left(-8b(3bB - Ac) + 8bBcx^2 + \frac{5(7b^3B - 3Ab^2c)}{b+cx^2}\right) dx}{8bc^4}$$

$$= -\frac{(3bB - Ac)x}{c^4} + \frac{Bx^3}{3c^3} + \frac{b^2(bB - Ac)x}{4c^4 (b+cx^2)^2} - \frac{b(13bB - 9Ac)x}{8c^4 (b+cx^2)} + \frac{(5b(7bB - 3Ac)) \int \frac{1}{b+cx^2} dx}{8c^4}$$

$$= -\frac{(3bB - Ac)x}{c^4} + \frac{Bx^3}{3c^3} + \frac{b^2(bB - Ac)x}{4c^4 (b+cx^2)^2} - \frac{b(13bB - 9Ac)x}{8c^4 (b+cx^2)} + \frac{5\sqrt{b}(7bB - 3Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{9/2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96

$$\int \frac{x^{12}(A + Bx^2)}{(bx^2 + cx^4)^3} dx = \frac{-105b^3Bx + bc^2x^3(75A - 56Bx^2) + 5b^2cx(9A - 35Bx^2) + 8c^3x^5(3A + Bx^2)}{24c^4 (b+cx^2)^2} + \frac{5\sqrt{b}(7bB - 3Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{9/2}}$$

```
[In] Integrate[(x^12*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-105*b^3*B*x + b*c^2*x^3*(75*A - 56*B*x^2) + 5*b^2*c*x*(9*A - 35*B*x^2) +
8*c^3*x^5*(3*A + B*x^2))/(24*c^4*(b + c*x^2)^2) + (5*Sqrt[b]*(7*b*B - 3*A*c
)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(9/2))
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81

method	result
default	$\frac{\frac{1}{3}Bc\,x^3+Acx-3bBx}{c^4} - \frac{b\left(\frac{\left(-\frac{9}{8}A\,c^2+\frac{13}{8}Bbc\right)x^3-\frac{b(7Ac-11Bb)x}{8}}{\left(c\,x^2+b\right)^2}+\frac{5(3Ac-7Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{c^4}$
risch	$\frac{B\,x^3}{3c^3} + \frac{Ax}{c^3} - \frac{3bBx}{c^4} + \frac{\left(\frac{9}{8}Ab\,c^2-\frac{13}{8}B\,b^2c\right)x^3+\frac{b^2(7Ac-11Bb)x}{8}}{c^4\left(c\,x^2+b\right)^2} + \frac{15\sqrt{-bc}\,\ln\left(-\sqrt{-bc}\,x-b\right)A}{16c^4} - \frac{35\sqrt{-bc}\,\ln\left(-\sqrt{-bc}\,x-b\right)Bb}{16c^5} -$

```
[In] int(x^12*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/3*B*c*x^3+A*c*x-3*b*B*x)-b/c^4*(((-9/8*A*c^2+13/8*B*b*c)*x^3-1/8*b
*(7*A*c-11*B*b)*x)/(c*x^2+b)^2+5/8*(3*A*c-7*B*b)/(b*c)^(1/2)*arctan(c*x/(b*
c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.03

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \left[\frac{16\,Bc^3x^7 - 16\,(7\,Bbc^2 - 3\,Ac^3)x^5 - 50\,(7\,Bb^2c - 3\,Abc^2)x^3 - 15\,((7\,Bbc^2 - 3\,Ac^3)x^4 + 7\,Bb^3 - 3\,Ab^2c}{48\,(c^6x^4 + 2\,bc^5x^2 + b^2c^4)}\right.$$

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*B*c^3*x^7 - 16*(7*B*b*c^2 - 3*A*c^3)*x^5 - 50*(7*B*b^2*c - 3*A*b*
c^2)*x^3 - 15*((7*B*b*c^2 - 3*A*c^3)*x^4 + 7*B*b^3 - 3*A*b^2*c + 2*(7*B*b^2
*c - 3*A*b*c^2)*x^2)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2 +
 b)) - 30*(7*B*b^3 - 3*A*b^2*c)*x)/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4), 1/24*
(8*B*c^3*x^7 - 8*(7*B*b*c^2 - 3*A*c^3)*x^5 - 25*(7*B*b^2*c - 3*A*b*c^2)*x^3
 + 15*((7*B*b*c^2 - 3*A*c^3)*x^4 + 7*B*b^3 - 3*A*b^2*c + 2*(7*B*b^2*c - 3*A
*b*c^2)*x^2)*sqrt(b/c)*arctan(c*x*sqrt(b/c)/b) - 15*(7*B*b^3 - 3*A*b^2*c)*x
)/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)]
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.81

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{Bx^3}{3c^3} + x\left(\frac{A}{c^3} - \frac{3Bb}{c^4}\right) - \frac{5\sqrt{-\frac{b}{c^9}}(-3Ac+7Bb)\log\left(-\frac{5c^4\sqrt{-\frac{b}{c^9}}(-3Ac+7Bb)}{-15Ac+35Bb} + x\right)}{16} + \frac{5\sqrt{-\frac{b}{c^9}}(-3Ac+7Bb)\log\left(\frac{5c^4\sqrt{-\frac{b}{c^9}}(-3Ac+7Bb)}{-15Ac+35Bb} + x\right)}{16} + \frac{x^3\cdot(9Abc^2 - 13Bb^2c) + x(7Ab^2c - 11Bb^3)}{8b^2c^4 + 16bc^5x^2 + 8c^6x^4}$$

```
[In] integrate(x**12*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*x**3/(3*c**3) + x*(A/c**3 - 3*B*b/c**4) - 5*sqrt(-b/c**9)*(-3*A*c + 7*B*b
)*log(-5*c**4*sqrt(-b/c**9)*(-3*A*c + 7*B*b)/(-15*A*c + 35*B*b) + x)/16 + 5
*sqrt(-b/c**9)*(-3*A*c + 7*B*b)*log(5*c**4*sqrt(-b/c**9)*(-3*A*c + 7*B*b)/(
-15*A*c + 35*B*b) + x)/16 + (x**3*(9*A*b*c**2 - 13*B*b**2*c) + x*(7*A*b**2*
c - 11*B*b**3))/(8*b**2*c**4 + 16*b*c**5*x**2 + 8*c**6*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.02

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{(13\,Bb^2c-9\,Abc^2)x^3+(11\,Bb^3-7\,Ab^2c)x}{8\,(c^6x^4+2\,bc^5x^2+b^2c^4)} + \frac{5\,(7\,Bb^2-3\,Abc)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^4} + \frac{Bcx^3-3\,(3\,Bb-Ac)x}{3\,c^4}$$

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((13*B*b^2*c - 9*A*b*c^2)*x^3 + (11*B*b^3 - 7*A*b^2*c)*x)/(c^6*x^4 + 2
*b*c^5*x^2 + b^2*c^4) + 5/8*(7*B*b^2 - 3*A*b*c)*arctan(c*x/sqrt(b*c))/(sqrt
(b*c)*c^4) + 1/3*(B*c*x^3 - 3*(3*B*b - A*c)*x)/c^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.94

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{5\,(7\,Bb^2-3\,Abc)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^4} - \frac{13\,Bb^2cx^3-9\,Abc^2x^3+11\,Bb^3x-7\,Ab^2cx}{8\,(cx^2+b)^2c^4} + \frac{Bc^6x^3-9\,Bbc^5x+3\,Ac^6x}{3\,c^9}$$

```
[In] integrate(x^12*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 5/8*(7*B*b^2 - 3*A*b*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) - 1/8*(13*B*b
^2*c*x^3 - 9*A*b*c^2*x^3 + 11*B*b^3*x - 7*A*b^2*c*x)/((c*x^2 + b)^2*c^4) +
1/3*(B*c^6*x^3 - 9*B*b*c^5*x + 3*A*c^6*x)/c^9
```

Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.17

$$\int \frac{x^{12}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{x^3\left(\frac{9\,A\,b\,c^2}{8}-\frac{13\,B\,b^2\,c}{8}\right)-x\left(\frac{11\,B\,b^3}{8}-\frac{7\,A\,b^2\,c}{8}\right)}{b^2\,c^4+2\,b\,c^5\,x^2+c^6\,x^4}+x\left(\frac{A}{c^3}-\frac{3\,B\,b}{c^4}\right)+\frac{B\,x^3}{3\,c^3}+\frac{5\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c}\,x\,(3\,A\,c-7\,B\,b)}{7\,B\,b^2-3\,A\,b\,c}\right)\,(3\,A\,c-7\,B\,b)}{8\,c^{9/2}}$$

```
[In] int((x^12*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (x^3*((9*A*b*c^2)/8 - (13*B*b^2*c)/8) - x*((11*B*b^3)/8 - (7*A*b^2*c)/8))/(
b^2*c^4 + c^6*x^4 + 2*b*c^5*x^2) + x*(A/c^3 - (3*B*b)/c^4) + (B*x^3)/(3*c^3
) + (5*b^(1/2)*atan((b^(1/2)*c^(1/2)*x*(3*A*c - 7*B*b))/(7*B*b^2 - 3*A*b*c)
)*(3*A*c - 7*B*b))/(8*c^(9/2))
```

3.76 $\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 89

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{Bx^2}{2c^3}+\frac{b^2(bB-Ac)}{4c^4(b+cx^2)^2}-\frac{b(3bB-2Ac)}{2c^4(b+cx^2)}-\frac{(3bB-Ac)\log(b+cx^2)}{2c^4}$$

```
[Out] 1/2*B*x^2/c^3+1/4*b^2*(-A*c+B*b)/c^4/(c*x^2+b)^2-1/2*b*(-2*A*c+3*B*b)/c^4/(
c*x^2+b)-1/2*(-A*c+3*B*b)*ln(c*x^2+b)/c^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{b^2(bB-Ac)}{4c^4(b+cx^2)^2}-\frac{b(3bB-2Ac)}{2c^4(b+cx^2)}-\frac{(3bB-Ac)\log(b+cx^2)}{2c^4}+\frac{Bx^2}{2c^3}$$

```
[In] Int[(x^11*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (B*x^2)/(2*c^3) + (b^2*(b*B - A*c))/(4*c^4*(b + c*x^2)^2) - (b*(3*b*B - 2*A
*c))/(2*c^4*(b + c*x^2)) - ((3*b*B - A*c)*Log[b + c*x^2])/(2*c^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^5(A+Bx^2)}{(b+cx^2)^3}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(b+cx)^3}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{B}{c^3}-\frac{b^2(bB-Ac)}{c^3(b+cx)^3}+\frac{b(3bB-2Ac)}{c^3(b+cx)^2}+\frac{-3bB+Ac}{c^3(b+cx)}\right)dx, x, x^2\right)\\ &= \frac{Bx^2}{2c^3}+\frac{b^2(bB-Ac)}{4c^4\left(b+cx^2\right)^2}-\frac{b(3bB-2Ac)}{2c^4\left(b+cx^2\right)}-\frac{(3bB-Ac)\log\left(b+cx^2\right)}{2c^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \frac{x^{11}(A+Bx^2)}{\left(bx^2+cx^4\right)^3}\,dx = \frac{Bx^2}{2c^3}+\frac{b^3B-Ab^2c}{4c^4\left(b+cx^2\right)^2}+\frac{-3b^2B+2Abc}{2c^4\left(b+cx^2\right)}+\frac{(-3bB+Ac)\log\left(b+cx^2\right)}{2c^4}$$

```
[In] Integrate[(x^11*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (B*x^2)/(2*c^3) + (b^3*B - A*b^2*c)/(4*c^4*(b + c*x^2)^2) + (-3*b^2*B + 2*A
*b*c)/(2*c^4*(b + c*x^2)) + ((-3*b*B + A*c)*Log[b + c*x^2])/(2*c^4)
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

method	result
norman	$\frac{\frac{b(Ac-3Bb)x^7}{c^3}+\frac{B\,x^{11}}{2c}+\frac{b^2(3Ac-9Bb)x^5}{4c^4}}{x^5(c\,x^2+b)^2}+\frac{(Ac-3Bb)\ln\left(c\,x^2+b\right)}{2c^4}$
default	$\frac{B\,x^2}{2c^3}+\frac{\frac{(Ac-3Bb)\ln\left(c\,x^2+b\right)}{c}-\frac{b^2(Ac-Bb)}{2c\left(c\,x^2+b\right)^2}+\frac{b(2Ac-3Bb)}{c\left(c\,x^2+b\right)}}{2c^3}$
risch	$\frac{B\,x^2}{2c^3}+\frac{\left(Abc-\frac{3}{2}B\,b^2\right)x^2+\frac{b^2(3Ac-5Bb)}{4c}}{c^3(c\,x^2+b)^2}+\frac{\ln(c\,x^2+b)A}{2c^3}-\frac{3\ln(c\,x^2+b)Bb}{2c^4}$
parallelrisch	$\frac{2B\,c^3x^6+2A\ln(c\,x^2+b)x^4c^3-6B\ln(c\,x^2+b)x^4b\,c^2+4A\ln(c\,x^2+b)x^2b\,c^2-12B\ln(c\,x^2+b)x^2b^2c+4Ab\,c^2x^2-12B\,b^2c\,x^2+2A\ln(}{4c^4(c\,x^2+b)^2}$

```
[In] int(x^11*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (b*(A*c-3*B*b)/c^3*x^7+1/2*B/c*x^11+1/4*b^2*(3*A*c-9*B*b)/c^4*x^5)/x^5/(c*x
^2+b)^2+1/2*(A*c-3*B*b)/c^4*ln(c*x^2+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.60

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$
$$=\frac{2\,Bc^3x^6+4\,Bbc^2x^4-5\,Bb^3+3\,Ab^2c-4\,(Bb^2c-Abc^2)x^2-2\,((3\,Bbc^2-Ac^3)x^4+3\,Bb^3-Ab^2c+2\,(3\,Bl}{4\,(c^6x^4+2\,bc^5x^2+b^2c^4)}$$

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*B*c^3*x^6 + 4*B*b*c^2*x^4 - 5*B*b^3 + 3*A*b^2*c - 4*(B*b^2*c - A*b*c
^2)*x^2 - 2*((3*B*b*c^2 - A*c^3)*x^4 + 3*B*b^3 - A*b^2*c + 2*(3*B*b^2*c - A
*b*c^2)*x^2)*log(c*x^2 + b))/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{Bx^2}{2c^3}+\frac{3Ab^2c-5Bb^3+x^2\cdot(4Abc^2-6Bb^2c)}{4b^2c^4+8bc^5x^2+4c^6x^4}$$
$$-\frac{(-Ac+3Bb)\log(b+cx^2)}{2c^4}$$

```
[In] integrate(x**11*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*x**2/(2*c**3) + (3*A*b**2*c - 5*B*b**3 + x**2*(4*A*b*c**2 - 6*B*b**2*c))/
(4*b**2*c**4 + 8*b*c**5*x**2 + 4*c**6*x**4) - (-A*c + 3*B*b)*log(b + c*x**2
)/(2*c**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{5\,Bb^3-3\,Ab^2c+2\,(3\,Bb^2c-2\,Abc^2)x^2}{4\,(c^6x^4+2\,bc^5x^2+b^2c^4)} + \frac{Bx^2}{2\,c^3} - \frac{(3\,Bb-Ac)\log\left(cx^2+b\right)}{2\,c^4}$$

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(5*B*b^3 - 3*A*b^2*c + 2*(3*B*b^2*c - 2*A*b*c^2)*x^2)/(c^6*x^4 + 2*b*c
^5*x^2 + b^2*c^4) + 1/2*B*x^2/c^3 - 1/2*(3*B*b - A*c)*log(c*x^2 + b)/c^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04

$$\int \frac{x^{11}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{Bx^2}{2\,c^3} - \frac{(3\,Bb-Ac)\log\left(|cx^2+b|\right)}{2\,c^4} + \frac{9\,Bbc^2x^4-3\,Ac^3x^4+12\,Bb^2cx^2-2\,Abc^2x^2+4\,Bb^3}{4\,(cx^2+b)^2c^4}$$

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/c^3 - 1/2*(3*B*b - A*c)*log(abs(c*x^2 + b))/c^4 + 1/4*(9*B*b*c^2*
x^4 - 3*A*c^3*x^4 + 12*B*b^2*c*x^2 - 2*A*b*c^2*x^2 + 4*B*b^3)/((c*x^2 + b)^
2*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

$$\int \frac{x^{11}(A+Bx^2)}{\left(bx^2+cx^4\right)^3}\,dx=\frac{B\,x^2}{2\,c^3}-\frac{x^2\,\left(\frac{3\,B\,b^2}{2}-A\,b\,c\right)+\frac{5\,B\,b^3-3\,A\,b^2\,c}{4\,c}}{b^2\,c^3+2\,b\,c^4\,x^2+c^5\,x^4}+\frac{\ln\left(c\,x^2+b\right)\,\left(A\,c-3\,B\,b\right)}{2\,c^4}$$

```
[In] int((x^11*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (B*x^2)/(2*c^3) - (x^2*((3*B*b^2)/2 - A*b*c) + (5*B*b^3 - 3*A*b^2*c)/(4*c))
/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^2) + (log(b + c*x^2)*(A*c - 3*B*b))/(2*c^4)
```

3.77 $\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 95

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{Bx}{c^3}-\frac{b(bB-Ac)x}{4c^3(b+cx^2)^2}+\frac{(9bB-5Ac)x}{8c^3(b+cx^2)}-\frac{3(5bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{7/2}}$$

```
[Out] B*x/c^3-1/4*b*(-A*c+B*b)*x/c^3/(c*x^2+b)^2+1/8*(-5*A*c+9*B*b)*x/c^3/(c*x^2+
b)-3/8*(-A*c+5*B*b)*arctan(x*c^(1/2)/b^(1/2))/c^(7/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1598, 466, 1171, 396, 211}

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{3(5bB-Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{7/2}}+\frac{x(9bB-5Ac)}{8c^3(b+cx^2)}-\frac{bx(bB-Ac)}{4c^3(b+cx^2)^2}+\frac{Bx}{c^3}$$

```
[In] Int[(x^10*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (B*x)/c^3 - (b*(b*B - A*c)*x)/(4*c^3*(b + c*x^2)^2) + ((9*b*B - 5*A*c)*x)/(
8*c^3*(b + c*x^2)) - (3*(5*b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*Sqrt[
b]*c^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^4(A+Bx^2)}{(b+cx^2)^3}\,dx\\
&= -\frac{b(bB-Ac)x}{4c^3(b+cx^2)^2} - \frac{\int \frac{-b(bB-Ac)+4c(bB-Ac)x^2-4Bc^2x^4}{(b+cx^2)^2}\,dx}{4c^3}\\
&= -\frac{b(bB-Ac)x}{4c^3(b+cx^2)^2} + \frac{(9bB-5Ac)x}{8c^3(b+cx^2)} + \frac{\int \frac{-b(7bB-3Ac)+8bBcx^2}{b+cx^2}\,dx}{8bc^3}\\
&= \frac{Bx}{c^3} - \frac{b(bB-Ac)x}{4c^3(b+cx^2)^2} + \frac{(9bB-5Ac)x}{8c^3(b+cx^2)} - \frac{(3(5bB-Ac))\int \frac{1}{b+cx^2}\,dx}{8c^3}
\end{aligned}$$

$$= \frac{Bx}{c^3} - \frac{b(bB - Ac)x}{4c^3 (b + cx^2)^2} + \frac{(9bB - 5Ac)x}{8c^3 (b + cx^2)} - \frac{3(5bB - Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{7/2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \frac{x^{10}(A + Bx^2)}{(bx^2 + cx^4)^3}\, dx = \frac{x(15b^2B + c^2x^2(-5A + 8Bx^2) + b(-3Ac + 25Bcx^2))}{8c^3 (b + cx^2)^2} - \frac{3(5bB - Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{7/2}}$$

```
[In] Integrate[(x^10*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(15*b^2*B + c^2*x^2*(-5*A + 8*B*x^2) + b*(-3*A*c + 25*B*c*x^2)))/(8*c^3*
(b + c*x^2)^2) - (3*(5*b*B - A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*Sqrt[b]*c
^(7/2))
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

method	result
default	$\frac{Bx}{c^3} + \frac{\frac{\left(-\frac{5}{8}Ac^2+\frac{9}{8}Bbc\right)x^3-\frac{b(3Ac-7Bb)x}{8}}{\left(cx^2+b\right)^2}+\frac{3(Ac-5Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}}{c^3}$
risch	$\frac{Bx}{c^3} + \frac{(-\frac{5}{8}Ac^2+\frac{9}{8}Bbc)x^3-\frac{b(3Ac-7Bb)x}{8}}{c^3(cx^2+b)^2} - \frac{3\ln(cx+\sqrt{-bc})A}{16c^2\sqrt{-bc}} + \frac{15\ln(cx+\sqrt{-bc})Bb}{16c^3\sqrt{-bc}} + \frac{3\ln(-cx+\sqrt{-bc})A}{16c^2\sqrt{-bc}} - \frac{15\ln(-cx+\sqrt{}}{16c^3\sqrt{-}}$

```
[In] int(x^10*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] B*x/c^3+1/c^3*(((-5/8*A*c^2+9/8*B*b*c)*x^3-1/8*b*(3*A*c-7*B*b)*x)/(c*x^2+b)
^2+3/8*(A*c-5*B*b)/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 328, normalized size of antiderivative = 3.45

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \left[\frac{16\,Bbc^3x^5+10\,(5\,Bb^2c^2-Abc^3)x^3+3\,((5\,Bbc^2-Ac^3)x^4+5\,Bb^3-Ab^2c+2\,(5\,Bb^2c-Abc^2)x^2)\sqrt{-bc}\,\mathrm{lo}}{16\,(bc^6x^4+2\,b^2c^5x^2+b^3c^4)}\right.$$

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*B*b*c^3*x^5 + 10*(5*B*b^2*c^2 - A*b*c^3)*x^3 + 3*((5*B*b*c^2 - A*
c^3)*x^4 + 5*B*b^3 - A*b^2*c + 2*(5*B*b^2*c - A*b*c^2)*x^2)*sqrt(-b*c)*log(
(c*x^2 - 2*sqrt(-b*c)*x - b)/(c*x^2 + b)) + 6*(5*B*b^3*c - A*b^2*c^2)*x)/(b
*c^6*x^4 + 2*b^2*c^5*x^2 + b^3*c^4), 1/8*(8*B*b*c^3*x^5 + 5*(5*B*b^2*c^2 -
A*b*c^3)*x^3 - 3*((5*B*b*c^2 - A*c^3)*x^4 + 5*B*b^3 - A*b^2*c + 2*(5*B*b^2*
c - A*b*c^2)*x^2)*sqrt(b*c)*arctan(sqrt(b*c)*x/b) + 3*(5*B*b^3*c - A*b^2*c^
2)*x)/(b*c^6*x^4 + 2*b^2*c^5*x^2 + b^3*c^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(92) = 184.

Time = 0.58 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.04

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{Bx}{c^3} + \frac{3\sqrt{-\frac{1}{bc^7}}(-Ac+5Bb)\log\left(-\frac{3bc^3\sqrt{-\frac{1}{bc^7}}(-Ac+5Bb)}{-3Ac+15Bb}+x\right)}{16} - \frac{3\sqrt{-\frac{1}{bc^7}}(-Ac+5Bb)\log\left(\frac{3bc^3\sqrt{-\frac{1}{bc^7}}(-Ac+5Bb)}{-3Ac+15Bb}+x\right)}{16} + \frac{x^3(-5Ac^2+9Bbc)+x(-3Abc+7Bb^2)}{8b^2c^3+16bc^4x^2+8c^5x^4}$$

```
[In] integrate(x**10*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] B*x/c**3 + 3*sqrt(-1/(b*c**7))*(-A*c + 5*B*b)*log(-3*b*c**3*sqrt(-1/(b*c**7
))*(-A*c + 5*B*b)/(-3*A*c + 15*B*b) + x)/16 - 3*sqrt(-1/(b*c**7))*(-A*c + 5
*B*b)*log(3*b*c**3*sqrt(-1/(b*c**7))*(-A*c + 5*B*b)/(-3*A*c + 15*B*b) + x)/
16 + (x**3*(-5*A*c**2 + 9*B*b*c) + x*(-3*A*b*c + 7*B*b**2))/(8*b**2*c**3 +
16*b*c**4*x**2 + 8*c**5*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(9\,Bbc-5\,Ac^2)x^3+(7\,Bb^2-3\,Abc)x}{8\,(c^5x^4+2\,bc^4x^2+b^2c^3)} + \frac{Bx}{c^3} - \frac{3\,(5\,Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^3}$$

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((9*B*b*c - 5*A*c^2)*x^3 + (7*B*b^2 - 3*A*b*c)*x)/(c^5*x^4 + 2*b*c^4*x^
2 + b^2*c^3) + B*x/c^3 - 3/8*(5*B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)
*c^3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.84

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{Bx}{c^3} - \frac{3\,(5\,Bb-Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^3} + \frac{9\,Bbcx^3-5\,Ac^2x^3+7\,Bb^2x-3\,Abcx}{8\,(cx^2+b)^2c^3}$$

```
[In] integrate(x^10*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] B*x/c^3 - 3/8*(5*B*b - A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/8*(9*
B*b*c*x^3 - 5*A*c^2*x^3 + 7*B*b^2*x - 3*A*b*c*x)/((c*x^2 + b)^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \frac{x^{10}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{B\,x}{c^3} - \frac{x^3\left(\frac{5\,A\,c^2}{8}-\frac{9\,B\,b\,c}{8}\right)-x\left(\frac{7\,B\,b^2}{8}-\frac{3\,A\,b\,c}{8}\right)}{b^2\,c^3+2\,b\,c^4\,x^2+c^5\,x^4} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(A\,c-5\,B\,b)}{8\,\sqrt{b}\,c^{7/2}}$$

```
[In] int((x^10*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (B*x)/c^3 - (x^3*((5*A*c^2)/8 - (9*B*b*c)/8) - x*((7*B*b^2)/8 - (3*A*b*c)/8
))/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^2) + (3*atan((c^(1/2)*x)/b^(1/2))*(A*c -
5*B*b))/(8*b^(1/2)*c^(7/2))
```

3.78 $\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{b(bB-Ac)}{4c^3(b+cx^2)^2}+\frac{2bB-Ac}{2c^3(b+cx^2)}+\frac{B\log(b+cx^2)}{2c^3}$$

```
[Out] -1/4*b*(-A*c+B*b)/c^3/(c*x^2+b)^2+1/2*(-A*c+2*B*b)/c^3/(c*x^2+b)+1/2*B*ln(c
*x^2+b)/c^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{b(bB-Ac)}{4c^3(b+cx^2)^2}+\frac{2bB-Ac}{2c^3(b+cx^2)}+\frac{B\log(b+cx^2)}{2c^3}$$

```
[In] Int[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(b*(b*B - A*c))/(c^3*(b + c*x^2)^2) + (2*b*B - A*c)/(2*c^3*(b + c*x^2)
) + (B*Log[b + c*x^2])/(2*c^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^3(A+Bx^2)}{(b+cx^2)^3}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{(b+cx)^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b(bB-Ac)}{c^2(b+cx)^3}+\frac{-2bB+Ac}{c^2(b+cx)^2}+\frac{B}{c^2(b+cx)}\right)\,dx,x,x^2\right)\\ &= -\frac{b(bB-Ac)}{4c^3(b+cx^2)^2}+\frac{2bB-Ac}{2c^3(b+cx^2)}+\frac{B\log(b+cx^2)}{2c^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3b^2B-2Ac^2x^2-bc(A-4Bx^2)+2B(b+cx^2)^2\log(b+cx^2)}{4c^3(b+cx^2)^2}$$

```
[In] Integrate[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
[Out] (3*b^2*B - 2*A*c^2*x^2 - b*c*(A - 4*B*x^2) + 2*B*(b + c*x^2)^2*Log[b + c*x^
2])/(4*c^3*(b + c*x^2)^2)
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

method	result	size
risch	$\frac{-\frac{(Ac-2Bb)x^{2}}{2c^{2}}-\frac{b(Ac-3Bb)}{4c^{3}}}{(c\,x^{2}+b)^{2}}+\frac{B\ln(c\,x^{2}+b)}{2c^{3}}$	57
default	$\frac{B\ln(c\,x^{2}+b)}{2c^{3}}+\frac{b(Ac-Bb)}{4c^{3}(c\,x^{2}+b)^{2}}-\frac{Ac-2Bb}{2c^{3}(c\,x^{2}+b)}$	61
norman	$\frac{-\frac{(Ac-2Bb)x^{7}}{2c^{2}}-\frac{b(Ac-3Bb)x^{5}}{4c^{3}}}{x^{5}(c\,x^{2}+b)^{2}}+\frac{B\ln(c\,x^{2}+b)}{2c^{3}}$	63
parallelrisch	$-\frac{-2B\ln(c\,x^{2}+b)x^{4}c^{2}-4B\ln(c\,x^{2}+b)x^{2}bc+2A\,c^{2}x^{2}-4Bbc\,x^{2}-2B\ln(c\,x^{2}+b)b^{2}+Abc-3B\,b^{2}}{4c^{3}(c\,x^{2}+b)^{2}}$	90

```
[In] int(x^9*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
[Out] (-1/2*(A*c-2*B*b)/c^2*x^2-1/4*b*(A*c-3*B*b)/c^3)/(c*x^2+b)^2+1/2*B*ln(c*x^2
+b)/c^3
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.33

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3\,Bb^2-Abc+2\,(2\,Bbc-Ac^2)x^2+2\,(Bc^2x^4+2\,Bbcx^2+Bb^2)\log\left(cx^2+b\right)}{4\,(c^5x^4+2\,bc^4x^2+b^2c^3)}$$

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
[Out] 1/4*(3*B*b^2 - A*b*c + 2*(2*B*b*c - A*c^2)*x^2 + 2*(B*c^2*x^4 + 2*B*b*c*x^2
 + B*b^2)*log(c*x^2 + b))/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{B\log\left(b+cx^2\right)}{2c^3}+\frac{-Abc+3Bb^2+x^2(-2Ac^2+4Bbc)}{4b^2c^3+8bc^4x^2+4c^5x^4}$$

```
[In] integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
[Out] B*log(b + c*x**2)/(2*c**3) + (-A*b*c + 3*B*b**2 + x**2*(-2*A*c**2 + 4*B*b*c
))/(4*b**2*c**3 + 8*b*c**4*x**2 + 4*c**5*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3\,Bb^2 - Abc + 2\,(2\,Bbc - Ac^2)x^2}{4\,(c^5x^4 + 2\,bc^4x^2 + b^2c^3)} + \frac{B\log{(cx^2+b)}}{2\,c^3}$$

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(3*B*b^2 - A*b*c + 2*(2*B*b*c - A*c^2)*x^2)/(c^5*x^4 + 2*b*c^4*x^2 + b^
2*c^3) + 1/2*B*log(c*x^2 + b)/c^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{B\log{(|cx^2+b|)}}{2\,c^3} - \frac{3\,Bcx^4 + 2\,Bbx^2 + 2\,Acx^2 + Ab}{4\,(cx^2+b)^2c^2}$$

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*log(abs(c*x^2 + b))/c^3 - 1/4*(3*B*c*x^4 + 2*B*b*x^2 + 2*A*c*x^2 + A*
b)/((c*x^2 + b)^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{3\,B\,b^2-A\,b\,c}{4\,c^3} - \frac{x^2\,(A\,c-2\,B\,b)}{2\,c^2}}{b^2+2\,b\,c\,x^2+c^2\,x^4} + \frac{B\,\ln{(c\,x^2+b)}}{2\,c^3}$$

```
[In] int((x^9*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((3*B*b^2 - A*b*c)/(4*c^3) - (x^2*(A*c - 2*B*b))/(2*c^2))/(b^2 + c^2*x^4 +
2*b*c*x^2) + (B*log(b + c*x^2))/(2*c^3)
```

3.79 $\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 90

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^3} dx = \frac{(bB-Ac)x}{4c^2(b+cx^2)^2} - \frac{(5bB-Ac)x}{8bc^2(b+cx^2)} + \frac{(3bB+Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{3/2}c^{5/2}}$$

```
[Out] 1/4*(-A*c+B*b)*x/c^2/(c*x^2+b)^2-1/8*(-A*c+5*B*b)*x/b/c^2/(c*x^2+b)+1/8*(A*
c+3*B*b)*arctan(x*c^(1/2)/b^(1/2))/b^(3/2)/c^(5/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 466, 393, 211}

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^3} dx = \frac{(Ac+3bB)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{3/2}c^{5/2}} - \frac{x(5bB-Ac)}{8bc^2(b+cx^2)} + \frac{x(bB-Ac)}{4c^2(b+cx^2)^2}$$

```
[In] Int[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b*B - A*c)*x)/(4*c^2*(b + c*x^2)^2) - ((5*b*B - A*c)*x)/(8*b*c^2*(b + c*x
^2)) + ((3*b*B + A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(3/2)*c^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^2(A+Bx^2)}{(b+cx^2)^3}\,dx \\
&= \frac{(bB-Ac)x}{4c^2(b+cx^2)^2} - \frac{\int \frac{bB-Ac-4Bcx^2}{(b+cx^2)^2}\,dx}{4c^2} \\
&= \frac{(bB-Ac)x}{4c^2(b+cx^2)^2} - \frac{(5bB-Ac)x}{8bc^2(b+cx^2)} + \frac{(3bB+Ac)\int \frac{1}{b+cx^2}\,dx}{8bc^2} \\
&= \frac{(bB-Ac)x}{4c^2(b+cx^2)^2} - \frac{(5bB-Ac)x}{8bc^2(b+cx^2)} + \frac{(3bB+Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{3/2}c^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{\sqrt{c}x(-3b^2B+Ac^2x^2-bc(A+5Bx^2))}{b(b+cx^2)^2}+\frac{(3bB+Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}}{8c^{5/2}}$$

```
[In] Integrate[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((Sqrt[c]*x*(-3*b^2*B + A*c^2*x^2 - b*c*(A + 5*B*x^2)))/(b*(b + c*x^2)^2) +
 ((3*b*B + A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2))/(8*c^(5/2))
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\frac{(Ac-5Bb)x^3}{8bc}-\frac{(Ac+3Bb)x}{8c^2}}{(c\,x^2+b)^2}+\frac{(Ac+3Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8c^2b\sqrt{bc}}$	76
risch	$\frac{\frac{(Ac-5Bb)x^3}{8bc}-\frac{(Ac+3Bb)x}{8c^2}}{(c\,x^2+b)^2}-\frac{\ln(cx+\sqrt{-bc})A}{16\sqrt{-bc}\,cb}-\frac{3\ln(cx+\sqrt{-bc})B}{16\sqrt{-bc}\,c^2}+\frac{\ln(-cx+\sqrt{-bc})A}{16\sqrt{-bc}\,cb}+\frac{3\ln(-cx+\sqrt{-bc})B}{16\sqrt{-bc}\,c^2}$	146

```
[In] int(x^8*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(A*c-5*B*b)/b/c*x^3-1/8*(A*c+3*B*b)/c^2*x)/(c*x^2+b)^2+1/8*(A*c+3*B*b)
/c^2/b/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.34

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$
$$= \left[-\frac{2\,(5\,Bb^2c^2-Abc^3)x^3+((3\,Bbc^2+Ac^3)x^4+3\,Bb^3+Ab^2c+2\,(3\,Bb^2c+Abc^2)x^2)\sqrt{-bc}\log\left(\frac{cx^2-2\sqrt{-bc}x}{cx^2+b}\right.}{16\,(b^2c^5x^4+2\,b^3c^4x^2+b^4c^3)}\right.$$
$$-\frac{(5\,Bb^2c^2-Abc^3)x^3-((3\,Bbc^2+Ac^3)x^4+3\,Bb^3+Ab^2c+2\,(3\,Bb^2c+Abc^2)x^2)\sqrt{bc}\arctan\left(\frac{\sqrt{bc}x}{b}\right)+(3}{8\,(b^2c^5x^4+2\,b^3c^4x^2+b^4c^3)}$$

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*B*b^2*c^2 - A*b*c^3)*x^3 + ((3*B*b*c^2 + A*c^3)*x^4 + 3*B*b^3
+ A*b^2*c + 2*(3*B*b^2*c + A*b*c^2)*x^2)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*
c)*x - b)/(c*x^2 + b)) + 2*(3*B*b^3*c + A*b^2*c^2)*x)/(b^2*c^5*x^4 + 2*b^3*
c^4*x^2 + b^4*c^3), -1/8*((5*B*b^2*c^2 - A*b*c^3)*x^3 - ((3*B*b*c^2 + A*c^3
)*x^4 + 3*B*b^3 + A*b^2*c + 2*(3*B*b^2*c + A*b*c^2)*x^2)*sqrt(b*c)*arctan(s
qrt(b*c)*x/b) + (3*B*b^3*c + A*b^2*c^2)*x)/(b^2*c^5*x^4 + 2*b^3*c^4*x^2 + b
^4*c^3)]
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.72

$$\int \frac{x^8(A+Bx^2)}{\left(bx^2+cx^4\right)^3}\,dx = -\frac{\sqrt{-\frac{1}{b^3c^5}}(Ac+3Bb)\log\left(-b^2c^2\sqrt{-\frac{1}{b^3c^5}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{b^3c^5}}(Ac+3Bb)\log\left(b^2c^2\sqrt{-\frac{1}{b^3c^5}}+x\right)}{16} + \frac{x^3(Ac^2-5Bbc)+x(-Abc-3Bb^2)}{8b^3c^2+16b^2c^3x^2+8bc^4x^4}$$

```
[In] integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] -sqrt(-1/(b**3*c**5))*(A*c + 3*B*b)*log(-b**2*c**2*sqrt(-1/(b**3*c**5)) + x
)/16 + sqrt(-1/(b**3*c**5))*(A*c + 3*B*b)*log(b**2*c**2*sqrt(-1/(b**3*c**5)
) + x)/16 + (x**3*(A*c**2 - 5*B*b*c) + x*(-A*b*c - 3*B*b**2))/(8*b**3*c**2
+ 16*b**2*c**3*x**2 + 8*b*c**4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{x^8(A+Bx^2)}{\left(bx^2+cx^4\right)^3}\,dx = -\frac{(5\,Bbc-Ac^2)x^3+(3\,Bb^2+Abc)x}{8\,(bc^4x^4+2\,b^2c^3x^2+b^3c^2)} + \frac{(3\,Bb+Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}bc^2}$$

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*B*b*c - A*c^2)*x^3 + (3*B*b^2 + A*b*c)*x)/(b*c^4*x^4 + 2*b^2*c^3*x
^2 + b^3*c^2) + 1/8*(3*B*b + A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(3\,Bb+Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}bc^2} - \frac{5\,Bbcx^3-Ac^2x^3+3\,Bb^2x+Abcx}{8\,(cx^2+b)^2bc^2}$$

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*B*b + A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b*c^2) - 1/8*(5*B*b*c*x^
3 - A*c^2*x^3 + 3*B*b^2*x + A*b*c*x)/((c*x^2 + b)^2*b*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(A\,c+3\,B\,b)}{8\,b^{3/2}\,c^{5/2}} - \frac{\frac{x\,(A\,c+3\,B\,b)}{8\,c^2}-\frac{x^3\,(A\,c-5\,B\,b)}{8\,b\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}$$

```
[In] int((x^8*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (atan((c^(1/2)*x)/b^(1/2))*(A*c + 3*B*b))/(8*b^(3/2)*c^(5/2)) - ((x*(A*c +
3*B*b))/(8*c^2) - (x^3*(A*c - 5*B*b))/(8*b*c))/(b^2 + c^2*x^4 + 2*b*c*x^2)
```

3.80 $\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 32

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(A+Bx^2)^2}{4(bB-Ac)\,(b+cx^2)^2}$$

```
[Out] 1/4*(B*x^2+A)^2/(-A*c+B*b)/(c*x^2+b)^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 455, 37}

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(A+Bx^2)^2}{4\,(b+cx^2)^2\,(bB-Ac)}$$

```
[In] Int[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (A + B*x^2)^2/(4*(b*B - A*c)*(b + c*x^2)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x(A+Bx^2)}{(b+cx^2)^3}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{(b+cx)^3}\,dx, x, x^2\right)\\ &= \frac{(A+Bx^2)^2}{4(bB-Ac)\,(b+cx^2)^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{bB+c(A+2Bx^2)}{4c^2\,(b+cx^2)^2}$$

```
[In] Integrate[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
[Out] -1/4*(b*B + c*(A + 2*B*x^2))/(c^2*(b + c*x^2)^2)
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
gosper	$-\frac{2Bc\,x^2+Ac+Bb}{4(c\,x^2+b)^2c^2}$	29
parallelrisch	$-\frac{2Bc\,x^2+Ac+Bb}{4(c\,x^2+b)^2c^2}$	29
risch	$\frac{-\frac{B\,x^2}{2c}-\frac{Ac+Bb}{4c^2}}{(c\,x^2+b)^2}$	33
default	$-\frac{Ac-Bb}{4c^2(c\,x^2+b)^2}-\frac{B}{2c^2(c\,x^2+b)}$	39
norman	$\frac{-\frac{B\,x^7}{2c}-\frac{(Ac+Bb)x^5}{4c^2}}{x^5(c\,x^2+b)^2}$	39

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
[Out] -1/4*(2*B*c*x^2+A*c+B*b)/(c*x^2+b)^2/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{2\,Bcx^2+Bb+Ac}{4\,(c^4x^4+2\,bc^3x^2+b^2c^2)}$$

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
[Out] -1/4*(2*B*c*x^2 + B*b + A*c)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{-Ac-Bb-2Bcx^2}{4b^2c^2+8bc^3x^2+4c^4x^4}$$

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
[Out] (-A*c - B*b - 2*B*c*x**2)/(4*b**2*c**2 + 8*b*c**3*x**2 + 4*c**4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{2\,Bcx^2+Bb+Ac}{4\,(c^4x^4+2\,bc^3x^2+b^2c^2)}$$

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
[Out] -1/4*(2*B*c*x^2 + B*b + A*c)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{2\,Bcx^2+Bb+Ac}{4\,(cx^2+b)^2c^2}$$

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
[Out] -1/4*(2*B*c*x^2 + B*b + A*c)/((c*x^2 + b)^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.38

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{\frac{A\,c+B\,b}{4\,c^2}+\frac{B\,x^2}{2\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}$$

```
[In] int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
[Out] -((A*c + B*b)/(4*c^2) + (B*x^2)/(2*c))/(b^2 + c^2*x^4 + 2*b*c*x^2)
```

3.81 $\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 92

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{(bB-Ac)x}{4bc\,(b+cx^2)^2}+\frac{(bB+3Ac)x}{8b^2c\,(b+cx^2)}+\frac{(bB+3Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}c^{3/2}}$$

```
[Out] -1/4*(-A*c+B*b)*x/b/c/(c*x^2+b)^2+1/8*(3*A*c+B*b)*x/b^2/c/(c*x^2+b)+1/8*(3*
A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/b^(5/2)/c^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1598, 393, 205, 211}

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(3Ac+bB)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}c^{3/2}}+\frac{x(3Ac+bB)}{8b^2c\,(b+cx^2)}-\frac{x(bB-Ac)}{4bc\,(b+cx^2)^2}$$

```
[In] Int[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*((b*B - A*c)*x)/(b*c*(b + c*x^2)^2) + ((b*B + 3*A*c)*x)/(8*b^2*c*(b +
c*x^2)) + ((b*B + 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(5/2)*c^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
```

```
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{(b+cx^2)^3}\,dx \\
&= -\frac{(bB-Ac)x}{4bc\,(b+cx^2)^2} + \frac{(bB+3Ac)\int \frac{1}{(b+cx^2)^2}\,dx}{4bc} \\
&= -\frac{(bB-Ac)x}{4bc\,(b+cx^2)^2} + \frac{(bB+3Ac)x}{8b^2c\,(b+cx^2)} + \frac{(bB+3Ac)\int \frac{1}{b+cx^2}\,dx}{8b^2c} \\
&= -\frac{(bB-Ac)x}{4bc\,(b+cx^2)^2} + \frac{(bB+3Ac)x}{8b^2c\,(b+cx^2)} + \frac{(bB+3Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}c^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.91

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{x(-b^2B+3Ac^2x^2+bc(5A+Bx^2))}{8b^2c\,(b+cx^2)^2} + \frac{(bB+3Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}c^{3/2}}$$

```
[In] Integrate[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(-(b^2*B) + 3*A*c^2*x^2 + b*c*(5*A + B*x^2)))/(8*b^2*c*(b + c*x^2)^2) +
((b*B + 3*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(5/2)*c^(3/2))
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\frac{(3Ac+Bb)x^3}{8b^2}+\frac{(5Ac-Bb)x}{8bc}}{(c\,x^2+b)^2}+\frac{(3Ac+Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8b^2c\sqrt{bc}}$	77
risch	$\frac{\frac{(3Ac+Bb)x^3}{8b^2}+\frac{(5Ac-Bb)x}{8bc}}{(c\,x^2+b)^2}-\frac{3\ln\left(cx+\sqrt{-bc}\right)A}{16\sqrt{-bc}\,b^2}-\frac{\ln\left(cx+\sqrt{-bc}\right)B}{16\sqrt{-bc}\,cb}+\frac{3\ln\left(-cx+\sqrt{-bc}\right)A}{16\sqrt{-bc}\,b^2}+\frac{\ln\left(-cx+\sqrt{-bc}\right)B}{16\sqrt{-bc}\,cb}$	147

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(3*A*c+B*b)/b^2*x^3+1/8*(5*A*c-B*b)/b/c*x)/(c*x^2+b)^2+1/8*(3*A*c+B*b)
/b^2/c/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.26

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \left[\frac{2\,(Bb^2c^2+3\,Abc^3)x^3-((Bbc^2+3\,Ac^3)x^4+Bb^3+3\,Ab^2c+2\,(Bb^2c+3\,Abc^2)x^2)\sqrt{-bc}\log\left(\frac{cx^2-2\,\sqrt{-bc}x}{cx^2+b}\right.}{16\,(b^3c^4x^4+2\,b^4c^3x^2+b^5c^2)}\right.$$

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(B*b^2*c^2 + 3*A*b*c^3)*x^3 - ((B*b*c^2 + 3*A*c^3)*x^4 + B*b^3 + 3
*A*b^2*c + 2*(B*b^2*c + 3*A*b*c^2)*x^2)*sqrt(-b*c)*log((c*x^2 - 2*sqrt(-b*c
)*x - b)/(c*x^2 + b)) - 2*(B*b^3*c - 5*A*b^2*c^2)*x)/(b^3*c^4*x^4 + 2*b^4*c
^3*x^2 + b^5*c^2), 1/8*((B*b^2*c^2 + 3*A*b*c^3)*x^3 + ((B*b*c^2 + 3*A*c^3)*
x^4 + B*b^3 + 3*A*b^2*c + 2*(B*b^2*c + 3*A*b*c^2)*x^2)*sqrt(b*c)*arctan(sqr
t(b*c)*x/b) - (B*b^3*c - 5*A*b^2*c^2)*x)/(b^3*c^4*x^4 + 2*b^4*c^3*x^2 + b^5
*c^2)]
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.63

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{\sqrt{-\frac{1}{b^5c^3}}\cdot(3Ac+Bb)\log\left(-b^3c\sqrt{-\frac{1}{b^5c^3}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{b^5c^3}}\cdot(3Ac+Bb)\log\left(b^3c\sqrt{-\frac{1}{b^5c^3}}+x\right)}{16} + \frac{x^3\cdot(3Ac^2+Bbc)+x(5Abc-Bb^2)}{8b^4c+16b^3c^2x^2+8b^2c^3x^4}$$

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] -sqrt(-1/(b**5*c**3))*(3*A*c + B*b)*log(-b**3*c*sqrt(-1/(b**5*c**3)) + x)/1
6 + sqrt(-1/(b**5*c**3))*(3*A*c + B*b)*log(b**3*c*sqrt(-1/(b**5*c**3)) + x)
/16 + (x**3*(3*A*c**2 + B*b*c) + x*(5*A*b*c - B*b**2))/(8*b**4*c + 16*b**3*
c**2*x**2 + 8*b**2*c**3*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(Bbc+3\,Ac^2)x^3-(Bb^2-5\,Abc)x}{8\,(b^2c^3x^4+2\,b^3c^2x^2+b^4c)} + \frac{(Bb+3\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^2c}$$

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((B*b*c + 3*A*c^2)*x^3 - (B*b^2 - 5*A*b*c)*x)/(b^2*c^3*x^4 + 2*b^3*c^2*
x^2 + b^4*c) + 1/8*(B*b + 3*A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2*c)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.85

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(Bb+3\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^2c} + \frac{Bbcx^3+3\,Ac^2x^3-Bb^2x+5\,Abcx}{8\,(cx^2+b)^2b^2c}$$

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(B*b + 3*A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2*c) + 1/8*(B*b*c*x^3
+ 3*A*c^2*x^3 - B*b^2*x + 5*A*b*c*x)/((c*x^2 + b)^2*b^2*c)
```

Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{x^3\,(3\,A\,c+B\,b)}{8\,b^2}+\frac{x\,(5\,A\,c-B\,b)}{8\,b\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}+\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(3\,A\,c+B\,b)}{8\,b^{5/2}\,c^{3/2}}$$

```
[In] int((x^6*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^3*(3*A*c + B*b))/(8*b^2) + (x*(5*A*c - B*b))/(8*b*c))/(b^2 + c^2*x^4 +
2*b*c*x^2) + (atan((c^(1/2)*x)/b^(1/2))*(3*A*c + B*b))/(8*b^(5/2)*c^(3/2))
```

3.82 $\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^3}\,dx=-\frac{bB-Ac}{4bc\,(b+cx^2)^2}+\frac{A}{2b^2\,(b+cx^2)}+\frac{A\log(x)}{b^3}-\frac{A\log\left(b+cx^2\right)}{2b^3}$$

```
[Out] 1/4*(A*c-B*b)/b/c/(c*x^2+b)^2+1/2*A/b^2/(c*x^2+b)+A*ln(x)/b^3-1/2*A*ln(c*x^
2+b)/b^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^3}\,dx=-\frac{A\log\left(b+cx^2\right)}{2b^3}+\frac{A\log(x)}{b^3}+\frac{A}{2b^2\,(b+cx^2)}-\frac{bB-Ac}{4bc\,(b+cx^2)^2}$$

```
[In] Int[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(b*B - A*c)/(b*c*(b + c*x^2)^2) + A/(2*b^2*(b + c*x^2)) + (A*Log[x])/b
^3 - (A*Log[b + c*x^2])/(2*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{x\left(b+cx^2\right)^3}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x(b+cx)^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{b^3x}+\frac{bB-Ac}{b(b+cx)^3}-\frac{Ac}{b^2(b+cx)^2}-\frac{Ac}{b^3(b+cx)}\right)\,dx, x, x^2\right) \\
&= -\frac{bB-Ac}{4bc\left(b+cx^2\right)^2}+\frac{A}{2b^2\left(b+cx^2\right)}+\frac{A\log(x)}{b^3}-\frac{A\log\left(b+cx^2\right)}{2b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{x^5(A+Bx^2)}{\left(bx^2+cx^4\right)^3}\,dx = \frac{\frac{b(-b^2B+3Abc+2Ac^2x^2)}{c(b+cx^2)^2}+4A\log(x)-2A\log\left(b+cx^2\right)}{4b^3}$$

```
[In] Integrate[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
[Out] ((b*(-(b^2*B) + 3*A*b*c + 2*A*c^2*x^2))/(c*(b + c*x^2)^2) + 4*A*Log[x] - 2*
A*Log[b + c*x^2])/(4*b^3)
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

method	result
risch	$\frac{\frac{A c\,x^{2}}{2 b^{2}}+\frac{3 A c -B b}{4 b c}}{\left(c\,x^{2}+b \right)^{2}}+\frac{A \ln \left(x \right)}{b^{3}}-\frac{A \ln \left(c\,x^{2}+b \right)}{2 b^{3}}$
default	$\frac{A \ln \left(x \right)}{b^{3}}-\frac{-\frac{A b}{c\,x^{2}+b}+A \ln \left(c\,x^{2}+b \right)-\frac{b^{2} \left(A c -B b \right)}{2 c \left(c\,x^{2}+b \right)^{2}}}{2 b^{3}}$
norman	$\frac{-\frac{\left(2 A c -B b \right) x^{7}}{2 b^{2}}-\frac{c \left(3 A c -B b \right) x^{9}}{4 b^{3}}}{x^{5} \left(c\,x^{2}+b \right)^{2}}+\frac{A \ln \left(x \right)}{b^{3}}-\frac{A \ln \left(c\,x^{2}+b \right)}{2 b^{3}}$
parallelrisch	$\frac{4 A \ln \left(x \right) x^{4} c^{2}-2 A \ln \left(c\,x^{2}+b \right) x^{4} c^{2}-3 A\,c^{2} x^{4}+x^{4} B b c +8 A \ln \left(x \right) x^{2} b c -4 A \ln \left(c\,x^{2}+b \right) x^{2} b c -4 A b c\,x^{2}+2 b^{2} B\,x^{2}+4 A\,b^{2} \ln \left(x \right)-2 A \ln (}{4 b^{3} \left(c\,x^{2}+b \right)^{2}}$

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2/b^2*A*c*x^2+1/4*(3*A*c-B*b)/b/c)/(c*x^2+b)^2+A*ln(x)/b^3-1/2*A*ln(c*x^
2+b)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.75

$$\int \frac{x^5(A+Bx^2)}{\left(bx^2+cx^4\right)^3}\,dx$$
$$= \frac{2\,Abc^2x^2 - Bb^3 + 3\,Ab^2c - 2\,(Ac^3x^4 + 2\,Abc^2x^2 + Ab^2c)\log\left(cx^2+b\right) + 4\,(Ac^3x^4 + 2\,Abc^2x^2 + Ab^2c)\log\left(x\right.}{4\,(b^3c^3x^4 + 2\,b^4c^2x^2 + b^5c)}$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*A*b*c^2*x^2 - B*b^3 + 3*A*b^2*c - 2*(A*c^3*x^4 + 2*A*b*c^2*x^2 + A*b
^2*c)*log(c*x^2 + b) + 4*(A*c^3*x^4 + 2*A*b*c^2*x^2 + A*b^2*c)*log(x))/(b^3
*c^3*x^4 + 2*b^4*c^2*x^2 + b^5*c)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10

$$\int \frac{x^5(A+Bx^2)}{\left(bx^2+cx^4\right)^3}\,dx = \frac{A\log\left(x\right)}{b^3} - \frac{A\log\left(\frac{b}{c}+x^2\right)}{2b^3} + \frac{3Abc + 2Ac^2x^2 - Bb^2}{4b^4c + 8b^3c^2x^2 + 4b^2c^3x^4}$$

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] A*log(x)/b**3 - A*log(b/c + x**2)/(2*b**3) + (3*A*b*c + 2*A*c**2*x**2 - B*b
**2)/(4*b**4*c + 8*b**3*c**2*x**2 + 4*b**2*c**3*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.13

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{2\,Ac^2x^2 - Bb^2 + 3\,Abc}{4\,(b^2c^3x^4 + 2\,b^3c^2x^2 + b^4c)} - \frac{A\log\left(cx^2+b\right)}{2\,b^3} + \frac{A\log\left(x^2\right)}{2\,b^3}$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*A*c^2*x^2 - B*b^2 + 3*A*b*c)/(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c) -
 1/2*A*log(c*x^2 + b)/b^3 + 1/2*A*log(x^2)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{A\log\left(x^2\right)}{2\,b^3} - \frac{A\log\left(|cx^2+b|\right)}{2\,b^3} + \frac{3\,Ac^3x^4 + 8\,Abc^2x^2 - Bb^3 + 6\,Ab^2c}{4\,(cx^2+b)^2b^3c}$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*A*log(x^2)/b^3 - 1/2*A*log(abs(c*x^2 + b))/b^3 + 1/4*(3*A*c^3*x^4 + 8*A
*b*c^2*x^2 - B*b^3 + 6*A*b^2*c)/((c*x^2 + b)^2*b^3*c)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{3\,A\,c-B\,b}{4\,b\,c} + \frac{A\,c\,x^2}{2\,b^2}}{b^2 + 2\,b\,c\,x^2 + c^2\,x^4} - \frac{A\,\ln\left(c\,x^2+b\right)}{2\,b^3} + \frac{A\,\ln\left(x\right)}{b^3}$$

```
[In] int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((3*A*c - B*b)/(4*b*c) + (A*c*x^2)/(2*b^2))/(b^2 + c^2*x^4 + 2*b*c*x^2) - (
A*log(b + c*x^2))/(2*b^3) + (A*log(x))/b^3
```

3.83 $\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 96

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{A}{b^3x}+\frac{(bB-Ac)x}{4b^2(b+cx^2)^2}+\frac{(3bB-7Ac)x}{8b^3(b+cx^2)}+\frac{3(bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2}\sqrt{c}}$$

```
[Out] -A/b^3/x+1/4*(-A*c+B*b)*x/b^2/(c*x^2+b)^2+1/8*(-7*A*c+3*B*b)*x/b^3/(c*x^2+b
)+3/8*(-5*A*c+B*b)*arctan(x*c^(1/2)/b^(1/2))/b^(7/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1598, 467, 464, 211}

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3(bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2}\sqrt{c}}+\frac{x(3bB-7Ac)}{8b^3(b+cx^2)}-\frac{A}{b^3x}+\frac{x(bB-Ac)}{4b^2(b+cx^2)^2}$$

```
[In] Int[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(A/(b^3*x)) + ((b*B - A*c)*x)/(4*b^2*(b + c*x^2)^2) + ((3*b*B - 7*A*c)*x)/
(8*b^3*(b + c*x^2)) + (3*(b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(7
/2)*Sqrt[c])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{x^2\left(b+cx^2\right)^3}\,dx\\
&= \frac{(bB-Ac)x}{4b^2\left(b+cx^2\right)^2}-\frac{1}{4}\int \frac{-\frac{4A}{b}-\frac{3(bB-Ac)x^2}{b^2}}{x^2\left(b+cx^2\right)^2}\,dx\\
&= \frac{(bB-Ac)x}{4b^2\left(b+cx^2\right)^2}+\frac{(3bB-7Ac)x}{8b^3\left(b+cx^2\right)}+\frac{1}{8}\int \frac{\frac{8A}{b^2}+\frac{(3bB-7Ac)x^2}{b^3}}{x^2\left(b+cx^2\right)}\,dx\\
&= -\frac{A}{b^3x}+\frac{(bB-Ac)x}{4b^2\left(b+cx^2\right)^2}+\frac{(3bB-7Ac)x}{8b^3\left(b+cx^2\right)}+\frac{(3(bB-5Ac))\int \frac{1}{b+cx^2}\,dx}{8b^3}\\
&= -\frac{A}{b^3x}+\frac{(bB-Ac)x}{4b^2\left(b+cx^2\right)^2}+\frac{(3bB-7Ac)x}{8b^3\left(b+cx^2\right)}+\frac{3(bB-5Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2}\sqrt{c}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{A}{b^3x}+\frac{(bB-Ac)x}{4b^2\,(b+cx^2)^2}+\frac{(3bB-7Ac)x}{8b^3\,(b+cx^2)}+\frac{3(bB-5Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2}\sqrt{c}}$$

```
[In] Integrate[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(A/(b^3*x)) + ((b*B - A*c)*x)/(4*b^2*(b + c*x^2)^2) + ((3*b*B - 7*A*c)*x)/
(8*b^3*(b + c*x^2)) + (3*(b*B - 5*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(7
/2)*Sqrt[c])
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

method	result
default	$-\frac{A}{b^{3}x}-\frac{\frac{\left(\frac{7}{8}A\,c^{2}-\frac{3}{8}Bbc\right)x^{3}+\frac{b(9Ac-5Bb)x}{8}}{\left(c\,x^{2}+b\right)^{2}}+\frac{3(5Ac-Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}}{b^{3}}$
risch	$\frac{-\frac{3c(5Ac-Bb)x^{4}}{8b^{3}}-\frac{5(5Ac-Bb)x^{2}}{8b^{2}}-\frac{A}{b}}{x(c\,x^{2}+b)^{2}}-\frac{15\ln\left(-\sqrt{-bc}\,x-b\right)Ac}{16\sqrt{-bc}\,b^{3}}+\frac{3\ln\left(-\sqrt{-bc}\,x-b\right)B}{16\sqrt{-bc}\,b^{2}}+\frac{15\ln\left(-\sqrt{-bc}\,x+b\right)Ac}{16\sqrt{-bc}\,b^{3}}-\frac{3\ln\left(-\sqrt{-bc}\,x+b\right.}{16\sqrt{-bc}\,b^{2}}$

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -A/b^3/x-1/b^3*(((7/8*A*c^2-3/8*B*b*c)*x^3+1/8*b*(9*A*c-5*B*b)*x)/(c*x^2+b)
^2+3/8*(5*A*c-B*b)/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.38

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$

$$= \left[-\frac{16\,Ab^3c-6\,(Bb^2c^2-5\,Abc^3)x^4-10\,(Bb^3c-5\,Ab^2c^2)x^2-3\,((Bbc^2-5\,Ac^3)x^5+2\,(Bb^2c-5\,Abc^2)x^3}{16\,(b^4c^3x^5+2\,b^5c^2x^3+b^6cx)}\right.$$

$$-\frac{8\,Ab^3c-3\,(Bb^2c^2-5\,Abc^3)x^4-5\,(Bb^3c-5\,Ab^2c^2)x^2-3\,((Bbc^2-5\,Ac^3)x^5+2\,(Bb^2c-5\,Abc^2)x^3+(}{8\,(b^4c^3x^5+2\,b^5c^2x^3+b^6cx)}$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*A*b^3*c - 6*(B*b^2*c^2 - 5*A*b*c^3)*x^4 - 10*(B*b^3*c - 5*A*b^2*
c^2)*x^2 - 3*((B*b*c^2 - 5*A*c^3)*x^5 + 2*(B*b^2*c - 5*A*b*c^2)*x^3 + (B*b^
3 - 5*A*b^2*c)*x)*sqrt(-b*c)*log((c*x^2 + 2*sqrt(-b*c)*x - b)/(c*x^2 + b)))
/(b^4*c^3*x^5 + 2*b^5*c^2*x^3 + b^6*c*x), -1/8*(8*A*b^3*c - 3*(B*b^2*c^2 -
5*A*b*c^3)*x^4 - 5*(B*b^3*c - 5*A*b^2*c^2)*x^2 - 3*((B*b*c^2 - 5*A*c^3)*x^5
 + 2*(B*b^2*c - 5*A*b*c^2)*x^3 + (B*b^3 - 5*A*b^2*c)*x)*sqrt(b*c)*arctan(sq
rt(b*c)*x/b))/(b^4*c^3*x^5 + 2*b^5*c^2*x^3 + b^6*c*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(92) = 184$.

Time = 0.36 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.02

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{3\sqrt{-\frac{1}{b^7c}}(-5Ac+Bb)\log\left(-\frac{3b^4\sqrt{-\frac{1}{b^7c}}(-5Ac+Bb)}{-15Ac+3Bb}+x\right)}{16} + \frac{3\sqrt{-\frac{1}{b^7c}}(-5Ac+Bb)\log\left(\frac{3b^4\sqrt{-\frac{1}{b^7c}}(-5Ac+Bb)}{-15Ac+3Bb}+x\right)}{16} + \frac{-8Ab^2+x^4(-15Ac^2+3Bbc)+x^2(-25Abc+5Bb^2)}{8b^5x+16b^4cx^3+8b^3c^2x^5}$$

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*sqrt(-1/(b**7*c))*(-5*A*c + B*b)*log(-3*b**4*sqrt(-1/(b**7*c))*(-5*A*c +
 B*b)/(-15*A*c + 3*B*b) + x)/16 + 3*sqrt(-1/(b**7*c))*(-5*A*c + B*b)*log(3*
b**4*sqrt(-1/(b**7*c))*(-5*A*c + B*b)/(-15*A*c + 3*B*b) + x)/16 + (-8*A*b**
2 + x**4*(-15*A*c**2 + 3*B*b*c) + x**2*(-25*A*b*c + 5*B*b**2))/(8*b**5*x +
16*b**4*c*x**3 + 8*b**3*c**2*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3\,(Bbc-5\,Ac^2)x^4-8\,Ab^2+5\,(Bb^2-5\,Abc)x^2}{8\,(b^3c^2x^5+2\,b^4cx^3+b^5x)} + \frac{3\,(Bb-5\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^3}$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(B*b*c - 5*A*c^2)*x^4 - 8*A*b^2 + 5*(B*b^2 - 5*A*b*c)*x^2)/(b^3*c^2*
x^5 + 2*b^4*c*x^3 + b^5*x) + 3/8*(B*b - 5*A*c)*arctan(c*x/sqrt(b*c))/(sqrt(
b*c)*b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3\,(Bb-5\,Ac)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^3} - \frac{A}{b^3x} + \frac{3\,Bbcx^3-7\,Ac^2x^3+5\,Bb^2x-9\,Abcx}{8\,(cx^2+b)^2b^3}$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*(B*b - 5*A*c)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - A/(b^3*x) + 1/8*(
3*B*b*c*x^3 - 7*A*c^2*x^3 + 5*B*b^2*x - 9*A*b*c*x)/((c*x^2 + b)^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.18

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{\frac{A}{b}+\frac{5\,x^2\,(5\,A\,c-B\,b)}{8\,b^2}+\frac{3\,c\,x^4\,(5\,A\,c-B\,b)}{8\,b^3}}{b^2\,x+2\,b\,c\,x^3+c^2\,x^5} - \frac{3\,\mathrm{atan}\left(\frac{3\,\sqrt{c}\,x\,(5\,A\,c-B\,b)}{\sqrt{b}\,(15\,A\,c-3\,B\,b)}\right)\,(5\,A\,c-B\,b)}{8\,b^{7/2}\,\sqrt{c}}$$

```
[In] int((x^4*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] - (A/b + (5*x^2*(5*A*c - B*b))/(8*b^2) + (3*c*x^4*(5*A*c - B*b))/(8*b^3))/(
b^2*x + c^2*x^5 + 2*b*c*x^3) - (3*atan((3*c^(1/2)*x*(5*A*c - B*b))/(b^(1/2)
*(15*A*c - 3*B*b)))*(5*A*c - B*b))/(8*b^(7/2)*c^(1/2))
```

3.84 $\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 97

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3} dx = -\frac{A}{2b^3x^2} + \frac{bB-Ac}{4b^2(b+cx^2)^2} + \frac{bB-2Ac}{2b^3(b+cx^2)} + \frac{(bB-3Ac)\log(x)}{b^4} - \frac{(bB-3Ac)\log(b+cx^2)}{2b^4}$$

```
[Out] -1/2*A/b^3/x^2+1/4*(-A*c+B*b)/b^2/(c*x^2+b)^2+1/2*(-2*A*c+B*b)/b^3/(c*x^2+b
)+(-3*A*c+B*b)*ln(x)/b^4-1/2*(-3*A*c+B*b)*ln(c*x^2+b)/b^4
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3} dx = -\frac{(bB-3Ac)\log(b+cx^2)}{2b^4} + \frac{\log(x)(bB-3Ac)}{b^4} + \frac{bB-2Ac}{2b^3(b+cx^2)} - \frac{A}{2b^3x^2} + \frac{bB-Ac}{4b^2(b+cx^2)^2}$$

```
[In] Int[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/2*A/(b^3*x^2) + (b*B - A*c)/(4*b^2*(b + c*x^2)^2) + (b*B - 2*A*c)/(2*b^3
*(b + c*x^2)) + ((b*B - 3*A*c)*Log[x])/b^4 - ((b*B - 3*A*c)*Log[b + c*x^2])
/(2*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{x^3\left(b+cx^2\right)^3}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2(b+cx)^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{b^3x^2}+\frac{bB-3Ac}{b^4x}-\frac{c(bB-Ac)}{b^2(b+cx)^3}-\frac{c(bB-2Ac)}{b^3(b+cx)^2}-\frac{c(bB-3Ac)}{b^4(b+cx)}\right)\,dx, x, x^2\right) \\
&= -\frac{A}{2b^3x^2}+\frac{bB-Ac}{4b^2\left(b+cx^2\right)^2}+\frac{bB-2Ac}{2b^3\left(b+cx^2\right)}+\frac{(bB-3Ac)\log(x)}{b^4}-\frac{(bB-3Ac)\log\left(b+cx^2\right)}{2b^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.89

$$\int \frac{x^3(A+Bx^2)}{\left(bx^2+cx^4\right)^3}\,dx = \frac{-\frac{2Ab}{x^2}+\frac{b^2(bB-Ac)}{(b+cx^2)^2}+\frac{2b(bB-2Ac)}{b+cx^2}+4(bB-3Ac)\log(x)-2(bB-3Ac)\log\left(b+cx^2\right)}{4b^4}$$

```
[In] Integrate[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
[Out] ((-2*A*b)/x^2 + (b^2*(b*B - A*c))/(b + c*x^2)^2 + (2*b*(b*B - 2*A*c))/(b +
c*x^2) + 4*(b*B - 3*A*c)*Log[x] - 2*(b*B - 3*A*c)*Log[b + c*x^2])/(4*b^4)
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

method	result
norman	$\frac{\frac{c(3Ac-Bb)x^7}{b^3}-\frac{A\,x^3}{2b}+\frac{c^2(9Ac-3Bb)x^9}{4b^4}}{x^5(c\,x^2+b)^2}-\frac{(3Ac-Bb)\ln(x)}{b^4}+\frac{(3Ac-Bb)\ln(c\,x^2+b)}{2b^4}$
default	$-\frac{A}{2b^3x^2}+\frac{(-3Ac+Bb)\ln(x)}{b^4}+\frac{c\left(\frac{(3Ac-Bb)\ln(c\,x^2+b)}{c}-\frac{b^2(Ac-Bb)}{2c(c\,x^2+b)^2}-\frac{b(2Ac-Bb)}{c(c\,x^2+b)}\right)}{2b^4}$
risch	$\frac{-\frac{c(3Ac-Bb)x^4}{2b^3}-\frac{3(3Ac-Bb)x^2}{4b^2}-\frac{A}{2b}}{x^2(c\,x^2+b)^2}-\frac{3\ln(x)Ac}{b^4}+\frac{\ln(x)B}{b^3}+\frac{3\ln(-c\,x^2-b)Ac}{2b^4}-\frac{\ln(-c\,x^2-b)B}{2b^3}$
parallelrisch	$-\frac{12A\ln(x)x^6c^3-6A\ln(c\,x^2+b)x^6c^3-4B\ln(x)x^6b\,c^2+2B\ln(c\,x^2+b)x^6b\,c^2-9A\,c^3x^6+3x^6Bb\,c^2+24A\ln(x)x^4b\,c^2-12A\ln(c}{}$

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (c*(3*A*c-B*b)/b^3*x^7-1/2*A/b*x^3+1/4*c^2*(9*A*c-3*B*b)/b^4*x^9)/x^5/(c*x^
2+b)^2-(3*A*c-B*b)/b^4*ln(x)+1/2*(3*A*c-B*b)/b^4*ln(c*x^2+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(89) = 178.

Time = 0.24 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.03

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{2\,(Bb^2c-3\,Abc^2)x^4-2\,Ab^3+3\,(Bb^3-3\,Ab^2c)x^2-2\,((Bbc^2-3\,Ac^3)x^6+2\,(Bb^2c-3\,Abc^2)x^4+(Bb^3-}{4\,(b^4c^2x^6+2\,b^5cx}$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(B*b^2*c - 3*A*b*c^2)*x^4 - 2*A*b^3 + 3*(B*b^3 - 3*A*b^2*c)*x^2 - 2*
((B*b*c^2 - 3*A*c^3)*x^6 + 2*(B*b^2*c - 3*A*b*c^2)*x^4 + (B*b^3 - 3*A*b^2*c
)*x^2)*log(c*x^2 + b) + 4*((B*b*c^2 - 3*A*c^3)*x^6 + 2*(B*b^2*c - 3*A*b*c^2
)*x^4 + (B*b^3 - 3*A*b^2*c)*x^2)*log(x))/(b^4*c^2*x^6 + 2*b^5*c*x^4 + b^6*x
^2)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{-2Ab^2+x^4(-6Ac^2+2Bbc)+x^2(-9Abc+3Bb^2)}{4b^5x^2+8b^4cx^4+4b^3c^2x^6} + \frac{(-3Ac+Bb)\log(x)}{b^4} - \frac{(-3Ac+Bb)\log\left(\frac{b}{c}+x^2\right)}{2b^4}$$

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-2*A*b**2 + x**4*(-6*A*c**2 + 2*B*b*c) + x**2*(-9*A*b*c + 3*B*b**2))/(4*b*
*5*x**2 + 8*b**4*c*x**4 + 4*b**3*c**2*x**6) + (-3*A*c + B*b)*log(x)/b**4 -
(-3*A*c + B*b)*log(b/c + x**2)/(2*b**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.12

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{2\,(Bbc-3\,Ac^2)x^4-2\,Ab^2+3\,(Bb^2-3\,Abc)x^2}{4\,(b^3c^2x^6+2\,b^4cx^4+b^5x^2)} - \frac{(Bb-3\,Ac)\log(cx^2+b)}{2\,b^4} + \frac{(Bb-3\,Ac)\log(x^2)}{2\,b^4}$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(B*b*c - 3*A*c^2)*x^4 - 2*A*b^2 + 3*(B*b^2 - 3*A*b*c)*x^2)/(b^3*c^2*
x^6 + 2*b^4*c*x^4 + b^5*x^2) - 1/2*(B*b - 3*A*c)*log(c*x^2 + b)/b^4 + 1/2*(
B*b - 3*A*c)*log(x^2)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(Bb-3\,Ac)\log(|x|)}{b^4} - \frac{(Bbc-3\,Ac^2)\log(|cx^2+b|)}{2\,b^4c} + \frac{2\,(Bb^2c-3\,Abc^2)x^4-2\,Ab^3+3\,(Bb^3-3\,Ab^2c)x^2}{4\,(cx^2+b)^2b^4x^2}$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] (B*b - 3*A*c)*log(abs(x))/b^4 - 1/2*(B*b*c - 3*A*c^2)*log(abs(c*x^2 + b))/(
b^4*c) + 1/4*(2*(B*b^2*c - 3*A*b*c^2)*x^4 - 2*A*b^3 + 3*(B*b^3 - 3*A*b^2*c)
*x^2)/((c*x^2 + b)^2*b^4*x^2)
```

Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10

$$\int \frac{x^3(A+Bx^2)}{\left(bx^2+cx^4\right)^3}\,dx = \frac{\ln\left(c\,x^2+b\right)\,\left(3\,A\,c-B\,b\right)}{2\,b^4} - \frac{\frac{A}{2\,b}+\frac{3\,x^2\,\left(3\,A\,c-B\,b\right)}{4\,b^2}+\frac{c\,x^4\,\left(3\,A\,c-B\,b\right)}{2\,b^3}}{b^2\,x^2+2\,b\,c\,x^4+c^2\,x^6} - \frac{\ln\left(x\right)\,\left(3\,A\,c-B\,b\right)}{b^4}$$

```
[In] int((x^3*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (log(b + c*x^2)*(3*A*c - B*b))/(2*b^4) - (A/(2*b) + (3*x^2*(3*A*c - B*b))/(
4*b^2) + (c*x^4*(3*A*c - B*b))/(2*b^3))/(b^2*x^2 + c^2*x^6 + 2*b*c*x^4) - (
log(x)*(3*A*c - B*b))/b^4
```

3.85 $\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 117

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{A}{3b^3x^3}-\frac{bB-3Ac}{b^4x}-\frac{c(bB-Ac)x}{4b^3(b+cx^2)^2}-\frac{c(7bB-11Ac)x}{8b^4(b+cx^2)}-\frac{5\sqrt{c}(3bB-7Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}}$$

```
[Out] -1/3*A/b^3/x^3+(3*A*c-B*b)/b^4/x-1/4*c*(-A*c+B*b)*x/b^3/(c*x^2+b)^2-1/8*c*(
-11*A*c+7*B*b)*x/b^4/(c*x^2+b)-5/8*(-7*A*c+3*B*b)*arctan(x*c^(1/2)/b^(1/2))
*c^(1/2)/b^(9/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1598, 467, 1273, 1275, 211}

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{5\sqrt{c}(3bB-7Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}}-\frac{cx(7bB-11Ac)}{8b^4(b+cx^2)}-\frac{bB-3Ac}{b^4x}-\frac{cx(bB-Ac)}{4b^3(b+cx^2)^2}-\frac{A}{3b^3x^3}$$

```
[In] Int[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/3*A/(b^3*x^3) - (b*B - 3*A*c)/(b^4*x) - (c*(b*B - A*c)*x)/(4*b^3*(b + c*
x^2)^2) - (c*(7*b*B - 11*A*c)*x)/(8*b^4*(b + c*x^2)) - (5*Sqrt[c]*(3*b*B -
7*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{A+Bx^2}{x^4\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{c(bB-Ac)x}{4b^3\left(b+cx^2\right)^2} - \frac{1}{4}c\int \frac{-\frac{4A}{bc} - \frac{4(bB-Ac)x^2}{b^2c} + \frac{3(bB-Ac)x^4}{b^3}}{x^4\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{c(bB-Ac)x}{4b^3\left(b+cx^2\right)^2} - \frac{c(7bB-11Ac)x}{8b^4\left(b+cx^2\right)} - \frac{\int \frac{-8Abc-8c(bB-2Ac)x^2+\frac{c^2(7bB-11Ac)x^4}{b}}{x^4(b+cx^2)}\,dx}{8b^3c}$$

$$= -\frac{c(bB-Ac)x}{4b^3\left(b+cx^2\right)^2} - \frac{c(7bB-11Ac)x}{8b^4\left(b+cx^2\right)} - \frac{\int \left(-\frac{8Ac}{x^4}-\frac{8c(bB-3Ac)}{bx^2}+\frac{5c^2(3bB-7Ac)}{b(b+cx^2)}\right)dx}{8b^3c}$$

$$= -\frac{A}{3b^3x^3} - \frac{bB-3Ac}{b^4x} - \frac{c(bB-Ac)x}{4b^3\left(b+cx^2\right)^2} - \frac{c(7bB-11Ac)x}{8b^4\left(b+cx^2\right)} - \frac{(5c(3bB-7Ac))\int \frac{1}{b+cx^2}\,dx}{8b^4}$$

$$= -\frac{A}{3b^3x^3} - \frac{bB-3Ac}{b^4x} - \frac{c(bB-Ac)x}{4b^3\left(b+cx^2\right)^2} - \frac{c(7bB-11Ac)x}{8b^4\left(b+cx^2\right)} - \frac{5\sqrt{c}(3bB-7Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int \frac{x^2(A+Bx^2)}{\left(bx^2+cx^4\right)^3}\,dx = -\frac{A}{3b^3x^3} + \frac{-bB+3Ac}{b^4x} - \frac{c(bB-Ac)x}{4b^3\left(b+cx^2\right)^2} - \frac{\left(7bBc-11Ac^2\right)x}{8b^4\left(b+cx^2\right)} - \frac{5\sqrt{c}(3bB-7Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}}$$

```
[In] Integrate[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/3*A/(b^3*x^3) + (-(b*B) + 3*A*c)/(b^4*x) - (c*(b*B - A*c)*x)/(4*b^3*(b +
 c*x^2)^2) - ((7*b*B*c - 11*A*c^2)*x)/(8*b^4*(b + c*x^2)) - (5*Sqrt[c]*(3*b
*B - 7*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(9/2))
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

method	result
default	$-\frac{A}{3b^3x^3}-\frac{-3Ac+Bb}{b^4x}+\frac{c\left(\frac{\left(\frac{11}{8}Ac^2-\frac{7}{8}Bbc\right)x^3+\frac{b(13Ac-9Bb)x}{8}}{\left(cx^2+b\right)^2}+\frac{5(7Ac-3Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{b^4}$
risch	$\frac{\frac{5c^2(7Ac-3Bb)x^6}{8b^4}+\frac{25c(7Ac-3Bb)x^4}{24b^3}+\frac{(7Ac-3Bb)x^2}{3b^2}-\frac{A}{3b}}{x^3\left(cx^2+b\right)^2}+\frac{5\left(\sum_{\_R=\mathrm{RootOf}\left(b^9\_Z^2+49A^2c^3-42ABbc^2+9B^2b^2c\right)}\_R\ln\left(\left(3\_R^2b^9+98A\right.\right.\right.}{16}$

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*A/b^3/x^3-(-3*A*c+B*b)/b^4/x+1/b^4*c*(((11/8*A*c^2-7/8*B*b*c)*x^3+1/8*
b*(13*A*c-9*B*b)*x)/(c*x^2+b)^2+5/8*(7*A*c-3*B*b)/(b*c)^(1/2)*arctan(c*x/(b
*c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 368, normalized size of antiderivative = 3.15

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$
$$= \left[-\frac{30\,(3\,Bbc^2-7\,Ac^3)x^6+50\,(3\,Bb^2c-7\,Abc^2)x^4+16\,Ab^3+16\,(3\,Bb^3-7\,Ab^2c)x^2+15\,((3\,Bbc^2-7\,A}{48\,(b^4c^2x^7+2\,b^5cx^5+b^6x}\right.$$
$$-\frac{15\,(3\,Bbc^2-7\,Ac^3)x^6+25\,(3\,Bb^2c-7\,Abc^2)x^4+8\,Ab^3+8\,(3\,Bb^3-7\,Ab^2c)x^2+15\,((3\,Bbc^2-7\,Ac^3)}{24\,(b^4c^2x^7+2\,b^5cx^5+b^6x^3)}$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(30*(3*B*b*c^2 - 7*A*c^3)*x^6 + 50*(3*B*b^2*c - 7*A*b*c^2)*x^4 + 16*
A*b^3 + 16*(3*B*b^3 - 7*A*b^2*c)*x^2 + 15*((3*B*b*c^2 - 7*A*c^3)*x^7 + 2*(3
*B*b^2*c - 7*A*b*c^2)*x^5 + (3*B*b^3 - 7*A*b^2*c)*x^3)*sqrt(-c/b)*log((c*x^
2 + 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^
3), -1/24*(15*(3*B*b*c^2 - 7*A*c^3)*x^6 + 25*(3*B*b^2*c - 7*A*b*c^2)*x^4 +
8*A*b^3 + 8*(3*B*b^3 - 7*A*b^2*c)*x^2 + 15*((3*B*b*c^2 - 7*A*c^3)*x^7 + 2*(
3*B*b^2*c - 7*A*b*c^2)*x^5 + (3*B*b^3 - 7*A*b^2*c)*x^3)*sqrt(c/b)*arctan(x*
sqrt(c/b)))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(109) = 218.

Time = 0.40 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.93

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$
$$= \frac{5\sqrt{-\frac{c}{b^9}}(-7Ac+3Bb)\log\left(-\frac{5b^5\sqrt{-\frac{c}{b^9}}(-7Ac+3Bb)}{-35Ac^2+15Bbc}+x\right)}{16} - \frac{5\sqrt{-\frac{c}{b^9}}(-7Ac+3Bb)\log\left(\frac{5b^5\sqrt{-\frac{c}{b^9}}(-7Ac+3Bb)}{-35Ac^2+15Bbc}+x\right)}{16} + \frac{-8Ab^3+x^6\cdot(105Ac^3-45Bbc^2)+x^4\cdot(175Abc^2-75Bb^2c)+x^2\cdot(56Ab^2c-24Bb^3)}{24b^6x^3+48b^5cx^5+24b^4c^2x^7}$$

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] 5*sqrt(-c/b**9)*(-7*A*c + 3*B*b)*log(-5*b**5*sqrt(-c/b**9)*(-7*A*c + 3*B*b)
/(-35*A*c**2 + 15*B*b*c) + x)/16 - 5*sqrt(-c/b**9)*(-7*A*c + 3*B*b)*log(5*b
**5*sqrt(-c/b**9)*(-7*A*c + 3*B*b)/(-35*A*c**2 + 15*B*b*c) + x)/16 + (-8*A*
b**3 + x**6*(105*A*c**3 - 45*B*b*c**2) + x**4*(175*A*b*c**2 - 75*B*b**2*c)
+ x**2*(56*A*b**2*c - 24*B*b**3))/(24*b**6*x**3 + 48*b**5*c*x**5 + 24*b**4*
c**2*x**7)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.09

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$
$$= -\frac{15\,(3\,Bbc^2-7\,Ac^3)x^6+25\,(3\,Bb^2c-7\,Abc^2)x^4+8\,Ab^3+8\,(3\,Bb^3-7\,Ab^2c)x^2}{24\,(b^4c^2x^7+2\,b^5cx^5+b^6x^3)} - \frac{5\,(3\,Bbc-7\,Ac^2)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^4}$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*(15*(3*B*b*c^2 - 7*A*c^3)*x^6 + 25*(3*B*b^2*c - 7*A*b*c^2)*x^4 + 8*A*
b^3 + 8*(3*B*b^3 - 7*A*b^2*c)*x^2)/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3) -
5/8*(3*B*b*c - 7*A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{5\,(3\,Bbc-7\,Ac^2)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^4} - \frac{7\,Bbc^2x^3-11\,Ac^3x^3+9\,Bb^2cx-13\,Abc^2x}{8\,(cx^2+b)^2b^4} - \frac{3\,Bbx^2-9\,Acx^2+Ab}{3\,b^4x^3}$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -5/8*(3*B*b*c - 7*A*c^2)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4) - 1/8*(7*B*b
*c^2*x^3 - 11*A*c^3*x^3 + 9*B*b^2*c*x - 13*A*b*c^2*x)/((c*x^2 + b)^2*b^4) -
 1/3*(3*B*b*x^2 - 9*A*c*x^2 + A*b)/(b^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{x^2\,(7\,A\,c-3\,B\,b)}{3\,b^2}-\frac{A}{3\,b}+\frac{5\,c^2\,x^6\,(7\,A\,c-3\,B\,b)}{8\,b^4}+\frac{25\,c\,x^4\,(7\,A\,c-3\,B\,b)}{24\,b^3}}{b^2\,x^3+2\,b\,c\,x^5+c^2\,x^7} + \frac{5\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(7\,A\,c-3\,B\,b)}{8\,b^{9/2}}$$

```
[In] int((x^2*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^2*(7*A*c - 3*B*b))/(3*b^2) - A/(3*b) + (5*c^2*x^6*(7*A*c - 3*B*b))/(8*b
^4) + (25*c*x^4*(7*A*c - 3*B*b))/(24*b^3))/(b^2*x^3 + c^2*x^7 + 2*b*c*x^5)
+ (5*c^(1/2)*atan((c^(1/2)*x)/b^(1/2))*(7*A*c - 3*B*b))/(8*b^(9/2))
```

3.86 $\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{A}{4b^3x^4} - \frac{bB-3Ac}{2b^4x^2} - \frac{c(bB-Ac)}{4b^3(b+cx^2)^2} - \frac{c(2bB-3Ac)}{2b^4(b+cx^2)} - \frac{3c(bB-2Ac)\log(x)}{b^5} + \frac{3c(bB-2Ac)\log(b+cx^2)}{2b^5}$$

```
[Out] -1/4*A/b^3/x^4+1/2*(3*A*c-B*b)/b^4/x^2-1/4*c*(-A*c+B*b)/b^3/(c*x^2+b)^2-1/2
*c*(-3*A*c+2*B*b)/b^4/(c*x^2+b)-3*c*(-2*A*c+B*b)*ln(x)/b^5+3/2*c*(-2*A*c+B*
b)*ln(c*x^2+b)/b^5
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1598, 457, 78}

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3c(bB-2Ac)\log(b+cx^2)}{2b^5} - \frac{3c\log(x)(bB-2Ac)}{b^5} - \frac{c(2bB-3Ac)}{2b^4(b+cx^2)} - \frac{bB-3Ac}{2b^4x^2} - \frac{c(bB-Ac)}{4b^3(b+cx^2)^2} - \frac{A}{4b^3x^4}$$

```
[In] Int[(x*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*A/(b^3*x^4) - (b*B - 3*A*c)/(2*b^4*x^2) - (c*(b*B - A*c))/(4*b^3*(b +
c*x^2)^2) - (c*(2*b*B - 3*A*c))/(2*b^4*(b + c*x^2)) - (3*c*(b*B - 2*A*c)*Lo
g[x])/b^5 + (3*c*(b*B - 2*A*c)*Log[b + c*x^2])/(2*b^5)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{x^5\left(b+cx^2\right)^3}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^3(b+cx)^3}\,dx,x,x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{b^3x^3}+\frac{bB-3Ac}{b^4x^2}-\frac{3c(bB-2Ac)}{b^5x}+\frac{c^2(bB-Ac)}{b^3(b+cx)^3}+\frac{c^2(2bB-3Ac)}{b^4(b+cx)^2}\right.\right. \\
&\qquad \left.\left.+\frac{3c^2(bB-2Ac)}{b^5(b+cx)}\right)dx,x,x^2\right) \\
&= -\frac{A}{4b^3x^4}-\frac{bB-3Ac}{2b^4x^2}-\frac{c(bB-Ac)}{4b^3\left(b+cx^2\right)^2}-\frac{c(2bB-3Ac)}{2b^4\left(b+cx^2\right)} \\
&\quad -\frac{3c(bB-2Ac)\log(x)}{b^5}+\frac{3c(bB-2Ac)\log\left(b+cx^2\right)}{2b^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{-\frac{Ab^2}{x^4}-\frac{2b(bB-3Ac)}{x^2}+\frac{b^2c(-bB+Ac)}{(b+cx^2)^2}+\frac{2bc(-2bB+3Ac)}{b+cx^2}+12c(-bB+2Ac)\log(x)+6c(bB-2Ac)\log\left(b+cx^2\right)}{4b^5}$$

```
[In] Integrate[(x*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-((A*b^2)/x^4) - (2*b*(b*B - 3*A*c))/x^2 + (b^2*c*(-(b*B) + A*c))/(b + c*x
^2)^2 + (2*b*c*(-2*b*B + 3*A*c))/(b + c*x^2) + 12*c*(-(b*B) + 2*A*c)*Log[x]
 + 6*c*(b*B - 2*A*c)*Log[b + c*x^2])/(4*b^5)
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02

method	result
default	$-\frac{A}{4b^3x^4}-\frac{-3Ac+Bb}{2x^2b^4}+\frac{3c(2Ac-Bb)\ln(x)}{b^5}-\frac{c^2\left(-\frac{b(3Ac-2Bb)}{c\left(c\,x^2+b\right)}+\frac{(6Ac-3Bb)\ln\left(c\,x^2+b\right)}{c}-\frac{b^2(Ac-Bb)}{2c\left(c\,x^2+b\right)^2}\right)}{2b^5}$
norman	$\frac{-\frac{Ax}{4b}+\frac{(2Ac-Bb)x^3}{2b^2}-\frac{c\left(6A\,c^2-3Bbc\right)x^7}{b^4}-\frac{c^2\left(18A\,c^2-9Bbc\right)x^9}{4b^5}}{x^5\left(c\,x^2+b\right)^2}+\frac{3c(2Ac-Bb)\ln(x)}{b^5}-\frac{3c(2Ac-Bb)\ln\left(c\,x^2+b\right)}{2b^5}$
risch	$\frac{\frac{3c^2(2Ac-Bb)x^6}{2b^4}+\frac{9c(2Ac-Bb)x^4}{4b^3}+\frac{(2Ac-Bb)x^2}{2b^2}-\frac{A}{4b}}{x^4\left(c\,x^2+b\right)^2}+\frac{6c^2\ln(x)A}{b^5}-\frac{3c\ln(x)B}{b^4}-\frac{3c^2\ln\left(c\,x^2+b\right)A}{b^5}+\frac{3c\ln\left(c\,x^2+b\right)B}{2b^4}$
parallelrisch	$24A\ln(x)x^8c^4-12A\ln\left(c\,x^2+b\right)x^8c^4-12B\ln(x)x^8b\,c^3+6B\ln\left(c\,x^2+b\right)x^8b\,c^3-18A\,x^8c^4+9B\,x^8b\,c^3+48A\ln(x)x^6b\,c^3-24A\ln(c\,x$

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*A/b^3/x^4-1/2*(-3*A*c+B*b)/x^2/b^4+3*c*(2*A*c-B*b)/b^5*ln(x)-1/2/b^5*c
^2*(-b*(3*A*c-2*B*b)/c/(c*x^2+b)+(6*A*c-3*B*b)/c*ln(c*x^2+b)-1/2*b^2*(A*c-B
*b)/c/(c*x^2+b)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(111) = 222$.

Time = 0.25 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.89

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{6\,(Bb^2c^2-2\,Abc^3)x^6+Ab^4+9\,(Bb^3c-2\,Ab^2c^2)x^4+2\,(Bb^4-2\,Ab^3c)x^2-6\,((Bbc^3-2\,Ac^4)x^8+2\,(B$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*(B*b^2*c^2 - 2*A*b*c^3)*x^6 + A*b^4 + 9*(B*b^3*c - 2*A*b^2*c^2)*x^4
 + 2*(B*b^4 - 2*A*b^3*c)*x^2 - 6*((B*b*c^3 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2 -
2*A*b*c^3)*x^6 + (B*b^3*c - 2*A*b^2*c^2)*x^4)*log(c*x^2 + b) + 12*((B*b*c^3
 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2 - 2*A*b*c^3)*x^6 + (B*b^3*c - 2*A*b^2*c^2)*x
^4)*log(x))/(b^5*c^2*x^8 + 2*b^6*c*x^6 + b^7*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.12

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{-Ab^3+x^6\cdot(12Ac^3-6Bbc^2)+x^4\cdot(18Abc^2-9Bb^2c)+x^2\cdot(4Ab^2c-2Bb^3)}{4b^6x^4+8b^5cx^6+4b^4c^2x^8} - \frac{3c(-2Ac+Bb)\log{(x)}}{b^5} + \frac{3c(-2Ac+Bb)\log{\left(\frac{b}{c}+x^2\right)}}{2b^5}$$

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-A*b**3 + x**6*(12*A*c**3 - 6*B*b*c**2) + x**4*(18*A*b*c**2 - 9*B*b**2*c)
+ x**2*(4*A*b**2*c - 2*B*b**3))/(4*b**6*x**4 + 8*b**5*c*x**6 + 4*b**4*c**2*
x**8) - 3*c*(-2*A*c + B*b)*log(x)/b**5 + 3*c*(-2*A*c + B*b)*log(b/c + x**2)
/(2*b**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.13

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{6\,(Bbc^2-2\,Ac^3)x^6+9\,(Bb^2c-2\,Abc^2)x^4+Ab^3+2\,(Bb^3-2\,Ab^2c)x^2}{4\,(b^4c^2x^8+2\,b^5cx^6+b^6x^4)} + \frac{3\,(Bbc-2\,Ac^2)\log{(cx^2+b)}}{2\,b^5} - \frac{3\,(Bbc-2\,Ac^2)\log{(x^2)}}{2\,b^5}$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*(B*b*c^2 - 2*A*c^3)*x^6 + 9*(B*b^2*c - 2*A*b*c^2)*x^4 + A*b^3 + 2*(
B*b^3 - 2*A*b^2*c)*x^2)/(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4) + 3/2*(B*b*c
- 2*A*c^2)*log(c*x^2 + b)/b^5 - 3/2*(B*b*c - 2*A*c^2)*log(x^2)/b^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.09

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{3\,(Bbc-2\,Ac^2)\log{(|x|)}}{b^5} + \frac{3\,(Bbc^2-2\,Ac^3)\log{(|cx^2+b|)}}{2\,b^5c} - \frac{6\,Bbc^2x^6-12\,Ac^3x^6+9\,Bb^2cx^4-18\,Abc^2x^4+2\,Bb^3x^2-4\,Ab^2cx^2+Ab^3}{4\,(cx^4+bx^2)^2b^4}$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -3*(B*b*c - 2*A*c^2)*log(abs(x))/b^5 + 3/2*(B*b*c^2 - 2*A*c^3)*log(abs(c*x^
2 + b))/(b^5*c) - 1/4*(6*B*b*c^2*x^6 - 12*A*c^3*x^6 + 9*B*b^2*c*x^4 - 18*A*
b*c^2*x^4 + 2*B*b^3*x^2 - 4*A*b^2*c*x^2 + A*b^3)/((c*x^4 + b*x^2)^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.08

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{x^2\,(2\,A\,c-B\,b)}{2\,b^2}-\frac{A}{4\,b}+\frac{3\,c^2\,x^6\,(2\,A\,c-B\,b)}{2\,b^4}+\frac{9\,c\,x^4\,(2\,A\,c-B\,b)}{4\,b^3}}{b^2\,x^4+2\,b\,c\,x^6+c^2\,x^8} - \frac{\ln{(c\,x^2+b)}\,\left(6\,A\,c^2-3\,B\,b\,c\right)}{2\,b^5} + \frac{\ln{(x)}\,\left(6\,A\,c^2-3\,B\,b\,c\right)}{b^5}$$

```
[In] int((x*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^2*(2*A*c - B*b))/(2*b^2) - A/(4*b) + (3*c^2*x^6*(2*A*c - B*b))/(2*b^4)
+ (9*c*x^4*(2*A*c - B*b))/(4*b^3))/(b^2*x^4 + c^2*x^8 + 2*b*c*x^6) - (log(b
 + c*x^2)*(6*A*c^2 - 3*B*b*c))/(2*b^5) + (log(x)*(6*A*c^2 - 3*B*b*c))/b^5
```

3.87 $\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 140

$$\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^3} dx = -\frac{A}{5b^3x^5} - \frac{bB-3Ac}{3b^4x^3} + \frac{3c(bB-2Ac)}{b^5x} + \frac{c^2(bB-Ac)x}{4b^4\left(b+cx^2\right)^2} + \frac{c^2(11bB-15Ac)x}{8b^5\left(b+cx^2\right)} + \frac{7c^{3/2}(5bB-9Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}}$$

```
[Out] -1/5*A/b^3/x^5+1/3*(3*A*c-B*b)/b^4/x^3+3*c*(-2*A*c+B*b)/b^5/x+1/4*c^2*(-A*c
+B*b)*x/b^4/(c*x^2+b)^2+1/8*c^2*(-15*A*c+11*B*b)*x/b^5/(c*x^2+b)+7/8*c^(3/2
)*(-9*A*c+5*B*b)*arctan(x*c^(1/2)/b^(1/2))/b^(11/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1607, 467, 1819, 1816, 211}

$$\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^3} dx = \frac{7c^{3/2}(5bB-9Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}} + \frac{c^2x(11bB-15Ac)}{8b^5\left(b+cx^2\right)} + \frac{3c(bB-2Ac)}{b^5x} + \frac{c^2x(bB-Ac)}{4b^4\left(b+cx^2\right)^2} - \frac{bB-3Ac}{3b^4x^3} - \frac{A}{5b^3x^5}$$

```
[In] Int[(A + B*x^2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/5*A/(b^3*x^5) - (b*B - 3*A*c)/(3*b^4*x^3) + (3*c*(b*B - 2*A*c))/(b^5*x)
+ (c^2*(b*B - A*c)*x)/(4*b^4*(b + c*x^2)^2) + (c^2*(11*b*B - 15*A*c)*x)/(8*
```

```
b^5*(b + c*x^2)) + (7*c^(3/2)*(5*b*B - 9*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/
(8*b^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int \frac{A+Bx^2}{x^6\left(b+cx^2\right)^3}\,dx$$

$$= \frac{c^2(bB-Ac)x}{4b^4\left(b+cx^2\right)^2} - \frac{1}{4}c^2\int \frac{-\frac{4A}{bc^2} - \frac{4(bB-Ac)x^2}{b^2c^2} + \frac{4(bB-Ac)x^4}{b^3c} - \frac{3(bB-Ac)x^6}{b^4}}{x^6\left(b+cx^2\right)^2}\,dx$$

$$= \frac{c^2(bB-Ac)x}{4b^4(b+cx^2)^2} + \frac{c^2(11bB-15Ac)x}{8b^5(b+cx^2)} + \frac{c^2\int \frac{\frac{8A}{bc^2}+\frac{8(bB-2Ac)x^2}{b^2c^2}-\frac{8(2bB-3Ac)x^4}{b^3c}+\frac{(11bB-15Ac)x^6}{b^4}}{x^6(b+cx^2)}dx}{8b}$$

$$= \frac{c^2(bB-Ac)x}{4b^4(b+cx^2)^2} + \frac{c^2(11bB-15Ac)x}{8b^5(b+cx^2)} + \frac{c^2\int \left(\frac{8A}{b^2c^2x^6}+\frac{8(bB-3Ac)}{b^3c^2x^4}-\frac{24(bB-2Ac)}{b^4cx^2}+\frac{7(5bB-9Ac)}{b^4(b+cx^2)}\right)dx}{8b}$$

$$= -\frac{A}{5b^3x^5} - \frac{bB-3Ac}{3b^4x^3} + \frac{3c(bB-2Ac)}{b^5x} + \frac{c^2(bB-Ac)x}{4b^4(b+cx^2)^2} + \frac{c^2(11bB-15Ac)x}{8b^5(b+cx^2)} + \frac{(7c^2(5bB-9Ac))\int \frac{1}{b+cx^2}dx}{8b^5}$$

$$= -\frac{A}{5b^3x^5} - \frac{bB-3Ac}{3b^4x^3} + \frac{3c(bB-2Ac)}{b^5x} + \frac{c^2(bB-Ac)x}{4b^4(b+cx^2)^2} + \frac{c^2(11bB-15Ac)x}{8b^5(b+cx^2)} + \frac{7c^{3/2}(5bB-9Ac)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^3}dx = -\frac{A}{5b^3x^5} - \frac{bB-3Ac}{3b^4x^3} + \frac{3c(bB-2Ac)}{b^5x} + \frac{c^2(bB-Ac)x}{4b^4(b+cx^2)^2} + \frac{c^2(11bB-15Ac)x}{8b^5(b+cx^2)} + \frac{7c^{3/2}(5bB-9Ac)\arctan\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}}$$

```
[In] Integrate[(A + B*x^2)/(b*x^2 + c*x^4)^3,x]
[Out] -1/5*A/(b^3*x^5) - (b*B - 3*A*c)/(3*b^4*x^3) + (3*c*(b*B - 2*A*c))/(b^5*x)
+ (c^2*(b*B - A*c)*x)/(4*b^4*(b + c*x^2)^2) + (c^2*(11*b*B - 15*A*c)*x)/(8*
b^5*(b + c*x^2)) + (7*c^(3/2)*(5*b*B - 9*A*c)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/
(8*b^(11/2))
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.85

method	result
default	$-\frac{A}{5b^3x^5}-\frac{-3Ac+Bb}{3x^3b^4}-\frac{3c(2Ac-Bb)}{b^5x}-\frac{c^2\left(\frac{\left(\frac{15}{8}Ac^2-\frac{11}{8}Bbc\right)x^3+\frac{b(17Ac-13Bb)x}{8}}{\left(cx^2+b\right)^2}+\frac{7(9Ac-5Bb)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{b^5}$
risch	$\frac{-\frac{7c^3(9Ac-5Bb)x^8}{8b^5}-\frac{35c^2(9Ac-5Bb)x^6}{24b^4}-\frac{7c(9Ac-5Bb)x^4}{15b^3}+\frac{(9Ac-5Bb)x^2}{15b^2}-\frac{A}{5b}}{x^5(cx^2+b)^2}+\frac{7\left(\sum_{\_R=\mathrm{RootOf}\left(b^{11}\_Z^2+81A^2c^5-90ABbc^4+25B^2b^2c^3\right)}\right.}{}$

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*A/b^3/x^5-1/3*(-3*A*c+B*b)/x^3/b^4-3*c*(2*A*c-B*b)/b^5/x-1/b^5*c^2*(((
15/8*A*c^2-11/8*B*b*c)*x^3+1/8*b*(17*A*c-13*B*b)*x)/(c*x^2+b)^2+7/8*(9*A*c-
5*B*b)/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 426, normalized size of antiderivative = 3.04

$$\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^3}\,dx$$

$$=\left[\frac{210\,(5\,Bbc^3-9\,Ac^4)x^8+350\,(5\,Bb^2c^2-9\,Abc^3)x^6-48\,Ab^4+112\,(5\,Bb^3c-9\,Ab^2c^2)x^4-16\,(5\,Bb^4-9}{240\,(b^5c^2x}\right.$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(210*(5*B*b*c^3 - 9*A*c^4)*x^8 + 350*(5*B*b^2*c^2 - 9*A*b*c^3)*x^6 -
 48*A*b^4 + 112*(5*B*b^3*c - 9*A*b^2*c^2)*x^4 - 16*(5*B*b^4 - 9*A*b^3*c)*x^
2 - 105*((5*B*b*c^3 - 9*A*c^4)*x^9 + 2*(5*B*b^2*c^2 - 9*A*b*c^3)*x^7 + (5*B
*b^3*c - 9*A*b^2*c^2)*x^5)*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c
*x^2 + b)))/(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5), 1/120*(105*(5*B*b*c^3 -
9*A*c^4)*x^8 + 175*(5*B*b^2*c^2 - 9*A*b*c^3)*x^6 - 24*A*b^4 + 56*(5*B*b^3*c
 - 9*A*b^2*c^2)*x^4 - 8*(5*B*b^4 - 9*A*b^3*c)*x^2 + 105*((5*B*b*c^3 - 9*A*c
^4)*x^9 + 2*(5*B*b^2*c^2 - 9*A*b*c^3)*x^7 + (5*B*b^3*c - 9*A*b^2*c^2)*x^5)*
sqrt(c/b)*arctan(x*sqrt(c/b)))/(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)]
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.86

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^3}\,dx = -\frac{7\sqrt{-\frac{c^3}{b^{11}}}(-9Ac+5Bb)\log\left(-\frac{7b^6\sqrt{-\frac{c^3}{b^{11}}}(-9Ac+5Bb)}{-63Ac^3+35Bbc^2}+x\right)}{16} + \frac{7\sqrt{-\frac{c^3}{b^{11}}}(-9Ac+5Bb)\log\left(\frac{7b^6\sqrt{-\frac{c^3}{b^{11}}}(-9Ac+5Bb)}{-63Ac^3+35Bbc^2}+x\right)}{16} + \frac{-24Ab^4+x^8(-945Ac^4+525Bbc^3)+x^6(-1575Abc^3+875Bb^2c^2)+x^4(-504Ab^2c^2+280Bb^3c)+x^2\cdot(7}{120b^7x^5+240b^6cx^7+120b^5c^2x^9}$$

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] -7*sqrt(-c**3/b**11)*(-9*A*c + 5*B*b)*log(-7*b**6*sqrt(-c**3/b**11)*(-9*A*c
 + 5*B*b)/(-63*A*c**3 + 35*B*b*c**2) + x)/16 + 7*sqrt(-c**3/b**11)*(-9*A*c
+ 5*B*b)*log(7*b**6*sqrt(-c**3/b**11)*(-9*A*c + 5*B*b)/(-63*A*c**3 + 35*B*b
*c**2) + x)/16 + (-24*A*b**4 + x**8*(-945*A*c**4 + 525*B*b*c**3) + x**6*(-1
575*A*b*c**3 + 875*B*b**2*c**2) + x**4*(-504*A*b**2*c**2 + 280*B*b**3*c) +
x**2*(72*A*b**3*c - 40*B*b**4))/(120*b**7*x**5 + 240*b**6*c*x**7 + 120*b**5
*c**2*x**9)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^3}\,dx = \frac{105\,(5\,Bbc^3-9\,Ac^4)x^8+175\,(5\,Bb^2c^2-9\,Abc^3)x^6-24\,Ab^4+56\,(5\,Bb^3c-9\,Ab^2c^2)x^4-8\,(5\,Bb^4-9\,Ab^3c}{120\,(b^5c^2x^9+2\,b^6cx^7+b^7x^5)} + \frac{7\,(5\,Bbc^2-9\,Ac^3)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^5}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/120*(105*(5*B*b*c^3 - 9*A*c^4)*x^8 + 175*(5*B*b^2*c^2 - 9*A*b*c^3)*x^6 -
24*A*b^4 + 56*(5*B*b^3*c - 9*A*b^2*c^2)*x^4 - 8*(5*B*b^4 - 9*A*b^3*c)*x^2)/
(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5) + 7/8*(5*B*b*c^2 - 9*A*c^3)*arctan(c*
x/sqrt(b*c))/(sqrt(b*c)*b^5)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^3}\,dx = \frac{7\,(5\,Bbc^2-9\,Ac^3)\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^5} + \frac{11\,Bbc^3x^3-15\,Ac^4x^3+13\,Bb^2c^2x-17\,Abc^3x}{8\,(cx^2+b)^2b^5} + \frac{45\,Bbcx^4-90\,Ac^2x^4-5\,Bb^2x^2+15\,Abcx^2-3\,Ab^2}{15\,b^5x^5}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 7/8*(5*B*b*c^2 - 9*A*c^3)*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^5) + 1/8*(11*B
*b*c^3*x^3 - 15*A*c^4*x^3 + 13*B*b^2*c^2*x - 17*A*b*c^3*x)/((c*x^2 + b)^2*b
^5) + 1/15*(45*B*b*c*x^4 - 90*A*c^2*x^4 - 5*B*b^2*x^2 + 15*A*b*c*x^2 - 3*A*
b^2)/(b^5*x^5)
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^3}\,dx = -\frac{\frac{A}{5\,b}-\frac{x^2\,(9\,A\,c-5\,B\,b)}{15\,b^2}+\frac{35\,c^2\,x^6\,(9\,A\,c-5\,B\,b)}{24\,b^4}+\frac{7\,c^3\,x^8\,(9\,A\,c-5\,B\,b)}{8\,b^5}+\frac{7\,c\,x^4\,(9\,A\,c-5\,B\,b)}{15\,b^3}}{b^2\,x^5+2\,b\,c\,x^7+c^2\,x^9} - \frac{7\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)\,(9\,A\,c-5\,B\,b)}{8\,b^{11/2}}$$

```
[In] int((A + B*x^2)/(b*x^2 + c*x^4)^3,x)
```

```
[Out] - (A/(5*b) - (x^2*(9*A*c - 5*B*b))/(15*b^2) + (35*c^2*x^6*(9*A*c - 5*B*b))/
(24*b^4) + (7*c^3*x^8*(9*A*c - 5*B*b))/(8*b^5) + (7*c*x^4*(9*A*c - 5*B*b))/
(15*b^3))/(b^2*x^5 + c^2*x^9 + 2*b*c*x^7) - (7*c^(3/2)*atan((c^(1/2)*x)/b^(
1/2))*(9*A*c - 5*B*b))/(8*b^(11/2))
```

3.88 $\int \frac{A+Bx^2}{x(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 148

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^3}\,dx = -\frac{A}{6b^3x^6}-\frac{bB-3Ac}{4b^4x^4}+\frac{3c(bB-2Ac)}{2b^5x^2}+\frac{c^2(bB-Ac)}{4b^4\left(b+cx^2\right)^2}+\frac{c^2(3bB-4Ac)}{2b^5\left(b+cx^2\right)}+\frac{2c^2(3bB-5Ac)\log(x)}{b^6}-\frac{c^2(3bB-5Ac)\log\left(b+cx^2\right)}{b^6}$$

```
[Out] -1/6*A/b^3/x^6+1/4*(3*A*c-B*b)/b^4/x^4+3/2*c*(-2*A*c+B*b)/b^5/x^2+1/4*c^2*(
-A*c+B*b)/b^4/(c*x^2+b)^2+1/2*c^2*(-4*A*c+3*B*b)/b^5/(c*x^2+b)+2*c^2*(-5*A*
c+3*B*b)*ln(x)/b^6-c^2*(-5*A*c+3*B*b)*ln(c*x^2+b)/b^6
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1598, 457, 78}

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^3}\,dx = -\frac{c^2(3bB-5Ac)\log\left(b+cx^2\right)}{b^6}+\frac{2c^2\log(x)(3bB-5Ac)}{b^6}+\frac{c^2(3bB-4Ac)}{2b^5\left(b+cx^2\right)}+\frac{3c(bB-2Ac)}{2b^5x^2}+\frac{c^2(bB-Ac)}{4b^4\left(b+cx^2\right)^2}-\frac{bB-3Ac}{4b^4x^4}-\frac{A}{6b^3x^6}$$

```
[In] Int[(A + B*x^2)/(x*(b*x^2 + c*x^4)^3),x]
```

```
[Out] -1/6*A/(b^3*x^6) - (b*B - 3*A*c)/(4*b^4*x^4) + (3*c*(b*B - 2*A*c))/(2*b^5*x
^2) + (c^2*(b*B - A*c))/(4*b^4*(b + c*x^2)^2) + (c^2*(3*b*B - 4*A*c))/(2*b^
5*(b + c*x^2)) + (2*c^2*(3*b*B - 5*A*c)*Log[x])/b^6 - (c^2*(3*b*B - 5*A*c)*
Log[b + c*x^2])/b^6
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{x^7\left(b+cx^2\right)^3}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^4(b+cx)^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{b^3x^4}+\frac{bB-3Ac}{b^4x^3}-\frac{3c(bB-2Ac)}{b^5x^2}+\frac{2c^2(3bB-5Ac)}{b^6x}-\frac{c^3(bB-Ac)}{b^4(b+cx)^3}\right.\right. \\
&\qquad \left.\left. -\frac{c^3(3bB-4Ac)}{b^5(b+cx)^2}-\frac{2c^3(3bB-5Ac)}{b^6(b+cx)}\right)dx, x, x^2\right) \\
&= -\frac{A}{6b^3x^6}-\frac{bB-3Ac}{4b^4x^4}+\frac{3c(bB-2Ac)}{2b^5x^2}+\frac{c^2(bB-Ac)}{4b^4\left(b+cx^2\right)^2}+\frac{c^2(3bB-4Ac)}{2b^5\left(b+cx^2\right)} \\
&\quad +\frac{2c^2(3bB-5Ac)\log(x)}{b^6}-\frac{c^2(3bB-5Ac)\log\left(b+cx^2\right)}{b^6}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^3}\,dx$$
$$= \frac{-\frac{2Ab^3}{x^6}-\frac{3b^2(bB-3Ac)}{x^4}+\frac{18bc(bB-2Ac)}{x^2}+\frac{3b^2c^2(bB-Ac)}{(b+cx^2)^2}+\frac{6bc^2(3bB-4Ac)}{b+cx^2}+24c^2(3bB-5Ac)\log(x)+12c^2(-3bB+}{12b^6}$$

```
[In] Integrate[(A + B*x^2)/(x*(b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-2*A*b^3)/x^6 - (3*b^2*(b*B - 3*A*c))/x^4 + (18*b*c*(b*B - 2*A*c))/x^2 +
(3*b^2*c^2*(b*B - A*c))/(b + c*x^2)^2 + (6*b*c^2*(3*b*B - 4*A*c))/(b + c*x^
2) + 24*c^2*(3*b*B - 5*A*c)*Log[x] + 12*c^2*(-3*b*B + 5*A*c)*Log[b + c*x^2]
)/(12*b^6)
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.97

method	result
default	$-\frac{A}{6b^3x^6}-\frac{-3Ac+Bb}{4x^4b^4}-\frac{3c(2Ac-Bb)}{2b^5x^2}-\frac{2c^2(5Ac-3Bb)\ln(x)}{b^6}+\frac{c^3\left(\frac{(10Ac-6Bb)\ln\left(c\,x^2+b\right)}{c}-\frac{b^2(Ac-Bb)}{2c\left(c\,x^2+b\right)^2}-\frac{b(4Ac-3Bb)}{c\left(c\,x^2+b\right)}\right)}{2b^6}$
norman	$\frac{-\frac{A}{6b}+\frac{(5Ac-3Bb)x^2}{12b^2}-\frac{c(5Ac-3Bb)x^4}{3b^3}+\frac{2c\left(5A\,c^3-3Bb\,c^2\right)x^8}{b^5}+\frac{c^2\left(15A\,c^3-9Bb\,c^2\right)x^{10}}{2b^6}}{x^6\left(c\,x^2+b\right)^2}+\frac{c^2(5Ac-3Bb)\ln\left(c\,x^2+b\right)}{b^6}-\frac{2c^2(5Ac-3B}{b^6}$
risch	$\frac{-\frac{c^3(5Ac-3Bb)x^8}{b^5}-\frac{3c^2(5Ac-3Bb)x^6}{2b^4}-\frac{c(5Ac-3Bb)x^4}{3b^3}+\frac{(5Ac-3Bb)x^2}{12b^2}-\frac{A}{6b}}{x^6\left(c\,x^2+b\right)^2}-\frac{10c^3\ln(x)A}{b^6}+\frac{6c^2\ln(x)B}{b^5}+\frac{5c^3\ln\left(-c\,x^2-b\right)A}{b^6}-$
parallelrisch	$-\frac{120A\ln(x)x^{10}c^5-60A\ln\left(c\,x^2+b\right)x^{10}c^5-72B\ln(x)x^{10}b\,c^4+36B\ln\left(c\,x^2+b\right)x^{10}b\,c^4-90A\,x^{10}c^5+54B\,x^{10}b\,c^4+240A\ln(x)x^8b\,c}{}$

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*A/b^3/x^6-1/4*(-3*A*c+B*b)/x^4/b^4-3/2*c*(2*A*c-B*b)/b^5/x^2-2*c^2*(5*
A*c-3*B*b)/b^6*ln(x)+1/2/b^6*c^3*((10*A*c-6*B*b)/c*ln(c*x^2+b)-1/2*b^2*(A*c
-B*b)/c/(c*x^2+b)^2-b*(4*A*c-3*B*b)/c/(c*x^2+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.80

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^3}\,dx$$
$$= \frac{12\,(3\,Bb^2c^3 - 5\,Abc^4)x^8 + 18\,(3\,Bb^3c^2 - 5\,Ab^2c^3)x^6 - 2\,Ab^5 + 4\,(3\,Bb^4c - 5\,Ab^3c^2)x^4 - (3\,Bb^5 - 5\,Ab^4c)x}{}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*(3*B*b^2*c^3 - 5*A*b*c^4)*x^8 + 18*(3*B*b^3*c^2 - 5*A*b^2*c^3)*x^6
 - 2*A*b^5 + 4*(3*B*b^4*c - 5*A*b^3*c^2)*x^4 - (3*B*b^5 - 5*A*b^4*c)*x^2 -
12*((3*B*b*c^4 - 5*A*c^5)*x^10 + 2*(3*B*b^2*c^3 - 5*A*b*c^4)*x^8 + (3*B*b^3
*c^2 - 5*A*b^2*c^3)*x^6)*log(c*x^2 + b) + 24*((3*B*b*c^4 - 5*A*c^5)*x^10 +
2*(3*B*b^2*c^3 - 5*A*b*c^4)*x^8 + (3*B*b^3*c^2 - 5*A*b^2*c^3)*x^6)*log(x))/
(b^6*c^2*x^10 + 2*b^7*c*x^8 + b^8*x^6)
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^3}\,dx$$
$$= \frac{-2Ab^4 + x^8(-60Ac^4 + 36Bbc^3) + x^6(-90Abc^3 + 54Bb^2c^2) + x^4(-20Ab^2c^2 + 12Bb^3c) + x^2\cdot(5Ab^3c - 3B}{12b^7x^6 + 24b^6cx^8 + 12b^5c^2x^{10}}$$
$$+ \frac{2c^2(-5Ac + 3Bb)\log{(x)}}{b^6} - \frac{c^2(-5Ac + 3Bb)\log{\left(\frac{b}{c} + x^2\right)}}{b^6}$$

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-2*A*b**4 + x**8*(-60*A*c**4 + 36*B*b*c**3) + x**6*(-90*A*b*c**3 + 54*B*b*
*2*c**2) + x**4*(-20*A*b**2*c**2 + 12*B*b**3*c) + x**2*(5*A*b**3*c - 3*B*b*
*4))/(12*b**7*x**6 + 24*b**6*c*x**8 + 12*b**5*c**2*x**10) + 2*c**2*(-5*A*c
+ 3*B*b)*log(x)/b**6 - c**2*(-5*A*c + 3*B*b)*log(b/c + x**2)/b**6
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx^2}{x\,(bx^2+cx^4)^3}\,dx$$
$$= \frac{12\,(3\,Bbc^3-5\,Ac^4)x^8+18\,(3\,Bb^2c^2-5\,Abc^3)x^6-2\,Ab^4+4\,(3\,Bb^3c-5\,Ab^2c^2)x^4-(3\,Bb^4-5\,Ab^3c)x^2}{12\,(b^5c^2x^{10}+2\,b^6cx^8+b^7x^6)} - \frac{(3\,Bbc^2-5\,Ac^3)\log(cx^2+b)}{b^6} + \frac{(3\,Bbc^2-5\,Ac^3)\log(x^2)}{b^6}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(12*(3*B*b*c^3 - 5*A*c^4)*x^8 + 18*(3*B*b^2*c^2 - 5*A*b*c^3)*x^6 - 2*A
*b^4 + 4*(3*B*b^3*c - 5*A*b^2*c^2)*x^4 - (3*B*b^4 - 5*A*b^3*c)*x^2)/(b^5*c^
2*x^10 + 2*b^6*c*x^8 + b^7*x^6) - (3*B*b*c^2 - 5*A*c^3)*log(c*x^2 + b)/b^6
+ (3*B*b*c^2 - 5*A*c^3)*log(x^2)/b^6
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.36

$$\int \frac{A+Bx^2}{x\,(bx^2+cx^4)^3}\,dx$$
$$= \frac{(3\,Bbc^2-5\,Ac^3)\log(x^2)}{b^6} - \frac{(3\,Bbc^3-5\,Ac^4)\log(|cx^2+b|)}{b^6c} + \frac{18\,Bbc^4x^4-30\,Ac^5x^4+42\,Bb^2c^3x^2-68\,Abc^4x^2+25\,Bb^3c^2-39\,Ab^2c^3}{4\,(cx^2+b)^2b^6} - \frac{66\,Bbc^2x^6-110\,Ac^3x^6-18\,Bb^2cx^4+36\,Abc^2x^4+3\,Bb^3x^2-9\,Ab^2cx^2+2\,Ab^3}{12\,b^6x^6}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] (3*B*b*c^2 - 5*A*c^3)*log(x^2)/b^6 - (3*B*b*c^3 - 5*A*c^4)*log(abs(c*x^2 +
b))/(b^6*c) + 1/4*(18*B*b*c^4*x^4 - 30*A*c^5*x^4 + 42*B*b^2*c^3*x^2 - 68*A*
b*c^4*x^2 + 25*B*b^3*c^2 - 39*A*b^2*c^3)/((c*x^2 + b)^2*b^6) - 1/12*(66*B*b
*c^2*x^6 - 110*A*c^3*x^6 - 18*B*b^2*c*x^4 + 36*A*b*c^2*x^4 + 3*B*b^3*x^2 -
9*A*b^2*c*x^2 + 2*A*b^3)/(b^6*x^6)
```

Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2}{x\,(bx^2+cx^4)^3}\,dx = \frac{\ln\left(c\,x^2+b\right)\,\left(5\,A\,c^3-3\,B\,b\,c^2\right)}{b^6} - \frac{\frac{A}{6\,b}-\frac{x^2\,\left(5\,A\,c-3\,B\,b\right)}{12\,b^2}+\frac{3\,c^2\,x^6\,\left(5\,A\,c-3\,B\,b\right)}{2\,b^4}+\frac{c^3\,x^8\,\left(5\,A\,c-3\,B\,b\right)}{b^5}+\frac{c\,x^4\,\left(5\,A\,c-3\,B\,b\right)}{3\,b^3}}{b^2\,x^6+2\,b\,c\,x^8+c^2\,x^{10}} - \frac{\ln\left(x\right)\,\left(10\,A\,c^3-6\,B\,b\,c^2\right)}{b^6}$$

```
[In] int((A + B*x^2)/(x*(b*x^2 + c*x^4)^3),x)
```

```
[Out] (log(b + c*x^2)*(5*A*c^3 - 3*B*b*c^2))/b^6 - (A/(6*b) - (x^2*(5*A*c - 3*B*b
))/(12*b^2) + (3*c^2*x^6*(5*A*c - 3*B*b))/(2*b^4) + (c^3*x^8*(5*A*c - 3*B*b
))/b^5 + (c*x^4*(5*A*c - 3*B*b))/(3*b^3))/(b^2*x^6 + c^2*x^10 + 2*b*c*x^8)
- (log(x)*(10*A*c^3 - 6*B*b*c^2))/b^6
```

3.89 $\int x^7(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 218

$$\int x^7(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{7b^3(3bB-4Ac)\,(b+2cx^2)\,\sqrt{bx^2+cx^4}}{1024c^5} - \frac{7b^2(3bB-4Ac)\,(bx^2+cx^4)^{3/2}}{384c^4} + \frac{7b(3bB-4Ac)x^2(bx^2+cx^4)^{3/2}}{320c^3} - \frac{(3bB-4Ac)x^4(bx^2+cx^4)^{3/2}}{40c^2} + \frac{Bx^6(bx^2+cx^4)^{3/2}}{12c} - \frac{7b^5(3bB-4Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^{11/2}}$$

```
[Out] -7/384*b^2*(-4*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/c^4+7/320*b*(-4*A*c+3*B*b)*x^
2*(c*x^4+b*x^2)^(3/2)/c^3-1/40*(-4*A*c+3*B*b)*x^4*(c*x^4+b*x^2)^(3/2)/c^2+1
/12*B*x^6*(c*x^4+b*x^2)^(3/2)/c-7/1024*b^5*(-4*A*c+3*B*b)*arctanh(x^2*c^(1/
2)/(c*x^4+b*x^2)^(1/2))/c^(11/2)+7/1024*b^3*(-4*A*c+3*B*b)*(2*c*x^2+b)*(c*x
^4+b*x^2)^(1/2)/c^5
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2059, 808, 684, 654, 626, 634, 212}

$$\int x^7(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = -\frac{7b^5(3bB-4Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^{11/2}} + \frac{7b^3(b+2cx^2)\sqrt{bx^2+cx^4}(3bB-4Ac)}{1024c^5} - \frac{7b^2(bx^2+cx^4)^{3/2}(3bB-4Ac)}{384c^4} + \frac{7bx^2(bx^2+cx^4)^{3/2}(3bB-4Ac)}{320c^3} - \frac{x^4(bx^2+cx^4)^{3/2}(3bB-4Ac)}{40c^2} + \frac{Bx^6(bx^2+cx^4)^{3/2}}{12c}$$

```
[In] Int[x^7*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (7*b^3*(3*b*B - 4*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(1024*c^5) - (7*b
^2*(3*b*B - 4*A*c)*(b*x^2 + c*x^4)^(3/2))/(384*c^4) + (7*b*(3*b*B - 4*A*c)*
x^2*(b*x^2 + c*x^4)^(3/2))/(320*c^3) - ((3*b*B - 4*A*c)*x^4*(b*x^2 + c*x^4)
^(3/2))/(40*c^2) + (B*x^6*(b*x^2 + c*x^4)^(3/2))/(12*c) - (7*b^5*(3*b*B - 4
*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(1024*c^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x^3(A+Bx)\sqrt{bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{Bx^6\left(bx^2+cx^4\right)^{3/2}}{12c}+\frac{\left(3(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int x^3\sqrt{bx+cx^2}\,dx,x,x^2\right)}{12c}\\
&= -\frac{(3bB-4Ac)x^4\left(bx^2+cx^4\right)^{3/2}}{40c^2}+\frac{Bx^6\left(bx^2+cx^4\right)^{3/2}}{12c}\\
&\quad+\frac{(7b(3bB-4Ac))\text{Subst}\left(\int x^2\sqrt{bx+cx^2}\,dx,x,x^2\right)}{80c^2}\\
&= \frac{7b(3bB-4Ac)x^2\left(bx^2+cx^4\right)^{3/2}}{320c^3}-\frac{(3bB-4Ac)x^4\left(bx^2+cx^4\right)^{3/2}}{40c^2}\\
&\quad+\frac{Bx^6\left(bx^2+cx^4\right)^{3/2}}{12c}-\frac{\left(7b^2(3bB-4Ac)\right)\text{Subst}\left(\int x\sqrt{bx+cx^2}\,dx,x,x^2\right)}{128c^3}
\end{aligned}$$

$$= -\frac{7b^2(3bB-4Ac)\left(bx^2+cx^4\right)^{3/2}}{384c^4}+\frac{7b(3bB-4Ac)x^2\left(bx^2+cx^4\right)^{3/2}}{320c^3} - \frac{(3bB-4Ac)x^4\left(bx^2+cx^4\right)^{3/2}}{40c^2}+\frac{Bx^6\left(bx^2+cx^4\right)^{3/2}}{12c} + \frac{\left(7b^3(3bB-4Ac)\right)\mathrm{Subst}\left(\int\sqrt{bx+cx^2}\,dx,x,x^2\right)}{256c^4}$$

$$= \frac{7b^3(3bB-4Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^5}-\frac{7b^2(3bB-4Ac)\left(bx^2+cx^4\right)^{3/2}}{384c^4} + \frac{7b(3bB-4Ac)x^2\left(bx^2+cx^4\right)^{3/2}}{320c^3}-\frac{(3bB-4Ac)x^4\left(bx^2+cx^4\right)^{3/2}}{40c^2} + \frac{Bx^6\left(bx^2+cx^4\right)^{3/2}}{12c}-\frac{\left(7b^5(3bB-4Ac)\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{2048c^5}$$

$$= \frac{7b^3(3bB-4Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^5}-\frac{7b^2(3bB-4Ac)\left(bx^2+cx^4\right)^{3/2}}{384c^4} + \frac{7b(3bB-4Ac)x^2\left(bx^2+cx^4\right)^{3/2}}{320c^3}-\frac{(3bB-4Ac)x^4\left(bx^2+cx^4\right)^{3/2}}{40c^2} + \frac{Bx^6\left(bx^2+cx^4\right)^{3/2}}{12c}-\frac{\left(7b^5(3bB-4Ac)\right)\mathrm{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^5}$$

$$= \frac{7b^3(3bB-4Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^5}-\frac{7b^2(3bB-4Ac)\left(bx^2+cx^4\right)^{3/2}}{384c^4} + \frac{7b(3bB-4Ac)x^2\left(bx^2+cx^4\right)^{3/2}}{320c^3}-\frac{(3bB-4Ac)x^4\left(bx^2+cx^4\right)^{3/2}}{40c^2} + \frac{Bx^6\left(bx^2+cx^4\right)^{3/2}}{12c}-\frac{7b^5(3bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^{11/2}}$$

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.87

$$\int x^7\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx$$

$$= \frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}(315b^5B-210b^4c(2A+Bx^2)+64bc^4x^6(3A+2Bx^2)+56b^3c^2x^2(5A+3Bx^2)+256c^5\right.}{15360c^{11/2}}$$

```
[In] Integrate[x^7*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*(315*b^5*B - 210*b^4*c*(2*A + B*x^2) + 64*b
*c^4*x^6*(3*A + 2*B*x^2) + 56*b^3*c^2*x^2*(5*A + 3*B*x^2) + 256*c^5*x^8*(6*
A + 5*B*x^2) - 16*b^2*c^3*x^4*(14*A + 9*B*x^2)) + (210*b^5*(3*b*B - 4*A*c)*
ArcTanh[(Sqrt[c]*x)/(Sqrt[b] - Sqrt[b + c*x^2])])/(x*Sqrt[b + c*x^2])))/(15
360*c^(11/2))
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$\frac{\frac{7\left(A\,b^{5} c-\frac{3}{4} B\,b^{6}\right) \ln\left(\frac{2 c\,x^{2}+2 \sqrt{x^{2} \left(c\,x^{2}+b\right)}\, \sqrt{c}+b}{\sqrt{c}}\right)}{512}+\frac{7\left(\frac{192 x^{8} \left(\frac{5 x^{2} B}{6}+A\right) c^{\frac{11}{2}}}{35}+\left(-\frac{3\left(\frac{x^{2} B}{2}+A\right) b^{3} c^{\frac{3}{2}}}{2}+b^{2} x^{2} \left(\frac{3 x^{2} B}{5}+A\right) c^{\frac{5}{2}}-\frac{4 x^{4} \left(\frac{9 x^{2}}{1}\right.}{}\right.\right.}{384}}{c^{\frac{11}{2}}}$
risch	$-\frac{\left(-1280 B\,c^{5} x^{10}-1536 A\,c^{5} x^{8}-128 B b\,c^{4} x^{8}-192 A b\,c^{4} x^{6}+144 B\,b^{2} c^{3} x^{6}+224 A\,b^{2} c^{3} x^{4}-168 B\,b^{3} c^{2} x^{4}-280 A\,b^{3} c^{2} x^{2}+210 B\,b^{4}\right.}{15360 c^{5}}$
default	$\frac{\sqrt{x^{4} c+b\,x^{2}}\,\left(1280 B\,c^{\frac{9}{2}} \left(c\,x^{2}+b\right)^{\frac{3}{2}} x^{9}+1536 A\,c^{\frac{9}{2}} \left(c\,x^{2}+b\right)^{\frac{3}{2}} x^{7}-1152 B\,c^{\frac{7}{2}} \left(c\,x^{2}+b\right)^{\frac{3}{2}} b\,x^{7}-1344 A\,c^{\frac{7}{2}} \left(c\,x^{2}+b\right)^{\frac{3}{2}} b\,x^{5}+1008 B\,c^{\frac{5}{2}}\right.}{}$

```
[In] int(x^7*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 7/384*(3/4*(A*b^5*c-3/4*B*b^6)*ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2)*c^(1/2)+
b)/c^(1/2))+(192/35*x^8*(5/6*x^2*B+A)*c^(11/2)+(-3/2*(1/2*x^2*B+A)*b^3*c^(3
/2)+b^2*x^2*(3/5*x^2*B+A)*c^(5/2)-4/5*x^4*(9/14*x^2*B+A)*b*c^(7/2)+24/35*x^
6*(2/3*x^2*B+A)*c^(9/2)+9/8*B*c^(1/2)*b^4)*b)*(x^2*(c*x^2+b))^(1/2)-3/4*(A*
c-3/4*B*b)*ln(2)*b^5)/c^(11/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.69

$$\int x^{7}\left(A+Bx^{2}\right)\sqrt{bx^{2}+cx^{4}}\,dx$$

$$=\left[-\frac{105\,(3\,Bb^{6}-4\,Ab^{5}c)\sqrt{c}\log\left(-2\,cx^{2}-b-2\,\sqrt{cx^{4}+bx^{2}}\sqrt{c}\right)-2\,(1280\,Bc^{6}x^{10}+128\,(Bbc^{5}+12\,Ac^{6})x^{8}}{}\right.$$

```
[In] integrate(x^7*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/30720*(105*(3*B*b^6 - 4*A*b^5*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^
4 + b*x^2)*sqrt(c)) - 2*(1280*B*c^6*x^10 + 128*(B*b*c^5 + 12*A*c^6)*x^8 + 3
15*B*b^5*c - 420*A*b^4*c^2 - 48*(3*B*b^2*c^4 - 4*A*b*c^5)*x^6 + 56*(3*B*b^3
*c^3 - 4*A*b^2*c^4)*x^4 - 70*(3*B*b^4*c^2 - 4*A*b^3*c^3)*x^2)*sqrt(c*x^4 +
b*x^2))/c^6, 1/15360*(105*(3*B*b^6 - 4*A*b^5*c)*sqrt(-c)*arctan(sqrt(c*x^4
+ b*x^2)*sqrt(-c)/(c*x^2 + b)) + (1280*B*c^6*x^10 + 128*(B*b*c^5 + 12*A*c^6
)*x^8 + 315*B*b^5*c - 420*A*b^4*c^2 - 48*(3*B*b^2*c^4 - 4*A*b*c^5)*x^6 + 56
*(3*B*b^3*c^3 - 4*A*b^2*c^4)*x^4 - 70*(3*B*b^4*c^2 - 4*A*b^3*c^3)*x^2)*sqrt
(c*x^4 + b*x^2))/c^6]
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.58

$$\int x^7\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx$$

$$=\frac{A\left(\begin{cases}\frac{7b^5\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{256c^4}+\sqrt{bx^2+cx^4}\left(-\frac{7b^4}{128c^4}+\frac{7b^3x^2}{192c^3}-\frac{7b^2x^4}{240c^2}+\frac{bx^6}{40c}+\frac{x^8}{5}\right) & \text{for } c\\ \frac{2\left(bx^2\right)^{\frac{9}{2}}}{9b^4} & \text{for } b\\ 0 & \text{othe}\end{cases}\right.}{2}$$

$$+\frac{B\left(\begin{cases}-\frac{21b^6\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{1024c^5}+\sqrt{bx^2+cx^4}\cdot\left(\frac{21b^5}{512c^5}-\frac{7b^4x^2}{256c^4}+\frac{7b^3x^4}{320c^3}-\frac{3b^2x^6}{160c^2}+\frac{bx^8}{60c}\right.\\ \frac{2\left(bx^2\right)^{\frac{11}{2}}}{11b^5}\\ 0\end{cases}\right.}{2}$$

```
[In] integrate(x**7*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] A*Piecewise((7*b**5*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*
c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt
(c*(b/(2*c) + x**2)**2), True))/(256*c**4) + sqrt(b*x**2 + c*x**4)*(-7*b**4
/(128*c**4) + 7*b**3*x**2/(192*c**3) - 7*b**2*x**4/(240*c**2) + b*x**6/(40*
c) + x**8/5), Ne(c, 0)), (2*(b*x**2)**(9/2)/(9*b**4), Ne(b, 0)), (0, True))
/2 + B*Piecewise((-21*b**6*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**
4) + 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**
2)/sqrt(c*(b/(2*c) + x**2)**2), True))/(1024*c**5) + sqrt(b*x**2 + c*x**4)*
(21*b**5/(512*c**5) - 7*b**4*x**2/(256*c**4) + 7*b**3*x**4/(320*c**3) - 3*b
**2*x**6/(160*c**2) + b*x**8/(60*c) + x**10/6), Ne(c, 0)), (2*(b*x**2)**(11
/2)/(11*b**5), Ne(b, 0)), (0, True))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.47

$$\int x^7 (A + Bx^2) \sqrt{bx^2 + cx^4} \, dx$$
$$= \frac{1}{7680} \left(\frac{768 \, (cx^4 + bx^2)^{\frac{3}{2}} x^4}{c} - \frac{420 \sqrt{cx^4 + bx^2} b^3 x^2}{c^3} - \frac{672 \, (cx^4 + bx^2)^{\frac{3}{2}} bx^2}{c^2} + \frac{105 \, b^5 \log \left(2 \, cx^2 + b + 2 \sqrt{cx^4}\right.}{c^{\frac{9}{2}}}\right.$$
$$+ \frac{1}{30720} \left(\frac{2560 \, (cx^4 + bx^2)^{\frac{3}{2}} x^6}{c} - \frac{2304 \, (cx^4 + bx^2)^{\frac{3}{2}} bx^4}{c^2} + \frac{1260 \sqrt{cx^4 + bx^2} b^4 x^2}{c^4} + \frac{2016 \, (cx^4 + bx^2)^{\frac{3}{2}} b^2 x^2}{c^3}\right.$$

```
[In] integrate(x^7*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7680*(768*(c*x^4 + b*x^2)^(3/2)*x^4/c - 420*sqrt(c*x^4 + b*x^2)*b^3*x^2/c
^3 - 672*(c*x^4 + b*x^2)^(3/2)*b*x^2/c^2 + 105*b^5*log(2*c*x^2 + b + 2*sqrt
(c*x^4 + b*x^2)*sqrt(c))/c^(9/2) - 210*sqrt(c*x^4 + b*x^2)*b^4/c^4 + 560*(c
*x^4 + b*x^2)^(3/2)*b^2/c^3)*A + 1/30720*(2560*(c*x^4 + b*x^2)^(3/2)*x^6/c
- 2304*(c*x^4 + b*x^2)^(3/2)*b*x^4/c^2 + 1260*sqrt(c*x^4 + b*x^2)*b^4*x^2/c
^4 + 2016*(c*x^4 + b*x^2)^(3/2)*b^2*x^2/c^3 - 315*b^6*log(2*c*x^2 + b + 2*s
qrt(c*x^4 + b*x^2)*sqrt(c))/c^(11/2) + 630*sqrt(c*x^4 + b*x^2)*b^5/c^5 - 16
80*(c*x^4 + b*x^2)^(3/2)*b^3/c^4)*B
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.12

$$\int x^7 (A + Bx^2) \sqrt{bx^2 + cx^4} \, dx$$
$$= \frac{1}{15360} \left(2 \left(4 \left(2 \left(8 \left(10 \, Bx^2 \text{sgn}(x) + \frac{Bbc^9 \text{sgn}(x) + 12 \, Ac^{10} \text{sgn}(x)}{c^{10}} \right) x^2 - \frac{3 \, (3 \, Bb^2 c^8 \text{sgn}(x) - 4 \, Abc^9 \text{sgn}(x)}{c^{10}}\right.\right.\right.\right.$$
$$+ \frac{7 \, (3 \, Bb^6 \text{sgn}(x) - 4 \, Ab^5 c \text{sgn}(x)) \log \left(\left| -\sqrt{c} x + \sqrt{cx^2 + b} \right| \right)}{1024 \, c^{\frac{11}{2}}}$$
$$- \frac{7 \, (3 \, Bb^6 \log (|b|) - 4 \, Ab^5 c \log (|b|)) \text{sgn}(x)}{2048 \, c^{\frac{11}{2}}}$$

```
[In] integrate(x^7*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15360*(2*(4*(2*(8*(10*B*x^2*sgn(x) + (B*b*c^9*sgn(x) + 12*A*c^10*sgn(x))/
c^10)*x^2 - 3*(3*B*b^2*c^8*sgn(x) - 4*A*b*c^9*sgn(x))/c^10)*x^2 + 7*(3*B*b^
3*c^7*sgn(x) - 4*A*b^2*c^8*sgn(x))/c^10)*x^2 - 35*(3*B*b^4*c^6*sgn(x) - 4*A
*b^3*c^7*sgn(x))/c^10)*x^2 + 105*(3*B*b^5*c^5*sgn(x) - 4*A*b^4*c^6*sgn(x))/
```

```
c^10)*sqrt(c*x^2 + b)*x + 7/1024*(3*B*b^6*sgn(x) - 4*A*b^5*c*sgn(x))*log(ab
s(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(11/2) - 7/2048*(3*B*b^6*log(abs(b)) - 4
*A*b^5*c*log(abs(b)))*sgn(x)/c^(11/2)
```

Mupad [B] (verification not implemented)

Time = 10.30 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.33

$$\int x^7\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx=\frac{A\,x^4\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{10\,c}+\frac{B\,x^6\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{12\,c}-\frac{3\,B\,b\,\left(\frac{7\,b\,\left(\frac{5\,b\,\left(\frac{b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,|x|\,\sqrt{c\,x^2+b}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{24\,c^2}\right)}{8\,c}-\frac{x^2\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{4\,c}\right)}{10\,c}+\frac{x^4\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{5\,c}\right)}{8\,c}+\frac{7\,A\,b\,\left(\frac{5\,b\,\left(\frac{b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,|x|\,\sqrt{c\,x^2+b}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{24\,c^2}\right)}{8\,c}-\frac{x^2\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{4\,c}\right)}{20\,c}$$

```
[In] int(x^7*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (A*x^4*(b*x^2 + c*x^4)^(3/2))/(10*c) + (B*x^6*(b*x^2 + c*x^4)^(3/2))/(12*c)
 - (3*B*b*((7*b*((5*b*((b^3*log(b + 2*c*x^2 + 2*c^(1/2)*abs(x)*(b + c*x^2)^
(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(8*c^2*x^4 - 3*b^2 + 2*b*c*x^
2))/(24*c^2)))/(8*c) - (x^2*(b*x^2 + c*x^4)^(3/2))/(4*c)))/(10*c) + (x^4*(b
*x^2 + c*x^4)^(3/2))/(5*c)))/(8*c) + (7*A*b*((5*b*((b^3*log(b + 2*c*x^2 + 2
*c^(1/2)*abs(x)*(b + c*x^2)^(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(
8*c^2*x^4 - 3*b^2 + 2*b*c*x^2))/(24*c^2)))/(8*c) - (x^2*(b*x^2 + c*x^4)^(3/
2))/(4*c)))/(20*c)
```

3.90 $\int x^5(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 181

$$\int x^5\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx=-\frac{b^2(7bB-10Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^4}+\frac{b(7bB-10Ac)\left(bx^2+cx^4\right)^{3/2}}{96c^3}-\frac{(7bB-10Ac)x^2\left(bx^2+cx^4\right)^{3/2}}{80c^2}+\frac{Bx^4\left(bx^2+cx^4\right)^{3/2}}{10c}+\frac{b^4(7bB-10Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{9/2}}$$

```
[Out] 1/96*b*(-10*A*c+7*B*b)*(c*x^4+b*x^2)^(3/2)/c^3-1/80*(-10*A*c+7*B*b)*x^2*(c*
x^4+b*x^2)^(3/2)/c^2+1/10*B*x^4*(c*x^4+b*x^2)^(3/2)/c+1/256*b^4*(-10*A*c+7*
B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(9/2)-1/256*b^2*(-10*A*c+7*
B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^4
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used

$= \{2059, 808, 684, 654, 626, 634, 212\}$

$$\int x^5 (A + Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{b^4(7bB-10Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{9/2}} - \frac{b^2(b+2cx^2)\sqrt{bx^2+cx^4}(7bB-10Ac)}{256c^4} + \frac{b(bx^2+cx^4)^{3/2}(7bB-10Ac)}{96c^3} - \frac{x^2(bx^2+cx^4)^{3/2}(7bB-10Ac)}{80c^2} + \frac{Bx^4(bx^2+cx^4)^{3/2}}{10c}$$

```
[In] Int[x^5*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -1/256*(b^2*(7*b*B - 10*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/c^4 + (b*(7
*b*B - 10*A*c)*(b*x^2 + c*x^4)^(3/2))/(96*c^3) - ((7*b*B - 10*A*c)*x^2*(b*x
^2 + c*x^4)^(3/2))/(80*c^2) + (B*x^4*(b*x^2 + c*x^4)^(3/2))/(10*c) + (b^4*(
7*b*B - 10*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(256*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
```

```
1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int x^2(A+Bx)\sqrt{bx+cx^2}\,dx,x,x^2\right)$$

$$= \frac{Bx^4(bx^2+cx^4)^{3/2}}{10c} + \frac{\left(2(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int x^2\sqrt{bx+cx^2}\,dx,x,x^2\right)}{10c}$$

$$= -\frac{(7bB-10Ac)x^2(bx^2+cx^4)^{3/2}}{80c^2} + \frac{Bx^4(bx^2+cx^4)^{3/2}}{10c} + \frac{(b(7bB-10Ac))\text{Subst}\left(\int x\sqrt{bx+cx^2}\,dx,x,x^2\right)}{32c^2}$$

$$= \frac{b(7bB-10Ac)\,(bx^2+cx^4)^{3/2}}{96c^3} - \frac{(7bB-10Ac)x^2(bx^2+cx^4)^{3/2}}{80c^2} + \frac{Bx^4(bx^2+cx^4)^{3/2}}{10c} - \frac{(b^2(7bB-10Ac))\,\text{Subst}\left(\int\sqrt{bx+cx^2}\,dx,x,x^2\right)}{64c^3}$$

$$= -\frac{b^2(7bB-10Ac)\,(b+2cx^2)\sqrt{bx^2+cx^4}}{256c^4} + \frac{b(7bB-10Ac)\,(bx^2+cx^4)^{3/2}}{96c^3} - \frac{(7bB-10Ac)x^2(bx^2+cx^4)^{3/2}}{80c^2} + \frac{Bx^4(bx^2+cx^4)^{3/2}}{10c} + \frac{(b^4(7bB-10Ac))\,\text{Subst}\left(\int\frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{512c^4}$$

$$= -\frac{b^2(7bB-10Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^4}+\frac{b(7bB-10Ac)\left(bx^2+cx^4\right)^{3/2}}{96c^3}$$
$$-\frac{(7bB-10Ac)x^2(bx^2+cx^4)^{3/2}}{80c^2}+\frac{Bx^4(bx^2+cx^4)^{3/2}}{10c}$$
$$+\frac{(b^4(7bB-10Ac))\operatorname{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^4}$$
$$= -\frac{b^2(7bB-10Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^4}$$
$$+\frac{b(7bB-10Ac)\left(bx^2+cx^4\right)^{3/2}}{96c^3}-\frac{(7bB-10Ac)x^2(bx^2+cx^4)^{3/2}}{80c^2}$$
$$+\frac{Bx^4(bx^2+cx^4)^{3/2}}{10c}+\frac{b^4(7bB-10Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{9/2}}$$

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.94

$$\int x^5\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx$$
$$=\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}x(-105b^4B+16bc^3x^4(5A+3Bx^2)+96c^4x^6(5A+4Bx^2)+10b^3c(15A+7Bx^2)-4b^2\right.}{3840c^{9/2}x}$$

```
[In] Integrate[x^5*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*(-105*b^4*B + 16*b*c^3*x^4*(5*A + 3*B*x^2
) + 96*c^4*x^6*(5*A + 4*B*x^2) + 10*b^3*c*(15*A + 7*B*x^2) - 4*b^2*c^2*x^2*
(25*A + 14*B*x^2)) + (30*b^4*(7*b*B - 10*A*c)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[b]
 + Sqrt[b + c*x^2])])/Sqrt[b + c*x^2]))/(3840*c^(9/2)*x)
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{\frac{5\left(-\frac{1}{2}Ab^4c+\frac{7}{20}b^5B\right)\ln\left(\frac{2cx^2+2\sqrt{x^2(cx^2+b)}\sqrt{c}+b}{\sqrt{c}}\right)}{128}+\frac{5\left(b^3\left(\frac{7x^2B}{15}+A\right)c^{\frac{3}{2}}-\frac{2x^2\left(\frac{14x^2B}{25}+A\right)b^2c^{\frac{5}{2}}}{3}+\frac{8x^4b\left(\frac{3x^2B}{5}+A\right)c^{\frac{7}{2}}}{15}+\frac{16x^6\left(4\right.}{}\right.}{128}}{c^{\frac{9}{2}}}$
risch	$\frac{(384Bx^8c^4+480Ax^6c^4+48Bx^6bc^3+80Ax^4bc^3-56Bx^4b^2c^2-100Ax^2b^2c^2+70Bx^2b^3c+150Ab^3c-105Bb^4)\sqrt{x^2(cx^2+b)}}{3840c^4}$
default	$\sqrt{x^4c+bx^2}\left(384B(cx^2+b)^{\frac{3}{2}}c^{\frac{7}{2}}x^7+480A(cx^2+b)^{\frac{3}{2}}c^{\frac{7}{2}}x^5-336B(cx^2+b)^{\frac{3}{2}}c^{\frac{5}{2}}bx^5-400A(cx^2+b)^{\frac{3}{2}}c^{\frac{5}{2}}bx^3+280B(cx^2+b)^{\frac{3}{2}}\right.$

```
[In] int(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/128/c^(9/2)*((-1/2*A*b^4*c+7/20*b^5*B)*ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2
)*c^(1/2)+b)/c^(1/2))+(b^3*(7/15*x^2*B+A)*c^(3/2)-2/3*x^2*(14/25*x^2*B+A)*b
^2*c^(5/2)+8/15*x^4*b*(3/5*x^2*B+A)*c^(7/2)+16/5*x^6*(4/5*x^2*B+A)*c^(9/2)-
7/10*B*c^(1/2)*b^4)*(x^2*(c*x^2+b))^(1/2)+1/2*ln(2)*(A*c-7/10*B*b)*b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.77

$$\int x^5\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx$$
$$=\left[-\frac{15\,(7\,Bb^5-10\,Ab^4c)\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,(384\,Bc^5x^8+48\,(Bbc^4+10\,Ac^5)x^6-1}{7680\,c^5}\right.$$
$$-\frac{15\,(7\,Bb^5-10\,Ab^4c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-(384\,Bc^5x^8+48\,(Bbc^4+10\,Ac^5)x^6-105\,Bb^4c+150}{3840\,c^5}$$

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(7*B*b^5 - 10*A*b^4*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4
 + b*x^2)*sqrt(c)) - 2*(384*B*c^5*x^8 + 48*(B*b*c^4 + 10*A*c^5)*x^6 - 105*B
*b^4*c + 150*A*b^3*c^2 - 8*(7*B*b^2*c^3 - 10*A*b*c^4)*x^4 + 10*(7*B*b^3*c^2
 - 10*A*b^2*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^5, -1/3840*(15*(7*B*b^5 - 10*A
*b^4*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (384*B*
c^5*x^8 + 48*(B*b*c^4 + 10*A*c^5)*x^6 - 105*B*b^4*c + 150*A*b^3*c^2 - 8*(7*
B*b^2*c^3 - 10*A*b*c^4)*x^4 + 10*(7*B*b^3*c^2 - 10*A*b^2*c^3)*x^2)*sqrt(c*x
^4 + b*x^2))/c^5]
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.76

$$\int x^5(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$

$$= \frac{A\left(\begin{cases} -\frac{5b^4\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{128c^3}+\sqrt{bx^2+cx^4}\cdot\left(\frac{5b^3}{64c^3}-\frac{5b^2x^2}{96c^2}+\frac{bx^4}{24c}+\frac{x^6}{4}\right) & \text{for } c\neq 0\\ \frac{2(bx^2)^{\frac{7}{2}}}{7b^3} & \text{for } b\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{2}$$

$$+ \frac{B\left(\begin{cases} \frac{7b^5\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{256c^4}+\sqrt{bx^2+cx^4}\left(-\frac{7b^4}{128c^4}+\frac{7b^3x^2}{192c^3}-\frac{7b^2x^4}{240c^2}+\frac{bx^6}{40c}+\frac{x^8}{5}\right) & \text{fo}\\ \frac{2(bx^2)^{\frac{9}{2}}}{9b^4} & \text{fo}\\ 0 & \text{ot}\end{cases}\right)}{2}$$

```
[In] integrate(x**5*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] A*Piecewise((-5*b**4*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2
*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqr
t(c*(b/(2*c) + x**2)**2), True))/(128*c**3) + sqrt(b*x**2 + c*x**4)*(5*b**3
/(64*c**3) - 5*b**2*x**2/(96*c**2) + b*x**4/(24*c) + x**6/4), Ne(c, 0)), (2
*(b*x**2)**(7/2)/(7*b**3), Ne(b, 0)), (0, True))/2 + B*Piecewise((7*b**5*Pi
ecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*c*x**2)/sqrt(c), Ne(b*
*2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt(c*(b/(2*c) + x**2)**2
), True))/(256*c**4) + sqrt(b*x**2 + c*x**4)*(-7*b**4/(128*c**4) + 7*b**3*x
**2/(192*c**3) - 7*b**2*x**4/(240*c**2) + b*x**6/(40*c) + x**8/5), Ne(c, 0)
), (2*(b*x**2)**(9/2)/(9*b**4), Ne(b, 0)), (0, True))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.51

$$\int x^5(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$
$$=\frac{1}{768}\left(\frac{60\sqrt{cx^4+bx^2}b^2x^2}{c^2}+\frac{96\,(cx^4+bx^2)^{\frac{3}{2}}x^2}{c}-\frac{15\,b^4\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{7}{2}}}+\frac{30\,\sqrt{cx^4+bx^2}b}{c^3}\right.$$
$$+\frac{1}{7680}\left(\frac{768\,(cx^4+bx^2)^{\frac{3}{2}}x^4}{c}-\frac{420\,\sqrt{cx^4+bx^2}b^3x^2}{c^3}-\frac{672\,(cx^4+bx^2)^{\frac{3}{2}}bx^2}{c^2}+\frac{105\,b^5\log\left(2\,cx^2+b+2\,\sqrt{cx}\right.}{c^{\frac{9}{2}}}\right.$$

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/768*(60*sqrt(c*x^4 + b*x^2)*b^2*x^2/c^2 + 96*(c*x^4 + b*x^2)^(3/2)*x^2/c
- 15*b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 30*sqrt
(c*x^4 + b*x^2)*b^3/c^3 - 80*(c*x^4 + b*x^2)^(3/2)*b/c^2)*A + 1/7680*(768*(
c*x^4 + b*x^2)^(3/2)*x^4/c - 420*sqrt(c*x^4 + b*x^2)*b^3*x^2/c^3 - 672*(c*x
^4 + b*x^2)^(3/2)*b*x^2/c^2 + 105*b^5*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^
2)*sqrt(c))/c^(9/2) - 210*sqrt(c*x^4 + b*x^2)*b^4/c^4 + 560*(c*x^4 + b*x^2)
^(3/2)*b^2/c^3)*B
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.17

$$\int x^5(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$
$$=\frac{1}{3840}\left(2\left(4\left(6\left(8\,Bx^2\mathrm{sgn}(x)+\frac{Bbc^7\mathrm{sgn}(x)+10\,Ac^8\mathrm{sgn}(x)}{c^8}\right)x^2-\frac{7\,Bb^2c^6\mathrm{sgn}(x)-10\,Abc^7\mathrm{sgn}(x)}{c^8}\right)x^2+\right.\right.$$
$$-\frac{(7\,Bb^5\mathrm{sgn}(x)-10\,Ab^4c\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{256\,c^{\frac{9}{2}}}$$
$$+\frac{(7\,Bb^5\log(|b|)-10\,Ab^4c\log(|b|))\mathrm{sgn}(x)}{512\,c^{\frac{9}{2}}}$$

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*B*x^2*sgn(x) + (B*b*c^7*sgn(x) + 10*A*c^8*sgn(x))/c^8)*x
^2 - (7*B*b^2*c^6*sgn(x) - 10*A*b*c^7*sgn(x))/c^8)*x^2 + 5*(7*B*b^3*c^5*sgn
(x) - 10*A*b^2*c^6*sgn(x))/c^8)*x^2 - 15*(7*B*b^4*c^4*sgn(x) - 10*A*b^3*c^5
*sgn(x))/c^8)*sqrt(c*x^2 + b)*x - 1/256*(7*B*b^5*sgn(x) - 10*A*b^4*c*sgn(x)
)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(9/2) + 1/512*(7*B*b^5*log(abs(b
)) - 10*A*b^4*c*log(abs(b)))*sgn(x)/c^(9/2)
```

Mupad [B] (verification not implemented)

Time = 9.82 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.29

$$\int x^5(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$
$$= \frac{A\,x^2\,(c\,x^4+b\,x^2)^{3/2}}{8\,c} - \frac{5\,A\,b\left(\frac{b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,|x|\,\sqrt{c\,x^2+b}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{24\,c^2}\right)}{16\,c}$$
$$+ \frac{B\,x^4\,(c\,x^4+b\,x^2)^{3/2}}{10\,c}$$
$$+ \frac{7\,B\,b\left(\frac{5\,b\left(\frac{b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,|x|\,\sqrt{c\,x^2+b}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{24\,c^2}\right)}{8\,c}-\frac{x^2\,(c\,x^4+b\,x^2)^{3/2}}{4\,c}\right)}{20\,c}$$

```
[In] int(x^5*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (A*x^2*(b*x^2 + c*x^4)^(3/2))/(8*c) - (5*A*b*((b^3*log(b + 2*c*x^2 + 2*c^(1
/2)*abs(x)*(b + c*x^2)^(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(8*c^2
*x^4 - 3*b^2 + 2*b*c*x^2))/(24*c^2)))/(16*c) + (B*x^4*(b*x^2 + c*x^4)^(3/2)
)/(10*c) + (7*B*b*((5*b*((b^3*log(b + 2*c*x^2 + 2*c^(1/2)*abs(x)*(b + c*x^2
)^(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(8*c^2*x^4 - 3*b^2 + 2*b*c*
x^2))/(24*c^2)))/(8*c) - (x^2*(b*x^2 + c*x^4)^(3/2))/(4*c)))/(20*c)
```

3.91 $\int x^3(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 125

$$\int x^3(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{b(5bB-8Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^3} - \frac{(5bB-8Ac-6Bcx^2)(bx^2+cx^4)^{3/2}}{48c^2} - \frac{b^3(5bB-8Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}}$$

```
[Out] -1/48*(-6*B*c*x^2-8*A*c+5*B*b)*(c*x^4+b*x^2)^(3/2)/c^2-1/128*b^3*(-8*A*c+5*
B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(7/2)+1/128*b*(-8*A*c+5*B*b
)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2059, 793, 626, 634, 212}

$$\int x^3(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = -\frac{b^3(5bB-8Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}} + \frac{b(b+2cx^2)\sqrt{bx^2+cx^4}(5bB-8Ac)}{128c^3} - \frac{(bx^2+cx^4)^{3/2}(-8Ac+5bB-6Bcx^2)}{48c^2}$$

```
[In] Int[x^3*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (b*(5*b*B - 8*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(128*c^3) - ((5*b*B -
 8*A*c - 6*B*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(48*c^2) - (b^3*(5*b*B - 8*A*c)*
ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(128*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int x(A+Bx)\sqrt{bx+cx^2}\,dx, x, x^2\right)$$

$$= -\frac{(5bB-8Ac-6Bcx^2)\,(bx^2+cx^4)^{3/2}}{48c^2} + \frac{(b(5bB-8Ac))\text{Subst}\left(\int\sqrt{bx+cx^2}\,dx, x, x^2\right)}{32c^2}$$

$$= \frac{b(5bB-8Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3} - \frac{\left(5bB-8Ac-6Bcx^2\right)\left(bx^2+cx^4\right)^{3/2}}{48c^2} - \frac{\left(b^3(5bB-8Ac)\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{256c^3}$$

$$= \frac{b(5bB-8Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3} - \frac{\left(5bB-8Ac-6Bcx^2\right)\left(bx^2+cx^4\right)^{3/2}}{48c^2} - \frac{\left(b^3(5bB-8Ac)\right)\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^3}$$

$$= \frac{b(5bB-8Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3} - \frac{\left(5bB-8Ac-6Bcx^2\right)\left(bx^2+cx^4\right)^{3/2}}{48c^2} - \frac{b^3(5bB-8Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.18

$$\int x^3\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx$$

$$= \frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{c}(15b^3B+8bc^2x^2(2A+Bx^2)+16c^3x^4(4A+3Bx^2)-2b^2c(12A+5Bx^2))+\frac{6b^3(5bB-8Ac)}{}\right.}{384c^{7/2}}$$

```
[In] Integrate[x^3*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*(15*b^3*B + 8*b*c^2*x^2*(2*A + B*x^2) + 16*
c^3*x^4*(4*A + 3*B*x^2) - 2*b^2*c*(12*A + 5*B*x^2)) + (6*b^3*(5*b*B - 8*A*c
)*ArcTanh[(Sqrt[c]*x)/(Sqrt[b] - Sqrt[b + c*x^2])])/(x*Sqrt[b + c*x^2])))/(
384*c^(7/2))
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{(-48B c^3x^6-64A c^3x^4-8Bb c^2x^4-16Ab c^2x^2+10B b^2c x^2+24b^2Ac-15B b^3)\sqrt{x^2(c x^2+b)}}{384c^3}+\frac{b^3(8Ac-5Bb)\ln\left(\sqrt{c}x+\right.}{128c^{\frac{7}{2}}x\sqrt{}}$
pseudoelliptic	$-\frac{(-\frac{1}{2}A b^3c+\frac{5}{16}B b^4)\ln\left(\frac{2c x^2+2\sqrt{x^2(c x^2+b)}\sqrt{c}+b}{\sqrt{c}}\right)+\left(b^2\left(\frac{5x^2B}{12}+A\right)c^{\frac{3}{2}}-\frac{2x^2\left(\frac{x^2B}{2}+A\right)b c^{\frac{5}{2}}}{3}+(-2B x^6-\frac{8}{3}A x^4)c^{\frac{7}{2}}-\frac{5B}{}\right.}{16c^{\frac{7}{2}}}$
default	$\frac{\sqrt{x^4c+b x^2}\left(48B(c x^2+b)^{\frac{3}{2}}c^{\frac{5}{2}}x^5+64A(c x^2+b)^{\frac{3}{2}}c^{\frac{5}{2}}x^3-40B(c x^2+b)^{\frac{3}{2}}c^{\frac{3}{2}}b x^3-48A(c x^2+b)^{\frac{3}{2}}c^{\frac{3}{2}}bx+24A\sqrt{c x^2+b}\,c^{\frac{3}{2}}b^2x+\right.}{384x\sqrt{c x^2+b}\,c^{\frac{7}{2}}}$

```
[In] int(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/384*(-48*B*c^3*x^6-64*A*c^3*x^4-8*B*b*c^2*x^4-16*A*b*c^2*x^2+10*B*b^2*c*
x^2+24*A*b^2*c-15*B*b^3)/c^3*(x^2*(c*x^2+b))^(1/2)+1/128*b^3*(8*A*c-5*B*b)/
c^(7/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.18

$$\int x^3\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx$$

$$=\left[-\frac{3\,(5\,Bb^4-8\,Ab^3c)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,(48\,Bc^4x^6+15\,Bb^3c-24\,Ab^2c^2+8\,(Bb}{768\,c^4}\right.$$

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(5*B*b^4 - 8*A*b^3*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 +
b*x^2)*sqrt(c)) - 2*(48*B*c^4*x^6 + 15*B*b^3*c - 24*A*b^2*c^2 + 8*(B*b*c^3
+ 8*A*c^4)*x^4 - 2*(5*B*b^2*c^2 - 8*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^4,
 1/384*(3*(5*B*b^4 - 8*A*b^3*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c
)/(c*x^2 + b)) + (48*B*c^4*x^6 + 15*B*b^3*c - 24*A*b^2*c^2 + 8*(B*b*c^3 + 8
*A*c^4)*x^4 - 2*(5*B*b^2*c^2 - 8*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^4]
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.30

$$\int x^3(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$
$$= \frac{A\left(\begin{cases} \frac{b^3\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise} \end{cases}\right)}{16c^2} + \sqrt{bx^2+cx^4}\left(-\frac{b^2}{8c^2}+\frac{bx^2}{12c}+\frac{x^4}{3}\right) & \text{for } c\neq 0 \\ \frac{2(bx^2)^{\frac{5}{2}}}{5b^2} & \text{for } b\neq 0 \\ 0 & \text{otherwise} \end{cases}\right)}{2}$$
$$+ \frac{B\left(\begin{cases} -\frac{5b^4\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise} \end{cases}\right)}{128c^3} + \sqrt{bx^2+cx^4}\cdot\left(\frac{5b^3}{64c^3}-\frac{5b^2x^2}{96c^2}+\frac{bx^4}{24c}+\frac{x^6}{4}\right) & \text{for } c\neq 0 \\ \frac{2(bx^2)^{\frac{7}{2}}}{7b^3} & \text{for } b\neq 0 \\ 0 & \text{otherwise} \end{cases}\right)}{2}$$

```
[In] integrate(x**3*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] A*Piecewise((b**3*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*c*
x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt(c
*(b/(2*c) + x**2)**2), True))/(16*c**2) + sqrt(b*x**2 + c*x**4)*(-b**2/(8*c
**2) + b*x**2/(12*c) + x**4/3), Ne(c, 0)), (2*(b*x**2)**(5/2)/(5*b**2), Ne(
b, 0)), (0, True))/2 + B*Piecewise((-5*b**4*Piecewise((log(b + 2*sqrt(c)*sq
rt(b*x**2 + c*x**4) + 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*
log(b/(2*c) + x**2)/sqrt(c*(b/(2*c) + x**2)**2), True))/(128*c**3) + sqrt(b
*x**2 + c*x**4)*(5*b**3/(64*c**3) - 5*b**2*x**2/(96*c**2) + b*x**4/(24*c) +
 x**6/4), Ne(c, 0)), (2*(b*x**2)**(7/2)/(7*b**3), Ne(b, 0)), (0, True))/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(109) = 218$.

Time = 0.21 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.80

$$\int x^3 (A + Bx^2) \sqrt{bx^2 + cx^4}\, dx =$$

$$-\frac{1}{96}\left(\frac{12\sqrt{cx^4 + bx^2}bx^2}{c} - \frac{3\,b^3\log\left(2\,cx^2 + b + 2\,\sqrt{cx^4 + bx^2}\sqrt{c}\right)}{c^{\frac{5}{2}}} + \frac{6\,\sqrt{cx^4 + bx^2}b^2}{c^2} - \frac{16\,(cx^4 + bx^2)^{\frac{3}{2}}}{c}\right)$$

$$+\frac{1}{768}\left(\frac{60\,\sqrt{cx^4 + bx^2}b^2x^2}{c^2} + \frac{96\,(cx^4 + bx^2)^{\frac{3}{2}}x^2}{c} - \frac{15\,b^4\log\left(2\,cx^2 + b + 2\,\sqrt{cx^4 + bx^2}\sqrt{c}\right)}{c^{\frac{7}{2}}} + \frac{30\,\sqrt{cx^4 + \cdots}}{c^3}\right.$$

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/96*(12*sqrt(c*x^4 + b*x^2)*b*x^2/c - 3*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^
4 + b*x^2)*sqrt(c))/c^(5/2) + 6*sqrt(c*x^4 + b*x^2)*b^2/c^2 - 16*(c*x^4 + b
*x^2)^(3/2)/c)*A + 1/768*(60*sqrt(c*x^4 + b*x^2)*b^2*x^2/c^2 + 96*(c*x^4 +
b*x^2)^(3/2)*x^2/c - 15*b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)
)/c^(7/2) + 30*sqrt(c*x^4 + b*x^2)*b^3/c^3 - 80*(c*x^4 + b*x^2)^(3/2)*b/c^2
)*B
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.42

$$\int x^3 (A + Bx^2) \sqrt{bx^2 + cx^4}\, dx$$

$$= \frac{1}{384}\left(2\left(4\left(6\,Bx^2\mathrm{sgn}(x) + \frac{Bbc^5\mathrm{sgn}(x) + 8\,Ac^6\mathrm{sgn}(x)}{c^6}\right)x^2 - \frac{5\,Bb^2c^4\mathrm{sgn}(x) - 8\,Abc^5\mathrm{sgn}(x)}{c^6}\right)x^2 + \frac{3\,(5\,B\cdots}{}\right.$$

$$+ \frac{(5\,Bb^4\mathrm{sgn}(x) - 8\,Ab^3c\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}x + \sqrt{cx^2 + b}\right|\right)}{128\,c^{\frac{7}{2}}}$$

$$- \frac{(5\,Bb^4\log(|b|) - 8\,Ab^3c\log(|b|))\mathrm{sgn}(x)}{256\,c^{\frac{7}{2}}}$$

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*B*x^2*sgn(x) + (B*b*c^5*sgn(x) + 8*A*c^6*sgn(x))/c^6)*x^2 -
(5*B*b^2*c^4*sgn(x) - 8*A*b*c^5*sgn(x))/c^6)*x^2 + 3*(5*B*b^3*c^3*sgn(x) -
8*A*b^2*c^4*sgn(x))/c^6)*sqrt(c*x^2 + b)*x + 1/128*(5*B*b^4*sgn(x) - 8*A*b^
3*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(7/2) - 1/256*(5*B*b^4
*log(abs(b)) - 8*A*b^3*c*log(abs(b)))*sgn(x)/c^(7/2)
```

Mupad [B] (verification not implemented)

Time = 9.62 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.42

$$\int x^3\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx$$
$$=\frac{B\,x^2\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}{8\,c}-\frac{5\,B\,b\,\left(\frac{b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,\left|x\right|\,\sqrt{c\,x^2+b}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{24\,c^2}\right)}{16\,c}+\frac{A\,b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,\left|x\right|\,\sqrt{c\,x^2+b}\right)}{32\,c^{5/2}}+\frac{A\,\sqrt{c\,x^4+b\,x^2}\,\left(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4\right)}{48\,c^2}$$

```
[In] int(x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (B*x^2*(b*x^2 + c*x^4)^(3/2))/(8*c) - (5*B*b*((b^3*log(b + 2*c*x^2 + 2*c^(1
/2)*abs(x)*(b + c*x^2)^(1/2)))/(16*c^(5/2)) + ((b*x^2 + c*x^4)^(1/2)*(8*c^2
*x^4 - 3*b^2 + 2*b*c*x^2))/(24*c^2)))/(16*c) + (A*b^3*log(b + 2*c*x^2 + 2*c
^(1/2)*abs(x)*(b + c*x^2)^(1/2)))/(32*c^(5/2)) + (A*(b*x^2 + c*x^4)^(1/2)*(
8*c^2*x^4 - 3*b^2 + 2*b*c*x^2))/(48*c^2)
```

3.92 $\int x(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 107

$$\int x(A+Bx^2)\sqrt{bx^2+cx^4}\,dx=-\frac{(bB-2Ac)\,(b+2cx^2)\,\sqrt{bx^2+cx^4}}{16c^2}+\frac{B(bx^2+cx^4)^{3/2}}{6c}+\frac{b^2(bB-2Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}$$

```
[Out] 1/6*B*(c*x^4+b*x^2)^(3/2)/c+1/16*b^2*(-2*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x^
4+b*x^2)^(1/2))/c^(5/2)-1/16*(-2*A*c+B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c
^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2059, 654, 626, 634, 212}

$$\int x(A+Bx^2)\sqrt{bx^2+cx^4}\,dx=\frac{b^2(bB-2Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{(b+2cx^2)\,\sqrt{bx^2+cx^4}(bB-2Ac)}{16c^2}+\frac{B(bx^2+cx^4)^{3/2}}{6c}$$

```
[In] Int[x*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -1/16*((b*B - 2*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/c^2 + (B*(b*x^2 + c
*x^4)^(3/2))/(6*c) + (b^2*(b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
c*x^4]])/(16*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int (A+Bx)\sqrt{bx+cx^2}\,dx, x, x^2\right)\\
&= \frac{B\left(bx^2+cx^4\right)^{3/2}}{6c} + \frac{(-bB+2Ac)\text{Subst}\left(\int \sqrt{bx+cx^2}\,dx, x, x^2\right)}{4c}\\
&= -\frac{(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2} + \frac{B\left(bx^2+cx^4\right)^{3/2}}{6c}\\
&\quad + \frac{\left(b^2(bB-2Ac)\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{32c^2}
\end{aligned}$$

$$= -\frac{(bB-2Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{16c^2}+\frac{B(bx^2+cx^4)^{3/2}}{6c} + \frac{(b^2(bB-2Ac))\,\mathrm{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^2}$$

$$= -\frac{(bB-2Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{16c^2}+\frac{B(bx^2+cx^4)^{3/2}}{6c}+\frac{b^2(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.18

$$\int x(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$

$$= \frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{c}x(-3b^2B+2bc(3A+Bx^2)+4c^2x^2(3A+2Bx^2))+\frac{6b^2(bB-2Ac)\mathrm{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{b}+\sqrt{b+cx^2}}\right)}{\sqrt{b+cx^2}}\right)}{48c^{5/2}x}$$

```
[In] Integrate[x*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[c]*x*(-3*b^2*B + 2*b*c*(3*A + B*x^2) + 4*c^2*x
^2*(3*A + 2*B*x^2)) + (6*b^2*(b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[b] +
Sqrt[b + c*x^2])])/Sqrt[b + c*x^2]))/(48*c^(5/2)*x)
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08

method	result
risch	$\frac{(8Bc^2x^4+12Ac^2x^2+2Bbcx^2+6Abc-3Bb^2)\sqrt{x^2(cx^2+b)}}{48c^2}-\frac{b^2(2Ac-Bb)\ln\left(\sqrt{c}x+\sqrt{cx^2+b}\right)\sqrt{x^2(cx^2+b)}}{16c^{\frac{5}{2}}x\sqrt{cx^2+b}}$
pseudoelliptic	$\frac{(-\frac{1}{2}b^2Ac+\frac{1}{4}Bb^3)\ln\left(\frac{2cx^2+2\sqrt{x^2(cx^2+b)}\sqrt{c}+b}{\sqrt{c}}\right)+\left(b\left(\frac{x^2B}{3}+A\right)c^{\frac{3}{2}}+(\frac{4}{3}x^4B+2Ax^2)c^{\frac{5}{2}}-\frac{B\sqrt{c}b^2}{2}\right)\sqrt{x^2(cx^2+b)}+\frac{\ln(2)(A}{}}{8c^{\frac{5}{2}}}$
default	$\frac{\sqrt{x^4c+bx^2}\left(8B(cx^2+b)^{\frac{3}{2}}c^{\frac{3}{2}}x^3+12A(cx^2+b)^{\frac{3}{2}}c^{\frac{3}{2}}x-6B(cx^2+b)^{\frac{3}{2}}\sqrt{c}bx-6A\sqrt{cx^2+b}c^{\frac{3}{2}}bx+3B\sqrt{cx^2+b}\sqrt{c}b^2x-6A\ln(\right.}{48x\sqrt{cx^2+b}c^{\frac{5}{2}}}$

```
[In] int(x*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(8*B*c^2*x^4+12*A*c^2*x^2+2*B*b*c*x^2+6*A*b*c-3*B*b^2)/c^2*(x^2*(c*x^2
+b))^(1/2)-1/16*b^2*(2*A*c-B*b)/c^(5/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*(x^2*
(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.08

$$\int x\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx$$
$$=\left[-\frac{3\,(Bb^3-2\,Ab^2c)\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,(8\,Bc^3x^4-3\,Bb^2c+6\,Abc^2+2\,(Bbc^2+6\,A\right.}{96\,c^3}$$
$$-\frac{3\,(Bb^3-2\,Ab^2c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-(8\,Bc^3x^4-3\,Bb^2c+6\,Abc^2+2\,(Bbc^2+6\,Ac^3)x^2)\sqrt{cx^4+}}{48\,c^3}$$

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(B*b^3 - 2*A*b^2*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x
^2)*sqrt(c)) - 2*(8*B*c^3*x^4 - 3*B*b^2*c + 6*A*b*c^2 + 2*(B*b*c^2 + 6*A*c^
3)*x^2)*sqrt(c*x^4 + b*x^2))/c^3, -1/48*(3*(B*b^3 - 2*A*b^2*c)*sqrt(-c)*arc
tan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (8*B*c^3*x^4 - 3*B*b^2*c +
6*A*b*c^2 + 2*(B*b*c^2 + 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^3]
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.38

$$\int x(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$

$$= \frac{A\left(\begin{cases} -\frac{b^2\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{8c}+\left(\frac{b}{4c}+\frac{x^2}{2}\right)\sqrt{bx^2+cx^4} & \text{for } c\neq 0\\ \frac{2(bx^2)^{\frac{3}{2}}}{3b} & \text{for } b\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{2}$$
$$+\frac{B\left(\begin{cases} \frac{b^3\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{16c^2}+\sqrt{bx^2+cx^4}\left(-\frac{b^2}{8c^2}+\frac{bx^2}{12c}+\frac{x^4}{3}\right) & \text{for } c\neq 0\\ \frac{2(bx^2)^{\frac{5}{2}}}{5b^2} & \text{for } b\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{2}$$

```
[In] integrate(x*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] A*Piecewise((-b**2*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*c
*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt(
c*(b/(2*c) + x**2)**2), True))/(8*c) + (b/(4*c) + x**2/2)*sqrt(b*x**2 + c*x
**4), Ne(c, 0)), (2*(b*x**2)**(3/2)/(3*b), Ne(b, 0)), (0, True))/2 + B*Piec
ewise((b**3*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*c*x**2)/
sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt(c*(b/(2
*c) + x**2)**2), True))/(16*c**2) + sqrt(b*x**2 + c*x**4)*(-b**2/(8*c**2) +
 b*x**2/(12*c) + x**4/3), Ne(c, 0)), (2*(b*x**2)**(5/2)/(5*b**2), Ne(b, 0))
, (0, True))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.65

$$\int x(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$
$$= \frac{1}{16}\left(4\sqrt{cx^4+bx^2}x^2 - \frac{b^2\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{3}{2}}} + \frac{2\sqrt{cx^4+bx^2}b}{c}\right)A - \frac{1}{96}\left(\frac{12\sqrt{cx^4+bx^2}bx^2}{c} - \frac{3\,b^3\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{5}{2}}} + \frac{6\sqrt{cx^4+bx^2}b^2}{c^2} - \frac{16\,(cx^4+bx^2)^{\frac{3}{2}}}{c}\right)B$$

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*sqrt(c*x^4 + b*x^2)*x^2 - b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^
2)*sqrt(c))/c^(3/2) + 2*sqrt(c*x^4 + b*x^2)*b/c)*A - 1/96*(12*sqrt(c*x^4 +
b*x^2)*b*x^2/c - 3*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(
5/2) + 6*sqrt(c*x^4 + b*x^2)*b^2/c^2 - 16*(c*x^4 + b*x^2)^(3/2)/c)*B
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.31

$$\int x(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$
$$= \frac{1}{48}\left(2\left(4\,Bx^2\mathrm{sgn}(x) + \frac{Bbc^3\mathrm{sgn}(x)+6\,Ac^4\mathrm{sgn}(x)}{c^4}\right)x^2 - \frac{3\,(Bb^2c^2\mathrm{sgn}(x)-2\,Abc^3\mathrm{sgn}(x))}{c^4}\right)\sqrt{cx^2+b}x - \frac{(Bb^3\mathrm{sgn}(x)-2\,Ab^2c\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{16\,c^{\frac{5}{2}}} + \frac{(Bb^3\log(|b|)-2\,Ab^2c\log(|b|))\mathrm{sgn}(x)}{32\,c^{\frac{5}{2}}}$$

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*B*x^2*sgn(x) + (B*b*c^3*sgn(x) + 6*A*c^4*sgn(x))/c^4)*x^2 - 3*(B
*b^2*c^2*sgn(x) - 2*A*b*c^3*sgn(x))/c^4)*sqrt(c*x^2 + b)*x - 1/16*(B*b^3*sg
n(x) - 2*A*b^2*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(5/2) + 1
/32*(B*b^3*log(abs(b)) - 2*A*b^2*c*log(abs(b)))*sgn(x)/c^(5/2)
```

Mupad [B] (verification not implemented)

Time = 9.67 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.31

$$\int x(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{A\left(\frac{b}{4c}+\frac{x^2}{2}\right)\sqrt{c\,x^4+b\,x^2}}{2} + \frac{B\,b^3\,\ln\left(b+2\,c\,x^2+2\,\sqrt{c}\,|x|\,\sqrt{c\,x^2+b}\right)}{32\,c^{5/2}} - \frac{A\,b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{16\,c^{3/2}} + \frac{B\,\sqrt{c\,x^4+b\,x^2}\,(-3\,b^2+2\,b\,c\,x^2+8\,c^2\,x^4)}{48\,c^2}$$

```
[In] int(x*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (A*(b/(4*c) + x^2/2)*(b*x^2 + c*x^4)^(1/2))/2 + (B*b^3*log(b + 2*c*x^2 + 2*
c^(1/2)*abs(x)*(b + c*x^2)^(1/2)))/(32*c^(5/2)) - (A*b^2*log((b/2 + c*x^2)/
c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(16*c^(3/2)) + (B*(b*x^2 + c*x^4)^(1/2)*(
8*c^2*x^4 - 3*b^2 + 2*b*c*x^2))/(48*c^2)
```

3.93 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 100

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}\,dx = -\frac{(bB-4Ac)\sqrt{bx^2+cx^4}}{8c}+\frac{B(bx^2+cx^4)^{3/2}}{4cx^2} -\frac{b(bB-4Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}$$

```
[Out] 1/4*B*(c*x^4+b*x^2)^(3/2)/c/x^2-1/8*b*(-4*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x
^4+b*x^2)^(1/2))/c^(3/2)-1/8*(-4*A*c+B*b)*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2059, 808, 678, 634, 212}

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}\,dx = -\frac{b(bB-4Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}} -\frac{\sqrt{bx^2+cx^4}(bB-4Ac)}{8c}+\frac{B(bx^2+cx^4)^{3/2}}{4cx^2}$$

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x,x]
```

```
[Out] -1/8*((b*B - 4*A*c)*Sqrt[b*x^2 + c*x^4])/c + (B*(b*x^2 + c*x^4)^(3/2))/(4*c
*x^2) - (b*(b*B - 4*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(
3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x}\,dx,x,x^2\right)$$

$$= \frac{B(bx^2+cx^4)^{3/2}}{4cx^2} + \frac{\left(bB-Ac+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x}\,dx,x,x^2\right)}{4c}$$

$$= -\frac{(bB-4Ac)\sqrt{bx^2+cx^4}}{8c} + \frac{B(bx^2+cx^4)^{3/2}}{4cx^2} - \frac{(b(bB-4Ac))\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{16c}$$

$$= -\frac{(bB-4Ac)\sqrt{bx^2+cx^4}}{8c}+\frac{B(bx^2+cx^4)^{3/2}}{4cx^2}-\frac{(b(bB-4Ac))\text{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{8c}$$

$$= -\frac{(bB-4Ac)\sqrt{bx^2+cx^4}}{8c}+\frac{B(bx^2+cx^4)^{3/2}}{4cx^2}-\frac{b(bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

$$\int\frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}\,dx$$
$$=\frac{x\left(\sqrt{c}x(b+cx^2)\left(bB+4Ac+2Bcx^2\right)+b(bB-4Ac)\sqrt{b+cx^2}\log\left(-\sqrt{c}x+\sqrt{b+cx^2}\right)\right)}{8c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x,x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2)*(b*B + 4*A*c + 2*B*c*x^2) + b*(b*B - 4*A*c)*Sqrt[
b + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(8*c^(3/2)*Sqrt[x^2*(b + c
*x^2)])
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

method	result
risch	$\frac{(2Bc\,x^2+4Ac+Bb)\sqrt{x^2(c\,x^2+b)}}{8c}+\frac{b(4Ac-Bb)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)\sqrt{x^2(c\,x^2+b)}}{8c^{\frac{3}{2}}x\sqrt{c\,x^2+b}}$
default	$\frac{\sqrt{x^4c+b\,x^2}\left(2B\sqrt{c}\,(c\,x^2+b)^{\frac{3}{2}}x+4A\,c^{\frac{3}{2}}\sqrt{c\,x^2+b}\,x-B\sqrt{c}\,\sqrt{c\,x^2+b}\,bx+4A\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)bc-B\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)b^2\right)}{8c^{\frac{3}{2}}\sqrt{c\,x^2+b}\,x}$
pseudoelliptic	$\frac{4B\,c^{\frac{3}{2}}x^2\sqrt{x^2(c\,x^2+b)}+4A\ln\left(\frac{2c\,x^2+2\sqrt{x^2\left(c\,x^2+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)bc-4A\ln(2)bc+8A\,c^{\frac{3}{2}}\sqrt{x^2(c\,x^2+b)}-B\ln\left(\frac{2c\,x^2+2\sqrt{x^2\left(c\,x^2+b\right)}\,\sqrt{}}{\sqrt{c}}\right.}{16c^{\frac{3}{2}}}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(2*B*c*x^2+4*A*c+B*b)/c*(x^2*(c*x^2+b))^(1/2)+1/8*b*(4*A*c-B*b)/c^(3/2)
*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.72

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}\,dx$$
$$= \left[-\frac{(Bb^2-4\,Abc)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,(2\,Bc^2x^2+Bbc+4\,Ac^2)\sqrt{cx^4+bx^2}}{16\,c^2}, \frac{(Bb^2-\right.$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [-1/16*((B*b^2 - 4*A*b*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*
sqrt(c)) - 2*(2*B*c^2*x^2 + B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2))/c^2, 1/8*
((B*b^2 - 4*A*b*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)
) + (2*B*c^2*x^2 + B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2))/c^2]
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{x}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.28

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}\,dx$$
$$= \frac{1}{4}\left(\frac{b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{\sqrt{c}}+2\,\sqrt{cx^4+bx^2}\right)A$$
$$+\frac{1}{16}\left(4\,\sqrt{cx^4+bx^2}x^2-\frac{b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{3}{2}}}+\frac{2\,\sqrt{cx^4+bx^2}b}{c}\right)B$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*(b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) + 2*sqrt(c*
x^4 + b*x^2))*A + 1/16*(4*sqrt(c*x^4 + b*x^2)*x^2 - b^2*log(2*c*x^2 + b + 2
*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(3/2) + 2*sqrt(c*x^4 + b*x^2)*b/c)*B
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}\,dx = \frac{1}{8}\left(2\,Bx^2\mathrm{sgn}(x)+\frac{Bbc\mathrm{sgn}(x)+4\,Ac^2\mathrm{sgn}(x)}{c^2}\right)\sqrt{cx^2+b}x + \frac{(Bb^2\mathrm{sgn}(x)-4\,Abc\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{8\,c^{\frac{3}{2}}} - \frac{(Bb^2\log(|b|)-4\,Abc\log(|b|))\mathrm{sgn}(x)}{16\,c^{\frac{3}{2}}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/8*(2*B*x^2*sgn(x) + (B*b*c*sgn(x) + 4*A*c^2*sgn(x))/c^2)*sqrt(c*x^2 + b)*
x + 1/8*(B*b^2*sgn(x) - 4*A*b*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b
)))/c^(3/2) - 1/16*(B*b^2*log(abs(b)) - 4*A*b*c*log(abs(b)))*sgn(x)/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x}\,dx = \frac{A\sqrt{c\,x^4+b\,x^2}}{2}+\frac{B\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\sqrt{c\,x^4+b\,x^2}}{2} + \frac{A\,b\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{4\,\sqrt{c}} - \frac{B\,b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{16\,c^{3/2}}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x,x)
```

```
[Out] (A*(b*x^2 + c*x^4)^(1/2))/2 + (B*(b/(4*c) + x^2/2)*(b*x^2 + c*x^4)^(1/2))/2
 + (A*b*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(4*c^(1/2)) - (
B*b^2*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(16*c^(3/2))
```

3.94 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 97

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3}\,dx = \frac{(bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{A(bx^2+cx^4)^{3/2}}{bx^4} + \frac{(bB+2Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}$$

```
[Out] -A*(c*x^4+b*x^2)^(3/2)/b/x^4+1/2*(2*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(1/2)+1/2*(2*A*c+B*b)*(c*x^4+b*x^2)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2059, 806, 678, 634, 212}

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3}\,dx = \frac{(2Ac+bB)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}} + \frac{\sqrt{bx^2+cx^4}(2Ac+bB)}{2b} - \frac{A(bx^2+cx^4)^{3/2}}{bx^4}$$

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^3,x]
```

```
[Out] ((b*B + 2*A*c)*Sqrt[b*x^2 + c*x^4])/(2*b) - (A*(b*x^2 + c*x^4)^(3/2))/(b*x^4) + ((b*B + 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(2*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{bx^4} + \frac{\left(-2(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x}\,dx, x, x^2\right)}{b}$$

$$= \frac{(bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{A(bx^2+cx^4)^{3/2}}{bx^4} + \frac{1}{4}(bB+2Ac)\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$

$$= \frac{(bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{A(bx^2+cx^4)^{3/2}}{bx^4} + \frac{1}{2}(bB+2Ac)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)$$

$$= \frac{(bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{A(bx^2+cx^4)^{3/2}}{bx^4} + \frac{(bB+2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3}\,dx = \frac{\sqrt{x^2(b+cx^2)}\left(-2A+Bx^2+\frac{2(bB+2Ac)x\,\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{b}+\sqrt{b+cx^2}}\right)}{\sqrt{c}\sqrt{b+cx^2}}\right)}{2x^2}$$

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^3,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-2*A + B*x^2 + (2*(b*B + 2*A*c)*x*ArcTanh[(Sqrt[c]*
x)/(-Sqrt[b] + Sqrt[b + c*x^2])]))/(Sqrt[c]*Sqrt[b + c*x^2])))/(2*x^2)
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{(-x^2B+2A)\sqrt{x^2(c\,x^2+b)}}{2x^2}+\frac{\left(Ac+\frac{Bb}{2}\right)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)\sqrt{x^2(c\,x^2+b)}}{\sqrt{c}\,x\sqrt{c\,x^2+b}}$
pseudoelliptic	$\frac{-2\left(-\frac{x^2B}{2}+A\right)\sqrt{c}\,\sqrt{x^2(c\,x^2+b)}+x^2\left(-\ln(2)+\ln\left(\frac{2c\,x^2+2\sqrt{x^2\left(c\,x^2+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)\right)\left(Ac+\frac{Bb}{2}\right)}{2\sqrt{c}\,x^2}$
default	$-\frac{\sqrt{x^4c+b\,x^2}\left(-2A\sqrt{c\,x^2+b}\,c^{\frac{3}{2}}x^2-B\sqrt{c\,x^2+b}\,\sqrt{c}\,b\,x^2+2A(c\,x^2+b)^{\frac{3}{2}}\sqrt{c}-2A\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)bcx-B\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+\right.\right.}}{2x^2\sqrt{c\,x^2+b}\,b\sqrt{c}}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-B*x^2+2*A)/x^2*(x^2*(c*x^2+b))^(1/2)+(A*c+1/2*B*b)*ln(c^(1/2)*x+(c*x
^2+b)^(1/2))/c^(1/2)*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.66

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3}\,dx$$
$$= \left[\frac{(Bb+2\,Ac)\sqrt{c}x^2\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}(Bcx^2-2\,Ac)}{4\,cx^2},\right.$$
$$\left.-\frac{(Bb+2\,Ac)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-\sqrt{cx^4+bx^2}(Bcx^2-2\,Ac)}{2\,cx^2}\right]$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*((B*b + 2*A*c)*sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sq
rt(c)) + 2*sqrt(c*x^4 + b*x^2)*(B*c*x^2 - 2*A*c))/(c*x^2), -1/2*((B*b + 2*A
*c)*sqrt(-c)*x^2*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - sqrt(c*
x^4 + b*x^2)*(B*c*x^2 - 2*A*c))/(c*x^2)]
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{x^3}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3}\,dx = \frac{1}{2}\left(\sqrt{c}\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-\frac{2\,\sqrt{cx^4+bx^2}}{x^2}\right)A$$
$$+\frac{1}{4}\left(\frac{b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{\sqrt{c}}+2\,\sqrt{cx^4+bx^2}\right)B$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*sqrt(c*x^
4 + b*x^2)/x^2)*A + 1/4*(b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))
/sqrt(c) + 2*sqrt(c*x^4 + b*x^2))*B
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3}\,dx = \frac{1}{2}\sqrt{cx^2+b}Bx\mathrm{sgn}(x) + \frac{2\,Ab\sqrt{c}\mathrm{sgn}(x)}{\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b} - \frac{(Bb\mathrm{sgn}(x)+2\,Ac\mathrm{sgn}(x))\log\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right)}{4\,\sqrt{c}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + b)*B*x*sgn(x) + 2*A*b*sqrt(c)*sgn(x)/((sqrt(c)*x - sqrt(c*
x^2 + b))^2 - b) - 1/4*(B*b*sgn(x) + 2*A*c*sgn(x))*log((sqrt(c)*x - sqrt(c*
x^2 + b))^2)/sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^3}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^3}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^3,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^3, x)
```

3.95 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 80

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5}\,dx = -\frac{B\sqrt{bx^2+cx^4}}{x^2}-\frac{A(bx^2+cx^4)^{3/2}}{3bx^6}+B\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] -1/3*A*(c*x^4+b*x^2)^(3/2)/b/x^6+B*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))
*c^(1/2)-B*(c*x^4+b*x^2)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2059, 806, 676, 634, 212}

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5}\,dx = -\frac{A(bx^2+cx^4)^{3/2}}{3bx^6}+B\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{B\sqrt{bx^2+cx^4}}{x^2}$$

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^5,x]
```

```
[Out] -((B*Sqrt[b*x^2 + c*x^4])/x^2) - (A*(b*x^2 + c*x^4)^(3/2))/(3*b*x^6) + B*Sq
rt[c]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 676

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Dist[c*(p/(e^2*(m + p + 1))), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^3}\,dx,x,x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{3bx^6} + \frac{1}{2}B\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^2}\,dx,x,x^2\right)$$

$$= -\frac{B\sqrt{bx^2+cx^4}}{x^2} - \frac{A(bx^2+cx^4)^{3/2}}{3bx^6} + \frac{1}{2}(Bc)\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)$$

$$= -\frac{B\sqrt{bx^2+cx^4}}{x^2} - \frac{A(bx^2+cx^4)^{3/2}}{3bx^6} + (Bc)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)$$

$$= -\frac{B\sqrt{bx^2+cx^4}}{x^2} - \frac{A(bx^2+cx^4)^{3/2}}{3bx^6} + B\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.24

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5}\,dx$$
$$= -\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b+cx^2}(3bBx^2+A(b+cx^2)) + 3bB\sqrt{c}x^3\log\left(-\sqrt{c}x+\sqrt{b+cx^2}\right)\right)}{3bx^4\sqrt{b+cx^2}}$$

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^5,x]
```

```
[Out] -1/3*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b + c*x^2]*(3*b*B*x^2 + A*(b + c*x^2)) +
3*b*B*Sqrt[c]*x^3*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))/(b*x^4*Sqrt[b + c*x
^2])
```

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08

method	result	size
risch	$-\frac{\left(A c \,x^{2}+3 b B \,x^{2}+A b \right) \sqrt{x^{2} \left(c \,x^{2}+b \right)}}{3 x^{4} b}+\frac{B \sqrt{c}\, \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+b}\right) \sqrt{x^{2} \left(c \,x^{2}+b \right)}}{x \sqrt{c \,x^{2}+b}}$	86
pseudoelliptic	$\frac{3 x^{4} B \left(-\ln \left(2\right)+\ln \left(\frac{2 c \,x^{2}+2 \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, \sqrt{c}+b}{\sqrt{c}}\right)\right) b \sqrt{c}-2 \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, \left(\left(A c +3 B b \right) x^{2}+A b \right)}{6 b \,x^{4}}$	87
default	$-\frac{\sqrt{x^{4} c +b \,x^{2}}\, \left(-3 B \sqrt{c \,x^{2}+b}\, c^{\frac{3}{2}} x^{4}+3 B \left(c \,x^{2}+b \right)^{\frac{3}{2}} \sqrt{c}\, x^{2}-3 B \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+b}\right) b c \,x^{3}+A \left(c \,x^{2}+b \right)^{\frac{3}{2}} \sqrt{c}\right)}{3 x^{4} \sqrt{c \,x^{2}+b}\, b \sqrt{c}}$	109

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(A*c*x^2+3*B*b*x^2+A*b)/x^4/b*(x^2*(c*x^2+b))^(1/2)+B*c^(1/2)*ln(c^(1/
2)*x+(c*x^2+b)^(1/2))*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.00

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5}\,dx = \left[\frac{3\,Bb\sqrt{c}x^4\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}((3\,Bb+Ac)x^2+Ab)}{6\,bx^4}, -\frac{3\,Bb\sqrt{-c}x^4\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}((3\,Bb+Ac)x^2+Ab)}{3\,bx^4}\right]$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="fricas")
[Out] [1/6*(3*B*b*sqrt(c)*x^4*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) -
 2*sqrt(c*x^4 + b*x^2)*((3*B*b + A*c)*x^2 + A*b))/(b*x^4), -1/3*(3*B*b*sqrt
(-c)*x^4*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*
x^2)*((3*B*b + A*c)*x^2 + A*b))/(b*x^4)]
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{x^5}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**5,x)
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.20

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5}\,dx = \frac{1}{2}\left(\sqrt{c}\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-\frac{2\,\sqrt{cx^4+bx^2}}{x^2}\right)B - \frac{1}{3}\,A\left(\frac{\sqrt{cx^4+bx^2}c}{bx^2}+\frac{\sqrt{cx^4+bx^2}}{x^4}\right)$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*sqrt(c*x^
4 + b*x^2)/x^2)*B - 1/3*A*(sqrt(c*x^4 + b*x^2)*c/(b*x^2) + sqrt(c*x^4 + b*x
^2)/x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(68) = 136$.

Time = 0.44 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.04

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5}\,dx = -\frac{1}{2}B\sqrt{c}\log\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right)\mathrm{sgn}(x) + \frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4Bb\sqrt{c}\mathrm{sgn}(x)+3\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4Ac^{\frac{3}{2}}\mathrm{sgn}(x)-6\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2Bb^2\sqrt{c}\mathrm{sg}}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^3}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/2*B*sqrt(c)*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)*sgn(x) + 2/3*(3*(sqrt(c
)*x - sqrt(c*x^2 + b))^4*B*b*sqrt(c)*sgn(x) + 3*(sqrt(c)*x - sqrt(c*x^2 + b
))^4*A*c^(3/2)*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^2*sqrt(c)*sgn
(x) + 3*B*b^3*sqrt(c)*sgn(x) + A*b^2*c^(3/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x
^2 + b))^2 - b)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^5}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^5}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^5,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^5, x)
```

3.96 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 61

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7}\,dx=-\frac{A(bx^2+cx^4)^{3/2}}{5bx^8}-\frac{(5bB-2Ac)\,(bx^2+cx^4)^{3/2}}{15b^2x^6}$$

```
[Out] -1/5*A*(c*x^4+b*x^2)^(3/2)/b/x^8-1/15*(-2*A*c+5*B*b)*(c*x^4+b*x^2)^(3/2)/b^
2/x^6
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2059, 806, 664}

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7}\,dx=-\frac{(bx^2+cx^4)^{3/2}\,(5bB-2Ac)}{15b^2x^6}-\frac{A(bx^2+cx^4)^{3/2}}{5bx^8}$$

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^7,x]
```

```
[Out] -1/5*(A*(b*x^2 + c*x^4)^(3/2))/(b*x^8) - ((5*b*B - 2*A*c)*(b*x^2 + c*x^4)^(
3/2))/(15*b^2*x^6)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^4}\,dx, x, x^2\right) \\
&= -\frac{A(bx^2+cx^4)^{3/2}}{5bx^8} + \frac{\left(-4(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^3}\,dx, x, x^2\right)}{5b} \\
&= -\frac{A(bx^2+cx^4)^{3/2}}{5bx^8} - \frac{(5bB-2Ac)\,(bx^2+cx^4)^{3/2}}{15b^2x^6}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7}\,dx = -\frac{\left(x^2(b+cx^2)\right)^{3/2}(3Ab+5bBx^2-2Acx^2)}{15b^2x^8}$$

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^7,x]
[Out] -1/15*((x^2*(b + c*x^2))^(3/2)*(3*A*b + 5*b*B*x^2 - 2*A*c*x^2))/(b^2*x^8)
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

method	result	size
pseudoelliptic	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(c\,x^{2}+b\right)\left(\left(\frac{5x^{2}B}{3}+A\right)b-\frac{2Ac\,x^{2}}{3}\right)}{5b^{2}x^{6}}$	47
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-2Ac\,x^{2}+5bB\,x^{2}+3Ab\right)\sqrt{x^{4}c+b\,x^{2}}}{15b^{2}x^{6}}$	48
default	$-\frac{\left(c\,x^{2}+b\right)\left(-2Ac\,x^{2}+5bB\,x^{2}+3Ab\right)\sqrt{x^{4}c+b\,x^{2}}}{15b^{2}x^{6}}$	48
trager	$-\frac{\left(-2A\,c^{2}x^{4}+5x^{4}Bbc+Abc\,x^{2}+5b^{2}B\,x^{2}+3b^{2}A\right)\sqrt{x^{4}c+b\,x^{2}}}{15b^{2}x^{6}}$	62
risch	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(-2A\,c^{2}x^{4}+5x^{4}Bbc+Abc\,x^{2}+5b^{2}B\,x^{2}+3b^{2}A\right)}{15x^{6}b^{2}}$	62

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^7,x,method=_RETURNVERBOSE)
[Out] -1/5*(x^2*(c*x^2+b))^(1/2)*(c*x^2+b)*((5/3*x^2*B+A)*b-2/3*A*c*x^2)/b^2/x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7}\,dx = -\frac{((5\,Bbc-2\,Ac^2)x^4+3\,Ab^2+(5\,Bb^2+Abc)x^2)\sqrt{cx^4+bx^2}}{15\,b^2x^6}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="fricas")
[Out] -1/15*((5*B*b*c - 2*A*c^2)*x^4 + 3*A*b^2 + (5*B*b^2 + A*b*c)*x^2)*sqrt(c*x^
4 + b*x^2)/(b^2*x^6)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{x^7}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**7,x)
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(53) = 106$.

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.82

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7}\,dx = -\frac{1}{3}\,B\left(\frac{\sqrt{cx^4+bx^2}c}{bx^2}+\frac{\sqrt{cx^4+bx^2}}{x^4}\right) + \frac{1}{15}\,A\left(\frac{2\,\sqrt{cx^4+bx^2}c^2}{b^2x^2}-\frac{\sqrt{cx^4+bx^2}c}{bx^4}-\frac{3\,\sqrt{cx^4+bx^2}}{x^6}\right)$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/3*B*(sqrt(c*x^4 + b*x^2)*c/(b*x^2) + sqrt(c*x^4 + b*x^2)/x^4) + 1/15*A*(
2*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^2) - sqrt(c*x^4 + b*x^2)*c/(b*x^4) - 3*sqr
t(c*x^4 + b*x^2)/x^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(53) = 106$.

Time = 0.70 (sec) , antiderivative size = 250, normalized size of antiderivative = 4.10

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7}\,dx = \frac{2\left(15\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^8Bc^{\frac{3}{2}}\mathrm{sgn}(x)-30\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6Bbc^{\frac{3}{2}}\mathrm{sgn}(x)+30\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6Ac^{\frac{5}{2}}\mathrm{sgn}\right.}{}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 2/15*(15*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*c^(3/2)*sgn(x) - 30*(sqrt(c)*x -
 sqrt(c*x^2 + b))^6*B*b*c^(3/2)*sgn(x) + 30*(sqrt(c)*x - sqrt(c*x^2 + b))^6
*A*c^(5/2)*sgn(x) + 20*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^2*c^(3/2)*sgn(x)
 + 10*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b*c^(5/2)*sgn(x) - 10*(sqrt(c)*x -
sqrt(c*x^2 + b))^2*B*b^3*c^(3/2)*sgn(x) + 10*(sqrt(c)*x - sqrt(c*x^2 + b))^
2*A*b^2*c^(5/2)*sgn(x) + 5*B*b^4*c^(3/2)*sgn(x) - 2*A*b^3*c^(5/2)*sgn(x))/(
(sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^5
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.85

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^7}\,dx=\frac{(A\,c^2+B\,b\,c)\,\sqrt{c\,x^4+b\,x^2}}{5\,b^2\,x^2}-\frac{(5\,B\,b^2+A\,c\,b)\,\sqrt{c\,x^4+b\,x^2}}{15\,b^2\,x^4}-\frac{A\,\sqrt{c\,x^4+b\,x^2}}{5\,x^6}-\frac{c\,(A\,c+8\,B\,b)\,\sqrt{c\,x^4+b\,x^2}}{15\,b^2\,x^2}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^7,x)
```

```
[Out] ((A*c^2 + B*b*c)*(b*x^2 + c*x^4)^(1/2))/(5*b^2*x^2) - ((5*B*b^2 + A*b*c)*(b
*x^2 + c*x^4)^(1/2))/(15*b^2*x^4) - (A*(b*x^2 + c*x^4)^(1/2))/(5*x^6) - (c*
(A*c + 8*B*b)*(b*x^2 + c*x^4)^(1/2))/(15*b^2*x^2)
```

3.97 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9} \, dx = -\frac{A(bx^2+cx^4)^{3/2}}{7bx^{10}} - \frac{(7bB-4Ac)\,(bx^2+cx^4)^{3/2}}{35b^2x^8} + \frac{2c(7bB-4Ac)\,(bx^2+cx^4)^{3/2}}{105b^3x^6}$$

```
[Out] -1/7*A*(c*x^4+b*x^2)^(3/2)/b/x^10-1/35*(-4*A*c+7*B*b)*(c*x^4+b*x^2)^(3/2)/b
^2/x^8+2/105*c*(-4*A*c+7*B*b)*(c*x^4+b*x^2)^(3/2)/b^3/x^6
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2059, 806, 672, 664}

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9} \, dx = \frac{2c(bx^2+cx^4)^{3/2}\,(7bB-4Ac)}{105b^3x^6} - \frac{(bx^2+cx^4)^{3/2}\,(7bB-4Ac)}{35b^2x^8} - \frac{A(bx^2+cx^4)^{3/2}}{7bx^{10}}$$

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^9,x]
```

```
[Out] -1/7*(A*(b*x^2 + c*x^4)^(3/2))/(b*x^10) - ((7*b*B - 4*A*c)*(b*x^2 + c*x^4)^
(3/2))/(35*b^2*x^8) + (2*c*(7*b*B - 4*A*c)*(b*x^2 + c*x^4)^(3/2))/(105*b^3*
x^6)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{7bx^{10}} + \frac{\left(-5(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^4}\,dx, x, x^2\right)}{7b}$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{7bx^{10}} - \frac{(7bB-4Ac)\,(bx^2+cx^4)^{3/2}}{35b^2x^8} - \frac{(c(7bB-4Ac))\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^3}\,dx, x, x^2\right)}{35b^2}$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{7bx^{10}} - \frac{(7bB-4Ac)(bx^2+cx^4)^{3/2}}{35b^2x^8} + \frac{2c(7bB-4Ac)(bx^2+cx^4)^{3/2}}{105b^3x^6}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9}\,dx = \frac{(x^2(b+cx^2))^{3/2}(7bBx^2(-3b+2cx^2)+A(-15b^2+12bcx^2-8c^2x^4))}{105b^3x^{10}}$$

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^9,x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(7*b*B*x^2*(-3*b + 2*c*x^2) + A*(-15*b^2 + 12*b*c*
x^2 - 8*c^2*x^4)))/(105*b^3*x^10)
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

method	result	size
pseudoelliptic	$-\frac{\left(\left(\frac{7x^2B}{5}+A\right)b^2-\frac{4\left(\frac{7x^2B}{6}+A\right)x^2cb}{5}+\frac{8Ac^2x^4}{15}\right)\sqrt{x^2(cx^2+b)}\,(cx^2+b)}{7b^3x^8}$	66
gosper	$-\frac{(cx^2+b)(8Ac^2x^4-14x^4Bbc-12Abcx^2+21b^2Bx^2+15b^2A)\sqrt{x^4c+bx^2}}{105b^3x^8}$	70
default	$-\frac{(cx^2+b)(8Ac^2x^4-14x^4Bbc-12Abcx^2+21b^2Bx^2+15b^2A)\sqrt{x^4c+bx^2}}{105b^3x^8}$	70
trager	$-\frac{(8Ac^3x^6-14x^6Bbc^2-4Abc^2x^4+7x^4Bb^2c+3Ab^2cx^2+21b^3Bx^2+15b^3A)\sqrt{x^4c+bx^2}}{105b^3x^8}$	87
risch	$-\frac{\sqrt{x^2(cx^2+b)}\,(8Ac^3x^6-14x^6Bbc^2-4Abc^2x^4+7x^4Bb^2c+3Ab^2cx^2+21b^3Bx^2+15b^3A)}{105x^8b^3}$	87

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*((7/5*x^2*B+A)*b^2-4/5*(7/6*x^2*B+A)*x^2*c*b+8/15*A*c^2*x^4)*(x^2*(c*x
^2+b))^(1/2)*(c*x^2+b)/b^3/x^8
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9}\,dx = \frac{(2\,(7\,Bbc^2-4\,Ac^3)x^6-(7\,Bb^2c-4\,Abc^2)x^4-15\,Ab^3-3\,(7\,Bb^3+Ab^2c)x^2)\sqrt{cx^4+bx^2}}{105\,b^3x^8}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="fricas")
[Out] 1/105*(2*(7*B*b*c^2 - 4*A*c^3)*x^6 - (7*B*b^2*c - 4*A*b*c^2)*x^4 - 15*A*b^3
 - 3*(7*B*b^3 + A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^3*x^8)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{x^9}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**9,x)
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**9, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.68

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9}\,dx = \frac{1}{15}\,B\left(\frac{2\,\sqrt{cx^4+bx^2}c^2}{b^2x^2}-\frac{\sqrt{cx^4+bx^2}c}{bx^4}-\frac{3\,\sqrt{cx^4+bx^2}}{x^6}\right) - \frac{1}{105}\,A\left(\frac{8\,\sqrt{cx^4+bx^2}c^3}{b^3x^2}-\frac{4\,\sqrt{cx^4+bx^2}c^2}{b^2x^4}+\frac{3\,\sqrt{cx^4+bx^2}c}{bx^6}+\frac{15\,\sqrt{cx^4+bx^2}}{x^8}\right)$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="maxima")
[Out] 1/15*B*(2*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^2) - sqrt(c*x^4 + b*x^2)*c/(b*x^4)
 - 3*sqrt(c*x^4 + b*x^2)/x^6) - 1/105*A*(8*sqrt(c*x^4 + b*x^2)*c^3/(b^3*x^2
) - 4*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^4) + 3*sqrt(c*x^4 + b*x^2)*c/(b*x^6) +
 15*sqrt(c*x^4 + b*x^2)/x^8)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(84) = 168.

Time = 0.89 (sec) , antiderivative size = 310, normalized size of antiderivative = 3.23

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9}\,dx = \frac{4\left(105\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Bc^{\frac{5}{2}}\mathrm{sgn}(x)-175\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Bbc^{\frac{5}{2}}\mathrm{sgn}(x)+280\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Ac\right.}{}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] 4/105*(105*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*c^(5/2)*sgn(x) - 175*(sqrt(c)
*x - sqrt(c*x^2 + b))^8*B*b*c^(5/2)*sgn(x) + 280*(sqrt(c)*x - sqrt(c*x^2 +
b))^8*A*c^(7/2)*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^2*c^(5/2)*s
gn(x) + 140*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b*c^(7/2)*sgn(x) - 42*(sqrt(c
)*x - sqrt(c*x^2 + b))^4*B*b^3*c^(5/2)*sgn(x) + 84*(sqrt(c)*x - sqrt(c*x^2
+ b))^4*A*b^2*c^(7/2)*sgn(x) + 49*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^4*c^(
5/2)*sgn(x) - 28*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^3*c^(7/2)*sgn(x) - 7*B
*b^5*c^(5/2)*sgn(x) + 4*A*b^4*c^(7/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b)
)^2 - b)^7
```

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.67

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^9}\,dx = \frac{4\,A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b^2\,x^4} - \frac{B\,\sqrt{c\,x^4+b\,x^2}}{5\,x^6} - \frac{A\,c\,\sqrt{c\,x^4+b\,x^2}}{35\,b\,x^6} - \frac{B\,c\,\sqrt{c\,x^4+b\,x^2}}{15\,b\,x^4} - \frac{A\,\sqrt{c\,x^4+b\,x^2}}{7\,x^8} - \frac{8\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{105\,b^3\,x^2} + \frac{2\,B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{15\,b^2\,x^2}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^9,x)
```

```
[Out] (4*A*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b^2*x^4) - (B*(b*x^2 + c*x^4)^(1/2))/(
5*x^6) - (A*c*(b*x^2 + c*x^4)^(1/2))/(35*b*x^6) - (B*c*(b*x^2 + c*x^4)^(1/2
))/(15*b*x^4) - (A*(b*x^2 + c*x^4)^(1/2))/(7*x^8) - (8*A*c^3*(b*x^2 + c*x^4
)^(1/2))/(105*b^3*x^2) + (2*B*c^2*(b*x^2 + c*x^4)^(1/2))/(15*b^2*x^2)
```

3.98 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 133

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx = -\frac{A(bx^2+cx^4)^{3/2}}{9bx^{12}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{3/2}}{21b^2x^{10}} + \frac{4c(3bB-2Ac)(bx^2+cx^4)^{3/2}}{105b^3x^8} - \frac{8c^2(3bB-2Ac)(bx^2+cx^4)^{3/2}}{315b^4x^6}$$

```
[Out] -1/9*A*(c*x^4+b*x^2)^(3/2)/b/x^12-1/21*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/b
^2/x^10+4/105*c*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/b^3/x^8-8/315*c^2*(-2*A*
c+3*B*b)*(c*x^4+b*x^2)^(3/2)/b^4/x^6
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2059, 806, 672, 664}

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx = -\frac{8c^2(bx^2+cx^4)^{3/2}(3bB-2Ac)}{315b^4x^6} + \frac{4c(bx^2+cx^4)^{3/2}(3bB-2Ac)}{105b^3x^8} - \frac{(bx^2+cx^4)^{3/2}(3bB-2Ac)}{21b^2x^{10}} - \frac{A(bx^2+cx^4)^{3/2}}{9bx^{12}}$$

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^11,x]
```

```
[Out] -1/9*(A*(b*x^2 + c*x^4)^(3/2))/(b*x^12) - ((3*b*B - 2*A*c)*(b*x^2 + c*x^4)^
(3/2))/(21*b^2*x^10) + (4*c*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(3/2))/(105*b^3
*x^8) - (8*c^2*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(3/2))/(315*b^4*x^6)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{9bx^{12}} + \frac{\left(-6(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^5}\,dx, x, x^2\right)}{9b}$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{9bx^{12}} - \frac{(3bB-2Ac)\,(bx^2+cx^4)^{3/2}}{21b^2x^{10}} - \frac{(2c(3bB-2Ac))\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^4}\,dx,x,x^2\right)}{21b^2}$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{9bx^{12}} - \frac{(3bB-2Ac)\,(bx^2+cx^4)^{3/2}}{21b^2x^{10}} + \frac{4c(3bB-2Ac)\,(bx^2+cx^4)^{3/2}}{105b^3x^8} + \frac{(4c^2(3bB-2Ac))\,\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^3}\,dx,x,x^2\right)}{105b^3}$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{9bx^{12}} - \frac{(3bB-2Ac)\,(bx^2+cx^4)^{3/2}}{21b^2x^{10}} + \frac{4c(3bB-2Ac)\,(bx^2+cx^4)^{3/2}}{105b^3x^8} - \frac{8c^2(3bB-2Ac)\,(bx^2+cx^4)^{3/2}}{315b^4x^6}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.66

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx = -\frac{(x^2(b+cx^2))^{3/2}\,(3bBx^2(15b^2-12bcx^2+8c^2x^4)+A(35b^3-30b^2cx^2+24bc^2x^4-16c^3x^6))}{315b^4x^{12}}$$

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^11,x]
[Out] -1/315*((x^2*(b + c*x^2))^(3/2)*(3*b*B*x^2*(15*b^2 - 12*b*c*x^2 + 8*c^2*x^4
) + A*(35*b^3 - 30*b^2*c*x^2 + 24*b*c^2*x^4 - 16*c^3*x^6)))/(b^4*x^12)
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$-\frac{\left(\left(\frac{9x^2B}{7}+A\right)b^3-\frac{6x^2c\left(\frac{6x^2B}{5}+A\right)b^2}{7}+\frac{24c^2x^4\left(x^2B+A\right)b}{35}-\frac{16A\,c^3x^6}{35}\right)\sqrt{x^2(c\,x^2+b)}\,(c\,x^2+b)}{9b^4x^{10}}$
gosper	$-\frac{(c\,x^2+b)\,(-16A\,c^3x^6+24x^6Bb\,c^2+24Ab\,c^2x^4-36x^4B\,b^2c-30A\,b^2c\,x^2+45b^3B\,x^2+35b^3A)\sqrt{x^4c+b\,x^2}}{315b^4x^{10}}$
default	$-\frac{(c\,x^2+b)\,(-16A\,c^3x^6+24x^6Bb\,c^2+24Ab\,c^2x^4-36x^4B\,b^2c-30A\,b^2c\,x^2+45b^3B\,x^2+35b^3A)\sqrt{x^4c+b\,x^2}}{315b^4x^{10}}$
trager	$-\frac{(-16A\,x^8c^4+24B\,x^8b\,c^3+8A\,x^6b\,c^3-12B\,x^6b^2c^2-6A\,b^2c^2x^4+9B\,b^3c\,x^4+5A\,x^2b^3c+45B\,x^2b^4+35A\,b^4)\sqrt{x^4c+b\,x^2}}{315b^4x^{10}}$
risch	$-\frac{\sqrt{x^2(c\,x^2+b)}\,(-16A\,x^8c^4+24B\,x^8b\,c^3+8A\,x^6b\,c^3-12B\,x^6b^2c^2-6A\,b^2c^2x^4+9B\,b^3c\,x^4+5A\,x^2b^3c+45B\,x^2b^4+35A\,b^4)}{315x^{10}b^4}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*((9/7*x^2*B+A)*b^3-6/7*x^2*c*(6/5*x^2*B+A)*b^2+24/35*c^2*x^4*(B*x^2+A)
*b-16/35*A*c^3*x^6)*(x^2*(c*x^2+b))^(1/2)*(c*x^2+b)/b^4/x^10
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx = -\frac{(8\,(3\,Bbc^3-2\,Ac^4)x^8-4\,(3\,Bb^2c^2-2\,Abc^3)x^6+35\,Ab^4+3\,(3\,Bb^3c-2\,Ab^2c^2)x^4+5\,(9\,Bb^4+Ab^3c)x^2)}{315\,b^4x^{10}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/315*(8*(3*B*b*c^3 - 2*A*c^4)*x^8 - 4*(3*B*b^2*c^2 - 2*A*b*c^3)*x^6 + 35*
A*b^4 + 3*(3*B*b^3*c - 2*A*b^2*c^2)*x^4 + 5*(9*B*b^4 + A*b^3*c)*x^2)*sqrt(c
*x^4 + b*x^2)/(b^4*x^10)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{x^{11}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**11,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**11, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.57

$$\begin{aligned}\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx &= -\frac{1}{105}\,B\left(\frac{8\,\sqrt{cx^4+bx^2}c^3}{b^3x^2}-\frac{4\,\sqrt{cx^4+bx^2}c^2}{b^2x^4}+\frac{3\,\sqrt{cx^4+bx^2}c}{bx^6}+\frac{15\,\sqrt{cx^4+bx^2}}{x^8}\right)\\ &\quad+\frac{1}{315}\,A\left(\frac{16\,\sqrt{cx^4+bx^2}c^4}{b^4x^2}-\frac{8\,\sqrt{cx^4+bx^2}c^3}{b^3x^4}+\frac{6\,\sqrt{cx^4+bx^2}c^2}{b^2x^6}-\frac{5\,\sqrt{cx^4+bx^2}c}{bx^8}-\frac{35\,\sqrt{cx^4+bx^2}}{x^{10}}\right)\end{aligned}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] -1/105*B*(8*sqrt(c*x^4 + b*x^2)*c^3/(b^3*x^2) - 4*sqrt(c*x^4 + b*x^2)*c^2/(
b^2*x^4) + 3*sqrt(c*x^4 + b*x^2)*c/(b*x^6) + 15*sqrt(c*x^4 + b*x^2)/x^8) +
1/315*A*(16*sqrt(c*x^4 + b*x^2)*c^4/(b^4*x^2) - 8*sqrt(c*x^4 + b*x^2)*c^3/(
b^3*x^4) + 6*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^6) - 5*sqrt(c*x^4 + b*x^2)*c/(b
*x^8) - 35*sqrt(c*x^4 + b*x^2)/x^10)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(117) = 234$.

Time = 1.40 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.78

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx$$
$$= \frac{16\left(210\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Bc^{\frac{7}{2}}\mathrm{sgn}(x)-315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Bbc^{\frac{7}{2}}\mathrm{sgn}(x)+630\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{1}\right.}{}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] 16/315*(210*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*c^(7/2)*sgn(x) - 315*(sqrt(c
)*x - sqrt(c*x^2 + b))^10*B*b*c^(7/2)*sgn(x) + 630*(sqrt(c)*x - sqrt(c*x^2
+ b))^10*A*c^(9/2)*sgn(x) + 63*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^2*c^(7/2
)*sgn(x) + 378*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*b*c^(9/2)*sgn(x) - 42*(sqr
t(c)*x - sqrt(c*x^2 + b))^6*B*b^3*c^(7/2)*sgn(x) + 168*(sqrt(c)*x - sqrt(c*
x^2 + b))^6*A*b^2*c^(9/2)*sgn(x) + 108*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^
4*c^(7/2)*sgn(x) - 72*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^3*c^(9/2)*sgn(x)
- 27*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^5*c^(7/2)*sgn(x) + 18*(sqrt(c)*x -
 sqrt(c*x^2 + b))^2*A*b^4*c^(9/2)*sgn(x) + 3*B*b^6*c^(7/2)*sgn(x) - 2*A*b^5
*c^(9/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^9
```

Mupad [B] (verification not implemented)

Time = 9.76 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.58

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11}}\,dx = \frac{2\,A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b^2\,x^6}-\frac{B\,\sqrt{c\,x^4+b\,x^2}}{7\,x^8}-\frac{A\,c\,\sqrt{c\,x^4+b\,x^2}}{63\,b\,x^8}-\frac{B\,c\,\sqrt{c\,x^4+b\,x^2}}{35\,b\,x^6}-\frac{A\,\sqrt{c\,x^4+b\,x^2}}{9\,x^{10}}-\frac{8\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{315\,b^3\,x^4}+\frac{16\,A\,c^4\,\sqrt{c\,x^4+b\,x^2}}{315\,b^4\,x^2}+\frac{4\,B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b^2\,x^4}-\frac{8\,B\,c^3\,\sqrt{c\,x^4+b\,x^2}}{105\,b^3\,x^2}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^11,x)
```

```
[Out] (2*A*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b^2*x^6) - (B*(b*x^2 + c*x^4)^(1/2))/(
7*x^8) - (A*c*(b*x^2 + c*x^4)^(1/2))/(63*b*x^8) - (B*c*(b*x^2 + c*x^4)^(1/2
))/(35*b*x^6) - (A*(b*x^2 + c*x^4)^(1/2))/(9*x^10) - (8*A*c^3*(b*x^2 + c*x^
4)^(1/2))/(315*b^3*x^4) + (16*A*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^4*x^2) +
(4*B*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b^2*x^4) - (8*B*c^3*(b*x^2 + c*x^4)^(1
/2))/(105*b^3*x^2)
```

3.99 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 170

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}}\,dx = -\frac{A(bx^2+cx^4)^{3/2}}{11bx^{14}} - \frac{(11bB-8Ac)(bx^2+cx^4)^{3/2}}{99b^2x^{12}} + \frac{2c(11bB-8Ac)(bx^2+cx^4)^{3/2}}{231b^3x^{10}} - \frac{8c^2(11bB-8Ac)(bx^2+cx^4)^{3/2}}{1155b^4x^8} + \frac{16c^3(11bB-8Ac)(bx^2+cx^4)^{3/2}}{3465b^5x^6}$$

```
[Out] -1/11*A*(c*x^4+b*x^2)^(3/2)/b/x^14-1/99*(-8*A*c+11*B*b)*(c*x^4+b*x^2)^(3/2)
/b^2/x^12+2/231*c*(-8*A*c+11*B*b)*(c*x^4+b*x^2)^(3/2)/b^3/x^10-8/1155*c^2*(
-8*A*c+11*B*b)*(c*x^4+b*x^2)^(3/2)/b^4/x^8+16/3465*c^3*(-8*A*c+11*B*b)*(c*x
^4+b*x^2)^(3/2)/b^5/x^6
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used

= {2059, 806, 672, 664}

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}}\,dx = \frac{16c^3(bx^2+cx^4)^{3/2}(11bB-8Ac)}{3465b^5x^6} - \frac{8c^2(bx^2+cx^4)^{3/2}(11bB-8Ac)}{1155b^4x^8} + \frac{2c(bx^2+cx^4)^{3/2}(11bB-8Ac)}{231b^3x^{10}} - \frac{(bx^2+cx^4)^{3/2}(11bB-8Ac)}{99b^2x^{12}} - \frac{A(bx^2+cx^4)^{3/2}}{11bx^{14}}$$

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^13,x]
```

```
[Out] -1/11*(A*(b*x^2 + c*x^4)^(3/2))/(b*x^14) - ((11*b*B - 8*A*c)*(b*x^2 + c*x^4
)^(3/2))/(99*b^2*x^12) + (2*c*(11*b*B - 8*A*c)*(b*x^2 + c*x^4)^(3/2))/(231*
b^3*x^10) - (8*c^2*(11*b*B - 8*A*c)*(b*x^2 + c*x^4)^(3/2))/(1155*b^4*x^8) +
 (16*c^3*(11*b*B - 8*A*c)*(b*x^2 + c*x^4)^(3/2))/(3465*b^5*x^6)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^7}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{11bx^{14}} + \frac{\left(-7(-bB+Ac)+\frac{3}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^6}\,dx, x, x^2\right)}{11b}$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{11bx^{14}} - \frac{(11bB-8Ac)\,(bx^2+cx^4)^{3/2}}{99b^2x^{12}} - \frac{(c(11bB-8Ac))\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^5}\,dx, x, x^2\right)}{33b^2}$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{11bx^{14}} - \frac{(11bB-8Ac)\,(bx^2+cx^4)^{3/2}}{99b^2x^{12}} + \frac{2c(11bB-8Ac)\,(bx^2+cx^4)^{3/2}}{231b^3x^{10}} + \frac{(4c^2(11bB-8Ac))\,\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^4}\,dx, x, x^2\right)}{231b^3}$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{11bx^{14}} - \frac{(11bB-8Ac)\,(bx^2+cx^4)^{3/2}}{99b^2x^{12}} + \frac{2c(11bB-8Ac)\,(bx^2+cx^4)^{3/2}}{231b^3x^{10}} - \frac{8c^2(11bB-8Ac)\,(bx^2+cx^4)^{3/2}}{1155b^4x^8} - \frac{(8c^3(11bB-8Ac))\,\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^3}\,dx, x, x^2\right)}{1155b^4}$$

$$= -\frac{A(bx^2+cx^4)^{3/2}}{11bx^{14}} - \frac{(11bB-8Ac)\,(bx^2+cx^4)^{3/2}}{99b^2x^{12}} + \frac{2c(11bB-8Ac)\,(bx^2+cx^4)^{3/2}}{231b^3x^{10}} - \frac{8c^2(11bB-8Ac)\,(bx^2+cx^4)^{3/2}}{1155b^4x^8} + \frac{16c^3(11bB-8Ac)\,(bx^2+cx^4)^{3/2}}{3465b^5x^6}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.65

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}}\,dx$$
$$= \frac{(x^2(b+cx^2))^{3/2}\,(11bBx^2(-35b^3+30b^2cx^2-24bc^2x^4+16c^3x^6)+A(-315b^4+280b^3cx^2-240b^2c^2x^4+192\ldots}{3465b^5x^{14}}$$

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^13,x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(11*b*B*x^2*(-35*b^3 + 30*b^2*c*x^2 - 24*b*c^2*x^4
 + 16*c^3*x^6) + A*(-315*b^4 + 280*b^3*c*x^2 - 240*b^2*c^2*x^4 + 192*b*c^3*
x^6 - 128*c^4*x^8)))/(3465*b^5*x^14)
```

Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$-\frac{\left(\left(\frac{11x^2B}{9}+A\right)b^4-\frac{8x^2c\left(\frac{33x^2B}{28}+A\right)b^3}{9}+\frac{16x^4\left(\frac{11x^2B}{10}+A\right)c^2b^2}{21}-\frac{64\left(\frac{11x^2B}{12}+A\right)x^6c^3b}{105}+\frac{128Ax^8c^4}{315}\right)\sqrt{x^2(cx^2+b)}\,(cx^2+b)}{11b^5x^{12}}$
gosper	$-\frac{(cx^2+b)(128Ax^8c^4-176Bx^8bc^3-192Ax^6bc^3+264Bx^6b^2c^2+240Ab^2c^2x^4-330Bb^3cx^4-280Ax^2b^3c+385Bx^2b^4+315A\ldots}{3465b^5x^{12}}$
default	$-\frac{(cx^2+b)(128Ax^8c^4-176Bx^8bc^3-192Ax^6bc^3+264Bx^6b^2c^2+240Ab^2c^2x^4-330Bb^3cx^4-280Ax^2b^3c+385Bx^2b^4+315A\ldots}{3465b^5x^{12}}$
trager	$-\frac{(128Ax^{10}c^5-176Bbc^4x^{10}-64Ax^8bc^4+88Bb^2c^3x^8+48Ab^2c^3x^6-66Bb^3c^2x^6-40Ab^3c^2x^4+55Bx^4b^4c+35Ab^4cx^2+385b\ldots}{3465b^5x^{12}}$
risch	$-\frac{\sqrt{x^2(cx^2+b)}\,(128Ax^{10}c^5-176Bbc^4x^{10}-64Ax^8bc^4+88Bb^2c^3x^8+48Ab^2c^3x^6-66Bb^3c^2x^6-40Ab^3c^2x^4+55Bx^4b^4c+35\ldots}{3465x^{12}b^5}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] -1/11*((11/9*x^2*B+A)*b^4-8/9*x^2*c*(33/28*x^2*B+A)*b^3+16/21*x^4*(11/10*x^
2*B+A)*c^2*b^2-64/105*(11/12*x^2*B+A)*x^6*c^3*b+128/315*A*x^8*c^4)*(x^2*(c*
x^2+b))^(1/2)*(c*x^2+b)/b^5/x^12
```

Fricas [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}}\,dx = \frac{(16\,(11\,Bbc^4-8\,Ac^5)x^{10}-8\,(11\,Bb^2c^3-8\,Abc^4)x^8+6\,(11\,Bb^3c^2-8\,Ab^2c^3)x^6-315\,Ab^5-5\,(11\,Bb^4c-\dots}{3465\,b^5x^{12}}$$

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="fricas")

[Out] 1/3465*(16*(11*B*b*c^4 - 8*A*c^5)*x^10 - 8*(11*B*b^2*c^3 - 8*A*b*c^4)*x^8 + 6*(11*B*b^3*c^2 - 8*A*b^2*c^3)*x^6 - 315*A*b^5 - 5*(11*B*b^4*c - 8*A*b^3*c^2)*x^4 - 35*(11*B*b^5 + A*b^4*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^5*x^12)

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{x^{13}}\,dx$$

[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**13,x)

[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**13, x)

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.51

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}}\,dx = \frac{1}{315}\,B\left(\frac{16\,\sqrt{cx^4+bx^2}c^4}{b^4x^2}-\frac{8\,\sqrt{cx^4+bx^2}c^3}{b^3x^4}+\frac{6\,\sqrt{cx^4+bx^2}c^2}{b^2x^6}-\frac{5\,\sqrt{cx^4+bx^2}c}{bx^8}-\frac{35\,\sqrt{cx^4+bx^2}}{x^{10}}\right) - \frac{1}{3465}\,A\left(\frac{128\,\sqrt{cx^4+bx^2}c^5}{b^5x^2}-\frac{64\,\sqrt{cx^4+bx^2}c^4}{b^4x^4}+\frac{48\,\sqrt{cx^4+bx^2}c^3}{b^3x^6}-\frac{40\,\sqrt{cx^4+bx^2}c^2}{b^2x^8}+\frac{35\,\sqrt{cx^4+bx^2}\dots}{bx^{10}}\right.$$

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="maxima")

[Out] 1/315*B*(16*sqrt(c*x^4 + b*x^2)*c^4/(b^4*x^2) - 8*sqrt(c*x^4 + b*x^2)*c^3/(b^3*x^4) + 6*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^6) - 5*sqrt(c*x^4 + b*x^2)*c/(b*x^8) - 35*sqrt(c*x^4 + b*x^2)/x^10) - 1/3465*A*(128*sqrt(c*x^4 + b*x^2)*c^5/(b^5*x^2) - 64*sqrt(c*x^4 + b*x^2)*c^4/(b^4*x^4) + 48*sqrt(c*x^4 + b*x^2)*c^3/(b^3*x^6) - 40*sqrt(c*x^4 + b*x^2)*c^2/(b^2*x^8) + 35*sqrt(c*x^4 + b*x^2)*c/(b*x^10) + 315*sqrt(c*x^4 + b*x^2)/x^12)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(150) = 300.

Time = 1.85 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.53

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}}\,dx$$
$$= \frac{32\left(3465\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{14}Bc^{\frac{9}{2}}\mathrm{sgn}(x)-4851\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Bbc^{\frac{9}{2}}\mathrm{sgn}(x)+11088\left(\sqrt{c}x-\sqrt{cx^2+\right.}{}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^13,x, algorithm="giac")
```

```
[Out] 32/3465*(3465*(sqrt(c)*x - sqrt(c*x^2 + b))^14*B*c^(9/2)*sgn(x) - 4851*(sqr
t(c)*x - sqrt(c*x^2 + b))^12*B*b*c^(9/2)*sgn(x) + 11088*(sqrt(c)*x - sqrt(c
*x^2 + b))^12*A*c^(11/2)*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*b^
2*c^(9/2)*sgn(x) + 7392*(sqrt(c)*x - sqrt(c*x^2 + b))^10*A*b*c^(11/2)*sgn(x
) - 165*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^3*c^(9/2)*sgn(x) + 2640*(sqrt(c
)*x - sqrt(c*x^2 + b))^8*A*b^2*c^(11/2)*sgn(x) + 1815*(sqrt(c)*x - sqrt(c*x
^2 + b))^6*B*b^4*c^(9/2)*sgn(x) - 1320*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b^
3*c^(11/2)*sgn(x) - 605*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^5*c^(9/2)*sgn(x
) + 440*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^4*c^(11/2)*sgn(x) + 121*(sqrt(c
)*x - sqrt(c*x^2 + b))^2*B*b^6*c^(9/2)*sgn(x) - 88*(sqrt(c)*x - sqrt(c*x^2
+ b))^2*A*b^5*c^(11/2)*sgn(x) - 11*B*b^7*c^(9/2)*sgn(x) + 8*A*b^6*c^(11/2)*
sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^11
```

Mupad [B] (verification not implemented)

Time = 10.22 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.53

$$\begin{aligned}\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13}}\,dx &= \frac{8\,A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{693\,b^2\,x^8}-\frac{B\,\sqrt{c\,x^4+b\,x^2}}{9\,x^{10}}\\ &\quad-\frac{A\,c\,\sqrt{c\,x^4+b\,x^2}}{99\,b\,x^{10}}-\frac{B\,c\,\sqrt{c\,x^4+b\,x^2}}{63\,b\,x^8}-\frac{A\,\sqrt{c\,x^4+b\,x^2}}{11\,x^{12}}\\ &\quad-\frac{16\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^3\,x^6}+\frac{64\,A\,c^4\,\sqrt{c\,x^4+b\,x^2}}{3465\,b^4\,x^4}\\ &\quad-\frac{128\,A\,c^5\,\sqrt{c\,x^4+b\,x^2}}{3465\,b^5\,x^2}+\frac{2\,B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b^2\,x^6}\\ &\quad-\frac{8\,B\,c^3\,\sqrt{c\,x^4+b\,x^2}}{315\,b^3\,x^4}+\frac{16\,B\,c^4\,\sqrt{c\,x^4+b\,x^2}}{315\,b^4\,x^2}\end{aligned}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^13,x)
```

```
[Out] (8*A*c^2*(b*x^2 + c*x^4)^(1/2))/(693*b^2*x^8) - (B*(b*x^2 + c*x^4)^(1/2))/(
9*x^10) - (A*c*(b*x^2 + c*x^4)^(1/2))/(99*b*x^10) - (B*c*(b*x^2 + c*x^4)^(1
/2))/(63*b*x^8) - (A*(b*x^2 + c*x^4)^(1/2))/(11*x^12) - (16*A*c^3*(b*x^2 +
c*x^4)^(1/2))/(1155*b^3*x^6) + (64*A*c^4*(b*x^2 + c*x^4)^(1/2))/(3465*b^4*x
^4) - (128*A*c^5*(b*x^2 + c*x^4)^(1/2))/(3465*b^5*x^2) + (2*B*c^2*(b*x^2 +
c*x^4)^(1/2))/(105*b^2*x^6) - (8*B*c^3*(b*x^2 + c*x^4)^(1/2))/(315*b^3*x^4)
 + (16*B*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^4*x^2)
```

3.100 $\int x^4(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 131

$$\int x^4(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = -\frac{8b^2(2bB-3Ac)(bx^2+cx^4)^{3/2}}{315c^4x^3} + \frac{4b(2bB-3Ac)(bx^2+cx^4)^{3/2}}{105c^3x} - \frac{(2bB-3Ac)x(bx^2+cx^4)^{3/2}}{21c^2} + \frac{Bx^3(bx^2+cx^4)^{3/2}}{9c}$$

```
[Out] -8/315*b^2*(-3*A*c+2*B*b)*(c*x^4+b*x^2)^(3/2)/c^4/x^3+4/105*b*(-3*A*c+2*B*b
)*(c*x^4+b*x^2)^(3/2)/c^3/x-1/21*(-3*A*c+2*B*b)*x*(c*x^4+b*x^2)^(3/2)/c^2+1
/9*B*x^3*(c*x^4+b*x^2)^(3/2)/c
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2064, 2041, 2025}

$$\int x^4(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = -\frac{8b^2(bx^2+cx^4)^{3/2}(2bB-3Ac)}{315c^4x^3} + \frac{4b(bx^2+cx^4)^{3/2}(2bB-3Ac)}{105c^3x} - \frac{x(bx^2+cx^4)^{3/2}(2bB-3Ac)}{21c^2} + \frac{Bx^3(bx^2+cx^4)^{3/2}}{9c}$$

```
[In] Int[x^4*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-8*b^2*(2*b*B - 3*A*c)*(b*x^2 + c*x^4)^(3/2))/(315*c^4*x^3) + (4*b*(2*b*B
- 3*A*c)*(b*x^2 + c*x^4)^(3/2))/(105*c^3*x) - ((2*b*B - 3*A*c)*x*(b*x^2 + c
*x^4)^(3/2))/(21*c^2) + (B*x^3*(b*x^2 + c*x^4)^(3/2))/(9*c)
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Bx^3(bx^2+cx^4)^{3/2}}{9c} - \frac{(6bB-9Ac)\int x^4\sqrt{bx^2+cx^4}\,dx}{9c} \\
&= -\frac{(2bB-3Ac)x(bx^2+cx^4)^{3/2}}{21c^2} + \frac{Bx^3(bx^2+cx^4)^{3/2}}{9c} + \frac{(4b(2bB-3Ac))\int x^2\sqrt{bx^2+cx^4}\,dx}{21c^2} \\
&= \frac{4b(2bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{105c^3x} - \frac{(2bB-3Ac)x(bx^2+cx^4)^{3/2}}{21c^2} \\
&\quad + \frac{Bx^3(bx^2+cx^4)^{3/2}}{9c} - \frac{(8b^2(2bB-3Ac))\int\sqrt{bx^2+cx^4}\,dx}{105c^3} \\
&= -\frac{8b^2(2bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{315c^4x^3} + \frac{4b(2bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{105c^3x} \\
&\quad - \frac{(2bB-3Ac)x(bx^2+cx^4)^{3/2}}{21c^2} + \frac{Bx^3(bx^2+cx^4)^{3/2}}{9c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.63

$$\int x^4(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$
$$=\frac{(x^2(b+cx^2))^{3/2}\left(-16b^3B+24b^2c(A+Bx^2)-6bc^2x^2(6A+5Bx^2)+5c^3x^4(9A+7Bx^2)\right)}{315c^4x^3}$$

```
[In] Integrate[x^4*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
[Out] ((x^2*(b + c*x^2))^(3/2)*(-16*b^3*B + 24*b^2*c*(A + B*x^2) - 6*b*c^2*x^2*(6
*A + 5*B*x^2) + 5*c^3*x^4*(9*A + 7*B*x^2)))/(315*c^4*x^3)
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.69

method	result	size
gosper	$\frac{(c\,x^2+b)\left(35B\,c^3x^6+45A\,c^3x^4-30Bb\,c^2x^4-36Ab\,c^2x^2+24B\,b^2c\,x^2+24b^2Ac-16B\,b^3\right)\sqrt{x^4c+b\,x^2}}{315c^4x}$	91
default	$\frac{(c\,x^2+b)\left(35B\,c^3x^6+45A\,c^3x^4-30Bb\,c^2x^4-36Ab\,c^2x^2+24B\,b^2c\,x^2+24b^2Ac-16B\,b^3\right)\sqrt{x^4c+b\,x^2}}{315c^4x}$	91
trager	$\frac{\left(35B\,x^8c^4+45A\,x^6c^4+5B\,x^6b\,c^3+9A\,x^4b\,c^3-6B\,x^4b^2c^2-12A\,x^2b^2c^2+8B\,x^2b^3c+24A\,b^3c-16B\,b^4\right)\sqrt{x^4c+b\,x^2}}{315c^4x}$	108
risch	$\frac{\sqrt{x^2(c\,x^2+b)}\left(35B\,x^8c^4+45A\,x^6c^4+5B\,x^6b\,c^3+9A\,x^4b\,c^3-6B\,x^4b^2c^2-12A\,x^2b^2c^2+8B\,x^2b^3c+24A\,b^3c-16B\,b^4\right)}{315x\,c^4}$	108

```
[In] int(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/315*(c*x^2+b)*(35*B*c^3*x^6+45*A*c^3*x^4-30*B*b*c^2*x^4-36*A*b*c^2*x^2+24
*B*b^2*c*x^2+24*A*b^2*c-16*B*b^3)*(c*x^4+b*x^2)^(1/2)/c^4/x
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.81

$$\int x^4(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$
$$=\frac{(35\,Bc^4x^8+5\,(Bbc^3+9\,Ac^4)x^6-16\,Bb^4+24\,Ab^3c-3\,(2\,Bb^2c^2-3\,Abc^3)x^4+4\,(2\,Bb^3c-3\,Ab^2c^2)x^2)\sqrt{c}}{315\,c^4x}$$

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/315*(35*B*c^4*x^8 + 5*(B*b*c^3 + 9*A*c^4)*x^6 - 16*B*b^4 + 24*A*b^3*c - 3
*(2*B*b^2*c^2 - 3*A*b*c^3)*x^4 + 4*(2*B*b^3*c - 3*A*b^2*c^2)*x^2)*sqrt(c*x^
4 + b*x^2)/(c^4*x)
```

Sympy [F]

$$\int x^4 (A + Bx^2) \sqrt{bx^2 + cx^4} \, dx = \int x^4 \sqrt{x^2 (b + cx^2)} (A + Bx^2) \, dx$$

```
[In] integrate(x**4*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x**4*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.81

$$\int x^4 (A + Bx^2) \sqrt{bx^2 + cx^4} \, dx = \frac{(15 \, c^3 x^6 + 3 \, bc^2 x^4 - 4 \, b^2 c x^2 + 8 \, b^3) \sqrt{cx^2 + b} A}{105 \, c^3} + \frac{(35 \, c^4 x^8 + 5 \, bc^3 x^6 - 6 \, b^2 c^2 x^4 + 8 \, b^3 c x^2 - 16 \, b^4) \sqrt{cx^2 + b} B}{315 \, c^4}$$

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/105*(15*c^3*x^6 + 3*b*c^2*x^4 - 4*b^2*c*x^2 + 8*b^3)*sqrt(c*x^2 + b)*A/c^
3 + 1/315*(35*c^4*x^8 + 5*b*c^3*x^6 - 6*b^2*c^2*x^4 + 8*b^3*c*x^2 - 16*b^4)
*sqrt(c*x^2 + b)*B/c^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07

$$\int x^4 (A + Bx^2) \sqrt{bx^2 + cx^4} \, dx = \frac{8 \left(2 \, Bb^{\frac{9}{2}} - 3 \, Ab^{\frac{7}{2}} c\right) \mathrm{sgn}(x)}{315 \, c^4} + \frac{35 \, (cx^2 + b)^{\frac{9}{2}} B \mathrm{sgn}(x) - 135 \, (cx^2 + b)^{\frac{7}{2}} Bb \mathrm{sgn}(x) + 189 \, (cx^2 + b)^{\frac{5}{2}} Bb^2 \mathrm{sgn}(x) - 105 \, (cx^2 + b)^{\frac{3}{2}} Bb^3 \mathrm{sgn}(x}{315 \, c^4}$$

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] 8/315*(2*B*b^(9/2) - 3*A*b^(7/2)*c)*sgn(x)/c^4 + 1/315*(35*(c*x^2 + b)^(9/2
)*B*sgn(x) - 135*(c*x^2 + b)^(7/2)*B*b*sgn(x) + 189*(c*x^2 + b)^(5/2)*B*b^2
*sgn(x) - 105*(c*x^2 + b)^(3/2)*B*b^3*sgn(x) + 45*(c*x^2 + b)^(7/2)*A*c*sgn
(x) - 126*(c*x^2 + b)^(5/2)*A*b*c*sgn(x) + 105*(c*x^2 + b)^(3/2)*A*b^2*c*sg
n(x))/c^4
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.79

$$\int x^4(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$
$$=\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{B\,x^8}{9}-\frac{16\,B\,b^4-24\,A\,b^3\,c}{315\,c^4}+\frac{x^6\,\left(45\,A\,c^4+5\,B\,b\,c^3\right)}{315\,c^4}-\frac{4\,b^2\,x^2\,\left(3\,A\,c-2\,B\,b\right)}{315\,c^3}+\frac{b\,x^4\,\left(3\,A\,c-2\,B\,b\right)}{105\,c^2}\right)}{x}$$

```
[In] int(x^4*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*((B*x^8)/9 - (16*B*b^4 - 24*A*b^3*c)/(315*c^4) + (x^
6*(45*A*c^4 + 5*B*b*c^3))/(315*c^4) - (4*b^2*x^2*(3*A*c - 2*B*b))/(315*c^3)
 + (b*x^4*(3*A*c - 2*B*b))/(105*c^2)))/x
```

3.101 $\int x^2(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 94

$$\int x^2(A+Bx^2)\sqrt{bx^2+cx^4}\,dx=\frac{2b(4bB-7Ac)\left(bx^2+cx^4\right)^{3/2}}{105c^3x^3}-\frac{(4bB-7Ac)\left(bx^2+cx^4\right)^{3/2}}{35c^2x}+\frac{Bx(bx^2+cx^4)^{3/2}}{7c}$$

```
[Out] 2/105*b*(-7*A*c+4*B*b)*(c*x^4+b*x^2)^(3/2)/c^3/x^3-1/35*(-7*A*c+4*B*b)*(c*x
^4+b*x^2)^(3/2)/c^2/x+1/7*B*x*(c*x^4+b*x^2)^(3/2)/c
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2064, 2041, 2025}

$$\int x^2(A+Bx^2)\sqrt{bx^2+cx^4}\,dx=\frac{2b(bx^2+cx^4)^{3/2}(4bB-7Ac)}{105c^3x^3}-\frac{(bx^2+cx^4)^{3/2}(4bB-7Ac)}{35c^2x}+\frac{Bx(bx^2+cx^4)^{3/2}}{7c}$$

```
[In] Int[x^2*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*b*(4*b*B - 7*A*c)*(b*x^2 + c*x^4)^(3/2))/(105*c^3*x^3) - ((4*b*B - 7*A*c
)*(b*x^2 + c*x^4)^(3/2))/(35*c^2*x) + (B*x*(b*x^2 + c*x^4)^(3/2))/(7*c)
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
```

```
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{Bx(bx^2+cx^4)^{3/2}}{7c} - \frac{(4bB-7Ac)\int x^2\sqrt{bx^2+cx^4}\,dx}{7c}\\ &= -\frac{(4bB-7Ac)\,(bx^2+cx^4)^{3/2}}{35c^2x} + \frac{Bx(bx^2+cx^4)^{3/2}}{7c} + \frac{(2b(4bB-7Ac))\int\sqrt{bx^2+cx^4}\,dx}{35c^2}\\ &= \frac{2b(4bB-7Ac)\,(bx^2+cx^4)^{3/2}}{105c^3x^3} - \frac{(4bB-7Ac)\,(bx^2+cx^4)^{3/2}}{35c^2x} + \frac{Bx(bx^2+cx^4)^{3/2}}{7c}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.68

$$\int x^2\left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx = \frac{(x^2(b+cx^2))^{3/2}\,(8b^2B+3c^2x^2(7A+5Bx^2)-2bc(7A+6Bx^2))}{105c^3x^3}$$

```
[In] Integrate[x^2*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(8*b^2*B + 3*c^2*x^2*(7*A + 5*B*x^2) - 2*b*c*(7*A
+ 6*B*x^2)))/(105*c^3*x^3)
```

Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-15 B\,c^{2} x^{4}-21 A\,c^{2} x^{2}+12 B b c\,x^{2}+14 A b c-8 B\,b^{2}\right)\sqrt{x^{4} c+b\,x^{2}}}{105 c^{3} x}$	67
default	$-\frac{\left(c\,x^{2}+b\right)\left(-15 B\,c^{2} x^{4}-21 A\,c^{2} x^{2}+12 B b c\,x^{2}+14 A b c-8 B\,b^{2}\right)\sqrt{x^{4} c+b\,x^{2}}}{105 c^{3} x}$	67
trager	$-\frac{\left(-15 B\,c^{3} x^{6}-21 A\,c^{3} x^{4}-3 B b\,c^{2} x^{4}-7 A b\,c^{2} x^{2}+4 B\,b^{2} c\,x^{2}+14 b^{2} A c-8 B\,b^{3}\right)\sqrt{x^{4} c+b\,x^{2}}}{105 c^{3} x}$	84
risch	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(-15 B\,c^{3} x^{6}-21 A\,c^{3} x^{4}-3 B b\,c^{2} x^{4}-7 A b\,c^{2} x^{2}+4 B\,b^{2} c\,x^{2}+14 b^{2} A c-8 B\,b^{3}\right)}{105 x\,c^{3}}$	84

```
[In] int(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/105*(c*x^2+b)*(-15*B*c^2*x^4-21*A*c^2*x^2+12*B*b*c*x^2+14*A*b*c-8*B*b^2)
*(c*x^4+b*x^2)^(1/2)/c^3/x
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87

$$\int x^2(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{(15\,Bc^3x^6+3\,(Bbc^2+7\,Ac^3)x^4+8\,Bb^3-14\,Ab^2c-(4\,Bb^2c-7\,Abc^2)x^2)\sqrt{cx^4+bx^2}}{105\,c^3x}$$

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/105*(15*B*c^3*x^6 + 3*(B*b*c^2 + 7*A*c^3)*x^4 + 8*B*b^3 - 14*A*b^2*c - (4
*B*b^2*c - 7*A*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^3*x)
```

Sympy [F]

$$\int x^2(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \int x^2\sqrt{x^2\,(b+cx^2)}\,(A+Bx^2)\,dx$$

```
[In] integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x**2*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

$$\int x^2(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{(3\,c^2x^4+bcx^2-2\,b^2)\sqrt{cx^2+b}A}{15\,c^2} + \frac{(15\,c^3x^6+3\,bc^2x^4-4\,b^2cx^2+8\,b^3)\sqrt{cx^2+b}B}{105\,c^3}$$

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*c^2*x^4 + b*c*x^2 - 2*b^2)*sqrt(c*x^2 + b)*A/c^2 + 1/105*(15*c^3*x^
6 + 3*b*c^2*x^4 - 4*b^2*c*x^2 + 8*b^3)*sqrt(c*x^2 + b)*B/c^3
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12

$$\int x^2(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = -\frac{2\left(4\,Bb^{\frac{7}{2}}-7\,Ab^{\frac{5}{2}}c\right)\mathrm{sgn}(x)}{105\,c^3} + \frac{15\,(cx^2+b)^{\frac{7}{2}}B\mathrm{sgn}(x)-42\,(cx^2+b)^{\frac{5}{2}}Bb\mathrm{sgn}(x)+35\,(cx^2+b)^{\frac{3}{2}}Bb^2\mathrm{sgn}(x)+21\,(cx^2+b)^{\frac{5}{2}}Ac\mathrm{sgn}(x)-35}{105\,c^3}$$

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/105*(4*B*b^(7/2) - 7*A*b^(5/2)*c)*sgn(x)/c^3 + 1/105*(15*(c*x^2 + b)^(7/
2)*B*sgn(x) - 42*(c*x^2 + b)^(5/2)*B*b*sgn(x) + 35*(c*x^2 + b)^(3/2)*B*b^2*
sgn(x) + 21*(c*x^2 + b)^(5/2)*A*c*sgn(x) - 35*(c*x^2 + b)^(3/2)*A*b*c*sgn(x
))/c^3
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

$$\int x^2(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{\sqrt{c\,x^4+b\,x^2}\left(\frac{B\,x^6}{7}+\frac{8\,B\,b^3-14\,A\,b^2\,c}{105\,c^3}+\frac{x^4\,(21\,A\,c^3+3\,B\,b\,c^2)}{105\,c^3}+\frac{b\,x^2\,(7\,A\,c-4\,B\,b)}{105\,c^2}\right)}{x}$$

```
[In] int(x^2*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*((B*x^6)/7 + (8*B*b^3 - 14*A*b^2*c)/(105*c^3) + (x^4
*(21*A*c^3 + 3*B*b*c^2))/(105*c^3) + (b*x^2*(7*A*c - 4*B*b))/(105*c^2)))/x
```

3.102 $\int (A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 61

$$\int (A+Bx^2)\sqrt{bx^2+cx^4}\,dx=-\frac{(2bB-5Ac)\left(bx^2+cx^4\right)^{3/2}}{15c^2x^3}+\frac{B\left(bx^2+cx^4\right)^{3/2}}{5cx}$$

```
[Out] -1/15*(-5*A*c+2*B*b)*(c*x^4+b*x^2)^(3/2)/c^2/x^3+1/5*B*(c*x^4+b*x^2)^(3/2)/
c/x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1159, 2025}

$$\int (A+Bx^2)\sqrt{bx^2+cx^4}\,dx=\frac{B\left(bx^2+cx^4\right)^{3/2}}{5cx}-\frac{\left(bx^2+cx^4\right)^{3/2}(2bB-5Ac)}{15c^2x^3}$$

```
[In] Int[(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -1/15*((2*b*B - 5*A*c)*(b*x^2 + c*x^4)^(3/2))/(c^2*x^3) + (B*(b*x^2 + c*x^4
)^(3/2))/(5*c*x)
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
Simp[e*((b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 3)*x)), x] - Dist[(b*e*(2*p + 1)
- c*d*(4*p + 3))/(c*(4*p + 3)), Int[(b*x^2 + c*x^4)^p, x], x] /; FreeQ[{b,
c, d, e, p}, x] &&  !IntegerQ[p] && NeQ[4*p + 3, 0] && NeQ[b*e*(2*p + 1) -
c*d*(4*p + 3), 0]
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{B(bx^2+cx^4)^{3/2}}{5cx} - \frac{(2bB-5Ac)\int\sqrt{bx^2+cx^4}\,dx}{5c}\\ &= -\frac{(2bB-5Ac)\,(bx^2+cx^4)^{3/2}}{15c^2x^3} + \frac{B(bx^2+cx^4)^{3/2}}{5cx}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.67

$$\int (A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{(x^2(b+cx^2))^{3/2}\,(-2bB+5Ac+3Bcx^2)}{15c^2x^3}$$

```
[In] Integrate[(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
[Out] ((x^2*(b + c*x^2))^(3/2)*(-2*b*B + 5*A*c + 3*B*c*x^2))/(15*c^2*x^3)
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

method	result	size
gosper	$\frac{(c\,x^2+b)\,(3Bc\,x^2+5Ac-2Bb)\,\sqrt{x^4c+b\,x^2}}{15c^2x}$	45
default	$\frac{(c\,x^2+b)\,(3Bc\,x^2+5Ac-2Bb)\,\sqrt{x^4c+b\,x^2}}{15c^2x}$	45
trager	$\frac{(3B\,c^2x^4+5A\,c^2x^2+Bbc\,x^2+5Abc-2B\,b^2)\,\sqrt{x^4c+b\,x^2}}{15c^2x}$	59
risch	$\frac{\sqrt{x^2(c\,x^2+b)}\,(3B\,c^2x^4+5A\,c^2x^2+Bbc\,x^2+5Abc-2B\,b^2)}{15x\,c^2}$	59

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/15*(c*x^2+b)*(3*B*c*x^2+5*A*c-2*B*b)*(c*x^4+b*x^2)^(1/2)/c^2/x
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int \left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx=\frac{\left(3\,Bc^2x^4-2\,Bb^2+5\,Abc+\left(Bbc+5\,Ac^2\right)x^2\right)\sqrt{cx^4+bx^2}}{15\,c^2x}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/15*(3*B*c^2*x^4 - 2*B*b^2 + 5*A*b*c + (B*b*c + 5*A*c^2)*x^2)*sqrt(c*x^4 +
 b*x^2)/(c^2*x)
```

Sympy [F]

$$\int \left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx=\int \sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \left(A+Bx^2\right)\sqrt{bx^2+cx^4}\,dx=\frac{\left(cx^2+b\right)^{\frac{3}{2}}A}{3\,c}+\frac{\left(3\,c^2x^4+bcx^2-2\,b^2\right)\sqrt{cx^2+b}B}{15\,c^2}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*(c*x^2 + b)^(3/2)*A/c + 1/15*(3*c^2*x^4 + b*c*x^2 - 2*b^2)*sqrt(c*x^2 +
 b)*B/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$\int (A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{\left(2\,Bb^{\frac{5}{2}}-5\,Ab^{\frac{3}{2}}c\right)\mathrm{sgn}(x)}{15\,c^2} + \frac{3\,(cx^2+b)^{\frac{5}{2}}B\mathrm{sgn}(x)-5\,(cx^2+b)^{\frac{3}{2}}Bb\mathrm{sgn}(x)+5\,(cx^2+b)^{\frac{3}{2}}Ac\mathrm{sgn}(x)}{15\,c^2}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(2*B*b^(5/2) - 5*A*b^(3/2)*c)*sgn(x)/c^2 + 1/15*(3*(c*x^2 + b)^(5/2)*B
*sgn(x) - 5*(c*x^2 + b)^(3/2)*B*b*sgn(x) + 5*(c*x^2 + b)^(3/2)*A*c*sgn(x))/
c^2
```

Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int (A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{B\,x^4}{5}-\frac{2\,B\,b^2-5\,A\,b\,c}{15\,c^2}+\frac{x^2\,\left(5\,A\,c^2+B\,b\,c\right)}{15\,c^2}\right)}{x}$$

```
[In] int((A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*((B*x^4)/5 - (2*B*b^2 - 5*A*b*c)/(15*c^2) + (x^2*(5*
A*c^2 + B*b*c))/(15*c^2)))/x
```

3.103 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 78

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2} \, dx = \frac{A\sqrt{bx^2+cx^4}}{x} + \frac{B(bx^2+cx^4)^{3/2}}{3cx^3} - A\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] 1/3*B*(c*x^4+b*x^2)^(3/2)/c/x^3-A*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))*b^
(1/2)+A*(c*x^4+b*x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2064, 2046, 2033, 212}

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2} \, dx = -A\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right) + \frac{A\sqrt{bx^2+cx^4}}{x} + \frac{B(bx^2+cx^4)^{3/2}}{3cx^3}$$

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^2,x]
```

```
[Out] (A*Sqrt[b*x^2 + c*x^4])/x + (B*(b*x^2 + c*x^4)^(3/2))/(3*c*x^3) - A*Sqrt[b]
*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{B(bx^2+cx^4)^{3/2}}{3cx^3} + A\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx \\
&= \frac{A\sqrt{bx^2+cx^4}}{x} + \frac{B(bx^2+cx^4)^{3/2}}{3cx^3} + (Ab)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx \\
&= \frac{A\sqrt{bx^2+cx^4}}{x} + \frac{B(bx^2+cx^4)^{3/2}}{3cx^3} - (Ab)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right) \\
&= \frac{A\sqrt{bx^2+cx^4}}{x} + \frac{B(bx^2+cx^4)^{3/2}}{3cx^3} - A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2}\,dx = \frac{x\left((b+cx^2)(bB+3Ac+Bcx^2)-3A\sqrt{b}c\sqrt{b+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{3c\sqrt{x^2(b+cx^2)}}$$

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^2,x]
[Out] (x*((b + c*x^2)*(b*B + 3*A*c + B*c*x^2) - 3*A*Sqrt[b]*c*Sqrt[b + c*x^2]*Arc
Tanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(3*c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{\sqrt{x^4c+b\,x^2}\left(-3A\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)\sqrt{b}\,c+B\left(c\,x^2+b\right)^{\frac{3}{2}}+3A\sqrt{c\,x^2+b}\,c\right)}{3x\sqrt{c\,x^2+b}\,c}$	84

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/3*(c*x^4+b*x^2)^(1/2)*(-3*A*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b^(1/2)*c
+B*(c*x^2+b)^(3/2)+3*A*(c*x^2+b)^(1/2)*c)/x/(c*x^2+b)^(1/2)/c
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.04

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2}\,dx = \left[\frac{3\,A\sqrt{b}cx\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}(Bcx^2+Bb+3\,Ac)}{6\,cx}, \frac{3\,A\sqrt{-b}cx\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{}}{cx^3+bx}\right.}{}\right.$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="fricas")
[Out] [1/6*(3*A*sqrt(b)*c*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/
x^3) + 2*sqrt(c*x^4 + b*x^2)*(B*c*x^2 + B*b + 3*A*c))/(c*x), 1/3*(3*A*sqrt(
-b)*c*x*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b
*x^2)*(B*c*x^2 + B*b + 3*A*c))/(c*x)]
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2}\,dx = \int \frac{\sqrt{x^2(b+cx^2)}(A+Bx^2)}{x^2}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**2,x)
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**2, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^2}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="maxima")
[Out] A*integrate(sqrt(c*x^2 + b)/x, x) + 1/3*(c*x^2 + b)^(3/2)*B/c
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2}\,dx = \frac{Ab\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} - \frac{\left(3\,Abc\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+B\sqrt{-b}b^{\frac{3}{2}}+3\,A\sqrt{-b}\sqrt{b}c\right)\operatorname{sgn}(x)}{3\,\sqrt{-b}c} + \frac{(cx^2+b)^{\frac{3}{2}}Bc^2\operatorname{sgn}(x)+3\,\sqrt{cx^2+b}Ac^3\operatorname{sgn}(x)}{3\,c^3}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^2,x, algorithm="giac")
[Out] A*b*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - 1/3*(3*A*b*c*arctan(
sqrt(b)/sqrt(-b)) + B*sqrt(-b)*b^(3/2) + 3*A*sqrt(-b)*sqrt(b)*c)*sgn(x)/(sq
rt(-b)*c) + 1/3*((c*x^2 + b)^(3/2)*B*c^2*sgn(x) + 3*sqrt(c*x^2 + b)*A*c^3*s
gn(x))/c^3
```

Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^2}\,dx = \frac{A\sqrt{c\,x^4+b\,x^2}}{x} + \frac{B\,(c\,x^2+b)\,\sqrt{c\,x^4+b\,x^2}}{3\,c\,x} + \frac{A\,\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,1\mathrm{i}}{\sqrt{c}\,x}\right)\,\sqrt{c\,x^4+b\,x^2}\,1\mathrm{i}}{\sqrt{c}\,x^2\,\sqrt{\frac{b}{c\,x^2}+1}}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^2,x)
```

```
[Out] (A*(b*x^2 + c*x^4)^(1/2))/x + (B*(b + c*x^2)*(b*x^2 + c*x^4)^(1/2))/(3*c*x)
 + (A*b^(1/2)*asin((b^(1/2)*1i)/(c^(1/2)*x))*(b*x^2 + c*x^4)^(1/2)*1i)/(c^(
1/2)*x^2*(b/(c*x^2) + 1)^(1/2))
```

3.104 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 100

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4}\,dx = \frac{(2bB+Ac)\sqrt{bx^2+cx^4}}{2bx} - \frac{A(bx^2+cx^4)^{3/2}}{2bx^5} - \frac{(2bB+Ac)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}$$

```
[Out] -1/2*A*(c*x^4+b*x^2)^(3/2)/b/x^5-1/2*(A*c+2*B*b)*arctanh(x*b^(1/2)/(c*x^4+b
*x^2)^(1/2))/b^(1/2)+1/2*(A*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/b/x
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2063, 2046, 2033, 212}

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4}\,dx = -\frac{(Ac+2bB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}} + \frac{\sqrt{bx^2+cx^4}(Ac+2bB)}{2bx} - \frac{A(bx^2+cx^4)^{3/2}}{2bx^5}$$

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^4,x]
```

```
[Out] ((2*b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(2*b*x) - (A*(b*x^2 + c*x^4)^(3/2))/(2*
b*x^5) - ((2*b*B + A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*Sqrt[b
])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{A(bx^2+cx^4)^{3/2}}{2bx^5}-\frac{(-2bB-Ac)\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx}{2b}\\
&= \frac{(2bB+Ac)\sqrt{bx^2+cx^4}}{2bx}-\frac{A(bx^2+cx^4)^{3/2}}{2bx^5}-\frac{1}{2}(-2bB-Ac)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx\\
&= \frac{(2bB+Ac)\sqrt{bx^2+cx^4}}{2bx}-\frac{A(bx^2+cx^4)^{3/2}}{2bx^5}\\
&\quad -\frac{1}{2}(2bB+Ac)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)\\
&= \frac{(2bB+Ac)\sqrt{bx^2+cx^4}}{2bx}-\frac{A(bx^2+cx^4)^{3/2}}{2bx^5}-\frac{(2bB+Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4}\,dx = -\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b}(A-2Bx^2)\sqrt{b+cx^2}+(2bB+Ac)x^2\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{2\sqrt{b}x^3\sqrt{b+cx^2}}$$

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^4,x]
```

```
[Out] -1/2*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*(A - 2*B*x^2)*Sqrt[b + c*x^2] + (2*b*B
 + A*c)*x^2*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(Sqrt[b]*x^3*Sqrt[b + c*x^2]
)
```

Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

method	result	s
risch	$-\frac{A\sqrt{x^2(cx^2+b)}}{2x^3}+\frac{\left(B\sqrt{cx^2+b}-\frac{(Ac+2Bb)\ln\left(\frac{2b+2\sqrt{b}\sqrt{cx^2+b}}{x}\right)}{2\sqrt{b}}\right)\sqrt{x^2(cx^2+b)}}{x\sqrt{cx^2+b}}$	
default	$-\frac{\sqrt{x^4c+bx^2}\left(A\sqrt{b}\ln\left(\frac{2b+2\sqrt{b}\sqrt{cx^2+b}}{x}\right)cx^2+2Bb^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\sqrt{cx^2+b}}{x}\right)x^2-A\sqrt{cx^2+b}\,cx^2-2B\sqrt{cx^2+b}\,bx^2+A(cx^2+b)^{\frac{3}{2}}\right)}{2x^3\sqrt{cx^2+b}\,b}$	

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*A/x^3*(x^2*(c*x^2+b))^(1/2)+(B*(c*x^2+b)^(1/2)-1/2*(A*c+2*B*b)/b^(1/2)
*ln((2*b+2*b^(1/2)*(c*x^2+b)^(1/2))/x))*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(
1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.69

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4}\,dx = \left[\frac{(2\,Bb+Ac)\sqrt{b}x^3\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}(2\,Bbx^2-Ab)}{4\,bx^3}, \frac{(2\,Bb+Ac)\sqrt{-b}x^3\arctan\left(\right.}{}\right.$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/4*((2*B*b + A*c)*sqrt(b)*x^3*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)
*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(2*B*b*x^2 - A*b))/(b*x^3), 1/2*((2*
B*b + A*c)*sqrt(-b)*x^3*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x))
+ sqrt(c*x^4 + b*x^2)*(2*B*b*x^2 - A*b))/(b*x^3)]
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4}\,dx = \int \frac{\sqrt{x^2(b+cx^2)}(A+Bx^2)}{x^4}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**4, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^4}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^4, x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.76

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4}\,dx = \frac{2\sqrt{cx^2+b}Bc\operatorname{sgn}(x) + \frac{(2Bbc\operatorname{sgn}(x)+Ac^2\operatorname{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{\sqrt{cx^2+b}Ac\operatorname{sgn}(x)}{x^2}}{2c}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(c*x^2 + b)*B*c*sgn(x) + (2*B*b*c*sgn(x) + A*c^2*sgn(x))*arctan(
sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b) - sqrt(c*x^2 + b)*A*c*sgn(x)/x^2)/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^4}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^4}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^4,x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^4, x)
```

3.105 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 103

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6}\,dx = -\frac{(4bB-Ac)\sqrt{bx^2+cx^4}}{8bx^3} - \frac{A(bx^2+cx^4)^{3/2}}{4bx^7} - \frac{c(4bB-Ac)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}$$

```
[Out] -1/4*A*(c*x^4+b*x^2)^(3/2)/b/x^7-1/8*c*(-A*c+4*B*b)*arctanh(x*b^(1/2)/(c*x^
4+b*x^2)^(1/2))/b^(3/2)-1/8*(-A*c+4*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2063, 2045, 2033, 212}

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6}\,dx = -\frac{c(4bB-Ac)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}} - \frac{\sqrt{bx^2+cx^4}(4bB-Ac)}{8bx^3} - \frac{A(bx^2+cx^4)^{3/2}}{4bx^7}$$

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^6,x]
```

```
[Out] -1/8*((4*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(b*x^3) - (A*(b*x^2 + c*x^4)^(3/2)
)/(4*b*x^7) - (c*(4*b*B - A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8
*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A(bx^2+cx^4)^{3/2}}{4bx^7} - \frac{(-4bB+Ac)\int \frac{\sqrt{bx^2+cx^4}}{x^4}\,dx}{4b} \\
&= -\frac{(4bB-Ac)\sqrt{bx^2+cx^4}}{8bx^3} - \frac{A(bx^2+cx^4)^{3/2}}{4bx^7} + \frac{(c(4bB-Ac))\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b} \\
&= -\frac{(4bB-Ac)\sqrt{bx^2+cx^4}}{8bx^3} - \frac{A(bx^2+cx^4)^{3/2}}{4bx^7} - \frac{(c(4bB-Ac))\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)}{8b} \\
&= -\frac{(4bB-Ac)\sqrt{bx^2+cx^4}}{8bx^3} - \frac{A(bx^2+cx^4)^{3/2}}{4bx^7} - \frac{c(4bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6}\,dx = -\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b}\sqrt{b+cx^2}(2Ab+4bBx^2+Acx^2)+c(4bB-Ac)x^4\operatorname{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{8b^{3/2}x^5\sqrt{b+cx^2}}$$

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^6,x]
```

```
[Out] -1/8*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*Sqrt[b + c*x^2]*(2*A*b + 4*b*B*x^2 + A
*c*x^2) + c*(4*b*B - A*c)*x^4*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(b^(3/2)*x
^5*Sqrt[b + c*x^2])
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{(Acx^2+4bBx^2+2Ab)\sqrt{x^2(cx^2+b)}}{8x^5b}+\frac{(Ac-4Bb)c\ln\left(\frac{2b+2\sqrt{b}\sqrt{cx^2+b}}{x}\right)\sqrt{x^2(cx^2+b)}}{8b^{\frac{3}{2}}x\sqrt{cx^2+b}}$
default	$-\frac{\sqrt{x^4c+bx^2}\left(-A\sqrt{b}\ln\left(\frac{2b+2\sqrt{b}\sqrt{cx^2+b}}{x}\right)c^2x^4+4Bb^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\sqrt{cx^2+b}}{x}\right)cx^4+A\sqrt{cx^2+b}\,c^2x^4-4B\sqrt{cx^2+b}\,bcx^4-A(cx^2+b)\right.}{8x^5\sqrt{cx^2+b}\,b^2}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(A*c*x^2+4*B*b*x^2+2*A*b)/x^5/b*(x^2*(c*x^2+b))^(1/2)+1/8*(A*c-4*B*b)*
c/b^(3/2)*ln((2*b+2*b^(1/2)*(c*x^2+b)^(1/2))/x)*(x^2*(c*x^2+b))^(1/2)/x/(c*
x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.92

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6}\,dx = \left[-\frac{(4\,Bbc-Ac^2)\sqrt{b}x^5\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}(2\,Ab^2+(4\,Bb^2+Abc)x^2)}{16\,b^2x^5}, \frac{(4\,Bbc-A}{}\right.$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] [-1/16*((4*B*b*c - A*c^2)*sqrt(b)*x^5*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 +
b*x^2)*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(2*A*b^2 + (4*B*b^2 + A*b*c)*x
^2))/(b^2*x^5), 1/8*((4*B*b*c - A*c^2)*sqrt(-b)*x^5*arctan(sqrt(c*x^4 + b*x
^2)*sqrt(-b)/(c*x^3 + b*x)) - sqrt(c*x^4 + b*x^2)*(2*A*b^2 + (4*B*b^2 + A*b
*c)*x^2))/(b^2*x^5)]
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6}\,dx = \int \frac{\sqrt{x^2(b+cx^2)}(A+Bx^2)}{x^6}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**6, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^6}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^6, x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.28

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6}\,dx = \frac{\frac{(4\,Bbc^2\mathrm{sgn}(x)-Ac^3\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b} - \frac{4\,(cx^2+b)^{\frac{3}{2}}Bbc^2\mathrm{sgn}(x)-4\,\sqrt{cx^2+b}Bb^2c^2\mathrm{sgn}(x)+(cx^2+b)^{\frac{3}{2}}Ac^3\mathrm{sgn}(x)+\sqrt{cx^2+b}Abc^3\mathrm{sgn}(x)}{bc^2x^4}}{8\,c}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/8*((4*B*b*c^2*sgn(x) - A*c^3*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sq
rt(-b)*b) - (4*(c*x^2 + b)^(3/2)*B*b*c^2*sgn(x) - 4*sqrt(c*x^2 + b)*B*b^2*c
^2*sgn(x) + (c*x^2 + b)^(3/2)*A*c^3*sgn(x) + sqrt(c*x^2 + b)*A*b*c^3*sgn(x)
)/(b*c^2*x^4))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^6}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^6}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^6,x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^6, x)
```

3.106 $\int x^5(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 223

$$\int x^5\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{b^4(9bB-14Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{2048c^5}-\frac{b^2(9bB-14Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{768c^4}+\frac{b(9bB-14Ac)\left(bx^2+cx^4\right)^{5/2}}{240c^3}-\frac{(9bB-14Ac)x^2\left(bx^2+cx^4\right)^{5/2}}{168c^2}+\frac{Bx^4\left(bx^2+cx^4\right)^{5/2}}{14c}-\frac{b^6(9bB-14Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2048c^{11/2}}$$

```
[Out] -1/768*b^2*(-14*A*c+9*B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(3/2)/c^4+1/240*b*(-14
*A*c+9*B*b)*(c*x^4+b*x^2)^(5/2)/c^3-1/168*(-14*A*c+9*B*b)*x^2*(c*x^4+b*x^2)
^(5/2)/c^2+1/14*B*x^4*(c*x^4+b*x^2)^(5/2)/c-1/2048*b^6*(-14*A*c+9*B*b)*arct
anh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(11/2)+1/2048*b^4*(-14*A*c+9*B*b)*(2
*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^5
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used

= {2059, 808, 684, 654, 626, 634, 212}

$$\int x^5 (A + Bx^2) (bx^2 + cx^4)^{3/2} \, dx = -\frac{b^6(9bB - 14Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2048c^{11/2}} + \frac{b^4(b + 2cx^2)\sqrt{bx^2 + cx^4}(9bB - 14Ac)}{2048c^5} - \frac{b^2(b + 2cx^2)(bx^2 + cx^4)^{3/2}(9bB - 14Ac)}{768c^4} + \frac{b(bx^2 + cx^4)^{5/2}(9bB - 14Ac)}{240c^3} - \frac{x^2(bx^2 + cx^4)^{5/2}(9bB - 14Ac)}{168c^2} + \frac{Bx^4(bx^2 + cx^4)^{5/2}}{14c}$$

```
[In] Int[x^5*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (b^4*(9*b*B - 14*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(2048*c^5) - (b^2*
(9*b*B - 14*A*c)*(b + 2*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(768*c^4) + (b*(9*b*B
 - 14*A*c)*(b*x^2 + c*x^4)^(5/2))/(240*c^3) - ((9*b*B - 14*A*c)*x^2*(b*x^2
+ c*x^4)^(5/2))/(168*c^2) + (B*x^4*(b*x^2 + c*x^4)^(5/2))/(14*c) - (b^6*(9*
b*B - 14*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(2048*c^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
```

```
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int x^2(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx,x,x^2\right)$$

$$= \frac{Bx^4(bx^2+cx^4)^{5/2}}{14c} + \frac{\left(2(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int x^2(bx+cx^2)^{3/2}\,dx,x,x^2\right)}{14c}$$

$$= -\frac{(9bB-14Ac)x^2(bx^2+cx^4)^{5/2}}{168c^2} + \frac{Bx^4(bx^2+cx^4)^{5/2}}{14c} + \frac{(b(9bB-14Ac))\text{Subst}\left(\int x(bx+cx^2)^{3/2}\,dx,x,x^2\right)}{48c^2}$$

$$= \frac{b(9bB-14Ac)\left(bx^2+cx^4\right)^{5/2}}{240c^3} - \frac{(9bB-14Ac)x^2(bx^2+cx^4)^{5/2}}{168c^2} + \frac{Bx^4(bx^2+cx^4)^{5/2}}{14c} - \frac{\left(b^2(9bB-14Ac)\right)\text{Subst}\left(\int (bx+cx^2)^{3/2}\,dx,x,x^2\right)}{96c^3}$$

$$= -\frac{b^2(9bB-14Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{768c^4} + \frac{b(9bB-14Ac)\left(bx^2+cx^4\right)^{5/2}}{240c^3} - \frac{(9bB-14Ac)x^2\left(bx^2+cx^4\right)^{5/2}}{168c^2} + \frac{Bx^4\left(bx^2+cx^4\right)^{5/2}}{14c} + \frac{\left(b^4(9bB-14Ac)\right)\text{Subst}\left(\int\sqrt{bx+cx^2}\,dx,x,x^2\right)}{512c^4}$$

$$= \frac{b^4(9bB-14Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{2048c^5} - \frac{b^2(9bB-14Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{768c^4} + \frac{b(9bB-14Ac)\left(bx^2+cx^4\right)^{5/2}}{240c^3} - \frac{(9bB-14Ac)x^2\left(bx^2+cx^4\right)^{5/2}}{168c^2} + \frac{Bx^4\left(bx^2+cx^4\right)^{5/2}}{14c} - \frac{\left(b^6(9bB-14Ac)\right)\text{Subst}\left(\int\frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{4096c^5}$$

$$= \frac{b^4(9bB-14Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{2048c^5} - \frac{b^2(9bB-14Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{768c^4} + \frac{b(9bB-14Ac)\left(bx^2+cx^4\right)^{5/2}}{240c^3} - \frac{(9bB-14Ac)x^2\left(bx^2+cx^4\right)^{5/2}}{168c^2} + \frac{Bx^4\left(bx^2+cx^4\right)^{5/2}}{14c} - \frac{\left(b^6(9bB-14Ac)\right)\text{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{2048c^5}$$

$$= \frac{b^4(9bB-14Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{2048c^5} - \frac{b^2(9bB-14Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{768c^4} + \frac{b(9bB-14Ac)\left(bx^2+cx^4\right)^{5/2}}{240c^3} - \frac{(9bB-14Ac)x^2\left(bx^2+cx^4\right)^{5/2}}{168c^2} + \frac{Bx^4\left(bx^2+cx^4\right)^{5/2}}{14c} - \frac{b^6(9bB-14Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2048c^{11/2}}$$

Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.11

$$\int x^5\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(945b^6B-1470Ab^5c-630b^5Bcx^2+980Ab^4c^2x^2+504b^4Bc^2x^4-784Ab^3c^3x\right.}{21} - \frac{b^6(9bB-14Ac)\left(x^2\left(b+cx^2\right)\right)^{3/2}\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{b}+\sqrt{b+cx^2}}\right)}{1024c^{11/2}x^3\left(b+cx^2\right)^{3/2}}$$

```
[In] Integrate[x^5*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(945*b^6*B - 1470*A*b^5*c - 630*b^5*B*c*x^2 + 980*
A*b^4*c^2*x^2 + 504*b^4*B*c^2*x^4 - 784*A*b^3*c^3*x^4 - 432*b^3*B*c^3*x^6 +
```

```
672*A*b^2*c^4*x^6 + 384*b^2*B*c^4*x^8 + 23296*A*b*c^5*x^8 + 19200*b*B*c^5*
x^10 + 17920*A*c^6*x^10 + 15360*B*c^6*x^12))/(215040*c^5*x^2*(b + c*x^2)) -
 (b^6*(9*b*B - 14*A*c)*(x^2*(b + c*x^2))^(3/2)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[b
] + Sqrt[b + c*x^2])])/(1024*c^(11/2)*x^3*(b + c*x^2)^(3/2))
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{\frac{7\left(A b^6 c-\frac{9}{14} B b^7\right) \ln\left(\frac{2 c x^2+2\sqrt{x^2\left(c x^2+b\right)}\sqrt{c}+b}{\sqrt{c}}\right)}{2048}+\frac{7\left(\frac{832 x^8\left(\frac{75 x^2 B}{91}+A\right) b c^{\frac{11}{2}}}{35}+\frac{128 x^{10}\left(\frac{6 x^2 B}{7}+A\right) c^{\frac{13}{2}}}{7}+\left(-\frac{3\left(\frac{3 x^2 B}{7}+A\right) b^3 c^{\frac{3}{2}}}{2}+\cdots\right.\right.}{\cdots}}{c^{\frac{11}{2}}}$
risch	$-\frac{\left(-15360 B c^6 x^{12}-17920 A c^6 x^{10}-19200 B b c^5 x^{10}-23296 A b c^5 x^8-384 B b^2 c^4 x^8-672 A b^2 c^4 x^6+432 B b^3 c^3 x^6+784 A b^3 c^3 x^4 \cdots\right.}{215040 c^5}$
default	$\frac{\left(x^4 c+b x^2\right)^{\frac{3}{2}}\left(15360 B\left(c x^2+b\right)^{\frac{5}{2}} c^{\frac{9}{2}} x^9+17920 A\left(c x^2+b\right)^{\frac{5}{2}} c^{\frac{9}{2}} x^7-11520 B\left(c x^2+b\right)^{\frac{5}{2}} c^{\frac{7}{2}} b x^7-12544 A\left(c x^2+b\right)^{\frac{5}{2}} c^{\frac{7}{2}} b x^5+8064 \cdots\right.}{\cdots}$

```
[In] int(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 7/1536*(3/4*(A*b^6*c-9/14*B*b^7)*ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2)*c^(1/2
)+b)/c^(1/2))+(832/35*x^8*(75/91*x^2*B+A)*b*c^(11/2)+128/7*x^10*(6/7*x^2*B+
A)*c^(13/2)+(-3/2*(3/7*x^2*B+A)*b^3*c^(3/2)+b^2*x^2*(18/35*x^2*B+A)*c^(5/2)
-4/5*x^4*(27/49*x^2*B+A)*b*c^(7/2)+24/35*x^6*(4/7*x^2*B+A)*c^(9/2)+27/28*B*
c^(1/2)*b^4)*b^2)*(x^2*(c*x^2+b))^(1/2)-3/4*ln(2)*b^6*(A*c-9/14*B*b))/c^(11
/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.87

$$\int x^5\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\left[-\frac{105\left(9\,Bb^7-14\,Ab^6c\right)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\left(15360\,Bc^7x^{12}+1280\left(15\cdots\right.\right.}{\cdots}\right.$$

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/430080*(105*(9*B*b^7 - 14*A*b^6*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*
x^4 + b*x^2)*sqrt(c)) - 2*(15360*B*c^7*x^12 + 1280*(15*B*b*c^6 + 14*A*c^7)*
x^10 + 128*(3*B*b^2*c^5 + 182*A*b*c^6)*x^8 + 945*B*b^6*c - 1470*A*b^5*c^2 -
```

```
 48*(9*B*b^3*c^4 - 14*A*b^2*c^5)*x^6 + 56*(9*B*b^4*c^3 - 14*A*b^3*c^4)*x^4
- 70*(9*B*b^5*c^2 - 14*A*b^4*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^6, 1/215040*(
105*(9*B*b^7 - 14*A*b^6*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*
x^2 + b)) + (15360*B*c^7*x^12 + 1280*(15*B*b*c^6 + 14*A*c^7)*x^10 + 128*(3*
B*b^2*c^5 + 182*A*b*c^6)*x^8 + 945*B*b^6*c - 1470*A*b^5*c^2 - 48*(9*B*b^3*c
^4 - 14*A*b^2*c^5)*x^6 + 56*(9*B*b^4*c^3 - 14*A*b^3*c^4)*x^4 - 70*(9*B*b^5*
c^2 - 14*A*b^4*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^6]
```

Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 726, normalized size of antiderivative = 3.26

$$\int x^5(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{Ab\left(\begin{cases}\frac{7b^5\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{256c^4}+\sqrt{bx^2+cx^4}\left(-\frac{7b^4}{128c^4}+\frac{7b^3x^2}{192c^3}-\frac{7b^2x^4}{240c^2}+\frac{bx^6}{40c} \\ \frac{2(bx^2)^{\frac{9}{2}}}{9b^4} \\ 0\end{cases}\right.}{2}$$

$$+\frac{Ac\left(\begin{cases}-\frac{21b^6\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{1024c^5}+\sqrt{bx^2+cx^4}\cdot\left(\frac{21b^5}{512c^5}-\frac{7b^4x^2}{256c^4}+\frac{7b^3x^4}{320c^3}-\frac{3b^2x^6}{160c^2}+\frac{bx^8}{60c}+ \\ \frac{2(bx^2)^{\frac{11}{2}}}{11b^5} \\ 0\end{cases}\right.}{2}$$

$$+\frac{Bb\left(\begin{cases}-\frac{21b^6\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{1024c^5}+\sqrt{bx^2+cx^4}\cdot\left(\frac{21b^5}{512c^5}-\frac{7b^4x^2}{256c^4}+\frac{7b^3x^4}{320c^3}-\frac{3b^2x^6}{160c^2}+\frac{bx^8}{60c}+ \\ \frac{2(bx^2)^{\frac{11}{2}}}{11b^5} \\ 0\end{cases}\right.}{2}$$

$$+\frac{Bc\left(\begin{cases}\frac{33b^7\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{2048c^6}+\sqrt{bx^2+cx^4}\left(-\frac{33b^6}{1024c^6}+\frac{11b^5x^2}{512c^5}-\frac{11b^4x^4}{640c^4}+\frac{33b^3x^6}{2240c^3}-\frac{11b^2x^8}{840c^2} \\ \frac{2(bx^2)^{\frac{13}{2}}}{13b^6} \\ 0\end{cases}\right.}{2}$$

```
[In] integrate(x**5*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] A*b*Piecewise((7*b**5*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) +
2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sq
rt(c*(b/(2*c) + x**2)**2), True))/(256*c**4) + sqrt(b*x**2 + c*x**4)*(-7*b*
*4/(128*c**4) + 7*b**3*x**2/(192*c**3) - 7*b**2*x**4/(240*c**2) + b*x**6/(4
0*c) + x**8/5), Ne(c, 0)), (2*(b*x**2)**(9/2)/(9*b**4), Ne(b, 0)), (0, True
))/2 + A*c*Piecewise((-21*b**6*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c
*x**4) + 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) +
 x**2)/sqrt(c*(b/(2*c) + x**2)**2), True))/(1024*c**5) + sqrt(b*x**2 + c*x*
*4)*(21*b**5/(512*c**5) - 7*b**4*x**2/(256*c**4) + 7*b**3*x**4/(320*c**3) -
 3*b**2*x**6/(160*c**2) + b*x**8/(60*c) + x**10/6), Ne(c, 0)), (2*(b*x**2)*
*(11/2)/(11*b**5), Ne(b, 0)), (0, True))/2 + B*b*Piecewise((-21*b**6*Piecew
ise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*c*x**2)/sqrt(c), Ne(b**2/c
, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt(c*(b/(2*c) + x**2)**2), T
rue))/(1024*c**5) + sqrt(b*x**2 + c*x**4)*(21*b**5/(512*c**5) - 7*b**4*x**2
/(256*c**4) + 7*b**3*x**4/(320*c**3) - 3*b**2*x**6/(160*c**2) + b*x**8/(60*
c) + x**10/6), Ne(c, 0)), (2*(b*x**2)**(11/2)/(11*b**5), Ne(b, 0)), (0, Tru
e))/2 + B*c*Piecewise((33*b**7*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c
*x**4) + 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) +
 x**2)/sqrt(c*(b/(2*c) + x**2)**2), True))/(2048*c**6) + sqrt(b*x**2 + c*x*
*4)*(-33*b**6/(1024*c**6) + 11*b**5*x**2/(512*c**5) - 11*b**4*x**4/(640*c**
4) + 33*b**3*x**6/(2240*c**3) - 11*b**2*x**8/(840*c**2) + b*x**10/(84*c) +
x**12/7), Ne(c, 0)), (2*(b*x**2)**(13/2)/(13*b**6), Ne(b, 0)), (0, True))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.63

$$\int x^5(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx =$$

$$-\frac{1}{30720}\left(\frac{420\sqrt{cx^4+bx^2}b^4x^2}{c^3}-\frac{1120\,(cx^4+bx^2)^{\frac{3}{2}}b^2x^2}{c^2}-\frac{2560\,(cx^4+bx^2)^{\frac{5}{2}}x^2}{c}-\frac{105\,b^6\log\left(2\,cx^2+b+2\right.}{c^{\frac{9}{2}}}\right.$$

$$+\frac{1}{143360}\left(\frac{10240\,(cx^4+bx^2)^{\frac{5}{2}}x^4}{c}+\frac{1260\sqrt{cx^4+bx^2}b^5x^2}{c^4}-\frac{3360\,(cx^4+bx^2)^{\frac{3}{2}}b^3x^2}{c^3}-\frac{7680\,(cx^4+bx^2)^{\frac{5}{2}}bx^2}{c^2}\right.$$

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/30720*(420*sqrt(c*x^4 + b*x^2)*b^4*x^2/c^3 - 1120*(c*x^4 + b*x^2)^(3/2)*
b^2*x^2/c^2 - 2560*(c*x^4 + b*x^2)^(5/2)*x^2/c - 105*b^6*log(2*c*x^2 + b +
2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(9/2) + 210*sqrt(c*x^4 + b*x^2)*b^5/c^4 -
560*(c*x^4 + b*x^2)^(3/2)*b^3/c^3 + 1792*(c*x^4 + b*x^2)^(5/2)*b/c^2)*A + 1
/143360*(10240*(c*x^4 + b*x^2)^(5/2)*x^4/c + 1260*sqrt(c*x^4 + b*x^2)*b^5*x
```

```
^2/c^4 - 3360*(c*x^4 + b*x^2)^(3/2)*b^3*x^2/c^3 - 7680*(c*x^4 + b*x^2)^(5/2
)*b*x^2/c^2 - 315*b^7*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(1
1/2) + 630*sqrt(c*x^4 + b*x^2)*b^6/c^5 - 1680*(c*x^4 + b*x^2)^(3/2)*b^4/c^4
 + 5376*(c*x^4 + b*x^2)^(5/2)*b^2/c^3)*B
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.26

$$\int x^5(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{1}{215040}\left(2\left(4\left(2\left(8\left(10\left(12\,Bcx^2\mathrm{sgn}(x)+\frac{15\,Bbc^{12}\mathrm{sgn}(x)+14\,Ac^{13}\mathrm{sgn}(x)}{c^{12}}\right)x^2+\frac{3\,Bb^2c^{11}\mathrm{sg}}{}\right.\right.\right.\right.\right. + \frac{(9\,Bb^7\mathrm{sgn}(x)-14\,Ab^6c\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{2048\,c^{\frac{11}{2}}} - \frac{(9\,Bb^7\log(|b|)-14\,Ab^6c\log(|b|))\mathrm{sgn}(x)}{4096\,c^{\frac{11}{2}}}$$

```
[In] integrate(x^5*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/215040*(2*(4*(2*(8*(10*(12*B*c*x^2*sgn(x) + (15*B*b*c^12*sgn(x) + 14*A*c^
13*sgn(x))/c^12)*x^2 + (3*B*b^2*c^11*sgn(x) + 182*A*b*c^12*sgn(x))/c^12)*x^
2 - 3*(9*B*b^3*c^10*sgn(x) - 14*A*b^2*c^11*sgn(x))/c^12)*x^2 + 7*(9*B*b^4*c
^9*sgn(x) - 14*A*b^3*c^10*sgn(x))/c^12)*x^2 - 35*(9*B*b^5*c^8*sgn(x) - 14*A
*b^4*c^9*sgn(x))/c^12)*x^2 + 105*(9*B*b^6*c^7*sgn(x) - 14*A*b^5*c^8*sgn(x))
/c^12)*sqrt(c*x^2 + b)*x + 1/2048*(9*B*b^7*sgn(x) - 14*A*b^6*c*sgn(x))*log(
abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(11/2) - 1/4096*(9*B*b^7*log(abs(b)) -
 14*A*b^6*c*log(abs(b)))*sgn(x)/c^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^5(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \int x^5\,\left(B\,x^2+A\right)\,{\left(c\,x^4+b\,x^2\right)}^{3/2}\,dx$$

```
[In] int(x^5*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^5*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

3.107 $\int x^3(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 167

$$\int x^3(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=-\frac{b^3(7bB-12Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{1024c^4}+\frac{b(7bB-12Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{384c^3}-\frac{\left(7bB-12Ac-10Bcx^2\right)\left(bx^2+cx^4\right)^{5/2}}{120c^2}+\frac{b^5(7bB-12Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^{9/2}}$$

```
[Out] 1/384*b*(-12*A*c+7*B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(3/2)/c^3-1/120*(-10*B*c*
x^2-12*A*c+7*B*b)*(c*x^4+b*x^2)^(5/2)/c^2+1/1024*b^5*(-12*A*c+7*B*b)*arctan
h(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(9/2)-1/1024*b^3*(-12*A*c+7*B*b)*(2*c*
x^2+b)*(c*x^4+b*x^2)^(1/2)/c^4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2059, 793, 626, 634, 212}

$$\int x^3(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{b^5(7bB-12Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^{9/2}}-\frac{b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}(7bB-12Ac)}{1024c^4}+\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}(7bB-12Ac)}{384c^3}-\frac{\left(bx^2+cx^4\right)^{5/2}\left(-12Ac+7bB-10Bcx^2\right)}{120c^2}$$

```
[In] Int[x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -1/1024*(b^3*(7*b*B - 12*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/c^4 + (b*(
7*b*B - 12*A*c)*(b + 2*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(384*c^3) - ((7*b*B -
12*A*c - 10*B*c*x^2)*(b*x^2 + c*x^4)^(5/2))/(120*c^2) + (b^5*(7*b*B - 12*A*
c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(1024*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int x(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx,x,x^2\right)$$

$$= -\frac{(7bB-12Ac-10Bcx^2)(bx^2+cx^4)^{5/2}}{120c^2} + \frac{(b(7bB-12Ac))\text{Subst}\left(\int (bx+cx^2)^{3/2}\,dx,x,x^2\right)}{48c^2}$$

$$= \frac{b(7bB-12Ac)(b+2cx^2)(bx^2+cx^4)^{3/2}}{384c^3} - \frac{(7bB-12Ac-10Bcx^2)(bx^2+cx^4)^{5/2}}{120c^2} - \frac{(b^3(7bB-12Ac))\text{Subst}\left(\int \sqrt{bx+cx^2}\,dx,x,x^2\right)}{256c^3}$$

$$= -\frac{b^3(7bB-12Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{1024c^4} + \frac{b(7bB-12Ac)(b+2cx^2)(bx^2+cx^4)^{3/2}}{384c^3} - \frac{(7bB-12Ac-10Bcx^2)(bx^2+cx^4)^{5/2}}{120c^2} + \frac{(b^5(7bB-12Ac))\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{2048c^4}$$

$$= -\frac{b^3(7bB-12Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{1024c^4} + \frac{b(7bB-12Ac)(b+2cx^2)(bx^2+cx^4)^{3/2}}{384c^3} - \frac{(7bB-12Ac-10Bcx^2)(bx^2+cx^4)^{5/2}}{120c^2} + \frac{(b^5(7bB-12Ac))\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^4}$$

$$= -\frac{b^3(7bB-12Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{1024c^4} + \frac{b(7bB-12Ac)(b+2cx^2)(bx^2+cx^4)^{3/2}}{384c^3} - \frac{(7bB-12Ac-10Bcx^2)(bx^2+cx^4)^{5/2}}{120c^2} + \frac{b^5(7bB-12Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{1024c^{9/2}}$$

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.35

$$\int x^3(A+Bx^2)(bx^2+cx^4)^{3/2}\,dx = \frac{(x^2(b+cx^2))^{3/2}(-105b^5Bx+180Ab^4cx+70b^4Bcx^3-120Ab^3c^2x^3-56b^3Bc^2x^5+96Ab^2c^3x}{15360c^4x^3(b+cx^2)} + \frac{b^5(7bB-12Ac)(x^2(b+cx^2))^{3/2}\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{b}+\sqrt{b+cx^2}}\right)}{512c^{9/2}x^3(b+cx^2)^{3/2}}$$

```
[In] Integrate[x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(-105*b^5*B*x + 180*A*b^4*c*x + 70*b^4*B*c*x^3 - 1
20*A*b^3*c^2*x^3 - 56*b^3*B*c^2*x^5 + 96*A*b^2*c^3*x^5 + 48*b^2*B*c^3*x^7 +
 2112*A*b*c^4*x^7 + 1664*b*B*c^4*x^9 + 1536*A*c^5*x^9 + 1280*B*c^5*x^11))/(
15360*c^4*x^3*(b + c*x^2)) + (b^5*(7*b*B - 12*A*c)*(x^2*(b + c*x^2))^(3/2)*
ArcTanh[(Sqrt[c]*x)/(-Sqrt[b] + Sqrt[b + c*x^2])])/(512*c^(9/2)*x^3*(b + c*
x^2)^(3/2))
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{\frac{3\left(-\frac{1}{2}Ab^5c+\frac{7}{24}Bb^6\right)\ln\left(\frac{2cx^2+2\sqrt{x^2(cx^2+b)}\sqrt{c}+b}{\sqrt{c}}\right)}{256}+\frac{3\left(\frac{128x^8\left(\frac{5x^2B}{6}+A\right)c^{\frac{11}{2}}}{15}+\left(b^3\left(\frac{7x^2B}{18}+A\right)c^{\frac{3}{2}}-\frac{2x^2\left(\frac{7x^2B}{15}+A\right)b^2c^{\frac{5}{2}}}{3}+\frac{8x^4(}{}\right.\right.}{256}}{c^{\frac{9}{2}}}$
risch	$\frac{(1280Bc^5x^{10}+1536Ac^5x^8+1664Bbc^4x^8+2112Abc^4x^6+48Bb^2c^3x^6+96Ab^2c^3x^4-56Bb^3c^2x^4-120Ab^3c^2x^2+70Bb^4cx^2+1}{15360c^4}$
default	$\frac{(x^4c+bx^2)^{\frac{3}{2}}\left(1280B(cx^2+b)^{\frac{5}{2}}c^{\frac{7}{2}}x^7+1536A(cx^2+b)^{\frac{5}{2}}c^{\frac{7}{2}}x^5-896B(cx^2+b)^{\frac{5}{2}}c^{\frac{5}{2}}bx^5-960A(cx^2+b)^{\frac{5}{2}}c^{\frac{5}{2}}bx^3+560B(cx^2+\right.}{}$

```
[In] int(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/256/c^(9/2)*((-1/2*A*b^5*c+7/24*B*b^6)*ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2
)*c^(1/2)+b)/c^(1/2))+(128/15*x^8*(5/6*x^2*B+A)*c^(11/2)+(b^3*(7/18*x^2*B+A
)*c^(3/2)-2/3*x^2*(7/15*x^2*B+A)*b^2*c^(5/2)+8/15*x^4*(1/2*x^2*B+A)*b*c^(7/
2)+176/15*x^6*(26/33*x^2*B+A)*c^(9/2)-7/12*B*c^(1/2)*b^4)*b)*(x^2*(c*x^2+b)
)^(1/2)+1/2*ln(2)*(A*c-7/12*B*b)*b^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.21

$$\int x^3\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\left[-\frac{15\,(7\,Bb^6-12\,Ab^5c)\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,(1280\,Bc^6x^{10}+128\,(13\,Bb}{}\right.$$

$$-\frac{15\,(7\,Bb^6-12\,Ab^5c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-(1280\,Bc^6x^{10}+128\,(13\,Bbc^5+12\,Ac^6)x^8-105\,Bb^5c+1}{15360}$$

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(7*B*b^6 - 12*A*b^5*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^
4 + b*x^2)*sqrt(c)) - 2*(1280*B*c^6*x^10 + 128*(13*B*b*c^5 + 12*A*c^6)*x^8
- 105*B*b^5*c + 180*A*b^4*c^2 + 48*(B*b^2*c^4 + 44*A*b*c^5)*x^6 - 8*(7*B*b^
3*c^3 - 12*A*b^2*c^4)*x^4 + 10*(7*B*b^4*c^2 - 12*A*b^3*c^3)*x^2)*sqrt(c*x^4
 + b*x^2))/c^5, -1/15360*(15*(7*B*b^6 - 12*A*b^5*c)*sqrt(-c)*arctan(sqrt(c*
x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (1280*B*c^6*x^10 + 128*(13*B*b*c^5 + 1
```

```
2*A*c^6)*x^8 - 105*B*b^5*c + 180*A*b^4*c^2 + 48*(B*b^2*c^4 + 44*A*b*c^5)*x^
6 - 8*(7*B*b^3*c^3 - 12*A*b^2*c^4)*x^4 + 10*(7*B*b^4*c^2 - 12*A*b^3*c^3)*x^
2)*sqrt(c*x^4 + b*x^2))/c^5]
```

Sympy [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 672, normalized size of antiderivative = 4.02

$$\int x^3(A+Bx^2)(bx^2+cx^4)^{3/2}\,dx = \frac{Ab\left(\begin{cases}-\frac{5b^4\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{128c^3}+\sqrt{bx^2+cx^4}\cdot\left(\frac{5b^3}{64c^3}-\frac{5b^2x^2}{96c^2}+\frac{bx^4}{24c}+\frac{x^6}{4}\right) \\ \frac{2(bx^2)^{\frac{7}{2}}}{7b^3} \\ 0\end{cases}\right)}{2}$$

$$+\frac{Ac\left(\begin{cases}\frac{7b^5\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{256c^4}+\sqrt{bx^2+cx^4}\left(-\frac{7b^4}{128c^4}+\frac{7b^3x^2}{192c^3}-\frac{7b^2x^4}{240c^2}+\frac{bx^6}{40c}+\frac{x^8}{5}\right) & \text{for } c \\ \frac{2(bx^2)^{\frac{9}{2}}}{9b^4} & \text{for } b \\ 0 & \text{other}\end{cases}\right)}{2}$$

$$+\frac{Bb\left(\begin{cases}\frac{7b^5\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{256c^4}+\sqrt{bx^2+cx^4}\left(-\frac{7b^4}{128c^4}+\frac{7b^3x^2}{192c^3}-\frac{7b^2x^4}{240c^2}+\frac{bx^6}{40c}+\frac{x^8}{5}\right) & \text{for } c \\ \frac{2(bx^2)^{\frac{9}{2}}}{9b^4} & \text{for } b \\ 0 & \text{other}\end{cases}\right)}{2}$$

$$+\frac{Bc\left(\begin{cases}-\frac{21b^6\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{1024c^5}+\sqrt{bx^2+cx^4}\cdot\left(\frac{21b^5}{512c^5}-\frac{7b^4x^2}{256c^4}+\frac{7b^3x^4}{320c^3}-\frac{3b^2x^6}{160c^2}+\frac{bx^8}{60c}+\right. \\ \frac{2(bx^2)^{\frac{11}{2}}}{11b^5} \\ 0\end{cases}\right)}{2}$$

```
[In] integrate(x**3*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] A*b*Piecewise((-5*b**4*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) +
 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/s
qrt(c*(b/(2*c) + x**2)**2), True))/(128*c**3) + sqrt(b*x**2 + c*x**4)*(5*b*
*3/(64*c**3) - 5*b**2*x**2/(96*c**2) + b*x**4/(24*c) + x**6/4), Ne(c, 0)),
(2*(b*x**2)**(7/2)/(7*b**3), Ne(b, 0)), (0, True))/2 + A*c*Piecewise((7*b**
5*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*c*x**2)/sqrt(c), N
e(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt(c*(b/(2*c) + x**2
)**2), True))/(256*c**4) + sqrt(b*x**2 + c*x**4)*(-7*b**4/(128*c**4) + 7*b*
*3*x**2/(192*c**3) - 7*b**2*x**4/(240*c**2) + b*x**6/(40*c) + x**8/5), Ne(c
, 0)), (2*(b*x**2)**(9/2)/(9*b**4), Ne(b, 0)), (0, True))/2 + B*b*Piecewise
((7*b**5*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*c*x**2)/sqr
t(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt(c*(b/(2*c)
 + x**2)**2), True))/(256*c**4) + sqrt(b*x**2 + c*x**4)*(-7*b**4/(128*c**4)
 + 7*b**3*x**2/(192*c**3) - 7*b**2*x**4/(240*c**2) + b*x**6/(40*c) + x**8/5
), Ne(c, 0)), (2*(b*x**2)**(9/2)/(9*b**4), Ne(b, 0)), (0, True))/2 + B*c*Pi
ecewise((-21*b**6*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*c*
x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt(c
*(b/(2*c) + x**2)**2), True))/(1024*c**5) + sqrt(b*x**2 + c*x**4)*(21*b**5/
(512*c**5) - 7*b**4*x**2/(256*c**4) + 7*b**3*x**4/(320*c**3) - 3*b**2*x**6/
(160*c**2) + b*x**8/(60*c) + x**10/6), Ne(c, 0)), (2*(b*x**2)**(11/2)/(11*b
**5), Ne(b, 0)), (0, True))/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(147) = 294$.

Time = 0.23 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.89

$$\int x^3(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{1}{2560}\left(\frac{60\sqrt{cx^4+bx^2}b^3x^2}{c^2} - \frac{160\left(cx^4+bx^2\right)^{\frac{3}{2}}bx^2}{c} - \frac{15\,b^5\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{7}{2}}} \cdots\right.$$
$$- \frac{1}{30720}\left(\frac{420\sqrt{cx^4+bx^2}b^4x^2}{c^3} - \frac{1120\left(cx^4+bx^2\right)^{\frac{3}{2}}b^2x^2}{c^2} - \frac{2560\left(cx^4+bx^2\right)^{\frac{5}{2}}x^2}{c} - \frac{105\,b^6\log\left(2\,cx^2+b+2\cdots\right.}{c^{\frac{9}{2}}}\cdots\right.$$

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2560*(60*sqrt(c*x^4 + b*x^2)*b^3*x^2/c^2 - 160*(c*x^4 + b*x^2)^(3/2)*b*x^
2/c - 15*b^5*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 30*
sqrt(c*x^4 + b*x^2)*b^4/c^3 - 80*(c*x^4 + b*x^2)^(3/2)*b^2/c^2 + 256*(c*x^4
 + b*x^2)^(5/2)/c)*A - 1/30720*(420*sqrt(c*x^4 + b*x^2)*b^4*x^2/c^3 - 1120*
(c*x^4 + b*x^2)^(3/2)*b^2*x^2/c^2 - 2560*(c*x^4 + b*x^2)^(5/2)*x^2/c - 105*
b^6*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(9/2) + 210*sqrt(c*x
```

```
^4 + b*x^2)*b^5/c^4 - 560*(c*x^4 + b*x^2)^(3/2)*b^3/c^3 + 1792*(c*x^4 + b*x
^2)^(5/2)*b/c^2)*B
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.47

$$\int x^3(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,Bcx^2\mathrm{sgn}(x)+\frac{13\,Bbc^{10}\mathrm{sgn}(x)+12\,Ac^{11}\mathrm{sgn}(x)}{c^{10}}\right)x^2+\frac{3\,(Bb^2c^9\mathrm{sgn}(x)}{}\right.\right.\right.\right.$$
$$-\frac{(7\,Bb^6\mathrm{sgn}(x)-12\,Ab^5c\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{1024\,c^{\frac{9}{2}}}$$
$$+\frac{(7\,Bb^6\log(|b|)-12\,Ab^5c\log(|b|))\mathrm{sgn}(x)}{2048\,c^{\frac{9}{2}}}$$

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15360*(2*(4*(2*(8*(10*B*c*x^2*sgn(x) + (13*B*b*c^10*sgn(x) + 12*A*c^11*sg
n(x))/c^10)*x^2 + 3*(B*b^2*c^9*sgn(x) + 44*A*b*c^10*sgn(x))/c^10)*x^2 - (7*
B*b^3*c^8*sgn(x) - 12*A*b^2*c^9*sgn(x))/c^10)*x^2 + 5*(7*B*b^4*c^7*sgn(x) -
 12*A*b^3*c^8*sgn(x))/c^10)*x^2 - 15*(7*B*b^5*c^6*sgn(x) - 12*A*b^4*c^7*sgn
(x))/c^10)*sqrt(c*x^2 + b)*x - 1/1024*(7*B*b^6*sgn(x) - 12*A*b^5*c*sgn(x))*
log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(9/2) + 1/2048*(7*B*b^6*log(abs(b)
) - 12*A*b^5*c*log(abs(b)))*sgn(x)/c^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\int x^3\,\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}\,dx$$

```
[In] int(x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(x^3*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

3.108 $\int x(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 148

$$\int x(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{3b^2(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{(bB-2Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{10c}-\frac{3b^4(bB-2Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{7/2}}$$

```
[Out] -1/32*(-2*A*c+B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(3/2)/c^2+1/10*B*(c*x^4+b*x^2)
^(5/2)/c-3/256*b^4*(-2*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^
(7/2)+3/256*b^2*(-2*A*c+B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2059, 654, 626, 634, 212}

$$\int x(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=-\frac{3b^4(bB-2Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{7/2}}+\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}(bB-2Ac)}{256c^3}-\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}(bB-2Ac)}{32c^2}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{10c}$$

```
[In] Int[x*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (3*b^2*(b*B - 2*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(256*c^3) - ((b*B -
 2*A*c)*(b + 2*c*x^2)*(b*x^2 + c*x^4)^(3/2))/(32*c^2) + (B*(b*x^2 + c*x^4)^
(5/2))/(10*c) - (3*b^4*(b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x
^4]])/(256*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int (A+Bx)\left(bx+cx^2\right)^{3/2}\,dx, x, x^2\right)$$

$$= \frac{B(bx^2+cx^4)^{5/2}}{10c} + \frac{(-bB+2Ac)\text{Subst}\left(\int \left(bx+cx^2\right)^{3/2}\,dx, x, x^2\right)}{4c}$$

$$= -\frac{(bB-2Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{B\left(bx^2+cx^4\right)^{5/2}}{10c}$$
$$+\frac{\left(3b^2(bB-2Ac)\right)\text{Subst}\left(\int\sqrt{bx+cx^2}\,dx,x,x^2\right)}{64c^2}$$

$$=\frac{3b^2(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{(bB-2Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}$$
$$+\frac{B\left(bx^2+cx^4\right)^{5/2}}{10c}-\frac{\left(3b^4(bB-2Ac)\right)\text{Subst}\left(\int\frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{512c^3}$$

$$=\frac{3b^2(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{(bB-2Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}$$
$$+\frac{B\left(bx^2+cx^4\right)^{5/2}}{10c}-\frac{\left(3b^4(bB-2Ac)\right)\text{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^3}$$

$$=\frac{3b^2(bB-2Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{(bB-2Ac)\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}$$
$$+\frac{B\left(bx^2+cx^4\right)^{5/2}}{10c}-\frac{3b^4(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{7/2}}$$

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.34

$$\int x\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(15b^4B-30Ab^3c-10b^3Bcx^2+20Ab^2c^2x^2+8b^2Bc^2x^4+240Abc^3x^4+176b\right.}{1280c^3x^2\left(b+cx^2\right)}$$
$$-\frac{3b^4(bB-2Ac)\left(x^2\left(b+cx^2\right)\right)^{3/2}\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{b}+\sqrt{b+cx^2}}\right)}{128c^{7/2}x^3\left(b+cx^2\right)^{3/2}}$$

```
[In] Integrate[x*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*(15*b^4*B - 30*A*b^3*c - 10*b^3*B*c*x^2 + 20*A*b^2
*c^2*x^2 + 8*b^2*B*c^2*x^4 + 240*A*b*c^3*x^4 + 176*b*B*c^3*x^6 + 160*A*c^4*
x^6 + 128*B*c^4*x^8))/(1280*c^3*x^2*(b + c*x^2)) - (3*b^4*(b*B - 2*A*c)*(x^
2*(b + c*x^2))^(3/2)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[b] + Sqrt[b + c*x^2])])/(12
8*c^(7/2)*x^3*(b + c*x^2)^(3/2))
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$-\frac{3\left(\left(-\frac{1}{2}A b^4 c+\frac{1}{4}b^5 B\right)\ln\left(\frac{2c x^2+2\sqrt{x^2\left(c x^2+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)+\left(b^3\left(\frac{x^2 B}{3}+A\right)c^{\frac{3}{2}}-\frac{2x^2\left(\frac{2x^2 B}{5}+A\right)b^2 c^{\frac{5}{2}}}{3}-8x^4\left(\frac{11x^2 B}{15}+A\right)b\,c^{\frac{7}{2}}-\cdots\right.\right.}{128c^{\frac{7}{2}}}$
risch	$-\frac{\left(-128B x^8 c^4-160A x^6 c^4-176B x^6 b c^3-240A x^4 b c^3-8B x^4 b^2 c^2-20A x^2 b^2 c^2+10B x^2 b^3 c+30A b^3 c-15B b^4\right)\sqrt{x^2\left(c x^2+b\right)}}{1280c^3}$
default	$\frac{\left(x^4 c+b x^2\right)^{\frac{3}{2}}\left(128B\left(c x^2+b\right)^{\frac{5}{2}}c^{\frac{5}{2}}x^5+160A\left(c x^2+b\right)^{\frac{5}{2}}c^{\frac{5}{2}}x^3-80B\left(c x^2+b\right)^{\frac{5}{2}}c^{\frac{3}{2}}b\,x^3-80A\left(c x^2+b\right)^{\frac{5}{2}}c^{\frac{3}{2}}b x+20A\left(c x^2+b\right)^{\frac{3}{2}}c^{\frac{3}{2}}b\cdots\right.}{}$

```
[In] int(x*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/128/c^(7/2)*((-1/2*A*b^4*c+1/4*b^5*B)*ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2
)*c^(1/2)+b)/c^(1/2))+(b^3*(1/3*x^2*B+A)*c^(3/2)-2/3*x^2*(2/5*x^2*B+A)*b^2*
c^(5/2)-8*x^4*(11/15*x^2*B+A)*b*c^(7/2)-16/3*x^6*(4/5*x^2*B+A)*c^(9/2)-1/2*
B*c^(1/2)*b^4)*(x^2*(c*x^2+b))^(1/2)+1/2*ln(2)*(A*c-1/2*B*b)*b^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.14

$$\int x\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\left[-\frac{15\left(Bb^5-2\,Ab^4c\right)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\left(128\,Bc^5x^8+16\left(11\,Bbc^4+1\cdots\right.\right.}{25\cdots}\right.$$

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(B*b^5 - 2*A*b^4*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 +
b*x^2)*sqrt(c)) - 2*(128*B*c^5*x^8 + 16*(11*B*b*c^4 + 10*A*c^5)*x^6 + 15*B*
b^4*c - 30*A*b^3*c^2 + 8*(B*b^2*c^3 + 30*A*b*c^4)*x^4 - 10*(B*b^3*c^2 - 2*A
*b^2*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^4, 1/1280*(15*(B*b^5 - 2*A*b^4*c)*sqr
t(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (128*B*c^5*x^8 + 1
6*(11*B*b*c^4 + 10*A*c^5)*x^6 + 15*B*b^4*c - 30*A*b^3*c^2 + 8*(B*b^2*c^3 +
30*A*b*c^4)*x^4 - 10*(B*b^3*c^2 - 2*A*b^2*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^
4]
```

Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 614, normalized size of antiderivative = 4.15

$$\int x(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{Ab\left(\begin{cases}\frac{b^3\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{16c^2}+\sqrt{bx^2+cx^4}\left(-\frac{b^2}{8c^2}+\frac{bx^2}{12c}+\frac{x^4}{3}\right) & \text{for } c\neq 0\\ \frac{2(bx^2)^{\frac{5}{2}}}{5b^2} & \text{for } b\neq 0\\ 0 & \text{otherwis}\end{cases}\right.}{2}$$

$$+\frac{Ac\left(\begin{cases}-\frac{5b^4\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{128c^3}+\sqrt{bx^2+cx^4}\cdot\left(\frac{5b^3}{64c^3}-\frac{5b^2x^2}{96c^2}+\frac{bx^4}{24c}+\frac{x^6}{4}\right) & \text{for } c\neq 0\\ \frac{2(bx^2)^{\frac{7}{2}}}{7b^3} & \text{for } b\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{2}$$

$$+\frac{Bb\left(\begin{cases}-\frac{5b^4\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{128c^3}+\sqrt{bx^2+cx^4}\cdot\left(\frac{5b^3}{64c^3}-\frac{5b^2x^2}{96c^2}+\frac{bx^4}{24c}+\frac{x^6}{4}\right) & \text{for } c\neq 0\\ \frac{2(bx^2)^{\frac{7}{2}}}{7b^3} & \text{for } b\neq 0\\ 0 & \text{otherwise}\end{cases}\right)}{2}$$

$$+\frac{Bc\left(\begin{cases}\frac{7b^5\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{256c^4}+\sqrt{bx^2+cx^4}\left(-\frac{7b^4}{128c^4}+\frac{7b^3x^2}{192c^3}-\frac{7b^2x^4}{240c^2}+\frac{bx^6}{40c}+\frac{x^8}{5}\right) & \text{for } c\\ \frac{2(bx^2)^{\frac{9}{2}}}{9b^4} & \text{for } b\\ 0 & \text{othe}\end{cases}\right.}{2}$$

```
[In] integrate(x*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] A*b*Piecewise((b**3*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*
c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt
(c*(b/(2*c) + x**2)**2), True))/(16*c**2) + sqrt(b*x**2 + c*x**4)*(-b**2/(8
*c**2) + b*x**2/(12*c) + x**4/3), Ne(c, 0)), (2*(b*x**2)**(5/2)/(5*b**2), N
e(b, 0)), (0, True))/2 + A*c*Piecewise((-5*b**4*Piecewise((log(b + 2*sqrt(c
)*sqrt(b*x**2 + c*x**4) + 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x*
*2)*log(b/(2*c) + x**2)/sqrt(c*(b/(2*c) + x**2)**2), True))/(128*c**3) + sq
rt(b*x**2 + c*x**4)*(5*b**3/(64*c**3) - 5*b**2*x**2/(96*c**2) + b*x**4/(24*
c) + x**6/4), Ne(c, 0)), (2*(b*x**2)**(7/2)/(7*b**3), Ne(b, 0)), (0, True))
/2 + B*b*Piecewise((-5*b**4*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x*
*4) + 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x*
*2)/sqrt(c*(b/(2*c) + x**2)**2), True))/(128*c**3) + sqrt(b*x**2 + c*x**4)*
(5*b**3/(64*c**3) - 5*b**2*x**2/(96*c**2) + b*x**4/(24*c) + x**6/4), Ne(c,
0)), (2*(b*x**2)**(7/2)/(7*b**3), Ne(b, 0)), (0, True))/2 + B*c*Piecewise((
7*b**5*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*c*x**2)/sqrt(
c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt(c*(b/(2*c) +
 x**2)**2), True))/(256*c**4) + sqrt(b*x**2 + c*x**4)*(-7*b**4/(128*c**4) +
 7*b**3*x**2/(192*c**3) - 7*b**2*x**4/(240*c**2) + b*x**6/(40*c) + x**8/5),
 Ne(c, 0)), (2*(b*x**2)**(9/2)/(9*b**4), Ne(b, 0)), (0, True))/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(128) = 256$.

Time = 0.22 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.80

$$\int x(A+Bx^2)\,(bx^2+cx^4)^{3/2}\,dx=\frac{1}{256}\left(32\,(cx^4+bx^2)^{\frac{3}{2}}x^2-\frac{12\,\sqrt{cx^4+bx^2}b^2x^2}{c}+\frac{3\,b^4\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{5}{2}}}-\frac{6\,\sqrt{c}}{}\right.$$
$$+\frac{1}{2560}\left(\frac{60\,\sqrt{cx^4+bx^2}b^3x^2}{c^2}-\frac{160\,(cx^4+bx^2)^{\frac{3}{2}}bx^2}{c}-\frac{15\,b^5\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{7}{2}}}+\frac{30\,\sqrt{cx^4+bx^2}}{c^3}\right.$$

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/256*(32*(c*x^4 + b*x^2)^(3/2)*x^2 - 12*sqrt(c*x^4 + b*x^2)*b^2*x^2/c + 3*
b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2) - 6*sqrt(c*x^4
 + b*x^2)*b^3/c^2 + 16*(c*x^4 + b*x^2)^(3/2)*b/c)*A + 1/2560*(60*sqrt(c*x^4
 + b*x^2)*b^3*x^2/c^2 - 160*(c*x^4 + b*x^2)^(3/2)*b*x^2/c - 15*b^5*log(2*c*
x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 30*sqrt(c*x^4 + b*x^2)*b
^4/c^3 - 80*(c*x^4 + b*x^2)^(3/2)*b^2/c^2 + 256*(c*x^4 + b*x^2)^(5/2)/c)*B
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.40

$$\int x(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{1}{1280}\left(2\left(4\left(2\left(8\,Bcx^2\mathrm{sgn}(x)+\frac{11\,Bbc^8\mathrm{sgn}(x)+10\,Ac^9\mathrm{sgn}(x)}{c^8}\right)x^2+\frac{Bb^2c^7\mathrm{sgn}(x)+30\,Ab\ldots}{c^8}\right.\right.\right.$$
$$+\frac{3\left(Bb^5\mathrm{sgn}(x)-2\,Ab^4c\mathrm{sgn}(x)\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{256\,c^{\frac{7}{2}}}$$
$$-\frac{3\left(Bb^5\log\left(|b|\right)-2\,Ab^4c\log\left(|b|\right)\right)\mathrm{sgn}(x)}{512\,c^{\frac{7}{2}}}$$

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1280*(2*(4*(2*(8*B*c*x^2*sgn(x) + (11*B*b*c^8*sgn(x) + 10*A*c^9*sgn(x))/c
^8)*x^2 + (B*b^2*c^7*sgn(x) + 30*A*b*c^8*sgn(x))/c^8)*x^2 - 5*(B*b^3*c^6*sg
n(x) - 2*A*b^2*c^7*sgn(x))/c^8)*x^2 + 15*(B*b^4*c^5*sgn(x) - 2*A*b^3*c^6*sg
n(x))/c^8)*sqrt(c*x^2 + b)*x + 3/256*(B*b^5*sgn(x) - 2*A*b^4*c*sgn(x))*log(
abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(7/2) - 3/512*(B*b^5*log(abs(b)) - 2*A
*b^4*c*log(abs(b)))*sgn(x)/c^(7/2)
```

Mupad [B] (verification not implemented)

Time = 9.87 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.59

$$\int x(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{B\left(c\,x^4+b\,x^2\right)^{5/2}}{10\,c}+\frac{A\left(c\,x^4+b\,x^2\right)^{3/2}\left(c\,x^2+\frac{b}{2}\right)}{8\,c}$$
$$-\frac{3\,A\,b^2\left(\left(\frac{b}{4\,c}+\frac{x^2}{2}\right)\sqrt{c\,x^4+b\,x^2}-\frac{b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{8\,c^{3/2}}\right)}{32\,c}$$
$$-\frac{B\,b\left(\frac{x^2\,\left(c\,x^4+b\,x^2\right)^{3/2}}{4}-\frac{3\,b^2\left(\frac{\left(2\,c\,x^2+b\right)\,\sqrt{c\,x^4+b\,x^2}}{4\,c}-\frac{b^2\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{8\,c^{3/2}}\right)}{16\,c}+\frac{b\,\left(c\,x^4+b\,x^2\right)^{3/2}}{8\,c}\right)}{4\,c}$$

```
[In] int(x*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] (B*(b*x^2 + c*x^4)^(5/2))/(10*c) + (A*(b*x^2 + c*x^4)^(3/2)*(b/2 + c*x^2))/
(8*c) - (3*A*b^2*((b/(4*c) + x^2/2)*(b*x^2 + c*x^4)^(1/2) - (b^2*log((b/2 +
```

```
 c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(8*c^(3/2))))/(32*c) - (B*b*((x^2
*(b*x^2 + c*x^4)^(3/2))/4 - (3*b^2*(((b + 2*c*x^2)*(b*x^2 + c*x^4)^(1/2))/(
4*c) - (b^2*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(8*c^(3/2))
))/(16*c) + (b*(b*x^2 + c*x^4)^(3/2))/(8*c)))/(4*c)
```

3.109 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 144

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x}\,dx = -\frac{b(3bB-8Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{128c^2} - \frac{(3bB-8Ac)(bx^2+cx^4)^{3/2}}{48c} + \frac{B(bx^2+cx^4)^{5/2}}{8cx^2} + \frac{b^3(3bB-8Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}}$$

```
[Out] -1/48*(-8*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/c+1/8*B*(c*x^4+b*x^2)^(5/2)/c/x^2+
1/128*b^3*(-8*A*c+3*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(5/2)-1
/128*b*(-8*A*c+3*B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2059, 808, 678, 626, 634, 212}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x}\,dx = \frac{b^3(3bB-8Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}} - \frac{b(b+2cx^2)\sqrt{bx^2+cx^4}(3bB-8Ac)}{128c^2} - \frac{(bx^2+cx^4)^{3/2}(3bB-8Ac)}{48c} + \frac{B(bx^2+cx^4)^{5/2}}{8cx^2}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x,x]
```

```
[Out] -1/128*(b*(3*b*B - 8*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/c^2 - ((3*b*B
- 8*A*c)*(b*x^2 + c*x^4)^(3/2))/(48*c) + (B*(b*x^2 + c*x^4)^(5/2))/(8*c*x^2
) + (b^3*(3*b*B - 8*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(128*c
^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x}\,dx, x, x^2\right)$$
$$= \frac{B\left(bx^2+cx^4\right)^{5/2}}{8cx^2} + \frac{\left(bB-Ac+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x}\,dx, x, x^2\right)}{8c}$$
$$= -\frac{(3bB-8Ac)\left(bx^2+cx^4\right)^{3/2}}{48c} + \frac{B\left(bx^2+cx^4\right)^{5/2}}{8cx^2} - \frac{(b(3bB-8Ac))\text{Subst}\left(\int \sqrt{bx+cx^2}\,dx, x, x^2\right)}{32c}$$
$$= -\frac{b(3bB-8Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2} - \frac{(3bB-8Ac)\left(bx^2+cx^4\right)^{3/2}}{48c} + \frac{B\left(bx^2+cx^4\right)^{5/2}}{8cx^2} + \frac{\left(b^3(3bB-8Ac)\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{256c^2}$$
$$= -\frac{b(3bB-8Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2} - \frac{(3bB-8Ac)\left(bx^2+cx^4\right)^{3/2}}{48c} + \frac{B\left(bx^2+cx^4\right)^{5/2}}{8cx^2} + \frac{\left(b^3(3bB-8Ac)\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^2}$$
$$= -\frac{b(3bB-8Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2} - \frac{(3bB-8Ac)\left(bx^2+cx^4\right)^{3/2}}{48c} + \frac{B\left(bx^2+cx^4\right)^{5/2}}{8cx^2} + \frac{b^3(3bB-8Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.09

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x}\,dx = \frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}\left(\frac{\sqrt{c}x\left(-9b^3B+6b^2c\left(4A+Bx^2\right)+16c^3x^4\left(4A+3Bx^2\right)+8bc^2x^2\left(14A+9Bx^2\right)\right)}{b+cx^2}\right.}{384c^{5/2}x^3}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x,x]
```

```
[Out] ((x^2*(b + c*x^2))^(3/2)*((Sqrt[c]*x*(-9*b^3*B + 6*b^2*c*(4*A + B*x^2) + 16
*c^3*x^4*(4*A + 3*B*x^2) + 8*b*c^2*x^2*(14*A + 9*B*x^2)))/(b + c*x^2) + (6*
b^3*(3*b*B - 8*A*c)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[b] + Sqrt[b + c*x^2])])/(b +
 c*x^2)^(3/2)))/(384*c^(5/2)*x^3)
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.97

method	result
risch	$\frac{\left(48B\,c^{3}x^{6}+64A\,c^{3}x^{4}+72Bb\,c^{2}x^{4}+112Ab\,c^{2}x^{2}+6B\,b^{2}c\,x^{2}+24b^{2}Ac-9B\,b^{3}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{384c^{2}}-\frac{b^{3}\left(8Ac-3Bb\right)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+\cdots}\right.}{128c^{\frac{5}{2}}x\sqrt{c\,x^{2}+\cdots}}$
pseudoelliptic	$\frac{\frac{\left(-A\,b^{3}c+\frac{3}{8}B\,b^{4}\right)\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)}{2}+\left(b^{2}\left(\frac{x^{2}B}{4}+A\right)c^{\frac{3}{2}}+\frac{14x^{2}\left(\frac{9x^{2}B}{14}+A\right)b\,c^{\frac{5}{2}}}{3}+2\left(B\,x^{6}+\frac{4}{3}A\,x^{4}\right)c^{\frac{7}{2}}-\frac{3B\sqrt{c}\,b^{3}}{8}\right)\sqrt{\cdots}}{16c^{\frac{5}{2}}}$
default	$\frac{\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}\left(48B\left(c\,x^{2}+b\right)^{\frac{5}{2}}c^{\frac{3}{2}}x^{3}+64A\left(c\,x^{2}+b\right)^{\frac{5}{2}}c^{\frac{3}{2}}x-24B\left(c\,x^{2}+b\right)^{\frac{5}{2}}\sqrt{c}\,bx-16A\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{\frac{3}{2}}bx+6B\left(c\,x^{2}+b\right)^{\frac{3}{2}}\sqrt{c}\,b^{2}x-\cdots\right.}{384x^{3}\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{\frac{5}{2}}}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/384/c^2*(48*B*c^3*x^6+64*A*c^3*x^4+72*B*b*c^2*x^4+112*A*b*c^2*x^2+6*B*b^2
*c*x^2+24*A*b^2*c-9*B*b^3)*(x^2*(c*x^2+b))^(1/2)-1/128*b^3*(8*A*c-3*B*b)/c^
(5/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.91

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x}\,dx=\left[-\frac{3\,(3\,Bb^4-8\,Ab^3c)\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,(48\,Bc^4x^6\cdots}{\cdots}\right.$$

$$-\frac{3\,(3\,Bb^4-8\,Ab^3c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-(48\,Bc^4x^6-9\,Bb^3c+24\,Ab^2c^2+8\,(9\,Bbc^3+8\,Ac^4)x^4+2\,(\cdots}{384\,c^3}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(3*B*b^4 - 8*A*b^3*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 +
b*x^2)*sqrt(c)) - 2*(48*B*c^4*x^6 - 9*B*b^3*c + 24*A*b^2*c^2 + 8*(9*B*b*c^3
 + 8*A*c^4)*x^4 + 2*(3*B*b^2*c^2 + 56*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^
3, -1/384*(3*(3*B*b^4 - 8*A*b^3*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt
(-c)/(c*x^2 + b)) - (48*B*c^4*x^6 - 9*B*b^3*c + 24*A*b^2*c^2 + 8*(9*B*b*c^3
 + 8*A*c^4)*x^4 + 2*(3*B*b^2*c^2 + 56*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^
3]
```

Sympy [A] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 551, normalized size of antiderivative = 3.83

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x}\,dx = \frac{Ab\left(\begin{cases} -\frac{b^2\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for}\ \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise} \end{cases}\right)}{8c} + \left(\frac{b}{4c}+\frac{x^2}{2}\right)\sqrt{bx^2+c\cdots} \\ \frac{2(bx^2)^{\frac{3}{2}}}{3b} \\ 0 \end{cases}\right.}{2}$$

$$+\frac{Ac\left(\begin{cases} \frac{b^3\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for}\ \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise} \end{cases}\right)}{16c^2} + \sqrt{bx^2+cx^4}\left(-\frac{b^2}{8c^2}+\frac{bx^2}{12c}+\frac{x^4}{3}\right) & \text{for}\ c\neq 0 \\ \frac{2(bx^2)^{\frac{5}{2}}}{5b^2} & \text{for}\ b\neq 0 \\ 0 & \text{otherwise} \end{cases}\right)}{2}$$

$$+\frac{Bb\left(\begin{cases} \frac{b^3\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for}\ \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise} \end{cases}\right)}{16c^2} + \sqrt{bx^2+cx^4}\left(-\frac{b^2}{8c^2}+\frac{bx^2}{12c}+\frac{x^4}{3}\right) & \text{for}\ c\neq 0 \\ \frac{2(bx^2)^{\frac{5}{2}}}{5b^2} & \text{for}\ b\neq 0 \\ 0 & \text{otherwise} \end{cases}\right)}{2}$$

$$+\frac{Bc\left(\begin{cases} -\frac{5b^4\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for}\ \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise} \end{cases}\right)}{128c^3} + \sqrt{bx^2+cx^4}\cdot\left(\frac{5b^3}{64c^3}-\frac{5b^2x^2}{96c^2}+\frac{bx^4}{24c}+\frac{x^6}{4}\right) & \text{for}\ c\neq 0 \\ \frac{2(bx^2)^{\frac{7}{2}}}{7b^3} & \text{for}\ b\neq 0 \\ 0 & \text{otherwise} \end{cases}\right.}{2}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x,x)
```

```
[Out] A*b*Piecewise((-b**2*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2
*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqr
t(c*(b/(2*c) + x**2)**2), True))/(8*c) + (b/(4*c) + x**2/2)*sqrt(b*x**2 + c
*x**4), Ne(c, 0)), (2*(b*x**2)**(3/2)/(3*b), Ne(b, 0)), (0, True))/2 + A*c*
Piecewise((b**3*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*c*x*
*2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt(c*(
b/(2*c) + x**2)**2), True))/(16*c**2) + sqrt(b*x**2 + c*x**4)*(-b**2/(8*c**
2) + b*x**2/(12*c) + x**4/3), Ne(c, 0)), (2*(b*x**2)**(5/2)/(5*b**2), Ne(b,
 0)), (0, True))/2 + B*b*Piecewise((b**3*Piecewise((log(b + 2*sqrt(c)*sqrt(
b*x**2 + c*x**4) + 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log
(b/(2*c) + x**2)/sqrt(c*(b/(2*c) + x**2)**2), True))/(16*c**2) + sqrt(b*x**
2 + c*x**4)*(-b**2/(8*c**2) + b*x**2/(12*c) + x**4/3), Ne(c, 0)), (2*(b*x**
2)**(5/2)/(5*b**2), Ne(b, 0)), (0, True))/2 + B*c*Piecewise((-5*b**4*Piecew
ise((log(b + 2*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*c*x**2)/sqrt(c), Ne(b**2/c
, 0)), ((b/(2*c) + x**2)*log(b/(2*c) + x**2)/sqrt(c*(b/(2*c) + x**2)**2), T
rue))/(128*c**3) + sqrt(b*x**2 + c*x**4)*(5*b**3/(64*c**3) - 5*b**2*x**2/(9
6*c**2) + b*x**4/(24*c) + x**6/4), Ne(c, 0)), (2*(b*x**2)**(7/2)/(7*b**3),
Ne(b, 0)), (0, True))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.50

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x}\,dx = \frac{1}{96}\left(12\sqrt{cx^4+bx^2}bx^2 - \frac{3\,b^3\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{3}{2}}} + 16\,(cx^4 \right.$$
$$+\frac{1}{256}\left(32\,(cx^4+bx^2)^{\frac{3}{2}}x^2 - \frac{12\sqrt{cx^4+bx^2}b^2x^2}{c} + \frac{3\,b^4\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{5}{2}}} - \frac{6\sqrt{cx^4+bx^2}b^3}{c^2} + \right.$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/96*(12*sqrt(c*x^4 + b*x^2)*b*x^2 - 3*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4 +
 b*x^2)*sqrt(c))/c^(3/2) + 16*(c*x^4 + b*x^2)^(3/2) + 6*sqrt(c*x^4 + b*x^2)
*b^2/c)*A + 1/256*(32*(c*x^4 + b*x^2)^(3/2)*x^2 - 12*sqrt(c*x^4 + b*x^2)*b^
2*x^2/c + 3*b^4*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2) -
6*sqrt(c*x^4 + b*x^2)*b^3/c^2 + 16*(c*x^4 + b*x^2)^(3/2)*b/c)*B
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.24

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x}\,dx = \frac{1}{384}\left(2\left(4\left(6\,Bcx^2\mathrm{sgn}(x)+\frac{9\,Bbc^6\mathrm{sgn}(x)+8\,Ac^7\mathrm{sgn}(x)}{c^6}\right)x^2+\frac{3\,Bb^2c^5\mathrm{sg}}{}\right.\right.$$
$$-\frac{(3\,Bb^4\mathrm{sgn}(x)-8\,Ab^3c\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{128\,c^{\frac{5}{2}}}$$
$$+\frac{(3\,Bb^4\log(|b|)-8\,Ab^3c\log(|b|))\mathrm{sgn}(x)}{256\,c^{\frac{5}{2}}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*B*c*x^2*sgn(x) + (9*B*b*c^6*sgn(x) + 8*A*c^7*sgn(x))/c^6)*x^
2 + (3*B*b^2*c^5*sgn(x) + 56*A*b*c^6*sgn(x))/c^6)*x^2 - 3*(3*B*b^3*c^4*sgn(
x) - 8*A*b^2*c^5*sgn(x))/c^6)*sqrt(c*x^2 + b)*x - 1/128*(3*B*b^4*sgn(x) - 8
*A*b^3*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(5/2) + 1/256*(3*
B*b^4*log(abs(b)) - 8*A*b^3*c*log(abs(b)))*sgn(x)/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x, x)
```

3.110 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^3} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^3} \, dx = \frac{(bB-6Ac)(b+2cx^2)\sqrt{bx^2+cx^4}}{16c} + \frac{(bB-6Ac)(bx^2+cx^4)^{3/2}}{6b} + \frac{A(bx^2+cx^4)^{5/2}}{bx^4} - \frac{b^2(bB-6Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}}$$

```
[Out] 1/6*(-6*A*c+B*b)*(c*x^4+b*x^2)^(3/2)/b+A*(c*x^4+b*x^2)^(5/2)/b/x^4-1/16*b^2
*(-6*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(3/2)+1/16*(-6*A*c
+B*b)*(2*c*x^2+b)*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2059, 806, 678, 626, 634, 212}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^3} \, dx = -\frac{b^2(bB-6Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}} + \frac{(bx^2+cx^4)^{3/2}(bB-6Ac)}{6b} + \frac{(b+2cx^2)\sqrt{bx^2+cx^4}(bB-6Ac)}{16c} + \frac{A(bx^2+cx^4)^{5/2}}{bx^4}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^3,x]
```

```
[Out] ((b*B - 6*A*c)*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(16*c) + ((b*B - 6*A*c)*(
b*x^2 + c*x^4)^(3/2))/(6*b) + (A*(b*x^2 + c*x^4)^(5/2))/(b*x^4) - (b^2*(b*B
 - 6*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^2}\,dx, x, x^2\right)$$

$$= \frac{A\left(bx^2+cx^4\right)^{5/2}}{bx^4} - \frac{\left(-2(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x}\,dx, x, x^2\right)}{b}$$

$$= \frac{(bB-6Ac)\left(bx^2+cx^4\right)^{3/2}}{6b} + \frac{A\left(bx^2+cx^4\right)^{5/2}}{bx^4} - \frac{1}{4}(-bB+6Ac)\text{Subst}\left(\int \sqrt{bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{(bB-6Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c} + \frac{(bB-6Ac)\left(bx^2+cx^4\right)^{3/2}}{6b} + \frac{A\left(bx^2+cx^4\right)^{5/2}}{bx^4} - \frac{\left(b^2(bB-6Ac)\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{32c}$$

$$= \frac{(bB-6Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c} + \frac{(bB-6Ac)\left(bx^2+cx^4\right)^{3/2}}{6b} + \frac{A\left(bx^2+cx^4\right)^{5/2}}{bx^4} - \frac{\left(b^2(bB-6Ac)\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{16c}$$

$$= \frac{(bB-6Ac)\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c} + \frac{(bB-6Ac)\left(bx^2+cx^4\right)^{3/2}}{6b} + \frac{A\left(bx^2+cx^4\right)^{5/2}}{bx^4} - \frac{b^2(bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^3}\,dx = \frac{x\left(\sqrt{c}x\left(b+cx^2\right)\left(3b^2B+4c^2x^2\left(3A+2Bx^2\right)+2bc\left(15A+7Bx^2\right)\right)+3b^2(bB\right.}{48c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^3,x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2)*(3*b^2*B + 4*c^2*x^2*(3*A + 2*B*x^2) + 2*b*c*(15*
A + 7*B*x^2)) + 3*b^2*(b*B - 6*A*c)*Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt
[b + c*x^2]]))/(48*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.85

method	result
risch	$\frac{(8Bc^2x^4+12Ac^2x^2+14Bbcx^2+30Abc+3Bb^2)\sqrt{x^2(cx^2+b)}}{48c}+\frac{b^2(6Ac-Bb)\ln\left(\sqrt{c}x+\sqrt{cx^2+b}\right)\sqrt{x^2(cx^2+b)}}{16c^{\frac{3}{2}}x\sqrt{cx^2+b}}$
default	$\frac{(x^4c+bx^2)^{\frac{3}{2}}\left(8B\sqrt{c}\,(cx^2+b)^{\frac{5}{2}}x+12Ac^{\frac{3}{2}}(cx^2+b)^{\frac{3}{2}}x-2B(cx^2+b)^{\frac{3}{2}}\sqrt{c}\,bx+18A\sqrt{cx^2+b}\,c^{\frac{3}{2}}bx-3B\sqrt{cx^2+b}\,\sqrt{c}\,b^2x+18A\right.}{48x^3(cx^2+b)^{\frac{3}{2}}c^{\frac{3}{2}}}$
pseudoelliptic	$\frac{16Bc^{\frac{5}{2}}x^4\sqrt{x^2(cx^2+b)}+24Ac^{\frac{5}{2}}\sqrt{x^2(cx^2+b)}\,x^2+28Bc^{\frac{3}{2}}\sqrt{x^2(cx^2+b)}\,bx^2+60Ac^{\frac{3}{2}}\sqrt{x^2(cx^2+b)}\,b-18A\ln(2)b^2c+18A\ln}{96c^{\frac{3}{2}}}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/c*(8*B*c^2*x^4+12*A*c^2*x^2+14*B*b*c*x^2+30*A*b*c+3*B*b^2)*(x^2*(c*x^2
+b))^(1/2)+1/16*b^2*(6*A*c-B*b)/c^(3/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*(x^2*
(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.64

$$\int\frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^3}\,dx=\left[-\frac{3\,(Bb^3-6\,Ab^2c)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,(8\,Bc^3x^4\right.}{96\,c^2}\right.$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(B*b^3 - 6*A*b^2*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x
^2)*sqrt(c)) - 2*(8*B*c^3*x^4 + 3*B*b^2*c + 30*A*b*c^2 + 2*(7*B*b*c^2 + 6*A
*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^2, 1/48*(3*(B*b^3 - 6*A*b^2*c)*sqrt(-c)*a
rctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (8*B*c^3*x^4 + 3*B*b^2*c
+ 30*A*b*c^2 + 2*(7*B*b*c^2 + 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^2]
```

Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^3}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}(A+Bx^2)}{x^3}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**3,x)
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.23

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^3}\,dx = \frac{1}{16}\left(\frac{3\,b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{\sqrt{c}}+6\,\sqrt{cx^4+bx^2}b+\frac{4\,(cx^4+bx^2}{x^2}\right.$$
$$+\frac{1}{96}\left(12\,\sqrt{cx^4+bx^2}bx^2-\frac{3\,b^3\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{3}{2}}}+16\,\left(cx^4+bx^2\right)^{\frac{3}{2}}+\frac{6\,\sqrt{cx^4+bx^2}b^2}{c}\right)B$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="maxima")
[Out] 1/16*(3*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) + 6*sq
rt(c*x^4 + b*x^2)*b + 4*(c*x^4 + b*x^2)^(3/2)/x^2)*A + 1/96*(12*sqrt(c*x^4
+ b*x^2)*b*x^2 - 3*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(
3/2) + 16*(c*x^4 + b*x^2)^(3/2) + 6*sqrt(c*x^4 + b*x^2)*b^2/c)*B
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^3}\,dx = \frac{1}{48}\left(2\left(4\,Bcx^2\mathrm{sgn}(x)+\frac{7\,Bbc^4\mathrm{sgn}(x)+6\,Ac^5\mathrm{sgn}(x)}{c^4}\right)x^2+\frac{3\,(Bb^2c^3\mathrm{sgn}(x)\right.$$
$$+\frac{(Bb^3\mathrm{sgn}(x)-6\,Ab^2c\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{16\,c^{\frac{3}{2}}}$$
$$-\frac{(Bb^3\log(|b|)-6\,Ab^2c\log(|b|))\mathrm{sgn}(x)}{32\,c^{\frac{3}{2}}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^3,x, algorithm="giac")
[Out] 1/48*(2*(4*B*c*x^2*sgn(x) + (7*B*b*c^4*sgn(x) + 6*A*c^5*sgn(x))/c^4)*x^2 +
3*(B*b^2*c^3*sgn(x) + 10*A*b*c^4*sgn(x))/c^4)*sqrt(c*x^2 + b)*x + 1/16*(B*b
^3*sgn(x) - 6*A*b^2*c*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/c^(3/2
) - 1/32*(B*b^3*log(abs(b)) - 6*A*b^2*c*log(abs(b)))*sgn(x)/c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^3}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^3}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^3,x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^3, x)
```

3.111 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^5} dx$

Optimal result

Integrand size = 26, antiderivative size = 128

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^5} dx = \frac{3}{8}(bB+4Ac)\sqrt{bx^2+cx^4} + \frac{(bB+4Ac)(bx^2+cx^4)^{3/2}}{4bx^2} - \frac{A(bx^2+cx^4)^{5/2}}{bx^6} + \frac{3b(bB+4Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}$$

```
[Out] 1/4*(4*A*c+B*b)*(c*x^4+b*x^2)^(3/2)/b/x^2-A*(c*x^4+b*x^2)^(5/2)/b/x^6+3/8*b
*(4*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(1/2)+3/8*(4*A*c+B*
b)*(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2059, 806, 678, 634, 212}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^5} dx = \frac{3b(4Ac+bB)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}} + \frac{(bx^2+cx^4)^{3/2}(4Ac+bB)}{4bx^2} + \frac{3}{8}\sqrt{bx^2+cx^4}(4Ac+bB) - \frac{A(bx^2+cx^4)^{5/2}}{bx^6}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^5,x]
```

```
[Out] (3*(b*B + 4*A*c)*Sqrt[b*x^2 + c*x^4])/8 + ((b*B + 4*A*c)*(b*x^2 + c*x^4)^(3
/2))/(4*b*x^2) - (A*(b*x^2 + c*x^4)^(5/2))/(b*x^6) + (3*b*(b*B + 4*A*c)*Arc
Tanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{bx^6} + \frac{\left(-3(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^2}\,dx, x, x^2\right)}{b}$$

$$= \frac{(bB+4Ac)\,(bx^2+cx^4)^{3/2}}{4bx^2} - \frac{A(bx^2+cx^4)^{5/2}}{bx^6} + \frac{1}{8}(3(bB+4Ac))\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x}\,dx, x, x^2\right)$$

$$= \frac{3}{8}(bB+4Ac)\sqrt{bx^2+cx^4} + \frac{(bB+4Ac)\,(bx^2+cx^4)^{3/2}}{4bx^2} - \frac{A(bx^2+cx^4)^{5/2}}{bx^6} + \frac{1}{16}(3b(bB+4Ac))\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$

$$= \frac{3}{8}(bB+4Ac)\sqrt{bx^2+cx^4} + \frac{(bB+4Ac)\,(bx^2+cx^4)^{3/2}}{4bx^2} - \frac{A(bx^2+cx^4)^{5/2}}{bx^6} + \frac{1}{8}(3b(bB+4Ac))\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)$$

$$= \frac{3}{8}(bB+4Ac)\sqrt{bx^2+cx^4} + \frac{(bB+4Ac)\,(bx^2+cx^4)^{3/2}}{4bx^2} - \frac{A(bx^2+cx^4)^{5/2}}{bx^6} + \frac{3b(bB+4Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^5}\,dx = \frac{\sqrt{c}(b+cx^2)\,(-8Ab+5bBx^2+4Acx^2+2Bcx^4)+6b(bB+4Ac)x\sqrt{b+cx^2}}{8\sqrt{c}\sqrt{x^2\,(b+cx^2)}}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^5,x]
```

```
[Out] (Sqrt[c]*(b + c*x^2)*(-8*A*b + 5*b*B*x^2 + 4*A*c*x^2 + 2*B*c*x^4) + 6*b*(b*
B + 4*A*c)*x*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/(-Sqrt[b] + Sqrt[b + c*x^2
])])/(8*Sqrt[c]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{(-2Bc\,x^4-4Ac\,x^2-5bB\,x^2+8Ab)\sqrt{x^2(c\,x^2+b)}}{8x^2}+\frac{3b(4Ac+Bb)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)\sqrt{x^2(c\,x^2+b)}}{8\sqrt{c}\,x\sqrt{c\,x^2+b}}$
pseudoelliptic	$\frac{\frac{x^2\left(\frac{x^2B}{2}+A\right)\sqrt{x^2\left(c\,x^2+b\right)}\,c^{\frac{3}{2}}}{2}+\frac{3\Big(-\frac{4\sqrt{x^2\left(c\,x^2+b\right)}\left(-\frac{5x^2B}{8}+A\right)\sqrt{c}}{3}+\Big(-\ln(2)+\ln\Big(\frac{2c\,x^2+2\sqrt{x^2\left(c\,x^2+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\Big)\Big)x^2\left(Ac+\frac{Bb}{4}\right)\Big)}{4}}{x^2\sqrt{c}}$
default	$-\frac{(x^4c+b\,x^2)^{\frac{3}{2}}\Big(-8A(c\,x^2+b)^{\frac{3}{2}}c^{\frac{3}{2}}x^2-2B(c\,x^2+b)^{\frac{3}{2}}\sqrt{c}\,b\,x^2+8A(c\,x^2+b)^{\frac{5}{2}}\sqrt{c}-12A\sqrt{c\,x^2+b}\,c^{\frac{3}{2}}b\,x^2-3B\sqrt{c\,x^2+b}\,\sqrt{c}\,b^2x^2}{8x^4(c\,x^2+b)^{\frac{3}{2}}b\sqrt{c}}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-2*B*c*x^4-4*A*c*x^2-5*B*b*x^2+8*A*b)/x^2*(x^2*(c*x^2+b))^(1/2)+3/8*b
*(4*A*c+B*b)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))/c^(1/2)*(x^2*(c*x^2+b))^(1/2)/x/
(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.63

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^5}\,dx = \left[\frac{3\,(Bb^2+4\,Abc)\sqrt{c}x^2\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\,(2\,Bc^2x^4-}{16\,cx^2}\right.$$

$$\left.-\frac{3\,(Bb^2+4\,Abc)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-(2\,Bc^2x^4-8\,Abc+(5\,Bbc+4\,Ac^2)x^2)\sqrt{cx^4+bx^2}}{8\,cx^2}\right]$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(B*b^2 + 4*A*b*c)*sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*
x^2)*sqrt(c)) + 2*(2*B*c^2*x^4 - 8*A*b*c + (5*B*b*c + 4*A*c^2)*x^2)*sqrt(c*
x^4 + b*x^2))/(c*x^2), -1/8*(3*(B*b^2 + 4*A*b*c)*sqrt(-c)*x^2*arctan(sqrt(c
*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (2*B*c^2*x^4 - 8*A*b*c + (5*B*b*c + 4
*A*c^2)*x^2)*sqrt(c*x^4 + b*x^2))/(c*x^2)]
```

Sympy [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^5}\,dx = \int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}(A+Bx^2)}{x^5}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**5,x)
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.16

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^5}\,dx = \frac{1}{4}\left(3\,b\sqrt{c}\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right) - \frac{6\,\sqrt{cx^4+bx^2}b}{x^2} + \frac{2\,(cx^4+b\cdots}{x^4}\right.$$
$$+\frac{1}{16}\left(\frac{3\,b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{\sqrt{c}} + 6\,\sqrt{cx^4+bx^2}b + \frac{4\,(cx^4+bx^2)^{\frac{3}{2}}}{x^2}\right)B$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="maxima")
[Out] 1/4*(3*b*sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 6*sqrt(
c*x^4 + b*x^2)*b/x^2 + 2*(c*x^4 + b*x^2)^(3/2)/x^4)*A + 1/16*(3*b^2*log(2*c
*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) + 6*sqrt(c*x^4 + b*x^2)*b
 + 4*(c*x^4 + b*x^2)^(3/2)/x^2)*B
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^5}\,dx = \frac{2\,Ab^2\sqrt{c}\mathrm{sgn}(x)}{\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b}$$
$$+\frac{1}{8}\left(2\,Bcx^2\mathrm{sgn}(x)+\frac{5\,Bbc^2\mathrm{sgn}(x)+4\,Ac^3\mathrm{sgn}(x)}{c^2}\right)\sqrt{cx^2+b}x$$
$$-\frac{3\,(Bb^2\mathrm{sgn}(x)+4\,Abc\mathrm{sgn}(x))\log\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right)}{16\,\sqrt{c}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^5,x, algorithm="giac")
[Out] 2*A*b^2*sqrt(c)*sgn(x)/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b) + 1/8*(2*B*c*x
^2*sgn(x) + (5*B*b*c^2*sgn(x) + 4*A*c^3*sgn(x))/c^2)*sqrt(c*x^2 + b)*x - 3/
16*(B*b^2*sgn(x) + 4*A*b*c*sgn(x))*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)/sqr
t(c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^5}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^5}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^5,x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^5, x)
```

3.112 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^7}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 136

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^7}\,dx = \frac{c(3bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{(3bB+2Ac)(bx^2+cx^4)^{3/2}}{3bx^4} - \frac{A(bx^2+cx^4)^{5/2}}{3bx^8} + \frac{1}{2}\sqrt{c}(3bB+2Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] -1/3*(2*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^4-1/3*A*(c*x^4+b*x^2)^(5/2)/b/x^
8+1/2*(2*A*c+3*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))*c^(1/2)+1/2*c*
(2*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2059, 806, 676, 678, 634, 212}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^7}\,dx = \frac{1}{2}\sqrt{c}(2Ac+3bB)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right) - \frac{(bx^2+cx^4)^{3/2}(2Ac+3bB)}{3bx^4} + \frac{c\sqrt{bx^2+cx^4}(2Ac+3bB)}{2b} - \frac{A(bx^2+cx^4)^{5/2}}{3bx^8}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^7,x]
```

```
[Out] (c*(3*b*B + 2*A*c)*Sqrt[b*x^2 + c*x^4])/(2*b) - ((3*b*B + 2*A*c)*(b*x^2 + c
*x^4)^(3/2))/(3*b*x^4) - (A*(b*x^2 + c*x^4)^(5/2))/(3*b*x^8) + (Sqrt[c]*(3*
b*B + 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 676

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Dist[c*(p/(e^2*(m + p + 1))), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
```

```
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{3bx^8} + \frac{\left(-4(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^3}\,dx, x, x^2\right)}{3b}$$

$$= -\frac{(3bB+2Ac)\left(bx^2+cx^4\right)^{3/2}}{3bx^4} - \frac{A(bx^2+cx^4)^{5/2}}{3bx^8} + \frac{\left(c\left(-4(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\right)\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x}\,dx, x, x^2\right)}{b}$$

$$= \frac{c(3bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{(3bB+2Ac)\left(bx^2+cx^4\right)^{3/2}}{3bx^4} - \frac{A(bx^2+cx^4)^{5/2}}{3bx^8} + \frac{1}{4}(c(3bB+2Ac))\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$

$$= \frac{c(3bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{(3bB+2Ac)\left(bx^2+cx^4\right)^{3/2}}{3bx^4} - \frac{A(bx^2+cx^4)^{5/2}}{3bx^8} + \frac{1}{2}(c(3bB+2Ac))\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)$$

$$= \frac{c(3bB+2Ac)\sqrt{bx^2+cx^4}}{2b} - \frac{(3bB+2Ac)\left(bx^2+cx^4\right)^{3/2}}{3bx^4} - \frac{A(bx^2+cx^4)^{5/2}}{3bx^8} + \frac{1}{2}\sqrt{c}(3bB+2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^7}\,dx = \frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b+cx^2}(-6bBx^2+3Bcx^4-2A(b+4cx^2))+6\sqrt{c}(3bB+\right.}{6x^4\sqrt{b+cx^2}}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^7,x]
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[b + c*x^2]*(-6*b*B*x^2 + 3*B*c*x^4 - 2*A*(b +
4*c*x^2)) + 6*Sqrt[c]*(3*b*B + 2*A*c)*x^3*ArcTanh[(Sqrt[c]*x)/(-Sqrt[b] + S
qrt[b + c*x^2])]))/(6*x^4*Sqrt[b + c*x^2])
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{\left(-3Bc\,x^{4}+8Ac\,x^{2}+6bB\,x^{2}+2Ab\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{6x^{4}}+\frac{\left(2Ac+3Bb\right)\sqrt{c}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{2x\sqrt{c\,x^{2}+b}}$
pseudoelliptic	$\frac{3x^{4}\left(3\sqrt{c}\,Bb+2A\,c^{\frac{3}{2}}\right)\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)+2\left(3Bc\,x^{4}+2(-4Ac-3Bb)x^{2}-2Ab\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}-6x^{4}\ln(2)\left(\frac{3\sqrt{c}\,Bb}{2}\right.}{12x^{4}}$
default	$-\frac{\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}\left(-4A\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{\frac{5}{2}}x^{4}-6B\left(c\,x^{2}+b\right)^{\frac{3}{2}}c^{\frac{3}{2}}b\,x^{4}+4A\left(c\,x^{2}+b\right)^{\frac{5}{2}}c^{\frac{3}{2}}x^{2}-6A\sqrt{c\,x^{2}+b}\,c^{\frac{5}{2}}b\,x^{4}+6B\left(c\,x^{2}+b\right)^{\frac{5}{2}}\sqrt{c}\,b\,x\right.}{6x^{6}\left(c\,x^{2}+b\right)^{\frac{3}{2}}b^{2}}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-3*B*c*x^4+8*A*c*x^2+6*B*b*x^2+2*A*b)/x^4*(x^2*(c*x^2+b))^(1/2)+1/2*(
2*A*c+3*B*b)*c^(1/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*(x^2*(c*x^2+b))^(1/2)/x/
(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.39

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^7}\,dx=\left[\frac{3\,(3\,Bb+2\,Ac)\sqrt{c}x^4\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\,(3\,Bcx^4-}{12\,x^4}\right.$$
$$\left.-\frac{3\,(3\,Bb+2\,Ac)\sqrt{-c}x^4\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-(3\,Bcx^4-2\,(3\,Bb+4\,Ac)x^2-2\,Ab)\sqrt{cx^4+bx^2}}{6\,x^4}\right]$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(3*B*b + 2*A*c)*sqrt(c)*x^4*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^
2)*sqrt(c)) + 2*(3*B*c*x^4 - 2*(3*B*b + 4*A*c)*x^2 - 2*A*b)*sqrt(c*x^4 + b*
x^2))/x^4, -1/6*(3*(3*B*b + 2*A*c)*sqrt(-c)*x^4*arctan(sqrt(c*x^4 + b*x^2)*
sqrt(-c)/(c*x^2 + b)) - (3*B*c*x^4 - 2*(3*B*b + 4*A*c)*x^2 - 2*A*b)*sqrt(c*
x^4 + b*x^2))/x^4]
```

Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^7}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}(A+Bx^2)}{x^7}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**7,x)
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**7, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.23

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^7}\,dx = \frac{1}{6}\left(3\,c^{\frac{3}{2}}\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right) - \frac{7\,\sqrt{cx^4+bx^2}c}{x^2} - \frac{\sqrt{cx^4+bx^2}}{x^4}\right.$$
$$+\frac{1}{4}\left(3\,b\sqrt{c}\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right) - \frac{6\,\sqrt{cx^4+bx^2}b}{x^2} + \frac{2\,(cx^4+bx^2)^{\frac{3}{2}}}{x^4}\right)B$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="maxima")
[Out] 1/6*(3*c^(3/2)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 7*sqrt(c*
x^4 + b*x^2)*c/x^2 - sqrt(c*x^4 + b*x^2)*b/x^4 - (c*x^4 + b*x^2)^(3/2)/x^6)
*A + 1/4*(3*b*sqrt(c)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 6*
sqrt(c*x^4 + b*x^2)*b/x^2 + 2*(c*x^4 + b*x^2)^(3/2)/x^4)*B
```

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.65

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^7}\,dx = \frac{1}{2}\sqrt{cx^2+b}Bcx\mathrm{sgn}(x)$$
$$-\frac{1}{4}\left(3\,Bb\sqrt{c}\mathrm{sgn}(x)+2\,Ac^{\frac{3}{2}}\mathrm{sgn}(x)\right)\log\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right)$$
$$+\frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4Bb^2\sqrt{c}\mathrm{sgn}(x)+6\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4Abc^{\frac{3}{2}}\mathrm{sgn}(x)-6\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2Bb^3\sqrt{c}\mathrm{sg}\right.}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^3}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + b)*B*c*x*sgn(x) - 1/4*(3*B*b*sqrt(c)*sgn(x) + 2*A*c^(3/2)*
sgn(x))*log((sqrt(c)*x - sqrt(c*x^2 + b))^2) + 2/3*(3*(sqrt(c)*x - sqrt(c*x
^2 + b))^4*B*b^2*sqrt(c)*sgn(x) + 6*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b*c^(
3/2)*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^3*sqrt(c)*sgn(x) - 6*(s
qrt(c)*x - sqrt(c*x^2 + b))^2*A*b^2*c^(3/2)*sgn(x) + 3*B*b^4*sqrt(c)*sgn(x)
 + 4*A*b^3*c^(3/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^7}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^7}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^7,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^7, x)
```

3.113 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^9}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 104

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^9}\,dx = -\frac{Bc\sqrt{bx^2+cx^4}}{x^2} - \frac{B(bx^2+cx^4)^{3/2}}{3x^6} - \frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}} + Bc^{3/2}\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] -1/3*B*(c*x^4+b*x^2)^(3/2)/x^6-1/5*A*(c*x^4+b*x^2)^(5/2)/b/x^10+B*c^(3/2)*a
rctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))-B*c*(c*x^4+b*x^2)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2059, 806, 676, 634, 212}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^9}\,dx = -\frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}} + Bc^{3/2}\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right) - \frac{Bc\sqrt{bx^2+cx^4}}{x^2} - \frac{B(bx^2+cx^4)^{3/2}}{3x^6}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^9,x]
```

```
[Out] -((B*c*Sqrt[b*x^2 + c*x^4])/x^2) - (B*(b*x^2 + c*x^4)^(3/2))/(3*x^6) - (A*(
b*x^2 + c*x^4)^(5/2))/(5*b*x^10) + B*c^(3/2)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x
^2 + c*x^4]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 676

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Dist[c*(p/(e^2*(m + p + 1))), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}} + \frac{1}{2}B\text{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{B(bx^2+cx^4)^{3/2}}{3x^6} - \frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}} + \frac{1}{2}(Bc)\text{Subst}\left(\int \frac{\sqrt{bx+cx^2}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{Bc\sqrt{bx^2+cx^4}}{x^2} - \frac{B(bx^2+cx^4)^{3/2}}{3x^6} - \frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}} + \frac{1}{2}\left(Bc^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{Bc\sqrt{bx^2+cx^4}}{x^2} - \frac{B(bx^2+cx^4)^{3/2}}{3x^6} - \frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}} + \left(Bc^2\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)$$

$$= -\frac{Bc\sqrt{bx^2+cx^4}}{x^2} - \frac{B(bx^2+cx^4)^{3/2}}{3x^6} - \frac{A(bx^2+cx^4)^{5/2}}{5bx^{10}} + Bc^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^9}\,dx = -\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b+cx^2}\left(3A(b+cx^2)^2+5bBx^2(b+4cx^2)\right)+15bBc^{3/2}x^5\log\left(-\sqrt{c}x+\sqrt{b+cx^2}\right)\right)}{15bx^6\sqrt{b+cx^2}}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^9,x]
[Out] -1/15*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b + c*x^2]*(3*A*(b + c*x^2)^2 + 5*b*B*x^
2*(b + 4*c*x^2)) + 15*b*B*c^(3/2)*x^5*Log[-(Sqrt[c]*x) + Sqrt[b + c*x^2]]))
/(b*x^6*Sqrt[b + c*x^2])
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{5x^6\left(-\ln(2)+\ln\left(\frac{2cx^2+2\sqrt{x^2(cx^2+b)}\sqrt{c}+b}{\sqrt{c}}\right)\right)Bbc^{\frac{3}{2}}-2\left(\left(\frac{5x^2B}{3}+A\right)b^2+2x^2\left(\frac{10x^2B}{3}+A\right)cb+Ac^2x^4\right)\sqrt{x^2(cx^2+b)}}{10bx^6}$
risch	$-\frac{(3Ac^2x^4+20x^4Bbc+6Abcx^2+5b^2Bx^2+3b^2A)\sqrt{x^2(cx^2+b)}}{15x^6b}+\frac{Bc^{\frac{3}{2}}\ln\left(\sqrt{c}x+\sqrt{cx^2+b}\right)\sqrt{x^2(cx^2+b)}}{x\sqrt{cx^2+b}}$
default	$-\frac{(x^4c+bx^2)^{\frac{3}{2}}(-10B(cx^2+b)^{\frac{3}{2}}c^{\frac{5}{2}}x^6+10B(cx^2+b)^{\frac{5}{2}}c^{\frac{3}{2}}x^4-15B\sqrt{cx^2+b}\,c^{\frac{5}{2}}bx^6-15B\ln(\sqrt{c}x+\sqrt{cx^2+b})b^2c^2x^5+5B(cx}{15x^8(cx^2+b)^{\frac{3}{2}}b^2\sqrt{c}}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*(5*x^6*(-ln(2)+ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2)*c^(1/2)+b)/c^(1/2))
)*B*b*c^(3/2)-2*((5/3*x^2*B+A)*b^2+2*x^2*(10/3*x^2*B+A)*c*b+A*c^2*x^4)*(x^2
*(c*x^2+b))^(1/2))/b/x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.99

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^9}\,dx = \left[\frac{15\,Bbc^{\frac{3}{2}}x^6\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\left((20\,Bbc+3\,Ac^2)x^4+\right.}{30\,bx^6}\right.$$
$$\left.-\frac{15\,Bb\sqrt{-c}cx^6\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)+\left((20\,Bbc+3\,Ac^2)x^4+3\,Ab^2+(5\,Bb^2+6\,Abc)x^2\right)\sqrt{cx^4+bx^2}}{15\,bx^6}\right]$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/30*(15*B*b*c^(3/2)*x^6*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c))
 - 2*((20*B*b*c + 3*A*c^2)*x^4 + 3*A*b^2 + (5*B*b^2 + 6*A*b*c)*x^2)*sqrt(c*
x^4 + b*x^2))/(b*x^6), -1/15*(15*B*b*sqrt(-c)*c*x^6*arctan(sqrt(c*x^4 + b*x
^2)*sqrt(-c)/(c*x^2 + b)) + ((20*B*b*c + 3*A*c^2)*x^4 + 3*A*b^2 + (5*B*b^2
+ 6*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b*x^6)]
```

Sympy [F]

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^9}\,dx = \int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}(A+Bx^2)}{x^9}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**9,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**9, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(88) = 176$.

Time = 0.21 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.70

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^9}\,dx = \frac{1}{6}\left(3\,c^{\frac{3}{2}}\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right) - \frac{7\,\sqrt{cx^4+bx^2}c}{x^2} - \frac{\sqrt{cx^4+bx^2}}{x^4}\right.$$
$$\left. - \frac{1}{10}\,A\left(\frac{2\,\sqrt{cx^4+bx^2}c^2}{bx^2} - \frac{\sqrt{cx^4+bx^2}c}{x^4} - \frac{3\,\sqrt{cx^4+bx^2}b}{x^6} + \frac{5\,(cx^4+bx^2)^{\frac{3}{2}}}{x^8}\right)\right.$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] 1/6*(3*c^(3/2)*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 7*sqrt(c*
x^4 + b*x^2)*c/x^2 - sqrt(c*x^4 + b*x^2)*b/x^4 - (c*x^4 + b*x^2)^(3/2)/x^6)
*B - 1/10*A*(2*sqrt(c*x^4 + b*x^2)*c^2/(b*x^2) - sqrt(c*x^4 + b*x^2)*c/x^4
- 3*sqrt(c*x^4 + b*x^2)*b/x^6 + 5*(c*x^4 + b*x^2)^(3/2)/x^8)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(88) = 176$.

Time = 0.83 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.44

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^9}\,dx = -\frac{1}{2}\,Bc^{\frac{3}{2}}\log\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right)\operatorname{sgn}(x)$$
$$+ \frac{2\left(30\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^8 Bbc^{\frac{3}{2}}\operatorname{sgn}(x) + 15\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^8 Ac^{\frac{5}{2}}\operatorname{sgn}(x) - 90\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6 Bb^2c^{\frac{3}{2}}\operatorname{sg}\right.}{}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/2*B*c^(3/2)*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)*sgn(x) + 2/15*(30*(sqrt
(c)*x - sqrt(c*x^2 + b))^8*B*b*c^(3/2)*sgn(x) + 15*(sqrt(c)*x - sqrt(c*x^2
+ b))^8*A*c^(5/2)*sgn(x) - 90*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^2*c^(3/2)
*sgn(x) + 110*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^3*c^(3/2)*sgn(x) + 30*(sq
rt(c)*x - sqrt(c*x^2 + b))^4*A*b^2*c^(5/2)*sgn(x) - 70*(sqrt(c)*x - sqrt(c*
x^2 + b))^2*B*b^4*c^(3/2)*sgn(x) + 20*B*b^5*c^(3/2)*sgn(x) + 3*A*b^4*c^(5/2
)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^9}\,dx=\int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^9}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^9,x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^9, x)
```

3.114 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11}} dx$

Optimal result

Integrand size = 26, antiderivative size = 61

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11}} dx = -\frac{A(bx^2+cx^4)^{5/2}}{7bx^{12}} - \frac{(7bB-2Ac)(bx^2+cx^4)^{5/2}}{35b^2x^{10}}$$

```
[Out] -1/7*A*(c*x^4+b*x^2)^(5/2)/b/x^12-1/35*(-2*A*c+7*B*b)*(c*x^4+b*x^2)^(5/2)/b
^2/x^10
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2059, 806, 664}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11}} dx = -\frac{(bx^2+cx^4)^{5/2}(7bB-2Ac)}{35b^2x^{10}} - \frac{A(bx^2+cx^4)^{5/2}}{7bx^{12}}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^11,x]
```

```
[Out] -1/7*(A*(b*x^2 + c*x^4)^(5/2))/(b*x^12) - ((7*b*B - 2*A*c)*(b*x^2 + c*x^4)^
(5/2))/(35*b^2*x^10)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{7bx^{12}} + \frac{\left(-6(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{\left(bx+cx^2\right)^{3/2}}{x^5}\,dx, x, x^2\right)}{7b}$$

$$= -\frac{A\left(bx^2+cx^4\right)^{5/2}}{7bx^{12}} - \frac{(7bB-2Ac)\left(bx^2+cx^4\right)^{5/2}}{35b^2x^{10}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{11}}\,dx = \frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}\left(-5Ab-7bBx^2+2Acx^2\right)}{35b^2x^{12}}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^11,x]
[Out] ((x^2*(b + c*x^2))^(5/2)*(-5*A*b - 7*b*B*x^2 + 2*A*c*x^2))/(35*b^2*x^12)
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.79

method	result	size
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-2Ac\,x^{2}+7bB\,x^{2}+5Ab\right)\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}}{35b^{2}x^{10}}$	48
default	$-\frac{\left(c\,x^{2}+b\right)\left(-2Ac\,x^{2}+7bB\,x^{2}+5Ab\right)\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}}{35b^{2}x^{10}}$	48
pseudoelliptic	$-\frac{\left(\left(\frac{7x^{2}B}{5}+A\right)b-\frac{2Ac\,x^{2}}{5}\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(c\,x^{2}+b\right)^{2}}{7x^{8}b^{2}}$	49
trager	$-\frac{\left(-2A\,c^{3}x^{6}+7x^{6}Bb\,c^{2}+Ab\,c^{2}x^{4}+14x^{4}B\,b^{2}c+8A\,b^{2}c\,x^{2}+7b^{3}B\,x^{2}+5b^{3}A\right)\sqrt{x^{4}c+b\,x^{2}}}{35b^{2}x^{8}}$	86
risch	$-\frac{\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\left(-2A\,c^{3}x^{6}+7x^{6}Bb\,c^{2}+Ab\,c^{2}x^{4}+14x^{4}B\,b^{2}c+8A\,b^{2}c\,x^{2}+7b^{3}B\,x^{2}+5b^{3}A\right)}{35x^{8}b^{2}}$	86

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^11,x,method=_RETURNVERBOSE)
[Out] -1/35*(c*x^2+b)*(-2*A*c*x^2+7*B*b*x^2+5*A*b)*(c*x^4+b*x^2)^(3/2)/b^2/x^10
```

Fricas [A] (verification not implemented)

none

Time = 0.76 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{11}}\,dx = -\frac{((7\,Bbc^2-2\,Ac^3)x^6+(14\,Bb^2c+Abc^2)x^4+5\,Ab^3+(7\,Bb^3+8\,Ab^2c)x^2)\sqrt{cx^4+bx^2}}{35\,b^2x^8}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="fricas")
[Out] -1/35*((7*B*b*c^2 - 2*A*c^3)*x^6 + (14*B*b^2*c + A*b*c^2)*x^4 + 5*A*b^3 + (
7*B*b^3 + 8*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^2*x^8)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{11}}\,dx = \int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}(A+Bx^2)}{x^{11}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**11,x)
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**11, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(53) = 106$.

Time = 0.23 (sec) , antiderivative size = 193, normalized size of antiderivative = 3.16

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11}}\,dx = -\frac{1}{10}\,B\left(\frac{2\sqrt{cx^4+bx^2}c^2}{bx^2} - \frac{\sqrt{cx^4+bx^2}c}{x^4} - \frac{3\sqrt{cx^4+bx^2}b}{x^6} + \frac{5\,(cx^4+bx^2)^{\frac{3}{2}}}{x^8}\right) + \frac{1}{140}\,A\left(\frac{8\sqrt{cx^4+bx^2}c^3}{b^2x^2} - \frac{4\sqrt{cx^4+bx^2}c^2}{bx^4} + \frac{3\sqrt{cx^4+bx^2}c}{x^6} + \frac{15\sqrt{cx^4+bx^2}b}{x^8} - \frac{35\,(cx^4+bx^2)^{\frac{3}{2}}}{x^{10}}\right)$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="maxima")
[Out] -1/10*B*(2*sqrt(c*x^4 + b*x^2)*c^2/(b*x^2) - sqrt(c*x^4 + b*x^2)*c/x^4 - 3*
sqrt(c*x^4 + b*x^2)*b/x^6 + 5*(c*x^4 + b*x^2)^(3/2)/x^8) + 1/140*A*(8*sqrt(
c*x^4 + b*x^2)*c^3/(b^2*x^2) - 4*sqrt(c*x^4 + b*x^2)*c^2/(b*x^4) + 3*sqrt(c
*x^4 + b*x^2)*c/x^6 + 15*sqrt(c*x^4 + b*x^2)*b/x^8 - 35*(c*x^4 + b*x^2)^(3/
2)/x^10)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(53) = 106$.

Time = 1.19 (sec) , antiderivative size = 370, normalized size of antiderivative = 6.07

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11}}\,dx = \frac{2\left(35\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Bc^{\frac{5}{2}}\mathrm{sgn}(x) - 70\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Bbc^{\frac{5}{2}}\mathrm{sgn}(\right.}{}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^11,x, algorithm="giac")
[Out] 2/35*(35*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*c^(5/2)*sgn(x) - 70*(sqrt(c)*x
- sqrt(c*x^2 + b))^10*B*b*c^(5/2)*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + b))
^10*A*c^(7/2)*sgn(x) + 105*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^2*c^(5/2)*sg
n(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*b*c^(7/2)*sgn(x) - 140*(sqrt(c)
*x - sqrt(c*x^2 + b))^6*B*b^3*c^(5/2)*sgn(x) + 140*(sqrt(c)*x - sqrt(c*x^2
+ b))^6*A*b^2*c^(7/2)*sgn(x) + 77*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^4*c^(
5/2)*sgn(x) + 28*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^3*c^(7/2)*sgn(x) - 14*
(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^5*c^(5/2)*sgn(x) + 14*(sqrt(c)*x - sqrt
(c*x^2 + b))^2*A*b^4*c^(7/2)*sgn(x) + 7*B*b^6*c^(5/2)*sgn(x) - 2*A*b^5*c^(7
/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^7
```

Mupad [B] (verification not implemented)

Time = 9.82 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.56

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11}}\,dx = \frac{2\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{35\,b^2\,x^2} - \frac{8\,A\,c\,\sqrt{c\,x^4+b\,x^2}}{35\,x^6} - \frac{B\,b\,\sqrt{c\,x^4+b\,x^2}}{5\,x^6} - \frac{2\,B\,c\,\sqrt{c\,x^4+b\,x^2}}{5\,x^4} - \frac{A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{35\,b\,x^4} - \frac{A\,b\,\sqrt{c\,x^4+b\,x^2}}{7\,x^8} - \frac{B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{5\,b\,x^2}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^11,x)
```

```
[Out] (2*A*c^3*(b*x^2 + c*x^4)^(1/2))/(35*b^2*x^2) - (8*A*c*(b*x^2 + c*x^4)^(1/2)
)/(35*x^6) - (B*b*(b*x^2 + c*x^4)^(1/2))/(5*x^6) - (2*B*c*(b*x^2 + c*x^4)^(
1/2))/(5*x^4) - (A*c^2*(b*x^2 + c*x^4)^(1/2))/(35*b*x^4) - (A*b*(b*x^2 + c*
x^4)^(1/2))/(7*x^8) - (B*c^2*(b*x^2 + c*x^4)^(1/2))/(5*b*x^2)
```

3.115 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13}}\,dx = -\frac{A(bx^2+cx^4)^{5/2}}{9bx^{14}} - \frac{(9bB-4Ac)(bx^2+cx^4)^{5/2}}{63b^2x^{12}} + \frac{2c(9bB-4Ac)(bx^2+cx^4)^{5/2}}{315b^3x^{10}}$$

```
[Out] -1/9*A*(c*x^4+b*x^2)^(5/2)/b/x^14-1/63*(-4*A*c+9*B*b)*(c*x^4+b*x^2)^(5/2)/b
^2/x^12+2/315*c*(-4*A*c+9*B*b)*(c*x^4+b*x^2)^(5/2)/b^3/x^10
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2059, 806, 672, 664}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13}}\,dx = \frac{2c(bx^2+cx^4)^{5/2}(9bB-4Ac)}{315b^3x^{10}} - \frac{(bx^2+cx^4)^{5/2}(9bB-4Ac)}{63b^2x^{12}} - \frac{A(bx^2+cx^4)^{5/2}}{9bx^{14}}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^13,x]
```

```
[Out] -1/9*(A*(b*x^2 + c*x^4)^(5/2))/(b*x^14) - ((9*b*B - 4*A*c)*(b*x^2 + c*x^4)^
(5/2))/(63*b^2*x^12) + (2*c*(9*b*B - 4*A*c)*(b*x^2 + c*x^4)^(5/2))/(315*b^3
*x^10)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\,(bx+cx^2)^{3/2}}{x^7}\,dx,x,x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{9bx^{14}} + \frac{\left(-7(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^6}\,dx,x,x^2\right)}{9b}$$

$$=-\frac{A(bx^2+cx^4)^{5/2}}{9bx^{14}}-\frac{(9bB-4Ac)(bx^2+cx^4)^{5/2}}{63b^2x^{12}}-\frac{(c(9bB-4Ac))\text{Subst}\left(\int\frac{(bx+cx^2)^{3/2}}{x^5}\,dx,x,x^2\right)}{63b^2}$$

$$=-\frac{A(bx^2+cx^4)^{5/2}}{9bx^{14}}-\frac{(9bB-4Ac)(bx^2+cx^4)^{5/2}}{63b^2x^{12}}+\frac{2c(9bB-4Ac)(bx^2+cx^4)^{5/2}}{315b^3x^{10}}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

$$\int\frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13}}\,dx=\frac{(x^2(b+cx^2))^{5/2}(9bBx^2(-5b+2cx^2)+A(-35b^2+20bcx^2-8c^2x^4))}{315b^3x^{14}}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^13,x]
[Out] ((x^2*(b + c*x^2))^(5/2)*(9*b*B*x^2*(-5*b + 2*c*x^2) + A*(-35*b^2 + 20*b*c*
x^2 - 8*c^2*x^4)))/(315*b^3*x^14)
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.71

method	result	s
pseudoelliptic	$-\frac{\sqrt{x^2(cx^2+b)}\,(cx^2+b)^2\left(\left(\frac{9x^2B}{7}+A\right)b^2-\frac{4x^2\left(\frac{9x^2B}{10}+A\right)cb}{7}+\frac{8Ac^2x^4}{35}\right)}{9x^{10}b^3}$	6
gosper	$-\frac{(cx^2+b)(8Ac^2x^4-18x^4Bbc-20Abcx^2+45b^2Bx^2+35b^2A)(x^4c+bx^2)^{\frac{3}{2}}}{315b^3x^{12}}$	7
default	$-\frac{(cx^2+b)(8Ac^2x^4-18x^4Bbc-20Abcx^2+45b^2Bx^2+35b^2A)(x^4c+bx^2)^{\frac{3}{2}}}{315b^3x^{12}}$	7
trager	$-\frac{(8Ax^8c^4-18Bx^8bc^3-4Ax^6bc^3+9Bx^6b^2c^2+3Ab^2c^2x^4+72Bb^3cx^4+50Ax^2b^3c+45Bx^2b^4+35Ab^4)\sqrt{x^4c+bx^2}}{315b^3x^{10}}$	1
risch	$-\frac{\sqrt{x^2(cx^2+b)}\,(8Ax^8c^4-18Bx^8bc^3-4Ax^6bc^3+9Bx^6b^2c^2+3Ab^2c^2x^4+72Bb^3cx^4+50Ax^2b^3c+45Bx^2b^4+35Ab^4)}{315x^{10}b^3}$	1

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^13,x,method=_RETURNVERBOSE)
[Out] -1/9*(x^2*(c*x^2+b))^(1/2)*(c*x^2+b)^2*((9/7*x^2*B+A)*b^2-4/7*x^2*(9/10*x^2
*B+A)*c*b+8/35*A*c^2*x^4)/x^10/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{13}}\,dx = \frac{(2\,(9\,Bbc^3-4\,Ac^4)x^8-(9\,Bb^2c^2-4\,Abc^3)x^6-35\,Ab^4-3\,(24\,Bb^3c+Ab^2c}{315\,b^3x^{10}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] 1/315*(2*(9*B*b*c^3 - 4*A*c^4)*x^8 - (9*B*b^2*c^2 - 4*A*b*c^3)*x^6 - 35*A*b
^4 - 3*(24*B*b^3*c + A*b^2*c^2)*x^4 - 5*(9*B*b^4 + 10*A*b^3*c)*x^2)*sqrt(c*
x^4 + b*x^2)/(b^3*x^10)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{13}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}\,(A+Bx^2)}{x^{13}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**13,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**13, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(84) = 168.

Time = 0.22 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.51

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{13}}\,dx = \frac{1}{140}\,B\left(\frac{8\,\sqrt{cx^4+bx^2}c^3}{b^2x^2}-\frac{4\,\sqrt{cx^4+bx^2}c^2}{bx^4}+\frac{3\,\sqrt{cx^4+bx^2}c}{x^6}+\frac{15\,\sqrt{cx^4+b}}{x^8}\right.$$
$$-\frac{1}{630}\,A\left(\frac{16\,\sqrt{cx^4+bx^2}c^4}{b^3x^2}-\frac{8\,\sqrt{cx^4+bx^2}c^3}{b^2x^4}+\frac{6\,\sqrt{cx^4+bx^2}c^2}{bx^6}-\frac{5\,\sqrt{cx^4+bx^2}c}{x^8}-\frac{35\,\sqrt{cx^4+bx^2}b}{x^{10}}+\frac{105\,(c}{}\right.$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] 1/140*B*(8*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^2) - 4*sqrt(c*x^4 + b*x^2)*c^2/(b
*x^4) + 3*sqrt(c*x^4 + b*x^2)*c/x^6 + 15*sqrt(c*x^4 + b*x^2)*b/x^8 - 35*(c*
x^4 + b*x^2)^(3/2)/x^10) - 1/630*A*(16*sqrt(c*x^4 + b*x^2)*c^4/(b^3*x^2) -
8*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^4) + 6*sqrt(c*x^4 + b*x^2)*c^2/(b*x^6) - 5
*sqrt(c*x^4 + b*x^2)*c/x^8 - 35*sqrt(c*x^4 + b*x^2)*b/x^10 + 105*(c*x^4 + b
*x^2)^(3/2)/x^12)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(84) = 168.

Time = 1.54 (sec) , antiderivative size = 430, normalized size of antiderivative = 4.48

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13}}\,dx = \frac{4\left(315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{14}Bc^{\frac{7}{2}}\mathrm{sgn}(x)-315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Bbc^{\frac{7}{2}}\mathrm{sg}\right.}{}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] 4/315*(315*(sqrt(c)*x - sqrt(c*x^2 + b))^14*B*c^(7/2)*sgn(x) - 315*(sqrt(c)
*x - sqrt(c*x^2 + b))^12*B*b*c^(7/2)*sgn(x) + 840*(sqrt(c)*x - sqrt(c*x^2 +
 b))^12*A*c^(9/2)*sgn(x) + 315*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*b^2*c^(7/
2)*sgn(x) + 1260*(sqrt(c)*x - sqrt(c*x^2 + b))^10*A*b*c^(9/2)*sgn(x) - 819*
(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^3*c^(7/2)*sgn(x) + 1764*(sqrt(c)*x - sq
rt(c*x^2 + b))^8*A*b^2*c^(9/2)*sgn(x) + 441*(sqrt(c)*x - sqrt(c*x^2 + b))^6
*B*b^4*c^(7/2)*sgn(x) + 504*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b^3*c^(9/2)*s
gn(x) - 9*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^5*c^(7/2)*sgn(x) + 144*(sqrt(
c)*x - sqrt(c*x^2 + b))^4*A*b^4*c^(9/2)*sgn(x) + 81*(sqrt(c)*x - sqrt(c*x^2
 + b))^2*B*b^6*c^(7/2)*sgn(x) - 36*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^5*c^
(9/2)*sgn(x) - 9*B*b^7*c^(7/2)*sgn(x) + 4*A*b^6*c^(9/2)*sgn(x))/((sqrt(c)*x
 - sqrt(c*x^2 + b))^2 - b)^9
```

Mupad [B] (verification not implemented)

Time = 10.24 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.15

$$\begin{aligned}\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13}}\,dx &= \frac{4\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{315\,b^2\,x^4}-\frac{10\,A\,c\,\sqrt{c\,x^4+b\,x^2}}{63\,x^8}\\ &\quad-\frac{B\,b\,\sqrt{c\,x^4+b\,x^2}}{7\,x^8}-\frac{8\,B\,c\,\sqrt{c\,x^4+b\,x^2}}{35\,x^6}-\frac{A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b\,x^6}-\frac{A\,b\,\sqrt{c\,x^4+b\,x^2}}{9\,x^{10}}\\ &\quad-\frac{8\,A\,c^4\,\sqrt{c\,x^4+b\,x^2}}{315\,b^3\,x^2}-\frac{B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{35\,b\,x^4}+\frac{2\,B\,c^3\,\sqrt{c\,x^4+b\,x^2}}{35\,b^2\,x^2}\end{aligned}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^13,x)
```

```
[Out] (4*A*c^3*(b*x^2 + c*x^4)^(1/2))/(315*b^2*x^4) - (10*A*c*(b*x^2 + c*x^4)^(1/
2))/(63*x^8) - (B*b*(b*x^2 + c*x^4)^(1/2))/(7*x^8) - (8*B*c*(b*x^2 + c*x^4)
^(1/2))/(35*x^6) - (A*c^2*(b*x^2 + c*x^4)^(1/2))/(105*b*x^6) - (A*b*(b*x^2
+ c*x^4)^(1/2))/(9*x^10) - (8*A*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^3*x^2) -
(B*c^2*(b*x^2 + c*x^4)^(1/2))/(35*b*x^4) + (2*B*c^3*(b*x^2 + c*x^4)^(1/2))/
(35*b^2*x^2)
```

3.116 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15}} dx$

Optimal result

Integrand size = 26, antiderivative size = 133

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15}} dx = -\frac{A(bx^2+cx^4)^{5/2}}{11bx^{16}} - \frac{(11bB-6Ac)(bx^2+cx^4)^{5/2}}{99b^2x^{14}} + \frac{4c(11bB-6Ac)(bx^2+cx^4)^{5/2}}{693b^3x^{12}} - \frac{8c^2(11bB-6Ac)(bx^2+cx^4)^{5/2}}{3465b^4x^{10}}$$

```
[Out] -1/11*A*(c*x^4+b*x^2)^(5/2)/b/x^16-1/99*(-6*A*c+11*B*b)*(c*x^4+b*x^2)^(5/2)
/b^2/x^14+4/693*c*(-6*A*c+11*B*b)*(c*x^4+b*x^2)^(5/2)/b^3/x^12-8/3465*c^2*(
-6*A*c+11*B*b)*(c*x^4+b*x^2)^(5/2)/b^4/x^10
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2059, 806, 672, 664}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15}} dx = -\frac{8c^2(bx^2+cx^4)^{5/2}(11bB-6Ac)}{3465b^4x^{10}} + \frac{4c(bx^2+cx^4)^{5/2}(11bB-6Ac)}{693b^3x^{12}} - \frac{(bx^2+cx^4)^{5/2}(11bB-6Ac)}{99b^2x^{14}} - \frac{A(bx^2+cx^4)^{5/2}}{11bx^{16}}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^15,x]
```

```
[Out] -1/11*(A*(b*x^2 + c*x^4)^(5/2))/(b*x^16) - ((11*b*B - 6*A*c)*(b*x^2 + c*x^4
)^(5/2))/(99*b^2*x^14) + (4*c*(11*b*B - 6*A*c)*(b*x^2 + c*x^4)^(5/2))/(693*
b^3*x^12) - (8*c^2*(11*b*B - 6*A*c)*(b*x^2 + c*x^4)^(5/2))/(3465*b^4*x^10)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^8}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{11bx^{16}} + \frac{\left(-8(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^7}\,dx, x, x^2\right)}{11b}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{11bx^{16}} - \frac{(11bB-6Ac)\,(bx^2+cx^4)^{5/2}}{99b^2x^{14}} - \frac{(2c(11bB-6Ac))\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^6}\,dx, x, x^2\right)}{99b^2}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{11bx^{16}} - \frac{(11bB-6Ac)\,(bx^2+cx^4)^{5/2}}{99b^2x^{14}} + \frac{4c(11bB-6Ac)\,(bx^2+cx^4)^{5/2}}{693b^3x^{12}} + \frac{(4c^2(11bB-6Ac))\,\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^5}\,dx, x, x^2\right)}{693b^3}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{11bx^{16}} - \frac{(11bB-6Ac)\,(bx^2+cx^4)^{5/2}}{99b^2x^{14}} + \frac{4c(11bB-6Ac)\,(bx^2+cx^4)^{5/2}}{693b^3x^{12}} - \frac{8c^2(11bB-6Ac)\,(bx^2+cx^4)^{5/2}}{3465b^4x^{10}}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.67

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{15}}\,dx = -\frac{(x^2(b+cx^2))^{5/2}\,(11bBx^2(35b^2-20bcx^2+8c^2x^4)+3A(105b^3-70b^2cx^2+40bc^2x^4-16c^3x^6))}{3465b^4x^{16}}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^15,x]
```

```
[Out] -1/3465*((x^2*(b + c*x^2))^(5/2)*(11*b*B*x^2*(35*b^2 - 20*b*c*x^2 + 8*c^2*x
^4) + 3*A*(105*b^3 - 70*b^2*c*x^2 + 40*b*c^2*x^4 - 16*c^3*x^6)))/(b^4*x^16)
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$-\frac{\left(\left(\frac{11x^2B}{9}+A\right)b^3-\frac{2x^2\left(\frac{22x^2B}{21}+A\right)c\,b^2}{3}+\frac{8x^4\left(\frac{11x^2B}{15}+A\right)c^2b}{21}-\frac{16A\,c^3x^6}{105}\right)\sqrt{x^2(c\,x^2+b)}\,(c\,x^2+b)^2}{11x^{12}b^4}$
gosper	$-\frac{(c\,x^2+b)(-48A\,c^3x^6+88x^6Bb\,c^2+120Ab\,c^2x^4-220x^4B\,b^2c-210A\,b^2c\,x^2+385b^3B\,x^2+315b^3A)(x^4c+b\,x^2)^{\frac{3}{2}}}{3465b^4x^{14}}$
default	$-\frac{(c\,x^2+b)(-48A\,c^3x^6+88x^6Bb\,c^2+120Ab\,c^2x^4-220x^4B\,b^2c-210A\,b^2c\,x^2+385b^3B\,x^2+315b^3A)(x^4c+b\,x^2)^{\frac{3}{2}}}{3465b^4x^{14}}$
trager	$-\frac{(-48A\,x^{10}c^5+88Bb\,c^4x^{10}+24A\,x^8b\,c^4-44B\,b^2c^3x^8-18A\,b^2c^3x^6+33B\,b^3c^2x^6+15A\,b^3c^2x^4+550B\,b^4c\,x^4+420A\,b^4c\,x^2+}{3465b^4x^{12}}$
risch	$-\frac{\sqrt{x^2(c\,x^2+b)}\,(-48A\,x^{10}c^5+88Bb\,c^4x^{10}+24A\,x^8b\,c^4-44B\,b^2c^3x^8-18A\,b^2c^3x^6+33B\,b^3c^2x^6+15A\,b^3c^2x^4+550B\,b^4c\,x^4+}{3465x^{12}b^4}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^15,x,method=_RETURNVERBOSE)
```

```
[Out] -1/11*((11/9*x^2*B+A)*b^3-2/3*x^2*(22/21*x^2*B+A)*c*b^2+8/21*x^4*(11/15*x^2
*B+A)*c^2*b-16/105*A*c^3*x^6)*(x^2*(c*x^2+b))^(1/2)*(c*x^2+b)^2/x^12/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{15}}\,dx =$$
$$-\frac{(8\,(11\,Bbc^4-6\,Ac^5)x^{10}-4\,(11\,Bb^2c^3-6\,Abc^4)x^8+3\,(11\,Bb^3c^2-6\,Ab^2c^3)x^6+315\,Ab^5+5\,(110\,Bb^4c+}{3465\,b^4x^{12}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="fricas")
```

```
[Out] -1/3465*(8*(11*B*b*c^4 - 6*A*c^5)*x^10 - 4*(11*B*b^2*c^3 - 6*A*b*c^4)*x^8 +
 3*(11*B*b^3*c^2 - 6*A*b^2*c^3)*x^6 + 315*A*b^5 + 5*(110*B*b^4*c + 3*A*b^3*
c^2)*x^4 + 35*(11*B*b^5 + 12*A*b^4*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^4*x^12)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{15}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}\,(A+Bx^2)}{x^{15}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**15,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**15, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(117) = 234.

Time = 0.21 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.17

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15}}\,dx =$$
$$-\frac{1}{630}B\left(\frac{16\sqrt{cx^4+bx^2}c^4}{b^3x^2}-\frac{8\sqrt{cx^4+bx^2}c^3}{b^2x^4}+\frac{6\sqrt{cx^4+bx^2}c^2}{bx^6}-\frac{5\sqrt{cx^4+bx^2}c}{x^8}-\frac{35\sqrt{cx^4+bx^2}b}{x^{10}}+\frac{105\,(c\right.$$
$$+\frac{1}{9240}A\left(\frac{128\sqrt{cx^4+bx^2}c^5}{b^4x^2}-\frac{64\sqrt{cx^4+bx^2}c^4}{b^3x^4}+\frac{48\sqrt{cx^4+bx^2}c^3}{b^2x^6}-\frac{40\sqrt{cx^4+bx^2}c^2}{bx^8}+\frac{35\sqrt{cx^4+bx^2}c}{x^{10}}+\right.$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="maxima")
```

```
[Out] -1/630*B*(16*sqrt(c*x^4 + b*x^2)*c^4/(b^3*x^2) - 8*sqrt(c*x^4 + b*x^2)*c^3/
(b^2*x^4) + 6*sqrt(c*x^4 + b*x^2)*c^2/(b*x^6) - 5*sqrt(c*x^4 + b*x^2)*c/x^8
 - 35*sqrt(c*x^4 + b*x^2)*b/x^10 + 105*(c*x^4 + b*x^2)^(3/2)/x^12) + 1/9240
*A*(128*sqrt(c*x^4 + b*x^2)*c^5/(b^4*x^2) - 64*sqrt(c*x^4 + b*x^2)*c^4/(b^3
*x^4) + 48*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^6) - 40*sqrt(c*x^4 + b*x^2)*c^2/(
b*x^8) + 35*sqrt(c*x^4 + b*x^2)*c/x^10 + 315*sqrt(c*x^4 + b*x^2)*b/x^12 - 1
155*(c*x^4 + b*x^2)^(3/2)/x^14)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. 2(117) = 234.

Time = 1.76 (sec) , antiderivative size = 490, normalized size of antiderivative = 3.68

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15}}\,dx = \frac{16\left(2310\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{16}Bc^{\frac{9}{2}}\operatorname{sgn}(x)-1155\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{14}Bbc^{\frac{9}{2}}\right.}{}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^15,x, algorithm="giac")
```

```
[Out] 16/3465*(2310*(sqrt(c)*x - sqrt(c*x^2 + b))^16*B*c^(9/2)*sgn(x) - 1155*(sqr
t(c)*x - sqrt(c*x^2 + b))^14*B*b*c^(9/2)*sgn(x) + 6930*(sqrt(c)*x - sqrt(c*
x^2 + b))^14*A*c^(11/2)*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*b^2
*c^(9/2)*sgn(x) + 12474*(sqrt(c)*x - sqrt(c*x^2 + b))^12*A*b*c^(11/2)*sgn(x
) - 4851*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*b^3*c^(9/2)*sgn(x) + 15246*(sqr
t(c)*x - sqrt(c*x^2 + b))^10*A*b^2*c^(11/2)*sgn(x) + 2475*(sqrt(c)*x - sqrt
(c*x^2 + b))^8*B*b^4*c^(9/2)*sgn(x) + 4950*(sqrt(c)*x - sqrt(c*x^2 + b))^8*
A*b^3*c^(11/2)*sgn(x) + 495*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^5*c^(9/2)*s
gn(x) + 990*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b^4*c^(11/2)*sgn(x) + 605*(sq
```

```
rt(c)*x - sqrt(c*x^2 + b))^4*B*b^6*c^(9/2)*sgn(x) - 330*(sqrt(c)*x - sqrt(c
*x^2 + b))^4*A*b^5*c^(11/2)*sgn(x) - 121*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*
b^7*c^(9/2)*sgn(x) + 66*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^6*c^(11/2)*sgn(
x) + 11*B*b^8*c^(9/2)*sgn(x) - 6*A*b^7*c^(11/2)*sgn(x))/((sqrt(c)*x - sqrt(
c*x^2 + b))^2 - b)^11
```

Mupad [B] (verification not implemented)

Time = 10.82 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.92

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{15}}\,dx = \frac{2\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{385\,b^2\,x^6} - \frac{4\,A\,c\,\sqrt{c\,x^4+b\,x^2}}{33\,x^{10}} - \frac{B\,b\,\sqrt{c\,x^4+b\,x^2}}{9\,x^{10}} - \frac{10\,B\,c\,\sqrt{c\,x^4+b\,x^2}}{63\,x^8} - \frac{A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{231\,b\,x^8} - \frac{A\,b\,\sqrt{c\,x^4+b\,x^2}}{11\,x^{12}} - \frac{8\,A\,c^4\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^3\,x^4} + \frac{16\,A\,c^5\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^4\,x^2} - \frac{B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{105\,b\,x^6} + \frac{4\,B\,c^3\,\sqrt{c\,x^4+b\,x^2}}{315\,b^2\,x^4} - \frac{8\,B\,c^4\,\sqrt{c\,x^4+b\,x^2}}{315\,b^3\,x^2}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^15,x)
```

```
[Out] (2*A*c^3*(b*x^2 + c*x^4)^(1/2))/(385*b^2*x^6) - (4*A*c*(b*x^2 + c*x^4)^(1/2
))/(33*x^10) - (B*b*(b*x^2 + c*x^4)^(1/2))/(9*x^10) - (10*B*c*(b*x^2 + c*x^
4)^(1/2))/(63*x^8) - (A*c^2*(b*x^2 + c*x^4)^(1/2))/(231*b*x^8) - (A*b*(b*x^
2 + c*x^4)^(1/2))/(11*x^12) - (8*A*c^4*(b*x^2 + c*x^4)^(1/2))/(1155*b^3*x^4
) + (16*A*c^5*(b*x^2 + c*x^4)^(1/2))/(1155*b^4*x^2) - (B*c^2*(b*x^2 + c*x^4
)^(1/2))/(105*b*x^6) + (4*B*c^3*(b*x^2 + c*x^4)^(1/2))/(315*b^2*x^4) - (8*B
*c^4*(b*x^2 + c*x^4)^(1/2))/(315*b^3*x^2)
```

3.117 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17}} dx$

Optimal result

Integrand size = 26, antiderivative size = 170

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17}} dx = -\frac{A(bx^2+cx^4)^{5/2}}{13bx^{18}} - \frac{(13bB-8Ac)(bx^2+cx^4)^{5/2}}{143b^2x^{16}} + \frac{2c(13bB-8Ac)(bx^2+cx^4)^{5/2}}{429b^3x^{14}} - \frac{8c^2(13bB-8Ac)(bx^2+cx^4)^{5/2}}{3003b^4x^{12}} + \frac{16c^3(13bB-8Ac)(bx^2+cx^4)^{5/2}}{15015b^5x^{10}}$$

```
[Out] -1/13*A*(c*x^4+b*x^2)^(5/2)/b/x^18-1/143*(-8*A*c+13*B*b)*(c*x^4+b*x^2)^(5/2
)/b^2/x^16+2/429*c*(-8*A*c+13*B*b)*(c*x^4+b*x^2)^(5/2)/b^3/x^14-8/3003*c^2*
(-8*A*c+13*B*b)*(c*x^4+b*x^2)^(5/2)/b^4/x^12+16/15015*c^3*(-8*A*c+13*B*b)*(
c*x^4+b*x^2)^(5/2)/b^5/x^10
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2059, 806, 672, 664}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17}} dx = \frac{16c^3(bx^2+cx^4)^{5/2}(13bB-8Ac)}{15015b^5x^{10}} - \frac{8c^2(bx^2+cx^4)^{5/2}(13bB-8Ac)}{3003b^4x^{12}} + \frac{2c(bx^2+cx^4)^{5/2}(13bB-8Ac)}{429b^3x^{14}} - \frac{(bx^2+cx^4)^{5/2}(13bB-8Ac)}{143b^2x^{16}} - \frac{A(bx^2+cx^4)^{5/2}}{13bx^{18}}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^17,x]
```

```
[Out] -1/13*(A*(b*x^2 + c*x^4)^(5/2))/(b*x^18) - ((13*b*B - 8*A*c)*(b*x^2 + c*x^4
)^(5/2))/(143*b^2*x^16) + (2*c*(13*b*B - 8*A*c)*(b*x^2 + c*x^4)^(5/2))/(429
*b^3*x^14) - (8*c^2*(13*b*B - 8*A*c)*(b*x^2 + c*x^4)^(5/2))/(3003*b^4*x^12)
 + (16*c^3*(13*b*B - 8*A*c)*(b*x^2 + c*x^4)^(5/2))/(15015*b^5*x^10)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^9}\,dx,x,x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{13bx^{18}} + \frac{\left(-9(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^8}\,dx,x,x^2\right)}{13b}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{13bx^{18}} - \frac{(13bB-8Ac)\,(bx^2+cx^4)^{5/2}}{143b^2x^{16}} - \frac{(3c(13bB-8Ac))\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^7}\,dx,x,x^2\right)}{143b^2}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{13bx^{18}} - \frac{(13bB-8Ac)\,(bx^2+cx^4)^{5/2}}{143b^2x^{16}} + \frac{2c(13bB-8Ac)\,(bx^2+cx^4)^{5/2}}{429b^3x^{14}} + \frac{(4c^2(13bB-8Ac))\,\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^6}\,dx,x,x^2\right)}{429b^3}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{13bx^{18}} - \frac{(13bB-8Ac)\,(bx^2+cx^4)^{5/2}}{143b^2x^{16}} + \frac{2c(13bB-8Ac)\,(bx^2+cx^4)^{5/2}}{429b^3x^{14}} - \frac{8c^2(13bB-8Ac)\,(bx^2+cx^4)^{5/2}}{3003b^4x^{12}} - \frac{(8c^3(13bB-8Ac))\,\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^5}\,dx,x,x^2\right)}{3003b^4}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{13bx^{18}} - \frac{(13bB-8Ac)\,(bx^2+cx^4)^{5/2}}{143b^2x^{16}} + \frac{2c(13bB-8Ac)\,(bx^2+cx^4)^{5/2}}{429b^3x^{14}} - \frac{8c^2(13bB-8Ac)\,(bx^2+cx^4)^{5/2}}{3003b^4x^{12}} + \frac{16c^3(13bB-8Ac)\,(bx^2+cx^4)^{5/2}}{15015b^5x^{10}}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.65

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{17}}\,dx = \frac{(x^2(b+cx^2))^{5/2}\,(13bBx^2(-105b^3+70b^2cx^2-40bc^2x^4+16c^3x^6)+A(-11\ldots}{15015b^5x^{18}}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^17,x]
```

```
[Out] ((x^2*(b + c*x^2))^(5/2)*(13*b*B*x^2*(-105*b^3 + 70*b^2*c*x^2 - 40*b*c^2*x^
4 + 16*c^3*x^6) + A*(-1155*b^4 + 840*b^3*c*x^2 - 560*b^2*c^2*x^4 + 320*b*c^
3*x^6 - 128*c^4*x^8)))/(15015*b^5*x^18)
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$-\frac{\sqrt{x^2(cx^2+b)}\left(\left(\frac{13x^2B}{11}+A\right)b^4-\frac{8x^2\left(\frac{13x^2B}{12}+A\right)cb^3}{11}+\frac{16x^4\left(\frac{13x^2B}{14}+A\right)c^2b^2}{33}-\frac{64x^6\left(\frac{13x^2B}{20}+A\right)c^3b}{231}+\frac{128Ax^8c^4}{1155}\right)(cx^2+b}{13x^{14}b^5}$
gosper	$-\frac{(cx^2+b)(128Ax^8c^4-208Bx^8bc^3-320Ax^6bc^3+520Bx^6b^2c^2+560Ab^2c^2x^4-910Bb^3cx^4-840Ax^2b^3c+1365Bx^2b^4+11}{15015b^5x^{16}}$
default	$-\frac{(cx^2+b)(128Ax^8c^4-208Bx^8bc^3-320Ax^6bc^3+520Bx^6b^2c^2+560Ab^2c^2x^4-910Bb^3cx^4-840Ax^2b^3c+1365Bx^2b^4+11}{15015b^5x^{16}}$
trager	$-\frac{(128Ac^6x^{12}-208Bbc^5x^{12}-64Abc^5x^{10}+104Bb^2c^4x^{10}+48Ab^2c^4x^8-78Bb^3c^3x^8-40Ab^3c^3x^6+65Bb^4c^2x^6+35Ab^4c^2x^4}{15015b^5x^{14}}$
risch	$-\frac{\sqrt{x^2(cx^2+b)}(128Ac^6x^{12}-208Bbc^5x^{12}-64Abc^5x^{10}+104Bb^2c^4x^{10}+48Ab^2c^4x^8-78Bb^3c^3x^8-40Ab^3c^3x^6+65Bb^4c^2x}{15015x^{14}b^5}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^17,x,method=_RETURNVERBOSE)
```

```
[Out] -1/13*(x^2*(c*x^2+b))^(1/2)*((13/11*x^2*B+A)*b^4-8/11*x^2*(13/12*x^2*B+A)*c
*b^3+16/33*x^4*(13/14*x^2*B+A)*c^2*b^2-64/231*x^6*(13/20*x^2*B+A)*c^3*b+128
/1155*A*x^8*c^4)*(c*x^2+b)^2/x^14/b^5
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.92

$$\int\frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17}}\,dx=\frac{(16(13Bbc^5-8Ac^6)x^{12}-8(13Bb^2c^4-8Abc^5)x^{10}+6(13Bb^3c^3-8Ab}{}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="fricas")
```

```
[Out] 1/15015*(16*(13*B*b*c^5 - 8*A*c^6)*x^12 - 8*(13*B*b^2*c^4 - 8*A*b*c^5)*x^10
 + 6*(13*B*b^3*c^3 - 8*A*b^2*c^4)*x^8 - 1155*A*b^6 - 5*(13*B*b^4*c^2 - 8*A*
b^3*c^3)*x^6 - 35*(52*B*b^5*c + A*b^4*c^2)*x^4 - 105*(13*B*b^6 + 14*A*b^5*c
)*x^2)*sqrt(c*x^4 + b*x^2)/(b^5*x^14)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}(A+Bx^2)}{x^{17}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**17,x)
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**17, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(150) = 300.

Time = 0.22 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.98

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17}}\,dx = \frac{1}{9240}B\left(\frac{128\sqrt{cx^4+bx^2}c^5}{b^4x^2} - \frac{64\sqrt{cx^4+bx^2}c^4}{b^3x^4} + \frac{48\sqrt{cx^4+bx^2}c^3}{b^2x^6} - \frac{40\sqrt{c}}{}\right.$$

$$-\frac{1}{30030}A\left(\frac{256\sqrt{cx^4+bx^2}c^6}{b^5x^2} - \frac{128\sqrt{cx^4+bx^2}c^5}{b^4x^4} + \frac{96\sqrt{cx^4+bx^2}c^4}{b^3x^6} - \frac{80\sqrt{cx^4+bx^2}c^3}{b^2x^8} + \frac{70\sqrt{cx^4+bx^2}c}{bx^{10}}\right.$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="maxima")
[Out] 1/9240*B*(128*sqrt(c*x^4 + b*x^2)*c^5/(b^4*x^2) - 64*sqrt(c*x^4 + b*x^2)*c^
4/(b^3*x^4) + 48*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^6) - 40*sqrt(c*x^4 + b*x^2)
*c^2/(b*x^8) + 35*sqrt(c*x^4 + b*x^2)*c/x^10 + 315*sqrt(c*x^4 + b*x^2)*b/x^
12 - 1155*(c*x^4 + b*x^2)^(3/2)/x^14) - 1/30030*A*(256*sqrt(c*x^4 + b*x^2)*
c^6/(b^5*x^2) - 128*sqrt(c*x^4 + b*x^2)*c^5/(b^4*x^4) + 96*sqrt(c*x^4 + b*x
^2)*c^4/(b^3*x^6) - 80*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^8) + 70*sqrt(c*x^4 +
b*x^2)*c^2/(b*x^10) - 63*sqrt(c*x^4 + b*x^2)*c/x^12 - 693*sqrt(c*x^4 + b*x^
2)*b/x^14 + 3003*(c*x^4 + b*x^2)^(3/2)/x^16)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(150) = 300.

Time = 2.16 (sec) , antiderivative size = 550, normalized size of antiderivative = 3.24

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17}}\,dx = \frac{32\left(15015\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{18}Bc^{\frac{11}{2}}\operatorname{sgn}(x) - 3003\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{16}Bbc\right.}{}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^17,x, algorithm="giac")
```

```
[Out] 32/15015*(15015*(sqrt(c)*x - sqrt(c*x^2 + b))^18*B*c^(11/2)*sgn(x) - 3003*(
sqrt(c)*x - sqrt(c*x^2 + b))^16*B*b*c^(11/2)*sgn(x) + 48048*(sqrt(c)*x - sq
rt(c*x^2 + b))^16*A*c^(13/2)*sgn(x) - 6006*(sqrt(c)*x - sqrt(c*x^2 + b))^14
*B*b^2*c^(11/2)*sgn(x) + 96096*(sqrt(c)*x - sqrt(c*x^2 + b))^14*A*b*c^(13/2
)*sgn(x) - 28314*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*b^3*c^(11/2)*sgn(x) + 1
09824*(sqrt(c)*x - sqrt(c*x^2 + b))^12*A*b^2*c^(13/2)*sgn(x) + 13728*(sqrt(
c)*x - sqrt(c*x^2 + b))^10*B*b^4*c^(11/2)*sgn(x) + 37752*(sqrt(c)*x - sqrt(
c*x^2 + b))^10*A*b^3*c^(13/2)*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2 + b))^8
*B*b^5*c^(11/2)*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*b^4*c^(13/2
)*sgn(x) + 3718*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^6*c^(11/2)*sgn(x) - 228
8*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*b^5*c^(13/2)*sgn(x) - 1014*(sqrt(c)*x -
 sqrt(c*x^2 + b))^4*B*b^7*c^(11/2)*sgn(x) + 624*(sqrt(c)*x - sqrt(c*x^2 + b
))^4*A*b^6*c^(13/2)*sgn(x) + 169*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^8*c^(1
1/2)*sgn(x) - 104*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^7*c^(13/2)*sgn(x) - 1
3*B*b^9*c^(11/2)*sgn(x) + 8*A*b^8*c^(13/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2
 + b))^2 - b)^13
```

Mupad [B] (verification not implemented)

Time = 11.41 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.80

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17}}\,dx = \frac{8\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{3003\,b^2\,x^8}-\frac{14\,A\,c\,\sqrt{c\,x^4+b\,x^2}}{143\,x^{12}} - \frac{B\,b\,\sqrt{c\,x^4+b\,x^2}}{11\,x^{12}}-\frac{4\,B\,c\,\sqrt{c\,x^4+b\,x^2}}{33\,x^{10}}-\frac{A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{429\,b\,x^{10}}-\frac{A\,b\,\sqrt{c\,x^4+b\,x^2}}{13\,x^{14}} - \frac{16\,A\,c^4\,\sqrt{c\,x^4+b\,x^2}}{5005\,b^3\,x^6}+\frac{64\,A\,c^5\,\sqrt{c\,x^4+b\,x^2}}{15015\,b^4\,x^4}-\frac{128\,A\,c^6\,\sqrt{c\,x^4+b\,x^2}}{15015\,b^5\,x^2} - \frac{B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{231\,b\,x^8}+\frac{2\,B\,c^3\,\sqrt{c\,x^4+b\,x^2}}{385\,b^2\,x^6}-\frac{8\,B\,c^4\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^3\,x^4}+\frac{16\,B\,c^5\,\sqrt{c\,x^4+b\,x^2}}{1155\,b^4\,x^2}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^17,x)
```

```
[Out] (8*A*c^3*(b*x^2 + c*x^4)^(1/2))/(3003*b^2*x^8) - (14*A*c*(b*x^2 + c*x^4)^(1
/2))/(143*x^12) - (B*b*(b*x^2 + c*x^4)^(1/2))/(11*x^12) - (4*B*c*(b*x^2 + c
*x^4)^(1/2))/(33*x^10) - (A*c^2*(b*x^2 + c*x^4)^(1/2))/(429*b*x^10) - (A*b*
(b*x^2 + c*x^4)^(1/2))/(13*x^14) - (16*A*c^4*(b*x^2 + c*x^4)^(1/2))/(5005*b
^3*x^6) + (64*A*c^5*(b*x^2 + c*x^4)^(1/2))/(15015*b^4*x^4) - (128*A*c^6*(b*
x^2 + c*x^4)^(1/2))/(15015*b^5*x^2) - (B*c^2*(b*x^2 + c*x^4)^(1/2))/(231*b*
x^8) + (2*B*c^3*(b*x^2 + c*x^4)^(1/2))/(385*b^2*x^6) - (8*B*c^4*(b*x^2 + c*
x^4)^(1/2))/(1155*b^3*x^4) + (16*B*c^5*(b*x^2 + c*x^4)^(1/2))/(1155*b^4*x^2
)
```

3.118 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{19}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 207

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{19}} \, dx = -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{5/2}}{39b^2x^{18}} + \frac{8c(3bB-2Ac)(bx^2+cx^4)^{5/2}}{429b^3x^{16}} - \frac{16c^2(3bB-2Ac)(bx^2+cx^4)^{5/2}}{1287b^4x^{14}} + \frac{64c^3(3bB-2Ac)(bx^2+cx^4)^{5/2}}{9009b^5x^{12}} - \frac{128c^4(3bB-2Ac)(bx^2+cx^4)^{5/2}}{45045b^6x^{10}}$$

```
[Out] -1/15*A*(c*x^4+b*x^2)^(5/2)/b/x^20-1/39*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(5/2)/
b^2/x^18+8/429*c*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(5/2)/b^3/x^16-16/1287*c^2*(-
2*A*c+3*B*b)*(c*x^4+b*x^2)^(5/2)/b^4/x^14+64/9009*c^3*(-2*A*c+3*B*b)*(c*x^4
+b*x^2)^(5/2)/b^5/x^12-128/45045*c^4*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(5/2)/b^6
/x^10
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2059, 806, 672, 664}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{19}} \, dx = -\frac{128c^4(bx^2+cx^4)^{5/2}(3bB-2Ac)}{45045b^6x^{10}} + \frac{64c^3(bx^2+cx^4)^{5/2}(3bB-2Ac)}{9009b^5x^{12}} - \frac{16c^2(bx^2+cx^4)^{5/2}(3bB-2Ac)}{1287b^4x^{14}} + \frac{8c(bx^2+cx^4)^{5/2}(3bB-2Ac)}{429b^3x^{16}} - \frac{(bx^2+cx^4)^{5/2}(3bB-2Ac)}{39b^2x^{18}} - \frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^19,x]
```

```
[Out] -1/15*(A*(b*x^2 + c*x^4)^(5/2))/(b*x^20) - ((3*b*B - 2*A*c)*(b*x^2 + c*x^4)
^(5/2))/(39*b^2*x^18) + (8*c*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(5/2))/(429*b^
3*x^16) - (16*c^2*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(5/2))/(1287*b^4*x^14) +
(64*c^3*(3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(5/2))/(9009*b^5*x^12) - (128*c^4*(
3*b*B - 2*A*c)*(b*x^2 + c*x^4)^(5/2))/(45045*b^6*x^10)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\,(bx+cx^2)^{3/2}}{x^{10}}\,dx, x, x^2\right)$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} + \frac{\left(-10(-bB+Ac)+\frac{5}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^9}\,dx, x, x^2\right)}{15b}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{5/2}}{39b^2x^{18}} - \frac{(4c(3bB-2Ac))\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^8}\,dx, x, x^2\right)}{39b^2}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{5/2}}{39b^2x^{18}} + \frac{8c(3bB-2Ac)(bx^2+cx^4)^{5/2}}{429b^3x^{16}} + \frac{(8c^2(3bB-2Ac))\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^7}\,dx, x, x^2\right)}{143b^3}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{5/2}}{39b^2x^{18}} + \frac{8c(3bB-2Ac)(bx^2+cx^4)^{5/2}}{429b^3x^{16}} - \frac{16c^2(3bB-2Ac)(bx^2+cx^4)^{5/2}}{1287b^4x^{14}} - \frac{(32c^3(3bB-2Ac))\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^6}\,dx, x, x^2\right)}{1287b^4}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{5/2}}{39b^2x^{18}} + \frac{8c(3bB-2Ac)(bx^2+cx^4)^{5/2}}{429b^3x^{16}} - \frac{16c^2(3bB-2Ac)(bx^2+cx^4)^{5/2}}{1287b^4x^{14}} + \frac{64c^3(3bB-2Ac)(bx^2+cx^4)^{5/2}}{9009b^5x^{12}} + \frac{(64c^4(3bB-2Ac))\text{Subst}\left(\int \frac{(bx+cx^2)^{3/2}}{x^5}\,dx, x, x^2\right)}{9009b^5}$$

$$= -\frac{A(bx^2+cx^4)^{5/2}}{15bx^{20}} - \frac{(3bB-2Ac)(bx^2+cx^4)^{5/2}}{39b^2x^{18}} + \frac{8c(3bB-2Ac)(bx^2+cx^4)^{5/2}}{429b^3x^{16}} - \frac{16c^2(3bB-2Ac)(bx^2+cx^4)^{5/2}}{1287b^4x^{14}} + \frac{64c^3(3bB-2Ac)(bx^2+cx^4)^{5/2}}{9009b^5x^{12}} - \frac{128c^4(3bB-2Ac)(bx^2+cx^4)^{5/2}}{45045b^6x^{10}}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.64

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{19}}\,dx = -\frac{(x^2(b+cx^2))^{5/2}\,(3bBx^2(1155b^4-840b^3cx^2+560b^2c^2x^4-320bc^3x^6+128c^4x^8)+A(3003b^5-2310b^4cx^2 \cdots}{45045b^6x^{20}}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^19,x]
```

```
[Out] -1/45045*((x^2*(b + c*x^2))^(5/2)*(3*b*B*x^2*(1155*b^4 - 840*b^3*c*x^2 + 56
0*b^2*c^2*x^4 - 320*b*c^3*x^6 + 128*c^4*x^8) + A*(3003*b^5 - 2310*b^4*c*x^2
 + 1680*b^3*c^2*x^4 - 1120*b^2*c^3*x^6 + 640*b*c^4*x^8 - 256*c^5*x^10)))/(b
^6*x^20)
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.60

method	result
pseudoelliptic	$-\frac{\sqrt{x^2(cx^2+b)}\,(cx^2+b)^2\left(\left(\frac{15x^2B}{13}+A\right)b^5-\frac{10x^2c\left(\frac{12x^2B}{11}+A\right)b^4}{13}+\frac{80c^2x^4\left(x^2B+A\right)b^3}{143}-\frac{160x^6\left(\frac{6x^2B}{7}+A\right)c^3b^2}{429}+\frac{640x^8c^4\left(\frac{3x \cdots}{300 \cdots}\right.\right.}{15x^{16}b^6}$
gosper	$-\frac{(cx^2+b)\left(-256Ac^5x^{10}+384Bbc^4x^{10}+640Ax^8bc^4-960Bb^2c^3x^8-1120Ab^2c^3x^6+1680Bb^3c^2x^6+1680Ab^3c^2x^4-2520B \cdots\right.}{45045x^{18}b^6}$
default	$-\frac{(cx^2+b)\left(-256Ac^5x^{10}+384Bbc^4x^{10}+640Ax^8bc^4-960Bb^2c^3x^8-1120Ab^2c^3x^6+1680Bb^3c^2x^6+1680Ab^3c^2x^4-2520B \cdots\right.}{45045x^{18}b^6}$
trager	$-\frac{\left(-256Ac^7x^{14}+384Bbc^6x^{14}+128Abc^6x^{12}-192Bb^2c^5x^{12}-96Ab^2c^5x^{10}+144Bb^3c^4x^{10}+80Ab^3c^4x^8-120Bb^4c^3x^8-70A \cdots\right.}{45045b^6x^{16}}$
risch	$-\frac{\sqrt{x^2(cx^2+b)}\left(-256Ac^7x^{14}+384Bbc^6x^{14}+128Abc^6x^{12}-192Bb^2c^5x^{12}-96Ab^2c^5x^{10}+144Bb^3c^4x^{10}+80Ab^3c^4x^8-120 \cdots\right.}{45045x^{16}b^6}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^19,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(x^2*(c*x^2+b))^(1/2)*(c*x^2+b)^2*((15/13*x^2*B+A)*b^5-10/13*x^2*c*(1
2/11*x^2*B+A)*b^4+80/143*c^2*x^4*(B*x^2+A)*b^3-160/429*x^6*(6/7*x^2*B+A)*c^
3*b^2+640/3003*x^8*c^4*(3/5*x^2*B+A)*b-256/3003*A*c^5*x^10)/x^16/b^6
```

Fricas [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{19}}\,dx = -\frac{(128\,(3\,Bbc^6-2\,Ac^7)x^{14}-64\,(3\,Bb^2c^5-2\,Abc^6)x^{12}+48\,(3\,Bb^3c^4-2\,Ab^2c^5)x^{10}-40\,(3\,Bb^4c^3-2\,Ab^3c^4)x}{45045}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^19,x, algorithm="fricas")
```

```
[Out] -1/45045*(128*(3*B*b*c^6 - 2*A*c^7)*x^14 - 64*(3*B*b^2*c^5 - 2*A*b*c^6)*x^1
2 + 48*(3*B*b^3*c^4 - 2*A*b^2*c^5)*x^10 - 40*(3*B*b^4*c^3 - 2*A*b^3*c^4)*x^
8 + 3003*A*b^7 + 35*(3*B*b^5*c^2 - 2*A*b^4*c^3)*x^6 + 63*(70*B*b^6*c + A*b^
5*c^2)*x^4 + 231*(15*B*b^7 + 16*A*b^6*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^6*x^16
)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{19}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}(A+Bx^2)}{x^{19}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**19,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**19, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(183) = 366.

Time = 0.22 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.86

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{19}}\,dx = -\frac{1}{30030}B\left(\frac{256\sqrt{cx^4+bx^2}c^6}{b^5x^2}-\frac{128\sqrt{cx^4+bx^2}c^5}{b^4x^4}+\frac{96\sqrt{cx^4+bx^2}c^4}{b^3x^6}-\frac{80\sqrt{cx^4+bx^2}c^3}{b^2x^8}+\frac{70\sqrt{cx^4+bx^2}c}{bx^{10}}\right. + \frac{1}{180180}A\left(\frac{1024\sqrt{cx^4+bx^2}c^7}{b^6x^2}-\frac{512\sqrt{cx^4+bx^2}c^6}{b^5x^4}+\frac{384\sqrt{cx^4+bx^2}c^5}{b^4x^6}-\frac{320\sqrt{cx^4+bx^2}c^4}{b^3x^8}+\frac{280\sqrt{cx^4+}}{b^2x^{10}}\right.$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^19,x, algorithm="maxima")
```

```
[Out] -1/30030*B*(256*sqrt(c*x^4 + b*x^2)*c^6/(b^5*x^2) - 128*sqrt(c*x^4 + b*x^2)
*c^5/(b^4*x^4) + 96*sqrt(c*x^4 + b*x^2)*c^4/(b^3*x^6) - 80*sqrt(c*x^4 + b*x
^2)*c^3/(b^2*x^8) + 70*sqrt(c*x^4 + b*x^2)*c^2/(b*x^10) - 63*sqrt(c*x^4 + b
*x^2)*c/x^12 - 693*sqrt(c*x^4 + b*x^2)*b/x^14 + 3003*(c*x^4 + b*x^2)^(3/2)/
x^16) + 1/180180*A*(1024*sqrt(c*x^4 + b*x^2)*c^7/(b^6*x^2) - 512*sqrt(c*x^4
 + b*x^2)*c^6/(b^5*x^4) + 384*sqrt(c*x^4 + b*x^2)*c^5/(b^4*x^6) - 320*sqrt(
c*x^4 + b*x^2)*c^4/(b^3*x^8) + 280*sqrt(c*x^4 + b*x^2)*c^3/(b^2*x^10) - 252
*sqrt(c*x^4 + b*x^2)*c^2/(b*x^12) + 231*sqrt(c*x^4 + b*x^2)*c/x^14 + 3003*s
qrt(c*x^4 + b*x^2)*b/x^16 - 15015*(c*x^4 + b*x^2)^(3/2)/x^18)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. $2(183) = 366$.

Time = 2.31 (sec) , antiderivative size = 582, normalized size of antiderivative = 2.81

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{19}}\,dx = \frac{256\left(18018\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{20}Bc^{\frac{13}{2}}\operatorname{sgn}(x)+60060\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{18}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^19,x, algorithm="giac")
```

```
[Out] 256/45045*(18018*(sqrt(c)*x - sqrt(c*x^2 + b))^20*B*c^(13/2)*sgn(x) + 60060
*(sqrt(c)*x - sqrt(c*x^2 + b))^18*A*c^(15/2)*sgn(x) - 12870*(sqrt(c)*x - sq
rt(c*x^2 + b))^16*B*b^2*c^(13/2)*sgn(x) + 128700*(sqrt(c)*x - sqrt(c*x^2 +
b))^16*A*b*c^(15/2)*sgn(x) - 32175*(sqrt(c)*x - sqrt(c*x^2 + b))^14*B*b^3*c
^(13/2)*sgn(x) + 141570*(sqrt(c)*x - sqrt(c*x^2 + b))^14*A*b^2*c^(15/2)*sgn
(x) + 15015*(sqrt(c)*x - sqrt(c*x^2 + b))^12*B*b^4*c^(13/2)*sgn(x) + 50050*
(sqrt(c)*x - sqrt(c*x^2 + b))^12*A*b^3*c^(15/2)*sgn(x) + 9009*(sqrt(c)*x -
sqrt(c*x^2 + b))^10*B*b^5*c^(13/2)*sgn(x) + 6006*(sqrt(c)*x - sqrt(c*x^2 +
b))^10*A*b^4*c^(15/2)*sgn(x) + 4095*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b^6*c
^(13/2)*sgn(x) - 2730*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*b^5*c^(15/2)*sgn(x)
 - 1365*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^7*c^(13/2)*sgn(x) + 910*(sqrt(c
)*x - sqrt(c*x^2 + b))^6*A*b^6*c^(15/2)*sgn(x) + 315*(sqrt(c)*x - sqrt(c*x^
2 + b))^4*B*b^8*c^(13/2)*sgn(x) - 210*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^7
*c^(15/2)*sgn(x) - 45*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^9*c^(13/2)*sgn(x)
 + 30*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^8*c^(15/2)*sgn(x) + 3*B*b^10*c^(1
3/2)*sgn(x) - 2*A*b^9*c^(15/2)*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b
)^15
```

Mupad [B] (verification not implemented)

Time = 12.10 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.72

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{19}}\,dx = \frac{2\,A\,c^3\,\sqrt{c\,x^4+b\,x^2}}{1287\,b^2\,x^{10}} - \frac{16\,A\,c\,\sqrt{c\,x^4+b\,x^2}}{195\,x^{14}} - \frac{B\,b\,\sqrt{c\,x^4+b\,x^2}}{13\,x^{14}} - \frac{14\,B\,c\,\sqrt{c\,x^4+b\,x^2}}{143\,x^{12}} - \frac{A\,c^2\,\sqrt{c\,x^4+b\,x^2}}{715\,b\,x^{12}} - \frac{A\,b\,\sqrt{c\,x^4+b\,x^2}}{15\,x^{16}} - \frac{16\,A\,c^4\,\sqrt{c\,x^4+b\,x^2}}{9009\,b^3\,x^8} + \frac{32\,A\,c^5\,\sqrt{c\,x^4+b\,x^2}}{15015\,b^4\,x^6} - \frac{128\,A\,c^6\,\sqrt{c\,x^4+b\,x^2}}{45045\,b^5\,x^4} + \frac{256\,A\,c^7\,\sqrt{c\,x^4+b\,x^2}}{45045\,b^6\,x^2} - \frac{B\,c^2\,\sqrt{c\,x^4+b\,x^2}}{429\,b\,x^{10}} + \frac{8\,B\,c^3\,\sqrt{c\,x^4+b\,x^2}}{3003\,b^2\,x^8} - \frac{16\,B\,c^4\,\sqrt{c\,x^4+b\,x^2}}{5005\,b^3\,x^6} + \frac{64\,B\,c^5\,\sqrt{c\,x^4+b\,x^2}}{15015\,b^4\,x^4} - \frac{128\,B\,c^6\,\sqrt{c\,x^4+b\,x^2}}{15015\,b^5\,x^2}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^19,x)
```

```
[Out] (2*A*c^3*(b*x^2 + c*x^4)^(1/2))/(1287*b^2*x^10) - (16*A*c*(b*x^2 + c*x^4)^(
1/2))/(195*x^14) - (B*b*(b*x^2 + c*x^4)^(1/2))/(13*x^14) - (14*B*c*(b*x^2 +
 c*x^4)^(1/2))/(143*x^12) - (A*c^2*(b*x^2 + c*x^4)^(1/2))/(715*b*x^12) - (A
*b*(b*x^2 + c*x^4)^(1/2))/(15*x^16) - (16*A*c^4*(b*x^2 + c*x^4)^(1/2))/(900
9*b^3*x^8) + (32*A*c^5*(b*x^2 + c*x^4)^(1/2))/(15015*b^4*x^6) - (128*A*c^6*
(b*x^2 + c*x^4)^(1/2))/(45045*b^5*x^4) + (256*A*c^7*(b*x^2 + c*x^4)^(1/2))/
(45045*b^6*x^2) - (B*c^2*(b*x^2 + c*x^4)^(1/2))/(429*b*x^10) + (8*B*c^3*(b*
x^2 + c*x^4)^(1/2))/(3003*b^2*x^8) - (16*B*c^4*(b*x^2 + c*x^4)^(1/2))/(5005
*b^3*x^6) + (64*B*c^5*(b*x^2 + c*x^4)^(1/2))/(15015*b^4*x^4) - (128*B*c^6*(
b*x^2 + c*x^4)^(1/2))/(15015*b^5*x^2)
```

3.119 $\int x^4(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 168

$$\int x^4\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{16b^3(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{15015c^5x^5}-\frac{8b^2(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3}+\frac{2b(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{429c^3x}-\frac{(8bB-13Ac)x(bx^2+cx^4)^{5/2}}{143c^2}+\frac{Bx^3(bx^2+cx^4)^{5/2}}{13c}$$

```
[Out] 16/15015*b^3*(-13*A*c+8*B*b)*(c*x^4+b*x^2)^(5/2)/c^5/x^5-8/3003*b^2*(-13*A*
c+8*B*b)*(c*x^4+b*x^2)^(5/2)/c^4/x^3+2/429*b*(-13*A*c+8*B*b)*(c*x^4+b*x^2)^
(5/2)/c^3/x-1/143*(-13*A*c+8*B*b)*x*(c*x^4+b*x^2)^(5/2)/c^2+1/13*B*x^3*(c*x
^4+b*x^2)^(5/2)/c
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2064, 2041, 2027, 2039}

$$\int x^4\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{16b^3\left(bx^2+cx^4\right)^{5/2}(8bB-13Ac)}{15015c^5x^5}-\frac{8b^2\left(bx^2+cx^4\right)^{5/2}(8bB-13Ac)}{3003c^4x^3}+\frac{2b\left(bx^2+cx^4\right)^{5/2}(8bB-13Ac)}{429c^3x}-\frac{x\left(bx^2+cx^4\right)^{5/2}(8bB-13Ac)}{143c^2}+\frac{Bx^3(bx^2+cx^4)^{5/2}}{13c}$$

```
[In] Int[x^4*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (16*b^3*(8*b*B - 13*A*c)*(b*x^2 + c*x^4)^(5/2))/(15015*c^5*x^5) - (8*b^2*(8
*b*B - 13*A*c)*(b*x^2 + c*x^4)^(5/2))/(3003*c^4*x^3) + (2*b*(8*b*B - 13*A*c
)*(b*x^2 + c*x^4)^(5/2))/(429*c^3*x) - ((8*b*B - 13*A*c)*x*(b*x^2 + c*x^4)^
(5/2))/(143*c^2) + (B*x^3*(b*x^2 + c*x^4)^(5/2))/(13*c)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Bx^3(bx^2+cx^4)^{5/2}}{13c} - \frac{(8bB-13Ac)\int x^4(bx^2+cx^4)^{3/2}\,dx}{13c} \\
&= -\frac{(8bB-13Ac)x(bx^2+cx^4)^{5/2}}{143c^2} + \frac{Bx^3(bx^2+cx^4)^{5/2}}{13c} \\
&\quad + \frac{(6b(8bB-13Ac))\int x^2(bx^2+cx^4)^{3/2}\,dx}{143c^2}
\end{aligned}$$

$$= \frac{2b(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{429c^3x} - \frac{(8bB-13Ac)x(bx^2+cx^4)^{5/2}}{143c^2} + \frac{Bx^3(bx^2+cx^4)^{5/2}}{13c} - \frac{(8b^2(8bB-13Ac))\int\left(bx^2+cx^4\right)^{3/2}\,dx}{429c^3}$$

$$= -\frac{8b^2(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3} + \frac{2b(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{429c^3x} - \frac{(8bB-13Ac)x(bx^2+cx^4)^{5/2}}{143c^2} + \frac{Bx^3(bx^2+cx^4)^{5/2}}{13c} + \frac{(16b^3(8bB-13Ac))\int\frac{\left(bx^2+cx^4\right)^{3/2}}{x^2}\,dx}{3003c^4}$$

$$= \frac{16b^3(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{15015c^5x^5} - \frac{8b^2(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3} + \frac{2b(8bB-13Ac)\left(bx^2+cx^4\right)^{5/2}}{429c^3x} - \frac{(8bB-13Ac)x(bx^2+cx^4)^{5/2}}{143c^2} + \frac{Bx^3(bx^2+cx^4)^{5/2}}{13c}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.67

$$\int x^4\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{x\left(b+cx^2\right)^3\left(128b^4B+105c^4x^6(13A+11Bx^2)-70bc^3x^4(13A+12Bx^2)+40b^2c^2x^2(13A+1\right.}{15015c^5\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[x^4*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b + c*x^2)^3*(128*b^4*B + 105*c^4*x^6*(13*A + 11*B*x^2) - 70*b*c^3*x^4*
(13*A + 12*B*x^2) + 40*b^2*c^2*x^2*(13*A + 14*B*x^2) - 16*b^3*c*(13*A + 20*
B*x^2)))/(15015*c^5*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.68

method	result
gosper	$-\frac{(c\,x^2+b)\left(-1155B\,x^8c^4-1365A\,x^6c^4+840B\,x^6b\,c^3+910A\,x^4b\,c^3-560B\,x^4b^2c^2-520A\,x^2b^2c^2+320B\,x^2b^3c+208A\,b^3c-128B\,b^4\right)(x^4}{15015c^5x^3}$
default	$-\frac{(c\,x^2+b)\left(-1155B\,x^8c^4-1365A\,x^6c^4+840B\,x^6b\,c^3+910A\,x^4b\,c^3-560B\,x^4b^2c^2-520A\,x^2b^2c^2+320B\,x^2b^3c+208A\,b^3c-128B\,b^4\right)(x^4}{15015c^5x^3}$
trager	$-\frac{(-1155B\,c^6x^{12}-1365A\,c^6x^{10}-1470Bb\,c^5x^{10}-1820Ab\,c^5x^8-35B\,b^2c^4x^8-65A\,b^2c^4x^6+40B\,b^3c^3x^6+78A\,b^3c^3x^4-48B\,b^4c^2x^4-104}{15015c^5x}$
risch	$-\frac{\sqrt{x^2(c\,x^2+b)}\,(-1155B\,c^6x^{12}-1365A\,c^6x^{10}-1470Bb\,c^5x^{10}-1820Ab\,c^5x^8-35B\,b^2c^4x^8-65A\,b^2c^4x^6+40B\,b^3c^3x^6+78A\,b^3c^3x^4-48}{15015x\,c^5}$

```
[In] int(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15015*(c*x^2+b)*(-1155*B*c^4*x^8-1365*A*c^4*x^6+840*B*b*c^3*x^6+910*A*b*
c^3*x^4-560*B*b^2*c^2*x^4-520*A*b^2*c^2*x^2+320*B*b^3*c*x^2+208*A*b^3*c-128
*B*b^4)*(c*x^4+b*x^2)^(3/2)/c^5/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.92

$$\int x^4(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{(1155\,Bc^6x^{12}+105\,(14\,Bbc^5+13\,Ac^6)x^{10}+35\,(Bb^2c^4+52\,Abc^5)x^8+128\,Bb^6-208\,Ab^5c-5}{15015\,c}$$

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15015*(1155*B*c^6*x^12 + 105*(14*B*b*c^5 + 13*A*c^6)*x^10 + 35*(B*b^2*c^4
 + 52*A*b*c^5)*x^8 + 128*B*b^6 - 208*A*b^5*c - 5*(8*B*b^3*c^3 - 13*A*b^2*c^
4)*x^6 + 6*(8*B*b^4*c^2 - 13*A*b^3*c^3)*x^4 - 8*(8*B*b^5*c - 13*A*b^4*c^2)*
x^2)*sqrt(c*x^4 + b*x^2)/(c^5*x)
```

Sympy [F]

$$\int x^4(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\int x^4\left(x^2(b+cx^2)\right)^{\frac{3}{2}}(A+Bx^2)\,dx$$

```
[In] integrate(x**4*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**4*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.89

$$\int x^4(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{(105\,c^5x^{10}+140\,bc^4x^8+5\,b^2c^3x^6-6\,b^3c^2x^4+8\,b^4cx^2-16\,b^5)\sqrt{cx^2+b}A}{1155\,c^4}+\frac{(1155\,c^6x^{12}+1470\,bc^5x^{10}+35\,b^2c^4x^8-40\,b^3c^3x^6+48\,b^4c^2x^4-64\,b^5cx^2+128\,b^6)\sqrt{cx^2+b}B}{15015\,c^5}$$

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/1155*(105*c^5*x^10 + 140*b*c^4*x^8 + 5*b^2*c^3*x^6 - 6*b^3*c^2*x^4 + 8*b^
4*c*x^2 - 16*b^5)*sqrt(c*x^2 + b)*A/c^4 + 1/15015*(1155*c^6*x^12 + 1470*b*c
^5*x^10 + 35*b^2*c^4*x^8 - 40*b^3*c^3*x^6 + 48*b^4*c^2*x^4 - 64*b^5*c*x^2 +
 128*b^6)*sqrt(c*x^2 + b)*B/c^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.04

$$\int x^4(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=-\frac{16\left(8\,Bb^{\frac{13}{2}}-13\,Ab^{\frac{11}{2}}c\right)\mathrm{sgn}(x)}{15015\,c^5}+\frac{1155\,(cx^2+b)^{\frac{13}{2}}B\mathrm{sgn}(x)-5460\,(cx^2+b)^{\frac{11}{2}}Bb\mathrm{sgn}(x)+10010\,(cx^2+b)^{\frac{9}{2}}Bb^2\mathrm{sgn}(x)-8580\,(cx^2+b)^{\frac{7}{2}}Bb^3}{}$$

```
[In] integrate(x^4*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -16/15015*(8*B*b^(13/2) - 13*A*b^(11/2)*c)*sgn(x)/c^5 + 1/15015*(1155*(c*x^
2 + b)^(13/2)*B*sgn(x) - 5460*(c*x^2 + b)^(11/2)*B*b*sgn(x) + 10010*(c*x^2
+ b)^(9/2)*B*b^2*sgn(x) - 8580*(c*x^2 + b)^(7/2)*B*b^3*sgn(x) + 3003*(c*x^2
 + b)^(5/2)*B*b^4*sgn(x) + 1365*(c*x^2 + b)^(11/2)*A*c*sgn(x) - 5005*(c*x^2
 + b)^(9/2)*A*b*c*sgn(x) + 6435*(c*x^2 + b)^(7/2)*A*b^2*c*sgn(x) - 3003*(c*
x^2 + b)^(5/2)*A*b^3*c*sgn(x))/c^5
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.85

$$\int x^4(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{128\,B\,b^6-208\,A\,b^5\,c}{15015\,c^5}+\frac{x^{10}\,\left(1365\,A\,c^6+1470\,B\,b\,c^5\right)}{15015\,c^5}+\frac{B\,c\,x^{12}}{13}+\frac{b^2\,x^6\,\left(13\,A\,c-8\,B\,b\right)}{3003\,c^2}-\frac{2\,b^3\,x^4\,\left(13\,A\,c-\right.}{5005\,c^3}\right.}{x}$$

```
[In] int(x^4*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*((128*B*b^6 - 208*A*b^5*c)/(15015*c^5) + (x^10*(1365
*A*c^6 + 1470*B*b*c^5))/(15015*c^5) + (B*c*x^12)/13 + (b^2*x^6*(13*A*c - 8*
B*b))/(3003*c^2) - (2*b^3*x^4*(13*A*c - 8*B*b))/(5005*c^3) + (8*b^4*x^2*(13
*A*c - 8*B*b))/(15015*c^4) + (b*x^8*(52*A*c + B*b))/(429*c)))/x
```

3.120 $\int x^2(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 131

$$\int x^2\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=-\frac{8b^2(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{3465c^4x^5}+\frac{4b(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{693c^3x^3}-\frac{(6bB-11Ac)\left(bx^2+cx^4\right)^{5/2}}{99c^2x}+\frac{Bx(bx^2+cx^4)^{5/2}}{11c}$$

```
[Out] -8/3465*b^2*(-11*A*c+6*B*b)*(c*x^4+b*x^2)^(5/2)/c^4/x^5+4/693*b*(-11*A*c+6*
B*b)*(c*x^4+b*x^2)^(5/2)/c^3/x^3-1/99*(-11*A*c+6*B*b)*(c*x^4+b*x^2)^(5/2)/c
^2/x+1/11*B*x*(c*x^4+b*x^2)^(5/2)/c
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2064, 2041, 2027, 2039}

$$\int x^2\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=-\frac{8b^2(bx^2+cx^4)^{5/2}(6bB-11Ac)}{3465c^4x^5}+\frac{4b(bx^2+cx^4)^{5/2}(6bB-11Ac)}{693c^3x^3}-\frac{(bx^2+cx^4)^{5/2}(6bB-11Ac)}{99c^2x}+\frac{Bx(bx^2+cx^4)^{5/2}}{11c}$$

```
[In] Int[x^2*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-8*b^2*(6*b*B - 11*A*c)*(b*x^2 + c*x^4)^(5/2))/(3465*c^4*x^5) + (4*b*(6*b*
B - 11*A*c)*(b*x^2 + c*x^4)^(5/2))/(693*c^3*x^3) - ((6*b*B - 11*A*c)*(b*x^2
 + c*x^4)^(5/2))/(99*c^2*x) + (B*x*(b*x^2 + c*x^4)^(5/2))/(11*c)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{Bx(bx^2+cx^4)^{5/2}}{11c} - \frac{(6bB-11Ac)\int x^2(bx^2+cx^4)^{3/2}\,dx}{11c} \\
&= -\frac{(6bB-11Ac)\,(bx^2+cx^4)^{5/2}}{99c^2x} + \frac{Bx(bx^2+cx^4)^{5/2}}{11c} + \frac{(4b(6bB-11Ac))\int (bx^2+cx^4)^{3/2}\,dx}{99c^2} \\
&= \frac{4b(6bB-11Ac)\,(bx^2+cx^4)^{5/2}}{693c^3x^3} - \frac{(6bB-11Ac)\,(bx^2+cx^4)^{5/2}}{99c^2x} \\
&\quad + \frac{Bx(bx^2+cx^4)^{5/2}}{11c} - \frac{(8b^2(6bB-11Ac))\int \frac{(bx^2+cx^4)^{3/2}}{x^2}\,dx}{693c^3}
\end{aligned}
$$

$$= -\frac{8b^2(6bB - 11Ac)\left(bx^2 + cx^4\right)^{5/2}}{3465c^4x^5} + \frac{4b(6bB - 11Ac)\left(bx^2 + cx^4\right)^{5/2}}{693c^3x^3} - \frac{(6bB - 11Ac)\left(bx^2 + cx^4\right)^{5/2}}{99c^2x} + \frac{Bx\left(bx^2 + cx^4\right)^{5/2}}{11c}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.72

$$\int x^2\left(A + Bx^2\right)\left(bx^2 + cx^4\right)^{3/2} dx = \frac{(b + cx^2)\left(x^2(b + cx^2)\right)^{3/2}\left(-48b^3B + 88Ab^2c + 120b^2Bcx^2 - 220Abc^2x^2 - 210bBc^2x^4 + 385A\right.}{3465c^4x^3}$$

```
[In] Integrate[x^2*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((b + c*x^2)*(x^2*(b + c*x^2))^(3/2)*(-48*b^3*B + 88*A*b^2*c + 120*b^2*B*c*
x^2 - 220*A*b*c^2*x^2 - 210*b*B*c^2*x^4 + 385*A*c^3*x^4 + 315*B*c^3*x^6))/(
3465*c^4*x^3)
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.69

method	result
gosper	$\frac{(c x^2+b)\left(315B c^3x^6+385A c^3x^4-210Bb c^2x^4-220Ab c^2x^2+120B b^2c x^2+88b^2Ac-48B b^3\right)\left(x^4c+b x^2\right)^{\frac{3}{2}}}{3465c^4x^3}$
default	$\frac{(c x^2+b)\left(315B c^3x^6+385A c^3x^4-210Bb c^2x^4-220Ab c^2x^2+120B b^2c x^2+88b^2Ac-48B b^3\right)\left(x^4c+b x^2\right)^{\frac{3}{2}}}{3465c^4x^3}$
trager	$\frac{(315B c^5x^{10}+385A c^5x^8+420Bb c^4x^8+550Ab c^4x^6+15B b^2c^3x^6+33A b^2c^3x^4-18B b^3c^2x^4-44A b^3c^2x^2+24B b^4c x^2+88A b^4c-48b^5}{3465c^4x}$
risch	$\frac{\sqrt{x^2(c x^2+b)}\,(315B c^5x^{10}+385A c^5x^8+420Bb c^4x^8+550Ab c^4x^6+15B b^2c^3x^6+33A b^2c^3x^4-18B b^3c^2x^4-44A b^3c^2x^2+24B b^4c x^2+}{3465x c^4}$

```
[In] int(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3465*(c*x^2+b)*(315*B*c^3*x^6+385*A*c^3*x^4-210*B*b*c^2*x^4-220*A*b*c^2*x
^2+120*B*b^2*c*x^2+88*A*b^2*c-48*B*b^3)*(c*x^4+b*x^2)^(3/2)/c^4/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00

$$\int x^2(A+Bx^2)(bx^2+cx^4)^{3/2}\,dx = \frac{(315\,Bc^5x^{10}+35\,(12\,Bbc^4+11\,Ac^5)x^8+5\,(3\,Bb^2c^3+110\,Abc^4)x^6-48\,Bb^5+88\,Ab^4c-3\,(6\,B}{3465\,c^4x}$$

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*B*c^5*x^10 + 35*(12*B*b*c^4 + 11*A*c^5)*x^8 + 5*(3*B*b^2*c^3 +
110*A*b*c^4)*x^6 - 48*B*b^5 + 88*A*b^4*c - 3*(6*B*b^3*c^2 - 11*A*b^2*c^3)*x
^4 + 4*(6*B*b^4*c - 11*A*b^3*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^4*x)
```

Sympy [F]

$$\int x^2(A+Bx^2)(bx^2+cx^4)^{3/2}\,dx = \int x^2(x^2(b+cx^2))^{\frac{3}{2}}(A+Bx^2)\,dx$$

```
[In] integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int x^2(A+Bx^2)(bx^2+cx^4)^{3/2}\,dx = \frac{(35\,c^4x^8+50\,bc^3x^6+3\,b^2c^2x^4-4\,b^3cx^2+8\,b^4)\sqrt{cx^2+b}A}{315\,c^3} + \frac{(105\,c^5x^{10}+140\,bc^4x^8+5\,b^2c^3x^6-6\,b^3c^2x^4+8\,b^4cx^2-16\,b^5)\sqrt{cx^2+b}B}{1155\,c^4}$$

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/315*(35*c^4*x^8 + 50*b*c^3*x^6 + 3*b^2*c^2*x^4 - 4*b^3*c*x^2 + 8*b^4)*sqr
t(c*x^2 + b)*A/c^3 + 1/1155*(105*c^5*x^10 + 140*b*c^4*x^8 + 5*b^2*c^3*x^6 -
 6*b^3*c^2*x^4 + 8*b^4*c*x^2 - 16*b^5)*sqrt(c*x^2 + b)*B/c^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07

$$\int x^2(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{8\left(6\,Bb^{\frac{11}{2}}-11\,Ab^{\frac{9}{2}}c\right)\mathrm{sgn}(x)}{3465\,c^4}$$
$$+\frac{315\,(cx^2+b)^{\frac{11}{2}}B\mathrm{sgn}(x)-1155\,(cx^2+b)^{\frac{9}{2}}Bb\mathrm{sgn}(x)+1485\,(cx^2+b)^{\frac{7}{2}}Bb^2\mathrm{sgn}(x)-693\,(cx^2+b)^{\frac{5}{2}}Bb^3\mathrm{sgn}(}{3465\,c^4}$$

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 8/3465*(6*B*b^(11/2) - 11*A*b^(9/2)*c)*sgn(x)/c^4 + 1/3465*(315*(c*x^2 + b)
^(11/2)*B*sgn(x) - 1155*(c*x^2 + b)^(9/2)*B*b*sgn(x) + 1485*(c*x^2 + b)^(7/
2)*B*b^2*sgn(x) - 693*(c*x^2 + b)^(5/2)*B*b^3*sgn(x) + 385*(c*x^2 + b)^(9/2
)*A*c*sgn(x) - 990*(c*x^2 + b)^(7/2)*A*b*c*sgn(x) + 693*(c*x^2 + b)^(5/2)*A
*b^2*c*sgn(x))/c^4
```

Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95

$$\int x^2(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{\sqrt{c\,x^4+b\,x^2}\left(\frac{x^8\,\left(385\,A\,c^5+420\,B\,b\,c^4\right)}{3465\,c^4}-\frac{48\,B\,b^5-88\,A\,b^4\,c}{3465\,c^4}+\frac{B\,c\,x^{10}}{11}+\frac{b^2\,x^4\,\left(11\,A\,c-6\,B\,b\right)}{1155\,c^2}-\frac{4\,b^3\,x^2\,\left(11\,A\,c-6\,B\right.}{3465\,c^3}\right.}{x}$$

```
[In] int(x^2*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*((x^8*(385*A*c^5 + 420*B*b*c^4))/(3465*c^4) - (48*B*
b^5 - 88*A*b^4*c)/(3465*c^4) + (B*c*x^10)/11 + (b^2*x^4*(11*A*c - 6*B*b))/(
1155*c^2) - (4*b^3*x^2*(11*A*c - 6*B*b))/(3465*c^3) + (b*x^6*(110*A*c + 3*B
*b))/(693*c)))/x
```

3.121 $\int (A+Bx^2) (bx^2+cx^4)^{3/2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 96

$$\int (A+Bx^2) (bx^2+cx^4)^{3/2} \, dx = \frac{2b(4bB-9Ac)(bx^2+cx^4)^{5/2}}{315c^3x^5} - \frac{(4bB-9Ac)(bx^2+cx^4)^{5/2}}{63c^2x^3} + \frac{B(bx^2+cx^4)^{5/2}}{9cx}$$

```
[Out] 2/315*b*(-9*A*c+4*B*b)*(c*x^4+b*x^2)^(5/2)/c^3/x^5-1/63*(-9*A*c+4*B*b)*(c*x
^4+b*x^2)^(5/2)/c^2/x^3+1/9*B*(c*x^4+b*x^2)^(5/2)/c/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1159, 2027, 2039}

$$\int (A+Bx^2) (bx^2+cx^4)^{3/2} \, dx = \frac{2b(bx^2+cx^4)^{5/2}(4bB-9Ac)}{315c^3x^5} - \frac{(bx^2+cx^4)^{5/2}(4bB-9Ac)}{63c^2x^3} + \frac{B(bx^2+cx^4)^{5/2}}{9cx}$$

```
[In] Int[(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*b*(4*b*B - 9*A*c)*(b*x^2 + c*x^4)^(5/2))/(315*c^3*x^5) - ((4*b*B - 9*A*c
)*(b*x^2 + c*x^4)^(5/2))/(63*c^2*x^3) + (B*(b*x^2 + c*x^4)^(5/2))/(9*c*x)
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
Simp[e*((b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 3)*x)), x] - Dist[(b*e*(2*p + 1)
```

```
- c*d*(4*p + 3))/(c*(4*p + 3)), Int[(b*x^2 + c*x^4)^p, x], x] /; FreeQ[{b,
c, d, e, p}, x] &&  !IntegerQ[p] && NeQ[4*p + 3, 0] && NeQ[b*e*(2*p + 1) -
c*d*(4*p + 3), 0]
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{B(bx^2+cx^4)^{5/2}}{9cx} - \frac{(4bB-9Ac)\int (bx^2+cx^4)^{3/2}\,dx}{9c}\\ &= -\frac{(4bB-9Ac)\,(bx^2+cx^4)^{5/2}}{63c^2x^3} + \frac{B(bx^2+cx^4)^{5/2}}{9cx} + \frac{(2b(4bB-9Ac))\int \frac{(bx^2+cx^4)^{3/2}}{x^2}\,dx}{63c^2}\\ &= \frac{2b(4bB-9Ac)\,(bx^2+cx^4)^{5/2}}{315c^3x^5} - \frac{(4bB-9Ac)\,(bx^2+cx^4)^{5/2}}{63c^2x^3} + \frac{B(bx^2+cx^4)^{5/2}}{9cx}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.73

$$\int (A+Bx^2)\,(bx^2+cx^4)^{3/2}\,dx = \frac{(b+cx^2)\,(x^2(b+cx^2))^{3/2}\,(8b^2B-18Abc-20bBcx^2+45Ac^2x^2+35Bc^2x^4)}{315c^3x^3}$$

```
[In] Integrate[(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
[Out] ((b + c*x^2)*(x^2*(b + c*x^2))^(3/2)*(8*b^2*B - 18*A*b*c - 20*b*B*c*x^2 + 4
5*A*c^2*x^2 + 35*B*c^2*x^4))/(315*c^3*x^3)
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.70

method	result	size
gosper	$-\frac{(c\,x^2+b)\,(-35B\,c^2x^4-45A\,c^2x^2+20Bbc\,x^2+18Abc-8B\,b^2)\,(x^4c+b\,x^2)^{\frac{3}{2}}}{315c^3x^3}$	67
default	$-\frac{(c\,x^2+b)\,(-35B\,c^2x^4-45A\,c^2x^2+20Bbc\,x^2+18Abc-8B\,b^2)\,(x^4c+b\,x^2)^{\frac{3}{2}}}{315c^3x^3}$	67
trager	$-\frac{(-35B\,x^8c^4-45A\,x^6c^4-50B\,x^6b\,c^3-72A\,x^4b\,c^3-3B\,x^4b^2c^2-9A\,x^2b^2c^2+4B\,x^2b^3c+18A\,b^3c-8B\,b^4)\sqrt{x^4c+b\,x^2}}{315c^3x}$	108
risch	$-\frac{\sqrt{x^2(c\,x^2+b)}\,(-35B\,x^8c^4-45A\,x^6c^4-50B\,x^6b\,c^3-72A\,x^4b\,c^3-3B\,x^4b^2c^2-9A\,x^2b^2c^2+4B\,x^2b^3c+18A\,b^3c-8B\,b^4)}{315x\,c^3}$	108

[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/315*(c*x^2+b)*(-35*B*c^2*x^4-45*A*c^2*x^2+20*B*b*c*x^2+18*A*b*c-8*B*b^2)
*(c*x^4+b*x^2)^(3/2)/c^3/x^3

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.10

$$\int (A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{(35\,Bc^4x^8+5\,(10\,Bbc^3+9\,Ac^4)x^6+8\,Bb^4-18\,Ab^3c+3\,(Bb^2c^2+24\,Abc^3)x^4-(4\,Bb^3c-9\,A}{315\,c^3x}$$

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")

[Out] 1/315*(35*B*c^4*x^8 + 5*(10*B*b*c^3 + 9*A*c^4)*x^6 + 8*B*b^4 - 18*A*b^3*c +
3*(B*b^2*c^2 + 24*A*b*c^3)*x^4 - (4*B*b^3*c - 9*A*b^2*c^2)*x^2)*sqrt(c*x^4
+ b*x^2)/(c^3*x)

Sympy [F]

$$\int (A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \int \left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)\,dx$$

[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)

[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.09

$$\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{(5\,c^3x^6+8\,bc^2x^4+b^2cx^2-2\,b^3)\sqrt{cx^2+b}A}{35\,c^2}+\frac{(35\,c^4x^8+50\,bc^3x^6+3\,b^2c^2x^4-4\,b^3cx^2+8\,b^4)\sqrt{cx^2+b}B}{315\,c^3}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/35*(5*c^3*x^6 + 8*b*c^2*x^4 + b^2*c*x^2 - 2*b^3)*sqrt(c*x^2 + b)*A/c^2 +
1/315*(35*c^4*x^8 + 50*b*c^3*x^6 + 3*b^2*c^2*x^4 - 4*b^3*c*x^2 + 8*b^4)*sqr
t(c*x^2 + b)*B/c^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.09

$$\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=-\frac{2\left(4\,Bb^{\frac{9}{2}}-9\,Ab^{\frac{7}{2}}c\right)\mathrm{sgn}(x)}{315\,c^3}+\frac{35\,(cx^2+b)^{\frac{9}{2}}B\mathrm{sgn}(x)-90\,(cx^2+b)^{\frac{7}{2}}Bb\mathrm{sgn}(x)+63\,(cx^2+b)^{\frac{5}{2}}Bb^2\mathrm{sgn}(x)+45\,(cx^2+b)^{\frac{7}{2}}Ac\mathrm{sgn}(x)-63\,(}{315\,c^3}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2/315*(4*B*b^(9/2) - 9*A*b^(7/2)*c)*sgn(x)/c^3 + 1/315*(35*(c*x^2 + b)^(9/
2)*B*sgn(x) - 90*(c*x^2 + b)^(7/2)*B*b*sgn(x) + 63*(c*x^2 + b)^(5/2)*B*b^2*
sgn(x) + 45*(c*x^2 + b)^(7/2)*A*c*sgn(x) - 63*(c*x^2 + b)^(5/2)*A*b*c*sgn(x
))/c^3
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.07

$$\int \left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{8\,B\,b^4-18\,A\,b^3\,c}{315\,c^3}+\frac{x^6\,\left(45\,A\,c^4+50\,B\,b\,c^3\right)}{315\,c^3}+\frac{B\,c\,x^8}{9}+\frac{b^2\,x^2\,\left(9\,A\,c-4\,B\,b\right)}{315\,c^2}+\frac{b\,x^4\,\left(24\,A\,c+B\,b\right)}{105\,c}\right)}{x}$$

```
[In] int((A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*((8*B*b^4 - 18*A*b^3*c)/(315*c^3) + (x^6*(45*A*c^4 +
 50*B*b*c^3))/(315*c^3) + (B*c*x^8)/9 + (b^2*x^2*(9*A*c - 4*B*b))/(315*c^2)
 + (b*x^4*(24*A*c + B*b))/(105*c)))/x
```

3.122 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^2} dx$

Optimal result

Integrand size = 26, antiderivative size = 61

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^2} dx = -\frac{(2bB-7Ac)(bx^2+cx^4)^{5/2}}{35c^2x^5} + \frac{B(bx^2+cx^4)^{5/2}}{7cx^3}$$

```
[Out] -1/35*(-7*A*c+2*B*b)*(c*x^4+b*x^2)^(5/2)/c^2/x^5+1/7*B*(c*x^4+b*x^2)^(5/2)/
c/x^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2064, 2039}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^2} dx = \frac{B(bx^2+cx^4)^{5/2}}{7cx^3} - \frac{(bx^2+cx^4)^{5/2}(2bB-7Ac)}{35c^2x^5}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^2,x]
```

```
[Out] -1/35*((2*b*B - 7*A*c)*(b*x^2 + c*x^4)^(5/2))/(c^2*x^5) + (B*(b*x^2 + c*x^4
)^(5/2))/(7*c*x^3)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{B(bx^2+cx^4)^{5/2}}{7cx^3} - \frac{(2bB-7Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^2}\,dx}{7c}\\ &= -\frac{(2bB-7Ac)\,(bx^2+cx^4)^{5/2}}{35c^2x^5} + \frac{B(bx^2+cx^4)^{5/2}}{7cx^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.67

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^2}\,dx = \frac{(x^2(b+cx^2))^{5/2}\,(-2bB+7Ac+5Bcx^2)}{35c^2x^5}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^2,x]
[Out] ((x^2*(b + c*x^2))^(5/2)*(-2*b*B + 7*A*c + 5*B*c*x^2))/(35*c^2*x^5)
```

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

method	result	size
gosper	$\frac{(c\,x^2+b)\,(5Bc\,x^2+7Ac-2Bb)\,(x^4c+b\,x^2)^{\frac{3}{2}}}{35c^2x^3}$	45
default	$\frac{(c\,x^2+b)\,(5Bc\,x^2+7Ac-2Bb)\,(x^4c+b\,x^2)^{\frac{3}{2}}}{35c^2x^3}$	45
trager	$\frac{(5B\,c^3x^6+7A\,c^3x^4+8Bb\,c^2x^4+14Ab\,c^2x^2+B\,b^2c\,x^2+7b^2Ac-2B\,b^3)\sqrt{x^4c+b\,x^2}}{35c^2x}$	83
risch	$\frac{\sqrt{x^2(c\,x^2+b)}\,(5B\,c^3x^6+7A\,c^3x^4+8Bb\,c^2x^4+14Ab\,c^2x^2+B\,b^2c\,x^2+7b^2Ac-2B\,b^3)}{35x\,c^2}$	83

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/35*(c*x^2+b)*(5*B*c*x^2+7*A*c-2*B*b)*(c*x^4+b*x^2)^(3/2)/c^2/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^2}\,dx = \frac{(5\,Bc^3x^6+(8\,Bbc^2+7\,Ac^3)x^4-2\,Bb^3+7\,Ab^2c+(Bb^2c+14\,Abc^2)x^2)\sqrt{c}}{35\,c^2x}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/35*(5*B*c^3*x^6 + (8*B*b*c^2 + 7*A*c^3)*x^4 - 2*B*b^3 + 7*A*b^2*c + (B*b^
2*c + 14*A*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^2*x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^2}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}\,(A+Bx^2)}{x^2}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**2,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^2}\,dx = \frac{(c^2x^4+2\,bcx^2+b^2)\sqrt{cx^2+b}A}{5\,c} + \frac{(5\,c^3x^6+8\,bc^2x^4+b^2cx^2-2\,b^3)\sqrt{cx^2+b}B}{35\,c^2}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(c*x^2 + b)*A/c + 1/35*(5*c^3*x^6 + 8*b
*c^2*x^4 + b^2*c*x^2 - 2*b^3)*sqrt(c*x^2 + b)*B/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^2}\,dx = \frac{\left(2\,Bb^{\frac{7}{2}} - 7\,Ab^{\frac{5}{2}}c\right)\mathrm{sgn}(x)}{35\,c^2} + \frac{5\,(cx^2+b)^{\frac{7}{2}}B\mathrm{sgn}(x) - 7\,(cx^2+b)^{\frac{5}{2}}Bb\mathrm{sgn}(x) + 7\,(cx^2+b)^{\frac{5}{2}}Ac\mathrm{sgn}(x)}{35\,c^2}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/35*(2*B*b^(7/2) - 7*A*b^(5/2)*c)*sgn(x)/c^2 + 1/35*(5*(c*x^2 + b)^(7/2)*B
*sgn(x) - 7*(c*x^2 + b)^(5/2)*B*b*sgn(x) + 7*(c*x^2 + b)^(5/2)*A*c*sgn(x))/
c^2
```

Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.36

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^2}\,dx = \frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{x^4\,(7\,A\,c^3+8\,B\,b\,c^2)}{35\,c^2} - \frac{2\,B\,b^3-7\,A\,b^2\,c}{35\,c^2} + \frac{B\,c\,x^6}{7} + \frac{b\,x^2\,(14\,A\,c+B\,b)}{35\,c}\right)}{x}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^2,x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*((x^4*(7*A*c^3 + 8*B*b*c^2))/(35*c^2) - (2*B*b^3 - 7
*A*b^2*c)/(35*c^2) + (B*c*x^6)/7 + (b*x^2*(14*A*c + B*b))/(35*c)))/x
```

3.123 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^4} dx$

Optimal result

Integrand size = 26, antiderivative size = 102

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^4} dx = \frac{Ab\sqrt{bx^2+cx^4}}{x} + \frac{A(bx^2+cx^4)^{3/2}}{3x^3} + \frac{B(bx^2+cx^4)^{5/2}}{5cx^5} - Ab^{3/2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] 1/3*A*(c*x^4+b*x^2)^(3/2)/x^3+1/5*B*(c*x^4+b*x^2)^(5/2)/c/x^5-A*b^(3/2)*arc
tanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))+A*b*(c*x^4+b*x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2064, 2046, 2033, 212}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^4} dx = -Ab^{3/2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right) + \frac{Ab\sqrt{bx^2+cx^4}}{x} + \frac{A(bx^2+cx^4)^{3/2}}{3x^3} + \frac{B(bx^2+cx^4)^{5/2}}{5cx^5}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^4,x]
```

```
[Out] (A*b*Sqrt[b*x^2 + c*x^4])/x + (A*(b*x^2 + c*x^4)^(3/2))/(3*x^3) + (B*(b*x^2
 + c*x^4)^(5/2))/(5*c*x^5) - A*b^(3/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x
^4]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{B(bx^2+cx^4)^{5/2}}{5cx^5} + A\int \frac{(bx^2+cx^4)^{3/2}}{x^4}\,dx \\
&= \frac{A(bx^2+cx^4)^{3/2}}{3x^3} + \frac{B(bx^2+cx^4)^{5/2}}{5cx^5} + (Ab)\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx \\
&= \frac{Ab\sqrt{bx^2+cx^4}}{x} + \frac{A(bx^2+cx^4)^{3/2}}{3x^3} + \frac{B(bx^2+cx^4)^{5/2}}{5cx^5} + (Ab^2)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx \\
&= \frac{Ab\sqrt{bx^2+cx^4}}{x} + \frac{A(bx^2+cx^4)^{3/2}}{3x^3} + \frac{B(bx^2+cx^4)^{5/2}}{5cx^5} \\
&\quad - (Ab^2)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right) \\
&= \frac{Ab\sqrt{bx^2+cx^4}}{x} + \frac{A(bx^2+cx^4)^{3/2}}{3x^3} + \frac{B(bx^2+cx^4)^{5/2}}{5cx^5} - Ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^4}\,dx = \frac{x\Big((b+cx^2)\left(3b^2B+c^2x^2(5A+3Bx^2)+b(20Ac+6Bcx^2)\right)-15Ab^{3/2}c\sqrt{b}}{15c\sqrt{x^2(b+cx^2)}}$$

[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^4,x]

[Out] (x*((b + c*x^2)*(3*b^2*B + c^2*x^2*(5*A + 3*B*x^2) + b*(20*A*c + 6*B*c*x^2)) - 15*A*b^(3/2)*c*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(15*c*Sqrt[x^2*(b + c*x^2)])

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{(x^4c+b\,x^2)^{\frac{3}{2}}\left(3B(c\,x^2+b)^{\frac{5}{2}}+5A(c\,x^2+b)^{\frac{3}{2}}c-15A\,b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)c+15A\sqrt{c\,x^2+b}\,bc\right)}{15x^3(c\,x^2+b)^{\frac{3}{2}}\,c}$	99

[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)

[Out] 1/15*(c*x^4+b*x^2)^(3/2)*(3*B*(c*x^2+b)^(5/2)+5*A*(c*x^2+b)^(3/2)*c-15*A*b^(3/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c+15*A*(c*x^2+b)^(1/2)*b*c)/x^3/(c*x^2+b)^(3/2)/c

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.02

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^4}\,dx = \left[\frac{15\,Ab^{\frac{3}{2}}cx\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,(3\,Bc^2x^4+3\,Bb^2+20\,Abc+(6}{30\,cx}\right.$$

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="fricas")

[Out] [1/30*(15*A*b^(3/2)*c*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3) + 2*(3*B*c^2*x^4 + 3*B*b^2 + 20*A*b*c + (6*B*b*c + 5*A*c^2)*x^2)*sqrt(c*x^4 + b*x^2))/(c*x), 1/15*(15*A*sqrt(-b)*b*c*x*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (3*B*c^2*x^4 + 3*B*b^2 + 20*A*b*c + (6*B*b*c + 5*A*c^2)*x^2)*sqrt(c*x^4 + b*x^2))/(c*x)]

Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^4}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}(A+Bx^2)}{x^4}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**4,x)
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**4, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^4}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^4}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^4, x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.37

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^4}\,dx = \frac{Ab^2\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} - \frac{\left(15\,Ab^2c\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+3\,B\sqrt{-b}b^{\frac{5}{2}}+20\,A\sqrt{-b}b^{\frac{3}{2}}c\right)\operatorname{sgn}(x)}{15\sqrt{-b}c} + \frac{3\,(cx^2+b)^{\frac{5}{2}}Bc^4\operatorname{sgn}(x)+5\,(cx^2+b)^{\frac{3}{2}}Ac^5\operatorname{sgn}(x)+15\sqrt{cx^2+b}Abc^5\operatorname{sgn}(x)}{15\,c^5}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^4,x, algorithm="giac")
[Out] A*b^2*arctan(sqrt(c*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - 1/15*(15*A*b^2*c*a
rctan(sqrt(b)/sqrt(-b)) + 3*B*sqrt(-b)*b^(5/2) + 20*A*sqrt(-b)*b^(3/2)*c)*s
gn(x)/(sqrt(-b)*c) + 1/15*(3*(c*x^2 + b)^(5/2)*B*c^4*sgn(x) + 5*(c*x^2 + b)
^(3/2)*A*c^5*sgn(x) + 15*sqrt(c*x^2 + b)*A*b*c^5*sgn(x))/c^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^4}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^4}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^4,x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^4, x)
```

3.124 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^6} dx$

Optimal result

Integrand size = 26, antiderivative size = 133

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^6} dx = \frac{(2bB+3Ac)\sqrt{bx^2+cx^4}}{2x} + \frac{(2bB+3Ac)(bx^2+cx^4)^{3/2}}{6bx^3} - \frac{A(bx^2+cx^4)^{5/2}}{2bx^7} - \frac{1}{2}\sqrt{b}(2bB+3Ac)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] 1/6*(3*A*c+2*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^3-1/2*A*(c*x^4+b*x^2)^(5/2)/b/x^7
-1/2*(3*A*c+2*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))*b^(1/2)+1/2*(3*A*
c+2*B*b)*(c*x^4+b*x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2063, 2046, 2033, 212}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^6} dx = -\frac{1}{2}\sqrt{b}(3Ac+2bB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right) + \frac{\sqrt{bx^2+cx^4}(3Ac+2bB)}{2x} + \frac{(bx^2+cx^4)^{3/2}(3Ac+2bB)}{6bx^3} - \frac{A(bx^2+cx^4)^{5/2}}{2bx^7}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^6,x]
```

```
[Out] ((2*b*B + 3*A*c)*Sqrt[b*x^2 + c*x^4])/(2*x) + ((2*b*B + 3*A*c)*(b*x^2 + c*x
^4)^(3/2))/(6*b*x^3) - (A*(b*x^2 + c*x^4)^(5/2))/(2*b*x^7) - (Sqrt[b]*(2*b*
B + 3*A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A(bx^2+cx^4)^{5/2}}{2bx^7} - \frac{(-2bB-3Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^4}\,dx}{2b} \\
&= \frac{(2bB+3Ac)\left(bx^2+cx^4\right)^{3/2}}{6bx^3} - \frac{A(bx^2+cx^4)^{5/2}}{2bx^7} - \frac{1}{2}(-2bB-3Ac)\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx \\
&= \frac{(2bB+3Ac)\sqrt{bx^2+cx^4}}{2x} + \frac{(2bB+3Ac)\left(bx^2+cx^4\right)^{3/2}}{6bx^3} \\
&\quad - \frac{A(bx^2+cx^4)^{5/2}}{2bx^7} + \frac{1}{2}(b(2bB+3Ac))\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx
\end{aligned}$$

$$= \frac{(2bB+3Ac)\sqrt{bx^2+cx^4}}{2x} + \frac{(2bB+3Ac)\left(bx^2+cx^4\right)^{3/2}}{6bx^3} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{2bx^7} - \frac{1}{2}(b(2bB+3Ac))\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)$$

$$= \frac{(2bB+3Ac)\sqrt{bx^2+cx^4}}{2x} + \frac{(2bB+3Ac)\left(bx^2+cx^4\right)^{3/2}}{6bx^3} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{2bx^7} - \frac{1}{2}\sqrt{b}(2bB+3Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.82

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^6}\,dx = \frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{b+cx^2}\left(-3Ab+8bBx^2+6Acx^2+2Bcx^4\right)-3\sqrt{b}(2bB\right.}{6x^3\sqrt{b+cx^2}}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^6,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[b + c*x^2]*(-3*A*b + 8*b*B*x^2 + 6*A*c*x^2 + 2
*B*c*x^4) - 3*Sqrt[b]*(2*b*B + 3*A*c)*x^2*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])
)/(6*x^3*Sqrt[b + c*x^2])
```

Maple [A] (verified)

Time = 2.69 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{bA\sqrt{x^2(c\,x^2+b)}}{2x^3}+\frac{\left(B\,c^2\left(\frac{x^2\sqrt{c\,x^2+b}}{3c}-\frac{2b\sqrt{c\,x^2+b}}{3c^2}\right)+A\sqrt{c\,x^2+b}\,c+2Bb\sqrt{c\,x^2+b}-\frac{\sqrt{b}\,(3Ac+2Bb)\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)}{2}\right)\sqrt{x^2(c\,x}}{x\sqrt{c\,x^2+b}}$
default	$-\frac{\left(x^4c+b\,x^2\right)^{\frac{3}{2}}\left(-3A\left(c\,x^2+b\right)^{\frac{3}{2}}c\,x^2+9A\,b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)c\,x^2-2B\left(c\,x^2+b\right)^{\frac{3}{2}}b\,x^2+6B\,b^{\frac{5}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)x^2+3A(c\,x^2\right.}{6x^5\left(c\,x^2+b\right)^{\frac{3}{2}}b}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*A/x^3*(x^2*(c*x^2+b))^(1/2)+(B*c^2*(1/3*x^2/c*(c*x^2+b)^(1/2)-2/3*b/
c^2*(c*x^2+b)^(1/2))+A*(c*x^2+b)^(1/2)*c+2*B*b*(c*x^2+b)^(1/2)-1/2*b^(1/2)*
(3*A*c+2*B*b)*ln((2*b+2*b^(1/2)*(c*x^2+b)^(1/2))/x))*(x^2*(c*x^2+b))^(1/2)/
x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.47

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^6}\,dx = \left[\frac{3\,(2\,Bb+3\,Ac)\sqrt{b}x^3\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,(2\,Bcx^4+2\,(4\,Bb+}{12\,x^3}\right.$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(2*B*b + 3*A*c)*sqrt(b)*x^3*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b
*x^2)*sqrt(b))/x^3) + 2*(2*B*c*x^4 + 2*(4*B*b + 3*A*c)*x^2 - 3*A*b)*sqrt(c*
x^4 + b*x^2))/x^3, 1/6*(3*(2*B*b + 3*A*c)*sqrt(-b)*x^3*arctan(sqrt(c*x^4 +
b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (2*B*c*x^4 + 2*(4*B*b + 3*A*c)*x^2 - 3*A*b
)*sqrt(c*x^4 + b*x^2))/x^3]
```

Sympy [F]

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^6}\,dx = \int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}(A+Bx^2)}{x^6}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**6,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**6, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^6}\,dx = \int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}(Bx^2+A)}{x^6}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^6, x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^6}\,dx = \frac{2\,(cx^2+b)^{\frac{3}{2}}Bc\mathrm{sgn}(x)+6\,\sqrt{cx^2+b}Bbc\mathrm{sgn}(x)+6\,\sqrt{cx^2+b}Ac^2\mathrm{sgn}(x)-}{6\,c}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/6*(2*(c*x^2 + b)^(3/2)*B*c*sgn(x) + 6*sqrt(c*x^2 + b)*B*b*c*sgn(x) + 6*sq
rt(c*x^2 + b)*A*c^2*sgn(x) - 3*sqrt(c*x^2 + b)*A*b*c*sgn(x)/x^2 + 3*(2*B*b^
2*c*sgn(x) + 3*A*b*c^2*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^6}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^6}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^6,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^6, x)
```

3.125 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^8} dx$

Optimal result

Integrand size = 26, antiderivative size = 135

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^8} dx = \frac{3c(4bB+Ac)\sqrt{bx^2+cx^4}}{8bx} - \frac{(4bB+Ac)(bx^2+cx^4)^{3/2}}{8bx^5} - \frac{A(bx^2+cx^4)^{5/2}}{4bx^9} - \frac{3c(4bB+Ac)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}$$

```
[Out] -1/8*(A*c+4*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^5-1/4*A*(c*x^4+b*x^2)^(5/2)/b/x^9-
3/8*c*(A*c+4*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(1/2)+3/8*c*(A*c
+4*B*b)*(c*x^4+b*x^2)^(1/2)/b/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2063, 2045, 2046, 2033, 212}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^8} dx = -\frac{3c(Ac+4bB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}} + \frac{3c\sqrt{bx^2+cx^4}(Ac+4bB)}{8bx} - \frac{(bx^2+cx^4)^{3/2}(Ac+4bB)}{8bx^5} - \frac{A(bx^2+cx^4)^{5/2}}{4bx^9}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^8,x]
```

```
[Out] (3*c*(4*b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(8*b*x) - ((4*b*B + A*c)*(b*x^2 + c
*x^4)^(3/2))/(8*b*x^5) - (A*(b*x^2 + c*x^4)^(5/2))/(4*b*x^9) - (3*c*(4*b*B
+ A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[b])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\text{integral} = -\frac{A(bx^2+cx^4)^{5/2}}{4bx^9} - \frac{(-4bB-Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^6}\,dx}{4b}$$

$$= -\frac{(4bB+Ac)\,(bx^2+cx^4)^{3/2}}{8bx^5} - \frac{A(bx^2+cx^4)^{5/2}}{4bx^9} + \frac{(3c(4bB+Ac))\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx}{8b}$$

$$
\begin{aligned}
&= \frac{3c(4bB+Ac)\sqrt{bx^2+cx^4}}{8bx} - \frac{(4bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{8bx^5} \\
&\quad - \frac{A\left(bx^2+cx^4\right)^{5/2}}{4bx^9} + \frac{1}{8}(3c(4bB+Ac))\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx \\
&= \frac{3c(4bB+Ac)\sqrt{bx^2+cx^4}}{8bx} - \frac{(4bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{8bx^5} - \frac{A\left(bx^2+cx^4\right)^{5/2}}{4bx^9} \\
&\quad - \frac{1}{8}(3c(4bB+Ac))\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right) \\
&= \frac{3c(4bB+Ac)\sqrt{bx^2+cx^4}}{8bx} - \frac{(4bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{8bx^5} \\
&\quad - \frac{A\left(bx^2+cx^4\right)^{5/2}}{4bx^9} - \frac{3c(4bB+Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.84

$$
\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^8}\,dx = -\frac{\sqrt{x^2\left(b+cx^2\right)}\left(\sqrt{b}\sqrt{b+cx^2}\left(2Ab+4bBx^2+5Acx^2-8Bcx^4\right)+3c(4bB+Ac)x^4\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{8\sqrt{b}x^5\sqrt{b+cx^2}}
$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^8,x]
[Out] -1/8*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*Sqrt[b + c*x^2]*(2*A*b + 4*b*B*x^2 + 5
*A*c*x^2 - 8*B*c*x^4) + 3*c*(4*b*B + A*c)*x^4*ArcTanh[Sqrt[b + c*x^2]/Sqrt[
b]]))/(Sqrt[b]*x^5*Sqrt[b + c*x^2])
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\left(5Ac\,x^2+4bB\,x^2+2Ab\right)\sqrt{x^2\left(c\,x^2+b\right)}}{8x^5}+\frac{c\left(8B\sqrt{c\,x^2+b}-\frac{(3Ac+12Bb)\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)}{\sqrt{b}}\right)\sqrt{x^2\left(c\,x^2+b\right)}}{8x\sqrt{c\,x^2+b}}$
default	$-\frac{\left(x^4c+b\,x^2\right)^{\frac{3}{2}}\left(-A\left(c\,x^2+b\right)^{\frac{3}{2}}c^2x^4+3A\,b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)c^2x^4-4B\left(c\,x^2+b\right)^{\frac{3}{2}}bc\,x^4+12B\,b^{\frac{5}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)c\,x^4+A(c\,x\right.}{8x^7\left(c\,x^2+b\right)^{\frac{3}{2}}b^2}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(5*A*c*x^2+4*B*b*x^2+2*A*b)/x^5*(x^2*(c*x^2+b))^(1/2)+1/8*c*(8*B*(c*x^
2+b)^(1/2)-(3*A*c+12*B*b)/b^(1/2)*ln((2*b+2*b^(1/2)*(c*x^2+b)^(1/2))/x))*(x
^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.61

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^8}\,dx = \left[\frac{3\,(4\,Bbc+Ac^2)\sqrt{b}x^5\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,(8\,Bbcx^4-2\,Ab^2\cdots}{16\,bx^5}\right.$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(4*B*b*c + A*c^2)*sqrt(b)*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 +
 b*x^2)*sqrt(b))/x^3) + 2*(8*B*b*c*x^4 - 2*A*b^2 - (4*B*b^2 + 5*A*b*c)*x^2)
*sqrt(c*x^4 + b*x^2))/(b*x^5), 1/8*(3*(4*B*b*c + A*c^2)*sqrt(-b)*x^5*arctan
(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (8*B*b*c*x^4 - 2*A*b^2 - (4*
B*b^2 + 5*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b*x^5)]
```

Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^8}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}\,(A+Bx^2)}{x^8}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**8,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**8, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^8}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^8}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^8, x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^8}\,dx = \frac{8\sqrt{cx^2+b}Bc^2\mathrm{sgn}(x) + \frac{3\,(4\,Bbc^2\mathrm{sgn}(x)+Ac^3\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{4\,(cx^2+b)^{\frac{3}{2}}Bbc^2}{}}{8\,c}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/8*(8*sqrt(c*x^2 + b)*B*c^2*sgn(x) + 3*(4*B*b*c^2*sgn(x) + A*c^3*sgn(x))*a
rctan(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b) - (4*(c*x^2 + b)^(3/2)*B*b*c^2*sgn
(x) - 4*sqrt(c*x^2 + b)*B*b^2*c^2*sgn(x) + 5*(c*x^2 + b)^(3/2)*A*c^3*sgn(x)
 - 3*sqrt(c*x^2 + b)*A*b*c^3*sgn(x))/(c^2*x^4))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^8}\,dx = \int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2)^{3/2}}{x^8}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^8,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^8, x)
```

3.126 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{10}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 140

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{10}}\,dx = -\frac{c(6bB-Ac)\sqrt{bx^2+cx^4}}{16bx^3} - \frac{(6bB-Ac)(bx^2+cx^4)^{3/2}}{24bx^7} - \frac{A(bx^2+cx^4)^{5/2}}{6bx^{11}} - \frac{c^2(6bB-Ac)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}}$$

```
[Out] -1/24*(-A*c+6*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^7-1/6*A*(c*x^4+b*x^2)^(5/2)/b/x^
11-1/16*c^2*(-A*c+6*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(3/2)-1/1
6*c*(-A*c+6*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2063, 2045, 2033, 212}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{10}}\,dx = -\frac{c^2(6bB-Ac)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}} - \frac{(bx^2+cx^4)^{3/2}(6bB-Ac)}{24bx^7} - \frac{c\sqrt{bx^2+cx^4}(6bB-Ac)}{16bx^3} - \frac{A(bx^2+cx^4)^{5/2}}{6bx^{11}}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^10,x]
```

```
[Out] -1/16*(c*(6*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(b*x^3) - ((6*b*B - A*c)*(b*x^2
 + c*x^4)^(3/2))/(24*b*x^7) - (A*(b*x^2 + c*x^4)^(5/2))/(6*b*x^11) - (c^2*(
6*b*B - A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(16*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A(bx^2+cx^4)^{5/2}}{6bx^{11}} - \frac{(-6bB+Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^8}\,dx}{6b} \\
&= -\frac{(6bB-Ac)\,(bx^2+cx^4)^{3/2}}{24bx^7} - \frac{A(bx^2+cx^4)^{5/2}}{6bx^{11}} + \frac{(c(6bB-Ac))\int \frac{\sqrt{bx^2+cx^4}}{x^4}\,dx}{8b} \\
&= -\frac{c(6bB-Ac)\sqrt{bx^2+cx^4}}{16bx^3} - \frac{(6bB-Ac)\,(bx^2+cx^4)^{3/2}}{24bx^7} \\
&\quad - \frac{A(bx^2+cx^4)^{5/2}}{6bx^{11}} + \frac{(c^2(6bB-Ac))\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{16b}
\end{aligned}$$

$$= -\frac{c(6bB-Ac)\sqrt{bx^2+cx^4}}{16bx^3} - \frac{(6bB-Ac)\left(bx^2+cx^4\right)^{3/2}}{24bx^7} - \frac{A(bx^2+cx^4)^{5/2}}{6bx^{11}} - \frac{(c^2(6bB-Ac))\,\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)}{16b}$$

$$= -\frac{c(6bB-Ac)\sqrt{bx^2+cx^4}}{16bx^3} - \frac{(6bB-Ac)\left(bx^2+cx^4\right)^{3/2}}{24bx^7} - \frac{A(bx^2+cx^4)^{5/2}}{6bx^{11}} - \frac{c^2(6bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.94

$$\int\frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{10}}\,dx = \frac{\sqrt{x^2\left(b+cx^2\right)}\left(-\sqrt{b}\sqrt{b+cx^2}(6bBx^2(2b+5cx^2)+A(8b^2+14bcx^2+3c^2\right.}{48b^{3/2}x^7\sqrt{b+cx^2}}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^10,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-(Sqrt[b]*Sqrt[b + c*x^2]*(6*b*B*x^2*(2*b + 5*c*x^2
) + A*(8*b^2 + 14*b*c*x^2 + 3*c^2*x^4))) + 3*c^2*(-6*b*B + A*c)*x^6*ArcTanh
[Sqrt[b + c*x^2]/Sqrt[b]]))/(48*b^(3/2)*x^7*Sqrt[b + c*x^2])
```

Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{\left(3A\,c^2x^4+30x^4Bbc+14Abc\,x^2+12b^2B\,x^2+8b^2A\right)\sqrt{x^2(c\,x^2+b)}}{48x^7b}+\frac{(Ac-6Bb)c^2\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)\sqrt{x^2(c\,x^2+b)}}{16b^{\frac{3}{2}}x\sqrt{c\,x^2+b}}$
default	$-\frac{(x^4c+b\,x^2)^{\frac{3}{2}}\left(A(c\,x^2+b)^{\frac{3}{2}}c^3x^6-3A\,b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)c^3x^6-6B(c\,x^2+b)^{\frac{3}{2}}b\,c^2x^6+18B\,b^{\frac{5}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)c^2x^6-A(c\right.}{48x^9}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(3*A*c^2*x^4+30*B*b*c*x^4+14*A*b*c*x^2+12*B*b^2*x^2+8*A*b^2)/x^7/b*(x
^2*(c*x^2+b))^(1/2)+1/16*(A*c-6*B*b)*c^2/b^(3/2)*ln((2*b+2*b^(1/2)*(c*x^2+b
)^(1/2))/x)*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.79

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{10}}\,dx = \left[-\frac{3\,(6\,Bbc^2-Ac^3)\sqrt{b}x^7\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,(3\,(10\,Bb^2c+Ab}{96\,b^2x^7}\right.$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(6*B*b*c^2 - A*c^3)*sqrt(b)*x^7*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^
4 + b*x^2)*sqrt(b))/x^3) + 2*(3*(10*B*b^2*c + A*b*c^2)*x^4 + 8*A*b^3 + 2*(6
*B*b^3 + 7*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b^2*x^7), 1/48*(3*(6*B*b*c^2
 - A*c^3)*sqrt(-b)*x^7*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) -
 (3*(10*B*b^2*c + A*b*c^2)*x^4 + 8*A*b^3 + 2*(6*B*b^3 + 7*A*b^2*c)*x^2)*sqr
t(c*x^4 + b*x^2))/(b^2*x^7)]
```

Sympy [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{10}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}\,(A+Bx^2)}{x^{10}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**10,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**10, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{10}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{10}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^10, x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.25

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{10}}\,dx = \frac{\frac{3\left(6\,Bbc^3\mathrm{sgn}(x)-Ac^4\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b} - \frac{30\left(cx^2+b\right)^{\frac{5}{2}}Bbc^3\mathrm{sgn}(x)-48\left(cx^2+b\right)^{\frac{3}{2}}Bb^2c^3\mathrm{sg}}{}}{4}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] 1/48*(3*(6*B*b*c^3*sgn(x) - A*c^4*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-b))/
(sqrt(-b)*b) - (30*(c*x^2 + b)^(5/2)*B*b*c^3*sgn(x) - 48*(c*x^2 + b)^(3/2)*
B*b^2*c^3*sgn(x) + 18*sqrt(c*x^2 + b)*B*b^3*c^3*sgn(x) + 3*(c*x^2 + b)^(5/2
)*A*c^4*sgn(x) + 8*(c*x^2 + b)^(3/2)*A*b*c^4*sgn(x) - 3*sqrt(c*x^2 + b)*A*b
^2*c^4*sgn(x))/(b*c^3*x^6))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{10}}\,dx = \int \frac{\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{10}}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^10,x)
```

```
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^10, x)
```

3.127 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{12}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 177

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{12}}\,dx = -\frac{c(8bB-3Ac)\sqrt{bx^2+cx^4}}{64bx^5} - \frac{c^2(8bB-3Ac)\sqrt{bx^2+cx^4}}{128b^2x^3} - \frac{(8bB-3Ac)(bx^2+cx^4)^{3/2}}{48bx^9} - \frac{A(bx^2+cx^4)^{5/2}}{8bx^{13}} + \frac{c^3(8bB-3Ac)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}}$$

```
[Out] -1/48*(-3*A*c+8*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^9-1/8*A*(c*x^4+b*x^2)^(5/2)/b/
x^13+1/128*c^3*(-3*A*c+8*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(5/2
)-1/64*c*(-3*A*c+8*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^5-1/128*c^2*(-3*A*c+8*B*b)*
(c*x^4+b*x^2)^(1/2)/b^2/x^3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2063, 2045, 2050, 2033, 212}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{12}}\,dx = \frac{c^3(8bB-3Ac)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}} - \frac{c^2\sqrt{bx^2+cx^4}(8bB-3Ac)}{128b^2x^3} - \frac{(bx^2+cx^4)^{3/2}(8bB-3Ac)}{48bx^9} - \frac{c\sqrt{bx^2+cx^4}(8bB-3Ac)}{64bx^5} - \frac{A(bx^2+cx^4)^{5/2}}{8bx^{13}}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^12,x]
```

```
[Out] -1/64*(c*(8*b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(b*x^5) - (c^2*(8*b*B - 3*A*c
)*Sqrt[b*x^2 + c*x^4])/(128*b^2*x^3) - ((8*b*B - 3*A*c)*(b*x^2 + c*x^4)^(3/
2))/(48*b*x^9) - (A*(b*x^2 + c*x^4)^(5/2))/(8*b*x^13) + (c^3*(8*b*B - 3*A*c
)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(128*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\text{integral} = -\frac{A(bx^2+cx^4)^{5/2}}{8bx^{13}} - \frac{(-8bB+3Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^{10}}\,dx}{8b}$$

$$= -\frac{(8bB-3Ac)\,(bx^2+cx^4)^{3/2}}{48bx^9} - \frac{A(bx^2+cx^4)^{5/2}}{8bx^{13}} + \frac{(c(8bB-3Ac))\int \frac{\sqrt{bx^2+cx^4}}{x^6}\,dx}{16b}$$

$$= -\frac{c(8bB-3Ac)\sqrt{bx^2+cx^4}}{64bx^5} - \frac{(8bB-3Ac)\,(bx^2+cx^4)^{3/2}}{48bx^9} - \frac{A(bx^2+cx^4)^{5/2}}{8bx^{13}} + \frac{(c^2(8bB-3Ac))\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{64b}$$

$$= -\frac{c(8bB-3Ac)\sqrt{bx^2+cx^4}}{64bx^5} - \frac{c^2(8bB-3Ac)\sqrt{bx^2+cx^4}}{128b^2x^3} - \frac{(8bB-3Ac)\,(bx^2+cx^4)^{3/2}}{48bx^9} - \frac{A(bx^2+cx^4)^{5/2}}{8bx^{13}} - \frac{(c^3(8bB-3Ac))\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{128b^2}$$

$$= -\frac{c(8bB-3Ac)\sqrt{bx^2+cx^4}}{64bx^5} - \frac{c^2(8bB-3Ac)\sqrt{bx^2+cx^4}}{128b^2x^3} - \frac{(8bB-3Ac)\,(bx^2+cx^4)^{3/2}}{48bx^9} - \frac{A(bx^2+cx^4)^{5/2}}{8bx^{13}} + \frac{(c^3(8bB-3Ac))\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)}{128b^2}$$

$$= -\frac{c(8bB-3Ac)\sqrt{bx^2+cx^4}}{64bx^5} - \frac{c^2(8bB-3Ac)\sqrt{bx^2+cx^4}}{128b^2x^3} - \frac{(8bB-3Ac)\,(bx^2+cx^4)^{3/2}}{48bx^9} - \frac{A(bx^2+cx^4)^{5/2}}{8bx^{13}} + \frac{c^3(8bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{12}}\,dx =$$

$$-\frac{\sqrt{x^2\,(b+cx^2)}\Big(\sqrt{b}\sqrt{b+cx^2}(8bBx^2(8b^2+14bcx^2+3c^2x^4)+A(48b^3+72b^2cx^2+6bc^2x^4-9c^3x^6))+3c^3(-}{384b^{5/2}x^9\sqrt{b+cx^2}}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^12,x]
```

```
[Out] -1/384*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*Sqrt[b + c*x^2]*(8*b*B*x^2*(8*b^2 +
14*b*c*x^2 + 3*c^2*x^4) + A*(48*b^3 + 72*b^2*c*x^2 + 6*b*c^2*x^4 - 9*c^3*x^
6)) + 3*c^3*(-8*b*B + 3*A*c)*x^8*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]))/(b^(5/2
)*x^9*Sqrt[b + c*x^2])
```

Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{(-9A\,c^3x^6+24x^6Bb\,c^2+6Ab\,c^2x^4+112x^4B\,b^2c+72A\,b^2c\,x^2+64b^3B\,x^2+48b^3A)\sqrt{x^2(c\,x^2+b)}}{384x^9b^2}-\frac{(3Ac-8Bb)c^3\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2}}{x}\right.}{128b^{\frac{5}{2}}x\sqrt{c\,x^2+}}$
default	$-\frac{(x^4c+b\,x^2)^{\frac{3}{2}}\left(-3A(c\,x^2+b)^{\frac{3}{2}}c^4x^8+9A\,b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)c^4x^8+8B(c\,x^2+b)^{\frac{3}{2}}b\,c^3x^8-24B\,b^{\frac{5}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)c^3x^8+\right.}{}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] -1/384*(-9*A*c^3*x^6+24*B*b*c^2*x^6+6*A*b*c^2*x^4+112*B*b^2*c*x^4+72*A*b^2*
c*x^2+64*B*b^3*x^2+48*A*b^3)/x^9/b^2*(x^2*(c*x^2+b))^(1/2)-1/128*(3*A*c-8*B
*b)*c^3/b^(5/2)*ln((2*b+2*b^(1/2)*(c*x^2+b)^(1/2))/x)*(x^2*(c*x^2+b))^(1/2)
/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.59 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.69

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{12}}\,dx=\left[-\frac{3\,(8\,Bbc^3-3\,Ac^4)\sqrt{b}x^9\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,(3\,(8\,Bb^2c^2-\right.$$

$$-\frac{3\,(8\,Bbc^3-3\,Ac^4)\sqrt{-b}x^9\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)+(3\,(8\,Bb^2c^2-3\,Abc^3)x^6+48\,Ab^4+2\,(56\,Bb^3c+3\,Ab^2c}{384\,b^3x^9}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(8*B*b*c^3 - 3*A*c^4)*sqrt(b)*x^9*log(-(c*x^3 + 2*b*x - 2*sqrt(c
*x^4 + b*x^2)*sqrt(b))/x^3) + 2*(3*(8*B*b^2*c^2 - 3*A*b*c^3)*x^6 + 48*A*b^4
 + 2*(56*B*b^3*c + 3*A*b^2*c^2)*x^4 + 8*(8*B*b^4 + 9*A*b^3*c)*x^2)*sqrt(c*x
^4 + b*x^2))/(b^3*x^9), -1/384*(3*(8*B*b*c^3 - 3*A*c^4)*sqrt(-b)*x^9*arctan
(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (3*(8*B*b^2*c^2 - 3*A*b*c^3)
*x^6 + 48*A*b^4 + 2*(56*B*b^3*c + 3*A*b^2*c^2)*x^4 + 8*(8*B*b^4 + 9*A*b^3*c
)*x^2)*sqrt(c*x^4 + b*x^2))/(b^3*x^9)]
```

Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{12}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}(A+Bx^2)}{x^{12}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**12,x)
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**12, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{12}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{12}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^12, x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{12}}\,dx = -\frac{\frac{3\,(8\,Bbc^4\mathrm{sgn}(x)-3\,Ac^5\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2} + \frac{24\,(cx^2+b)^{\frac{7}{2}}Bbc^4\mathrm{sgn}(x)+40\,(cx^2+b)^{\frac{5}{2}}Bb^2c^4\mathrm{sgn}(x)-88\,(cx^2+b)^{\frac{3}{2}}Bb^3c^4\mathrm{sgn}(x)+24\sqrt{cx^2+b}}{}}{384\,c}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^12,x, algorithm="giac")
[Out] -1/384*(3*(8*B*b*c^4*sgn(x) - 3*A*c^5*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-
b))/(sqrt(-b)*b^2) + (24*(c*x^2 + b)^(7/2)*B*b*c^4*sgn(x) + 40*(c*x^2 + b)^
(5/2)*B*b^2*c^4*sgn(x) - 88*(c*x^2 + b)^(3/2)*B*b^3*c^4*sgn(x) + 24*sqrt(c*
x^2 + b)*B*b^4*c^4*sgn(x) - 9*(c*x^2 + b)^(7/2)*A*c^5*sgn(x) + 33*(c*x^2 +
b)^(5/2)*A*b*c^5*sgn(x) + 33*(c*x^2 + b)^(3/2)*A*b^2*c^5*sgn(x) - 9*sqrt(c*
x^2 + b)*A*b^3*c^5*sgn(x))/(b^2*c^4*x^8))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{12}}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{12}}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^12,x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^12, x)
```

3.128 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{14}} dx$

Optimal result

Integrand size = 26, antiderivative size = 214

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{14}} dx = -\frac{c(2bB-Ac)\sqrt{bx^2+cx^4}}{32bx^7} - \frac{c^2(2bB-Ac)\sqrt{bx^2+cx^4}}{128b^2x^5} + \frac{3c^3(2bB-Ac)\sqrt{bx^2+cx^4}}{256b^3x^3} - \frac{(2bB-Ac)(bx^2+cx^4)^{3/2}}{16bx^{11}} - \frac{A(bx^2+cx^4)^{5/2}}{10bx^{15}} - \frac{3c^4(2bB-Ac)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{256b^{7/2}}$$

```
[Out] -1/16*(-A*c+2*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^11-1/10*A*(c*x^4+b*x^2)^(5/2)/b/
x^15-3/256*c^4*(-A*c+2*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(7/2)-
1/32*c*(-A*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^7-1/128*c^2*(-A*c+2*B*b)*(c*x^4
+b*x^2)^(1/2)/b^2/x^5+3/256*c^3*(-A*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2063, 2045, 2050, 2033, 212}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{14}} dx = -\frac{3c^4(2bB-Ac)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{256b^{7/2}} + \frac{3c^3\sqrt{bx^2+cx^4}(2bB-Ac)}{256b^3x^3} - \frac{c^2\sqrt{bx^2+cx^4}(2bB-Ac)}{128b^2x^5} - \frac{(bx^2+cx^4)^{3/2}(2bB-Ac)}{16bx^{11}} - \frac{c\sqrt{bx^2+cx^4}(2bB-Ac)}{32bx^7} - \frac{A(bx^2+cx^4)^{5/2}}{10bx^{15}}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^14,x]
```

```
[Out] -1/32*(c*(2*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(b*x^7) - (c^2*(2*b*B - A*c)*Sq
rt[b*x^2 + c*x^4])/(128*b^2*x^5) + (3*c^3*(2*b*B - A*c)*Sqrt[b*x^2 + c*x^4]
)/(256*b^3*x^3) - ((2*b*B - A*c)*(b*x^2 + c*x^4)^(3/2))/(16*b*x^11) - (A*(b
*x^2 + c*x^4)^(5/2))/(10*b*x^15) - (3*c^4*(2*b*B - A*c)*ArcTanh[(Sqrt[b]*x)
/Sqrt[b*x^2 + c*x^4]])/(256*b^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
```

0]

Rubi steps

$$\text{integral} = -\frac{A(bx^2+cx^4)^{5/2}}{10bx^{15}} - \frac{(-10bB+5Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^{12}}\,dx}{10b}$$

$$= -\frac{(2bB-Ac)\,(bx^2+cx^4)^{3/2}}{16bx^{11}} - \frac{A(bx^2+cx^4)^{5/2}}{10bx^{15}} + \frac{(3c(2bB-Ac))\int \frac{\sqrt{bx^2+cx^4}}{x^8}\,dx}{16b}$$

$$= -\frac{c(2bB-Ac)\sqrt{bx^2+cx^4}}{32bx^7} - \frac{(2bB-Ac)\,(bx^2+cx^4)^{3/2}}{16bx^{11}} - \frac{A(bx^2+cx^4)^{5/2}}{10bx^{15}} + \frac{(c^2(2bB-Ac))\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx}{32b}$$

$$= -\frac{c(2bB-Ac)\sqrt{bx^2+cx^4}}{32bx^7} - \frac{c^2(2bB-Ac)\sqrt{bx^2+cx^4}}{128b^2x^5} - \frac{(2bB-Ac)\,(bx^2+cx^4)^{3/2}}{16bx^{11}} - \frac{A(bx^2+cx^4)^{5/2}}{10bx^{15}} - \frac{(3c^3(2bB-Ac))\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{128b^2}$$

$$= -\frac{c(2bB-Ac)\sqrt{bx^2+cx^4}}{32bx^7} - \frac{c^2(2bB-Ac)\sqrt{bx^2+cx^4}}{128b^2x^5} + \frac{3c^3(2bB-Ac)\sqrt{bx^2+cx^4}}{256b^3x^3} - \frac{(2bB-Ac)\,(bx^2+cx^4)^{3/2}}{16bx^{11}} - \frac{A(bx^2+cx^4)^{5/2}}{10bx^{15}} + \frac{(3c^4(2bB-Ac))\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{256b^3}$$

$$= -\frac{c(2bB-Ac)\sqrt{bx^2+cx^4}}{32bx^7} - \frac{c^2(2bB-Ac)\sqrt{bx^2+cx^4}}{128b^2x^5} + \frac{3c^3(2bB-Ac)\sqrt{bx^2+cx^4}}{256b^3x^3} - \frac{(2bB-Ac)\,(bx^2+cx^4)^{3/2}}{16bx^{11}} - \frac{A(bx^2+cx^4)^{5/2}}{10bx^{15}} - \frac{(3c^4(2bB-Ac))\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)}{256b^3}$$

$$= -\frac{c(2bB-Ac)\sqrt{bx^2+cx^4}}{32bx^7} - \frac{c^2(2bB-Ac)\sqrt{bx^2+cx^4}}{128b^2x^5} + \frac{3c^3(2bB-Ac)\sqrt{bx^2+cx^4}}{256b^3x^3} - \frac{(2bB-Ac)\,(bx^2+cx^4)^{3/2}}{16bx^{11}} - \frac{A(bx^2+cx^4)^{5/2}}{10bx^{15}} - \frac{3c^4(2bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{256b^{7/2}}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{14}}\,dx = \frac{-\sqrt{b}(b+cx^2)\left(10bBx^2(16b^3+24b^2cx^2+2bc^2x^4-3c^3x^6)+A(128b^4+17\ldots\right.}{1280\ldots}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^14,x]
```

```
[Out] (-(Sqrt[b]*(b + c*x^2)*(10*b*B*x^2*(16*b^3 + 24*b^2*c*x^2 + 2*b*c^2*x^4 - 3
*c^3*x^6) + A*(128*b^4 + 176*b^3*c*x^2 + 8*b^2*c^2*x^4 - 10*b*c^3*x^6 + 15*
c^4*x^8))) + 15*c^4*(-2*b*B + A*c)*x^10*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*
x^2]/Sqrt[b]])/(1280*b^(7/2)*x^9*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 3.20 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{(15Ax^8c^4-30Bx^8bc^3-10Ax^6bc^3+20Bx^6b^2c^2+8Ab^2c^2x^4+240Bb^3cx^4+176Ax^2b^3c+160Bx^2b^4+128Ab^4)\sqrt{x^2(cx^2+b)}}{1280x^{11}b^3}+\ldots$
default	$\frac{(x^4c+bx^2)^{\frac{3}{2}}\left(15Ab^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\sqrt{cx^2+b}}{x}\right)c^5x^{10}-5A(cx^2+b)^{\frac{3}{2}}c^5x^{10}-30Bb^{\frac{5}{2}}\ln\left(\frac{2b+2\sqrt{b}\sqrt{cx^2+b}}{x}\right)c^4x^{10}+10B(cx^2+b)^{\frac{3}{2}}bc^4x^{10}\ldots\right.}{\ldots}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^14,x,method=_RETURNVERBOSE)
```

```
[Out] -1/1280*(15*A*c^4*x^8-30*B*b*c^3*x^8-10*A*b*c^3*x^6+20*B*b^2*c^2*x^6+8*A*b^
2*c^2*x^4+240*B*b^3*c*x^4+176*A*b^3*c*x^2+160*B*b^4*x^2+128*A*b^4)/x^11/b^3
*(x^2*(c*x^2+b))^(1/2)+3/256*(A*c-2*B*b)*c^4/b^(7/2)*ln((2*b+2*b^(1/2)*(c*x
^2+b)^(1/2))/x)*(x^2*(c*x^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.61

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{14}}\,dx = \left[-\frac{15\,(2\,Bbc^4-Ac^5)\sqrt{b}x^{11}\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)-2\,(15\,(2\,Bb^2c^3\ldots}{\ldots}\right.$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(2*B*b*c^4 - A*c^5)*sqrt(b)*x^11*log(-(c*x^3 + 2*b*x + 2*sqrt(
c*x^4 + b*x^2)*sqrt(b))/x^3) - 2*(15*(2*B*b^2*c^3 - A*b*c^4)*x^8 - 10*(2*B*
b^3*c^2 - A*b^2*c^3)*x^6 - 128*A*b^5 - 8*(30*B*b^4*c + A*b^3*c^2)*x^4 - 16*
(10*B*b^5 + 11*A*b^4*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b^4*x^11), 1/1280*(15*(2
*B*b*c^4 - A*c^5)*sqrt(-b)*x^11*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3
+ b*x)) + (15*(2*B*b^2*c^3 - A*b*c^4)*x^8 - 10*(2*B*b^3*c^2 - A*b^2*c^3)*x^
6 - 128*A*b^5 - 8*(30*B*b^4*c + A*b^3*c^2)*x^4 - 16*(10*B*b^5 + 11*A*b^4*c)
*x^2)*sqrt(c*x^4 + b*x^2))/(b^4*x^11)]
```

Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{14}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}(A+Bx^2)}{x^{14}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**14,x)
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**14, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{14}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{14}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^14, x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{14}}\,dx = \frac{\frac{15\,(2\,Bbc^5\mathrm{sgn}(x)-Ac^6\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^3} + \frac{30\,(cx^2+b)^{\frac{9}{2}}Bbc^5\mathrm{sgn}(x)-140\,(cx^2+b)^{\frac{7}{2}}Bb^2c^5\mathrm{sg}\cdots}{\cdots}}{\cdots}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^14,x, algorithm="giac")
[Out] 1/1280*(15*(2*B*b*c^5*sgn(x) - A*c^6*sgn(x))*arctan(sqrt(c*x^2 + b)/sqrt(-b
))/(sqrt(-b)*b^3) + (30*(c*x^2 + b)^(9/2)*B*b*c^5*sgn(x) - 140*(c*x^2 + b)^
(7/2)*B*b^2*c^5*sgn(x) + 140*(c*x^2 + b)^(3/2)*B*b^4*c^5*sgn(x) - 30*sqrt(c
*x^2 + b)*B*b^5*c^5*sgn(x) - 15*(c*x^2 + b)^(9/2)*A*c^6*sgn(x) + 70*(c*x^2
+ b)^(7/2)*A*b*c^6*sgn(x) - 128*(c*x^2 + b)^(5/2)*A*b^2*c^6*sgn(x) - 70*(c*
x^2 + b)^(3/2)*A*b^3*c^6*sgn(x) + 15*sqrt(c*x^2 + b)*A*b^4*c^6*sgn(x))/(b^3
*c^5*x^10))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{14}}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{14}}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^14,x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^14, x)
```

3.129 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{16}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 251

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{16}}\,dx = -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{c^2(12bB-7Ac)\sqrt{bx^2+cx^4}}{1920b^2x^7} + \frac{c^3(12bB-7Ac)\sqrt{bx^2+cx^4}}{1536b^3x^5} - \frac{c^4(12bB-7Ac)\sqrt{bx^2+cx^4}}{1024b^4x^3} - \frac{(12bB-7Ac)(bx^2+cx^4)^{3/2}}{120bx^{13}} - \frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}} + \frac{c^5(12bB-7Ac)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{1024b^{9/2}}$$

```
[Out] -1/120*(-7*A*c+12*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^13-1/12*A*(c*x^4+b*x^2)^(5/2
)/b/x^17+1/1024*c^5*(-7*A*c+12*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/
b^(9/2)-1/320*c*(-7*A*c+12*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^9-1/1920*c^2*(-7*A*
c+12*B*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^7+1/1536*c^3*(-7*A*c+12*B*b)*(c*x^4+b*x
^2)^(1/2)/b^3/x^5-1/1024*c^4*(-7*A*c+12*B*b)*(c*x^4+b*x^2)^(1/2)/b^4/x^3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used

$= \{2063, 2045, 2050, 2033, 212\}$

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{16}}\,dx = \frac{c^5(12bB-7Ac)\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{1024b^{9/2}} - \frac{c^4\sqrt{bx^2+cx^4}(12bB-7Ac)}{1024b^4x^3} + \frac{c^3\sqrt{bx^2+cx^4}(12bB-7Ac)}{1536b^3x^5} - \frac{c^2\sqrt{bx^2+cx^4}(12bB-7Ac)}{1920b^2x^7} - \frac{(bx^2+cx^4)^{3/2}(12bB-7Ac)}{120bx^{13}} - \frac{c\sqrt{bx^2+cx^4}(12bB-7Ac)}{320bx^9} - \frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^16,x]
```

```
[Out] -1/320*(c*(12*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(b*x^9) - (c^2*(12*b*B - 7*
A*c)*Sqrt[b*x^2 + c*x^4])/(1920*b^2*x^7) + (c^3*(12*b*B - 7*A*c)*Sqrt[b*x^2
 + c*x^4])/(1536*b^3*x^5) - (c^4*(12*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(102
4*b^4*x^3) - ((12*b*B - 7*A*c)*(b*x^2 + c*x^4)^(3/2))/(120*b*x^13) - (A*(b*
x^2 + c*x^4)^(5/2))/(12*b*x^17) + (c^5*(12*b*B - 7*A*c)*ArcTanh[(Sqrt[b]*x)
/Sqrt[b*x^2 + c*x^4]])/(1024*b^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
```

```
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}} - \frac{(-12bB+7Ac)\int \frac{(bx^2+cx^4)^{3/2}}{x^{14}}\,dx}{12b}\\
&= -\frac{(12bB-7Ac)\left(bx^2+cx^4\right)^{3/2}}{120bx^{13}} - \frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}} + \frac{(c(12bB-7Ac))\int \frac{\sqrt{bx^2+cx^4}}{x^{10}}\,dx}{40b}\\
&= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{(12bB-7Ac)\left(bx^2+cx^4\right)^{3/2}}{120bx^{13}}\\
&\quad - \frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}} + \frac{(c^2(12bB-7Ac))\int \frac{1}{x^6\sqrt{bx^2+cx^4}}\,dx}{320b}\\
&= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{c^2(12bB-7Ac)\sqrt{bx^2+cx^4}}{1920b^2x^7}\\
&\quad - \frac{(12bB-7Ac)\left(bx^2+cx^4\right)^{3/2}}{120bx^{13}} - \frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}}\\
&\quad - \frac{(c^3(12bB-7Ac))\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx}{384b^2}\\
&= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{c^2(12bB-7Ac)\sqrt{bx^2+cx^4}}{1920b^2x^7} + \frac{c^3(12bB-7Ac)\sqrt{bx^2+cx^4}}{1536b^3x^5}\\
&\quad - \frac{(12bB-7Ac)\left(bx^2+cx^4\right)^{3/2}}{120bx^{13}} - \frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}} + \frac{(c^4(12bB-7Ac))\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{512b^3}
\end{aligned}$$

$$= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{c^2(12bB-7Ac)\sqrt{bx^2+cx^4}}{1920b^2x^7} + \frac{c^3(12bB-7Ac)\sqrt{bx^2+cx^4}}{1536b^3x^5} - \frac{c^4(12bB-7Ac)\sqrt{bx^2+cx^4}}{1024b^4x^3} - \frac{(12bB-7Ac)\left(bx^2+cx^4\right)^{3/2}}{120bx^{13}} - \frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}} - \frac{(c^5(12bB-7Ac))\int\frac{1}{\sqrt{bx^2+cx^4}}\,dx}{1024b^4}$$

$$= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{c^2(12bB-7Ac)\sqrt{bx^2+cx^4}}{1920b^2x^7} + \frac{c^3(12bB-7Ac)\sqrt{bx^2+cx^4}}{1536b^3x^5} - \frac{c^4(12bB-7Ac)\sqrt{bx^2+cx^4}}{1024b^4x^3} - \frac{(12bB-7Ac)\left(bx^2+cx^4\right)^{3/2}}{120bx^{13}} - \frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}} + \frac{(c^5(12bB-7Ac))\,\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{1024b^4}$$

$$= -\frac{c(12bB-7Ac)\sqrt{bx^2+cx^4}}{320bx^9} - \frac{c^2(12bB-7Ac)\sqrt{bx^2+cx^4}}{1920b^2x^7} + \frac{c^3(12bB-7Ac)\sqrt{bx^2+cx^4}}{1536b^3x^5} - \frac{c^4(12bB-7Ac)\sqrt{bx^2+cx^4}}{1024b^4x^3} - \frac{(12bB-7Ac)\left(bx^2+cx^4\right)^{3/2}}{120bx^{13}} - \frac{A(bx^2+cx^4)^{5/2}}{12bx^{17}} + \frac{c^5(12bB-7Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{1024b^{9/2}}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.79

$$\int\frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{16}}\,dx = -\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b}\sqrt{b+cx^2}(12bBx^2(128b^4+176b^3cx^2+8b^2c^2x^4-10bc^3x^6+15c^4x^8)+A(1280b^5+1664\right.}{15360b^{9/2}x^{13}\sqrt{}}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^16,x]
```

```
[Out] -1/15360*(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*Sqrt[b + c*x^2]*(12*b*B*x^2*(128*b
^4 + 176*b^3*c*x^2 + 8*b^2*c^2*x^4 - 10*b*c^3*x^6 + 15*c^4*x^8) + A*(1280*b
^5 + 1664*b^4*c*x^2 + 48*b^3*c^2*x^4 - 56*b^2*c^3*x^6 + 70*b*c^4*x^8 - 105*
c^5*x^10)) + 15*c^5*(-12*b*B + 7*A*c)*x^12*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]]
))/(b^(9/2)*x^13*Sqrt[b + c*x^2])
```

Maple [A] (verified)

Time = 3.64 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{\left(-105 A \,c^{5} x^{10}+180 B b \,c^{4} x^{10}+70 A \,x^{8} b \,c^{4}-120 B \,b^{2} c^{3} x^{8}-56 A \,b^{2} c^{3} x^{6}+96 B \,b^{3} c^{2} x^{6}+48 A \,b^{3} c^{2} x^{4}+2112 B \,b^{4} c \,x^{4}+1664 A \,b^{4} c \,x^{2}+1536}{15360 x^{13} b^{4}}$
default	$-\frac{\left(x^{4} c+b \,x^{2}\right)^{\frac{3}{2}} \left(-35 A \left(c \,x^{2}+b \right)^{\frac{3}{2}} c^{6} x^{12}+105 A \,b^{\frac{3}{2}} \ln \left(\frac{2 b +2 \sqrt{b}\, \sqrt{c \,x^{2}+b}}{x}\right) c^{6} x^{12}+60 B \left(c \,x^{2}+b \right)^{\frac{3}{2}} b \,c^{5} x^{12}-180 B \,b^{\frac{5}{2}} \ln \left(\frac{2 b +2 \sqrt{b}\, \sqrt{c \,x^{2}+b}}{x}\right)}{}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^16,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15360*(-105*A*c^5*x^10+180*B*b*c^4*x^10+70*A*b*c^4*x^8-120*B*b^2*c^3*x^8
-56*A*b^2*c^3*x^6+96*B*b^3*c^2*x^6+48*A*b^3*c^2*x^4+2112*B*b^4*c*x^4+1664*A
*b^4*c*x^2+1536*B*b^5*x^2+1280*A*b^5)/x^13/b^4*(x^2*(c*x^2+b))^(1/2)-1/1024
*(7*A*c-12*B*b)*c^5/b^(9/2)*ln((2*b+2*b^(1/2)*(c*x^2+b)^(1/2))/x)*(x^2*(c*x
^2+b))^(1/2)/x/(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.57

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{16}}\,dx = \left[-\frac{15\,(12\,Bbc^5-7\,Ac^6)\sqrt{b}x^{13}\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,(15\,(12\,Bb^2c}{}\right.$$

$$-\frac{15\,(12\,Bbc^5-7\,Ac^6)\sqrt{-b}x^{13}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)+(15\,(12\,Bb^2c^4-7\,Abc^5)x^{10}-10\,(12\,Bb^3c^3-7\,Ab^2c^4)}{15360}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^16,x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(12*B*b*c^5 - 7*A*c^6)*sqrt(b)*x^13*log(-(c*x^3 + 2*b*x - 2*s
qrt(c*x^4 + b*x^2)*sqrt(b))/x^3) + 2*(15*(12*B*b^2*c^4 - 7*A*b*c^5)*x^10 -
10*(12*B*b^3*c^3 - 7*A*b^2*c^4)*x^8 + 1280*A*b^6 + 8*(12*B*b^4*c^2 - 7*A*b^
3*c^3)*x^6 + 48*(44*B*b^5*c + A*b^4*c^2)*x^4 + 128*(12*B*b^6 + 13*A*b^5*c)*
x^2)*sqrt(c*x^4 + b*x^2))/(b^5*x^13), -1/15360*(15*(12*B*b*c^5 - 7*A*c^6)*s
qrt(-b)*x^13*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + (15*(12*B
*b^2*c^4 - 7*A*b*c^5)*x^10 - 10*(12*B*b^3*c^3 - 7*A*b^2*c^4)*x^8 + 1280*A*b
^6 + 8*(12*B*b^4*c^2 - 7*A*b^3*c^3)*x^6 + 48*(44*B*b^5*c + A*b^4*c^2)*x^4 +
 128*(12*B*b^6 + 13*A*b^5*c)*x^2)*sqrt(c*x^4 + b*x^2))/(b^5*x^13)]
```

Sympy [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{16}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}\,(A+Bx^2)}{x^{16}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**16,x)
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**16, x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{16}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{16}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^16,x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^16, x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{16}}\,dx = -\frac{\frac{15\,(12\,Bbc^6\mathrm{sgn}(x)-7\,Ac^7\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4}+\frac{180\,(cx^2+b)^{\frac{11}{2}}Bbc^6\mathrm{sgn}(x)-1020\,(cx^2+b)^{\frac{9}{2}}Bb^2c^6\mathrm{sgn}(x)+2376\,(cx^2+b)^{\frac{7}{2}}Bb^3c^6\mathrm{sgn}(x)-}{}}{}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^16,x, algorithm="giac")
[Out] -1/15360*(15*(12*B*b*c^6*sgn(x) - 7*A*c^7*sgn(x))*arctan(sqrt(c*x^2 + b)/sq
rt(-b))/(sqrt(-b)*b^4) + (180*(c*x^2 + b)^(11/2)*B*b*c^6*sgn(x) - 1020*(c*x
^2 + b)^(9/2)*B*b^2*c^6*sgn(x) + 2376*(c*x^2 + b)^(7/2)*B*b^3*c^6*sgn(x) -
696*(c*x^2 + b)^(5/2)*B*b^4*c^6*sgn(x) - 1020*(c*x^2 + b)^(3/2)*B*b^5*c^6*s
gn(x) + 180*sqrt(c*x^2 + b)*B*b^6*c^6*sgn(x) - 105*(c*x^2 + b)^(11/2)*A*c^7
*sgn(x) + 595*(c*x^2 + b)^(9/2)*A*b*c^7*sgn(x) - 1386*(c*x^2 + b)^(7/2)*A*b
^2*c^7*sgn(x) + 1686*(c*x^2 + b)^(5/2)*A*b^3*c^7*sgn(x) + 595*(c*x^2 + b)^(
3/2)*A*b^4*c^7*sgn(x) - 105*sqrt(c*x^2 + b)*A*b^5*c^7*sgn(x))/(b^4*c^6*x^12
))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{16}}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{16}}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^16,x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^16, x)
```

3.130 $\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 176

$$\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{5b^2(7bB-8Ac)\sqrt{bx^2+cx^4}}{128c^4}+\frac{5b(7bB-8Ac)x^2\sqrt{bx^2+cx^4}}{192c^3} - \frac{(7bB-8Ac)x^4\sqrt{bx^2+cx^4}}{48c^2}+\frac{Bx^6\sqrt{bx^2+cx^4}}{8c} + \frac{5b^3(7bB-8Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{9/2}}$$

```
[Out] 5/128*b^3*(-8*A*c+7*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(9/2)-5
/128*b^2*(-8*A*c+7*B*b)*(c*x^4+b*x^2)^(1/2)/c^4+5/192*b*(-8*A*c+7*B*b)*x^2*
(c*x^4+b*x^2)^(1/2)/c^3-1/48*(-8*A*c+7*B*b)*x^4*(c*x^4+b*x^2)^(1/2)/c^2+1/8
*B*x^6*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2059, 808, 684, 654, 634, 212}

$$\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{5b^3(7bB-8Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{9/2}} - \frac{5b^2\sqrt{bx^2+cx^4}(7bB-8Ac)}{128c^4}+\frac{5bx^2\sqrt{bx^2+cx^4}(7bB-8Ac)}{192c^3} - \frac{x^4\sqrt{bx^2+cx^4}(7bB-8Ac)}{48c^2}+\frac{Bx^6\sqrt{bx^2+cx^4}}{8c}$$

```
[In] Int[(x^7*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-5*b^2*(7*b*B - 8*A*c)*Sqrt[b*x^2 + c*x^4])/(128*c^4) + (5*b*(7*b*B - 8*A*
c)*x^2*Sqrt[b*x^2 + c*x^4])/(192*c^3) - ((7*b*B - 8*A*c)*x^4*Sqrt[b*x^2 + c
*x^4])/(48*c^2) + (B*x^6*Sqrt[b*x^2 + c*x^4])/(8*c) + (5*b^3*(7*b*B - 8*A*c
)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(128*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
```

```
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^3(A+Bx)}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$

$$= \frac{Bx^6\sqrt{bx^2+cx^4}}{8c} + \frac{\left(3(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{8c}$$

$$= -\frac{(7bB-8Ac)x^4\sqrt{bx^2+cx^4}}{48c^2} + \frac{Bx^6\sqrt{bx^2+cx^4}}{8c} + \frac{(5b(7bB-8Ac))\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{96c^2}$$

$$= \frac{5b(7bB-8Ac)x^2\sqrt{bx^2+cx^4}}{192c^3} - \frac{(7bB-8Ac)x^4\sqrt{bx^2+cx^4}}{48c^2} + \frac{Bx^6\sqrt{bx^2+cx^4}}{8c} - \frac{(5b^2(7bB-8Ac))\text{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{128c^3}$$

$$= -\frac{5b^2(7bB-8Ac)\sqrt{bx^2+cx^4}}{128c^4} + \frac{5b(7bB-8Ac)x^2\sqrt{bx^2+cx^4}}{192c^3} - \frac{(7bB-8Ac)x^4\sqrt{bx^2+cx^4}}{48c^2} + \frac{Bx^6\sqrt{bx^2+cx^4}}{8c} + \frac{(5b^3(7bB-8Ac))\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{256c^4}$$

$$= -\frac{5b^2(7bB-8Ac)\sqrt{bx^2+cx^4}}{128c^4} + \frac{5b(7bB-8Ac)x^2\sqrt{bx^2+cx^4}}{192c^3} - \frac{(7bB-8Ac)x^4\sqrt{bx^2+cx^4}}{48c^2} + \frac{Bx^6\sqrt{bx^2+cx^4}}{8c} + \frac{(5b^3(7bB-8Ac))\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^4}$$

$$= -\frac{5b^2(7bB-8Ac)\sqrt{bx^2+cx^4}}{128c^4} + \frac{5b(7bB-8Ac)x^2\sqrt{bx^2+cx^4}}{192c^3} - \frac{(7bB-8Ac)x^4\sqrt{bx^2+cx^4}}{48c^2} + \frac{Bx^6\sqrt{bx^2+cx^4}}{8c} + \frac{5b^3(7bB-8Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{9/2}}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.88

$$\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{x\left(\sqrt{c}(b+cx^2)\left(-105b^3Bx+16c^3x^5(4A+3Bx^2)-8bc^2x^3(10A+7Bx^2)+10b^2cx(12A+7Bx^2)\right)+30b^3(\cdots\right.}{384c^{9/2}\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(x^7*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*(b + c*x^2)*(-105*b^3*B*x + 16*c^3*x^5*(4*A + 3*B*x^2) - 8*b*c^
2*x^3*(10*A + 7*B*x^2) + 10*b^2*c*x*(12*A + 7*B*x^2)) + 30*b^3*(7*b*B - 8*A
*c)*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/(-Sqrt[b] + Sqrt[b + c*x^2])]))/(38
4*c^(9/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{\frac{5\left(-\frac{1}{2}A\,b^3c+\frac{7}{16}B\,b^4\right)\ln\left(\frac{2c\,x^2+2\sqrt{x^2\left(c\,x^2+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)}{16}+\frac{5\left(b^2\left(\frac{7x^2B}{12}+A\right)c^{\frac{3}{2}}-\frac{2x^2\left(\frac{7x^2B}{10}+A\right)b\,c^{\frac{5}{2}}}{3}+\frac{8x^4\left(\frac{3x^2B}{4}+A\right)c^{\frac{7}{2}}}{15}-\frac{7B\sqrt{c}\,b^3}{8}\right)\cdots}{16}}{c^{\frac{9}{2}}}$
risch	$\frac{x^2\left(48B\,c^3x^6+64A\,c^3x^4-56Bb\,c^2x^4-80Ab\,c^2x^2+70B\,b^2c\,x^2+120b^2Ac-105B\,b^3\right)\left(c\,x^2+b\right)}{384c^4\sqrt{x^2\left(c\,x^2+b\right)}}-\frac{5b^3(8Ac-7Bb)\ln\left(\sqrt{c}\,x+\sqrt{c\,x\cdots}\right.}{128c^{\frac{9}{2}}\sqrt{x^2\left(c\,x^2+\cdots\right.}}$
default	$\frac{x\sqrt{c\,x^2+b}\left(48B\,c^{\frac{9}{2}}\sqrt{c\,x^2+b}\,x^7+64A\,c^{\frac{9}{2}}\sqrt{c\,x^2+b}\,x^5-56B\,c^{\frac{7}{2}}\sqrt{c\,x^2+b}\,b\,x^5-80A\,c^{\frac{7}{2}}\sqrt{c\,x^2+b}\,b\,x^3+70B\,c^{\frac{5}{2}}\sqrt{c\,x^2+b}\,b^2x^3+120A\cdots\right.}{384\sqrt{x^4c+b\,x^2}\,c^{\frac{11}{2}}}$

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/16/c^(9/2)*((-1/2*A*b^3*c+7/16*B*b^4)*ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2)
*c^(1/2)+b)/c^(1/2))+(b^2*(7/12*x^2*B+A)*c^(3/2)-2/3*x^2*(7/10*x^2*B+A)*b*c
^(5/2)+8/15*x^4*(3/4*x^2*B+A)*c^(7/2)-7/8*B*c^(1/2)*b^3)*(x^2*(c*x^2+b))^(1
/2)+1/2*ln(2)*b^3*(A*c-7/8*B*b))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.56

$$\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \left[-\frac{15\,(7\,Bb^4-8\,Ab^3c)\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,(48\,Bc^4x^6-105\,Bb^3c+120\,Ab^2c^2-8}{768\,c^5}\right.$$
$$-\frac{15\,(7\,Bb^4-8\,Ab^3c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-(48\,Bc^4x^6-105\,Bb^3c+120\,Ab^2c^2-8\,(7\,Bbc^3-8\,Ac}{384\,c^5}$$

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(15*(7*B*b^4 - 8*A*b^3*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 +
 b*x^2)*sqrt(c)) - 2*(48*B*c^4*x^6 - 105*B*b^3*c + 120*A*b^2*c^2 - 8*(7*B*b
*c^3 - 8*A*c^4)*x^4 + 10*(7*B*b^2*c^2 - 8*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2)
)/c^5, -1/384*(15*(7*B*b^4 - 8*A*b^3*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)
*sqrt(-c)/(c*x^2 + b)) - (48*B*c^4*x^6 - 105*B*b^3*c + 120*A*b^2*c^2 - 8*(7
*B*b*c^3 - 8*A*c^4)*x^4 + 10*(7*B*b^2*c^2 - 8*A*b*c^3)*x^2)*sqrt(c*x^4 + b*
x^2))/c^5]
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.27

$$\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\begin{cases} -\frac{5b^3\left(A-\frac{7Bb}{8c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{16c^3}+\sqrt{bx^2+cx^4}\left(\frac{Bx^6}{4c}+\frac{5b^2\left(A-\frac{7Bb}{8c}\right)}{8c^3}-\frac{5bx^2\left(A-\frac{7Bb}{8c}\right)}{12c^2}+\right. \\ \frac{\frac{2A\left(bx^2\right)^{\frac{7}{2}}}{7b^3}+\frac{2B\left(bx^2\right)^{\frac{9}{2}}}{9b^4}}{b} \\ \tilde{\infty}\left(\frac{Ax^8}{4}+\frac{Bx^{10}}{5}\right) \end{cases}}{2}$$

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Piecewise((-5*b**3*(A - 7*B*b/(8*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x*
*2 + c*x**4) + 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(
2*c) + x**2)/sqrt(c*(b/(2*c) + x**2)**2), True))/(16*c**3) + sqrt(b*x**2 +
c*x**4)*(B*x**6/(4*c) + 5*b**2*(A - 7*B*b/(8*c))/(8*c**3) - 5*b*x**2*(A - 7
*B*b/(8*c))/(12*c**2) + x**4*(A - 7*B*b/(8*c))/(3*c)), Ne(c, 0)), ((2*A*(b*
x**2)**(7/2)/(7*b**3) + 2*B*(b*x**2)**(9/2)/(9*b**4))/b, Ne(b, 0)), (zoo*(A
*x**8/4 + B*x**10/5), True))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.31

$$\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{1}{96}\left(\frac{16\sqrt{cx^4+bx^2}x^4}{c} - \frac{20\sqrt{cx^4+bx^2}bx^2}{c^2} - \frac{15\,b^3\log\left(2\,cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{7}{2}}} + \frac{30\sqrt{cx^4+bx^2}b^2}{c^3}\right)$$
$$+ \frac{1}{768}\left(\frac{96\sqrt{cx^4+bx^2}x^6}{c} - \frac{112\sqrt{cx^4+bx^2}bx^4}{c^2} + \frac{140\sqrt{cx^4+bx^2}b^2x^2}{c^3} + \frac{105\,b^4\log\left(2\,cx^2+b+2\sqrt{cx^4+\right.}}{c^{\frac{9}{2}}}\right.$$

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(16*sqrt(c*x^4 + b*x^2)*x^4/c - 20*sqrt(c*x^4 + b*x^2)*b*x^2/c^2 - 15*
b^3*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 30*sqrt(c*x^
4 + b*x^2)*b^2/c^3)*A + 1/768*(96*sqrt(c*x^4 + b*x^2)*x^6/c - 112*sqrt(c*x^
4 + b*x^2)*b*x^4/c^2 + 140*sqrt(c*x^4 + b*x^2)*b^2*x^2/c^3 + 105*b^4*log(2*
c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(9/2) - 210*sqrt(c*x^4 + b*x^2
)*b^3/c^4)*B
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.05

$$\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{1}{384}\left(2\left(4\,x^2\left(\frac{6\,Bx^2}{c\mathrm{sgn}\,(x)} - \frac{7\,Bbc^5\mathrm{sgn}(x) - 8\,Ac^6\mathrm{sgn}(x)}{c^7}\right) + \frac{5\,(7\,Bb^2c^4\mathrm{sgn}(x) - 8\,Abc^5\mathrm{sgn}(x))}{c^7}\right)x^2 - \frac{15\,(7\,\ldots}{}\right.\right.$$
$$+ \frac{5\,(7\,Bb^4\log\left(|b|\right) - 8\,Ab^3c\log\left(|b|\right))\mathrm{sgn}(x)}{256\,c^{\frac{9}{2}}} - \frac{5\,(7\,Bb^4 - 8\,Ab^3c)\log\left(\left|-\sqrt{c}x + \sqrt{cx^2+b}\right|\right)}{128\,c^{\frac{9}{2}}\mathrm{sgn}\,(x)}$$

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*x^2*(6*B*x^2/(c*sgn(x)) - (7*B*b*c^5*sgn(x) - 8*A*c^6*sgn(x))/c
^7) + 5*(7*B*b^2*c^4*sgn(x) - 8*A*b*c^5*sgn(x))/c^7)*x^2 - 15*(7*B*b^3*c^3*
sgn(x) - 8*A*b^2*c^4*sgn(x))/c^7)*sqrt(c*x^2 + b)*x + 5/256*(7*B*b^4*log(ab
s(b)) - 8*A*b^3*c*log(abs(b)))*sgn(x)/c^(9/2) - 5/128*(7*B*b^4 - 8*A*b^3*c)
*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(9/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{x^7\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

```
[In] int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.131 $\int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$

Optimal result

Integrand size = 26, antiderivative size = 139

$$\int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx = \frac{b(5bB-6Ac)\sqrt{bx^2+cx^4}}{16c^3} - \frac{(5bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{Bx^4\sqrt{bx^2+cx^4}}{6c} - \frac{b^2(5bB-6Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}}$$

```
[Out] -1/16*b^2*(-6*A*c+5*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(7/2)+1
/16*b*(-6*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/c^3-1/24*(-6*A*c+5*B*b)*x^2*(c*x^4
+b*x^2)^(1/2)/c^2+1/6*B*x^4*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2059, 808, 684, 654, 634, 212}

$$\int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx = -\frac{b^2(5bB-6Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}} + \frac{b\sqrt{bx^2+cx^4}(5bB-6Ac)}{16c^3} - \frac{x^2\sqrt{bx^2+cx^4}(5bB-6Ac)}{24c^2} + \frac{Bx^4\sqrt{bx^2+cx^4}}{6c}$$

```
[In] Int[(x^5*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (b*(5*b*B - 6*A*c)*Sqrt[b*x^2 + c*x^4])/(16*c^3) - ((5*b*B - 6*A*c)*x^2*Sqr
t[b*x^2 + c*x^4])/(24*c^2) + (B*x^4*Sqrt[b*x^2 + c*x^4])/(6*c) - (b^2*(5*b*
B - 6*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 808

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$

$$= \frac{Bx^4\sqrt{bx^2+cx^4}}{6c} + \frac{\left(2(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{6c}$$

$$= -\frac{(5bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{Bx^4\sqrt{bx^2+cx^4}}{6c} + \frac{(b(5bB-6Ac))\text{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{16c^2}$$

$$= \frac{b(5bB-6Ac)\sqrt{bx^2+cx^4}}{16c^3} - \frac{(5bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{Bx^4\sqrt{bx^2+cx^4}}{6c} - \frac{(b^2(5bB-6Ac))\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{32c^3}$$

$$= \frac{b(5bB-6Ac)\sqrt{bx^2+cx^4}}{16c^3} - \frac{(5bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{Bx^4\sqrt{bx^2+cx^4}}{6c} - \frac{(b^2(5bB-6Ac))\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^3}$$

$$= \frac{b(5bB-6Ac)\sqrt{bx^2+cx^4}}{16c^3} - \frac{(5bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^2} + \frac{Bx^4\sqrt{bx^2+cx^4}}{6c} - \frac{b^2(5bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{x\left(\sqrt{c}x(b+cx^2)\left(15b^2B+4c^2x^2(3A+2Bx^2)-2bc(9A+5Bx^2)\right)+6b^2(5bB-6Ac)\sqrt{b+cx^2}\text{arctanh}\left(\frac{\cdots}{\sqrt{b}-\cdots}\right.\right.}{48c^{7/2}\sqrt{x^2(b+cx^2)}}$$

```
[In] Integrate[(x^5*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2)*(15*b^2*B + 4*c^2*x^2*(3*A + 2*B*x^2) - 2*b*c*(9*
A + 5*B*x^2)) + 6*b^2*(5*b*B - 6*A*c)*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/(
Sqrt[b] - Sqrt[b + c*x^2])]))/(48*c^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{3\left(\left(-\frac{1}{2}b^2Ac+\frac{5}{12}B\,b^3\right)\ln\left(\frac{2c\,x^2+2\sqrt{x^2\left(c\,x^2+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)+\left(b\left(\frac{5x^2B}{9}+A\right)c^{\frac{3}{2}}-\frac{2x^2\left(\frac{2x^2B}{3}+A\right)c^{\frac{5}{2}}}{3}-\frac{5B\sqrt{c}\,b^2}{6}\right)\sqrt{x^2(c\,x^2+b)}}{8c^{\frac{7}{2}}}$
risch	$-\frac{x^2\left(-8B\,c^2x^4-12A\,c^2x^2+10Bbc\,x^2+18Abc-15B\,b^2\right)\left(c\,x^2+b\right)}{48c^3\sqrt{x^2(c\,x^2+b)}}+\frac{b^2(6Ac-5Bb)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)x\sqrt{c\,x^2+b}}{16c^{\frac{7}{2}}\sqrt{x^2(c\,x^2+b)}}$
default	$\frac{x\sqrt{c\,x^2+b}\,\left(8B\sqrt{c\,x^2+b}\,c^{\frac{7}{2}}x^5+12A\sqrt{c\,x^2+b}\,c^{\frac{7}{2}}x^3-10B\sqrt{c\,x^2+b}\,c^{\frac{5}{2}}b\,x^3-18A\sqrt{c\,x^2+b}\,c^{\frac{5}{2}}bx+15B\sqrt{c\,x^2+b}\,c^{\frac{3}{2}}b^2x+18A\ln\left(\right.\right.}{48\sqrt{x^4c+b\,x^2}\,c^{\frac{9}{2}}}$

[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -3/8/c^(7/2)*((-1/2*b^2*A*c+5/12*B*b^3)*ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2)*c^(1/2)+b)/c^(1/2))+(b*(5/9*x^2*B+A)*c^(3/2)-2/3*x^2*(2/3*x^2*B+A)*c^(5/2)-5/6*B*c^(1/2)*b^2)*(x^2*(c*x^2+b))^(1/2)+1/2*ln(2)*b^2*(A*c-5/6*B*b))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.63

$$\int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$

$$=\left[-\frac{3\,(5\,Bb^3-6\,Ab^2c)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,(8\,Bc^3x^4+15\,Bb^2c-18\,Abc^2-2\,(5\,Bbc\right.}{96\,c^4}$$

[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")

[Out] [-1/96*(3*(5*B*b^3 - 6*A*b^2*c)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*(8*B*c^3*x^4 + 15*B*b^2*c - 18*A*b*c^2 - 2*(5*B*b*c^2 - 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^4, 1/48*(3*(5*B*b^3 - 6*A*b^2*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (8*B*c^3*x^4 + 15*B*b^2*c - 18*A*b*c^2 - 2*(5*B*b*c^2 - 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/c^4]

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.45

$$\int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\begin{cases} \frac{3b^2\left(A-\frac{5Bb}{6c}\right)\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise} \end{cases}\right)}{8c^2} + \sqrt{bx^2+cx^4}\left(\frac{Bx^4}{3c}-\frac{3b\left(A-\frac{5Bb}{6c}\right)}{4c^2}+\frac{x^2\left(A-\frac{5Bb}{6c}\right)}{2c}\right) & \text{for } c \\ \frac{\frac{2A\left(bx^2\right)^{\frac{5}{2}}}{5b^2}+\frac{2B\left(bx^2\right)^{\frac{7}{2}}}{7b^3}}{b} & \text{for } b \\ \tilde{\infty}\left(\frac{Ax^6}{3}+\frac{Bx^8}{4}\right) & \text{other} \end{cases}}{2}$$

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Piecewise((3*b**2*(A - 5*B*b/(6*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**
2 + c*x**4) + 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2
*c) + x**2)/sqrt(c*(b/(2*c) + x**2)**2), True))/(8*c**2) + sqrt(b*x**2 + c*
x**4)*(B*x**4/(3*c) - 3*b*(A - 5*B*b/(6*c))/(4*c**2) + x**2*(A - 5*B*b/(6*c
))/(2*c)), Ne(c, 0)), ((2*A*(b*x**2)**(5/2)/(5*b**2) + 2*B*(b*x**2)**(7/2)/
(7*b**3))/b, Ne(b, 0)), (zoo*(A*x**6/3 + B*x**8/4), True))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.32

$$\int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{1}{16}\left(\frac{4\sqrt{cx^4+bx^2}x^2}{c}+\frac{3\,b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{5}{2}}}-\frac{6\,\sqrt{cx^4+bx^2}b}{c^2}\right)A + \frac{1}{96}\left(\frac{16\,\sqrt{cx^4+bx^2}x^4}{c}-\frac{20\,\sqrt{cx^4+bx^2}bx^2}{c^2}-\frac{15\,b^3\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{7}{2}}}+\frac{30\,\sqrt{cx^4+bx^2}b^2}{c^3}\right.$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*sqrt(c*x^4 + b*x^2)*x^2/c + 3*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 +
b*x^2)*sqrt(c))/c^(5/2) - 6*sqrt(c*x^4 + b*x^2)*b/c^2)*A + 1/96*(16*sqrt(c*
```

```
x^4 + b*x^2)*x^4/c - 20*sqrt(c*x^4 + b*x^2)*b*x^2/c^2 - 15*b^3*log(2*c*x^2
+ b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(7/2) + 30*sqrt(c*x^4 + b*x^2)*b^2/c
^3)*B
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.08

$$\int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$
$$=\frac{1}{48}\sqrt{cx^2+b}\left(2\,x^2\left(\frac{4\,Bx^2}{c\mathrm{sgn}\,(x)}-\frac{5\,Bbc^3\mathrm{sgn}(x)-6\,Ac^4\mathrm{sgn}(x)}{c^5}\right)+\frac{3\,(5\,Bb^2c^2\mathrm{sgn}(x)-6\,Abc^3\mathrm{sgn}(x))}{c^5}\right)x$$
$$-\frac{(5\,Bb^3\log(|b|)-6\,Ab^2c\log(|b|))\mathrm{sgn}(x)}{32\,c^{\frac{7}{2}}}+\frac{(5\,Bb^3-6\,Ab^2c)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{16\,c^{\frac{7}{2}}\mathrm{sgn}\,(x)}$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c*x^2 + b)*(2*x^2*(4*B*x^2/(c*sgn(x)) - (5*B*b*c^3*sgn(x) - 6*A*c
^4*sgn(x))/c^5) + 3*(5*B*b^2*c^2*sgn(x) - 6*A*b*c^3*sgn(x))/c^5)*x - 1/32*(
5*B*b^3*log(abs(b)) - 6*A*b^2*c*log(abs(b)))*sgn(x)/c^(7/2) + 1/16*(5*B*b^3
 - 6*A*b^2*c)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(7/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{x^5\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

```
[In] int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.132 $\int \frac{x^3(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 83

$$\int \frac{x^3(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=-\frac{(3bB-4Ac-2Bcx^2)\sqrt{bx^2+cx^4}}{8c^2}+\frac{b(3bB-4Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}$$

```
[Out] 1/8*b*(-4*A*c+3*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(5/2)-1/8*(
-2*B*c*x^2-4*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2059, 793, 634, 212}

$$\int \frac{x^3(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=\frac{b(3bB-4Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}-\frac{\sqrt{bx^2+cx^4}(-4Ac+3bB-2Bcx^2)}{8c^2}$$

```
[In] Int[(x^3*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -1/8*((3*b*B - 4*A*c - 2*B*c*x^2)*Sqrt[b*x^2 + c*x^4])/c^2 + (b*(3*b*B - 4*
A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\
&= -\frac{(3bB-4Ac-2Bcx^2)\sqrt{bx^2+cx^4}}{8c^2}+\frac{(b(3bB-4Ac))\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{16c^2}\\
&= -\frac{(3bB-4Ac-2Bcx^2)\sqrt{bx^2+cx^4}}{8c^2}+\frac{(b(3bB-4Ac))\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^2}\\
&= -\frac{(3bB-4Ac-2Bcx^2)\sqrt{bx^2+cx^4}}{8c^2}+\frac{b(3bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.30

$$\int \frac{x^3(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{x\left(\sqrt{c}x(b+cx^2)\left(-3bB+4Ac+2Bcx^2\right)+2b(3bB-4Ac)\sqrt{b+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{b}+\sqrt{b+cx^2}}\right)\right)}{8c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(x^3*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2)*(-3*b*B + 4*A*c + 2*B*c*x^2) + 2*b*(3*b*B - 4*A*c
)*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/(-Sqrt[b] + Sqrt[b + c*x^2])]))/(8*c^
(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.20

method	result
risch	$\frac{x^2\left(2Bc\,x^2+4Ac-3Bb\right)\left(c\,x^2+b\right)}{8c^2\sqrt{x^2(c\,x^2+b)}}-\frac{b(4Ac-3Bb)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)x\sqrt{c\,x^2+b}}{8c^{\frac{5}{2}}\sqrt{x^2(c\,x^2+b)}}$
default	$\frac{x\sqrt{c\,x^2+b}\left(2B\sqrt{c\,x^2+b}\,c^{\frac{5}{2}}x^3+4A\sqrt{c\,x^2+b}\,c^{\frac{5}{2}}x-3B\sqrt{c\,x^2+b}\,c^{\frac{3}{2}}bx-4A\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)b\,c^2+3B\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)b^2c\right)}{8\sqrt{x^4c+b\,x^2}\,c^{\frac{7}{2}}}$
pseudoelliptic	$\frac{4B\,c^{\frac{3}{2}}x^2\sqrt{x^2(c\,x^2+b)}+8A\,c^{\frac{3}{2}}\sqrt{x^2(c\,x^2+b)}-4A\ln\left(\frac{2c\,x^2+2\sqrt{x^2\left(c\,x^2+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)bc+4A\ln(2)bc-6Bb\sqrt{c}\,\sqrt{x^2(c\,x^2+b)}+3B\ln}{16c^{\frac{5}{2}}}$

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^2*(2*B*c*x^2+4*A*c-3*B*b)*(c*x^2+b)/c^2/(x^2*(c*x^2+b))^(1/2)-1/8*b*(
4*A*c-3*B*b)/c^(5/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*x/(x^2*(c*x^2+b))^(1/2)*
(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.13

$$\int \frac{x^3(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$
$$=\left[-\frac{(3\,Bb^2-4\,Abc)\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,(2\,Bc^2x^2-3\,Bbc+4\,Ac^2)\sqrt{cx^4+bx^2}}{16\,c^3},\right.$$
$$\left.-\frac{(3\,Bb^2-4\,Abc)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-(2\,Bc^2x^2-3\,Bbc+4\,Ac^2)\sqrt{cx^4+bx^2}}{8\,c^3}\right]$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*B*b^2 - 4*A*b*c)*sqrt(c)*log(-2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2
)*sqrt(c)) - 2*(2*B*c^2*x^2 - 3*B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2))/c^3,
-1/8*((3*B*b^2 - 4*A*b*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x
^2 + b)) - (2*B*c^2*x^2 - 3*B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2))/c^3]
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.05

$$\int \frac{x^3(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$
$$=\frac{\begin{cases} -\frac{b\left(A-\frac{3Bb}{4c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise}\end{cases}\right)}{2c}+\sqrt{bx^2+cx^4}\left(\frac{Bx^2}{2c}+\frac{A-\frac{3Bb}{4c}}{c}\right) & \text{for } c\neq 0\\ \frac{\frac{2A\left(bx^2\right)^{\frac{3}{2}}}{3b}+\frac{2B\left(bx^2\right)^{\frac{5}{2}}}{5b^2}}{b} & \text{for } b\neq 0\\ \tilde{\infty}\left(\frac{Ax^4}{2}+\frac{Bx^6}{3}\right) & \text{otherwise}\end{cases}}{2}$$

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Piecewise((-b*(A - 3*B*b/(4*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x**2 +
c*x**4) + 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x**2)*log(b/(2*c)
+ x**2)/sqrt(c*(b/(2*c) + x**2)**2), True))/(2*c) + sqrt(b*x**2 + c*x**4)*(
```

```
B*x**2/(2*c) + (A - 3*B*b/(4*c))/c), Ne(c, 0)), ((2*A*(b*x**2)**(3/2)/(3*b)
 + 2*B*(b*x**2)**(5/2)/(5*b**2))/b, Ne(b, 0)), (zoo*(A*x**4/2 + B*x**6/3),
True))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.61

$$\int \frac{x^3(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{1}{16}\left(\frac{4\sqrt{cx^4+bx^2}x^2}{c}+\frac{3\,b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{5}{2}}}-\frac{6\,\sqrt{cx^4+bx^2}b}{c^2}\right)B - \frac{1}{4}\,A\left(\frac{b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{3}{2}}}-\frac{2\,\sqrt{cx^4+bx^2}}{c}\right)$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*sqrt(c*x^4 + b*x^2)*x^2/c + 3*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 +
b*x^2)*sqrt(c))/c^(5/2) - 6*sqrt(c*x^4 + b*x^2)*b/c^2)*B - 1/4*A*(b*log(2*c
*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(3/2) - 2*sqrt(c*x^4 + b*x^2)/c
)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int \frac{x^3(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{1}{8}\,\sqrt{cx^2+b}x\left(\frac{2\,Bx^2}{c\mathrm{sgn}\left(x\right)}-\frac{3\,Bbc\mathrm{sgn}(x)-4\,Ac^2\mathrm{sgn}(x)}{c^3}\right) + \frac{(3\,Bb^2\log\left(|b|\right)-4\,Abc\log\left(|b|\right))\mathrm{sgn}(x)}{16\,c^{\frac{5}{2}}} - \frac{(3\,Bb^2-4\,Abc)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{8\,c^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^2 + b)*x*(2*B*x^2/(c*sgn(x)) - (3*B*b*c*sgn(x) - 4*A*c^2*sgn(x
))/c^3) + 1/16*(3*B*b^2*log(abs(b)) - 4*A*b*c*log(abs(b)))*sgn(x)/c^(5/2) -
 1/8*(3*B*b^2 - 4*A*b*c)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(5/2)*sg
n(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=\int \frac{x^3\,(B\,x^2+A)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

```
[In] int((x^3*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
[Out] int((x^3*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.133 $\int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 66

$$\int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=\frac{B\sqrt{bx^2+cx^4}}{2c}-\frac{(bB-2Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

```
[Out] -1/2*(-2*A*c+B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(3/2)+1/2*B*(c
*x^4+b*x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2059, 654, 634, 212}

$$\int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx=\frac{B\sqrt{bx^2+cx^4}}{2c}-\frac{(bB-2Ac)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

```
[In] Int[(x*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (B*Sqrt[b*x^2 + c*x^4])/(2*c) - ((b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b
*x^2 + c*x^4]])/(2*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{B\sqrt{bx^2+cx^4}}{2c} + \frac{(-bB+2Ac)\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{4c}\\
&= \frac{B\sqrt{bx^2+cx^4}}{2c} + \frac{(-bB+2Ac)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{2c}\\
&= \frac{B\sqrt{bx^2+cx^4}}{2c} - \frac{(bB-2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.38

$$
\int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{x\left(B\sqrt{c}x(b+cx^2)+2(bB-2Ac)\sqrt{b+cx^2}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{b}-\sqrt{b+cx^2}}\right)\right)}{2c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}
$$

```
[In] Integrate[(x*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(B*Sqrt[c]*x*(b + c*x^2) + 2*(b*B - 2*A*c)*Sqrt[b + c*x^2]*ArcTanh[(Sqrt
[c]*x)/(Sqrt[b] - Sqrt[b + c*x^2])]))/(2*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.27

method	result	size
risch	$\frac{B\,x^{2}\left(c\,x^{2}+b\right)}{2c\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{\left(2Ac-Bb\right)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)x\sqrt{c\,x^{2}+b}}{2c^{\frac{3}{2}}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	84
default	$\frac{x\sqrt{c\,x^{2}+b}\,\left(B\,c^{\frac{3}{2}}\sqrt{c\,x^{2}+b}\,x+2A\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)c^{2}-B\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)bc\right)}{2\sqrt{x^{4}c+b\,x^{2}}\,c^{\frac{5}{2}}}$	88
pseudoelliptic	$\frac{2A\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)c-2A\ln\left(2\right)c-B\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)b+B\ln\left(2\right)b+2B\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}}{4c^{\frac{3}{2}}}$	107

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*x^2/c*(c*x^2+b)/(x^2*(c*x^2+b))^(1/2)+1/2*(2*A*c-B*b)/c^(3/2)*ln(c^(1
/2)*x+(c*x^2+b)^(1/2))*x/(x^2*(c*x^2+b))^(1/2)*(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.98

$$\int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$
$$=\left[\frac{2\sqrt{cx^4+bx^2}Bc-(Bb-2\,Ac)\sqrt{c}\log\left(-2\,cx^2-b-2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{4\,c^2},\frac{\sqrt{cx^4+bx^2}Bc+(Bb-2\,Ac)\sqrt{-}}{2\,c^2}\right.$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*x^4 + b*x^2)*B*c - (B*b - 2*A*c)*sqrt(c)*log(-2*c*x^2 - b -
2*sqrt(c*x^4 + b*x^2)*sqrt(c)))/c^2, 1/2*(sqrt(c*x^4 + b*x^2)*B*c + (B*b -
2*A*c)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)))/c^2]
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.11

$$\int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\begin{cases} \frac{B\sqrt{bx^2+cx^4}}{c} + \left(A-\frac{Bb}{2c}\right)\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx^2+cx^4}+2cx^2\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x^2\right)\log\left(\frac{b}{2c}+x^2\right)}{\sqrt{c\left(\frac{b}{2c}+x^2\right)^2}} & \text{otherwise} \end{cases}\right) & \text{for } c\neq 0 \\ \frac{2A\sqrt{bx^2}+\frac{2B\left(bx^2\right)^{\frac{3}{2}}}{3b}}{b} & \text{for } b\neq 0 \\ \tilde{\infty}\left(Ax^2+\frac{Bx^4}{2}\right) & \text{otherwise} \end{cases}}{2}$$

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Piecewise((B*sqrt(b*x**2 + c*x**4)/c + (A - B*b/(2*c))*Piecewise((log(b + 2
*sqrt(c)*sqrt(b*x**2 + c*x**4) + 2*c*x**2)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*
c) + x**2)*log(b/(2*c) + x**2)/sqrt(c*(b/(2*c) + x**2)**2), True)), Ne(c, 0
)), ((2*A*sqrt(b*x**2) + 2*B*(b*x**2)**(3/2)/(3*b))/b, Ne(b, 0)), (zoo*(A*x
**2 + B*x**4/2), True))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.33

$$\int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{1}{4}\,B\left(\frac{b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{3}{2}}} - \frac{2\,\sqrt{cx^4+bx^2}}{c}\right) + \frac{A\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{2\,\sqrt{c}}$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*B*(b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(3/2) - 2*sqrt
(c*x^4 + b*x^2)/c) + 1/2*A*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))
/sqrt(c)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17

$$\int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{cx^2+b}Bx}{2\,c\mathrm{sgn}\,(x)} - \frac{(Bb\log\left(|b|\right)-2\,Ac\log\left(|b|\right))\mathrm{sgn}(x)}{4\,c^{\frac{3}{2}}} + \frac{(Bb-2\,Ac)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{2\,c^{\frac{3}{2}}\mathrm{sgn}\,(x)}$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + b)*B*x/(c*sgn(x)) - 1/4*(B*b*log(abs(b)) - 2*A*c*log(abs(b
)))*sgn(x)/c^(3/2) + 1/2*(B*b - 2*A*c)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)
))/(c^(3/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.62 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.35

$$\int \frac{x(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{B\,\sqrt{c\,x^4+b\,x^2}}{2\,c} + \frac{A\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{2\,\sqrt{c}} - \frac{B\,b\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{4\,c^{3/2}}$$

```
[In] int((x*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (B*(b*x^2 + c*x^4)^(1/2))/(2*c) + (A*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c
*x^4)^(1/2)))/(2*c^(1/2)) - (B*b*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4
)^(1/2)))/(4*c^(3/2))
```

3.134 $\int \frac{A+Bx^2}{x\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 57

$$\int \frac{A+Bx^2}{x\sqrt{bx^2+cx^4}}\,dx=-\frac{A\sqrt{bx^2+cx^4}}{bx^2}+\frac{B\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

```
[Out] B*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(1/2)-A*(c*x^4+b*x^2)^(1/2)/b/
x^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2059, 806, 634, 212}

$$\int \frac{A+Bx^2}{x\sqrt{bx^2+cx^4}}\,dx=\frac{B\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}-\frac{A\sqrt{bx^2+cx^4}}{bx^2}$$

```
[In] Int[(A + B*x^2)/(x*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -((A*Sqrt[b*x^2 + c*x^4])/(b*x^2)) + (B*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
c*x^4]])/Sqrt[c]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\
&= -\frac{A\sqrt{bx^2+cx^4}}{bx^2}+\frac{1}{2}B\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\
&= -\frac{A\sqrt{bx^2+cx^4}}{bx^2}+B\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x^2}{\sqrt{bx^2+cx^4}}\right)\\
&= -\frac{A\sqrt{bx^2+cx^4}}{bx^2}+\frac{B\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx^2}{x\sqrt{bx^2+cx^4}}\,dx = \frac{-A\sqrt{c}(b+cx^2)-bBx\sqrt{b+cx^2}\log\left(-\sqrt{c}x+\sqrt{b+cx^2}\right)}{b\sqrt{c}\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(A + B*x^2)/(x*Sqrt[b*x^2 + c*x^4]),x]
[Out] (-(A*Sqrt[c]*(b + c*x^2)) - b*B*x*Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[b
 + c*x^2]])/(b*Sqrt[c]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

method	result	size
default	$-\frac{\sqrt{c\,x^{2}+b}\,\left(-B\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)bx+A\sqrt{c\,x^{2}+b}\,\sqrt{c}\right)}{\sqrt{x^{4}c+b\,x^{2}}\,\sqrt{c}\,b}$	67
risch	$-\frac{A\left(c\,x^{2}+b\right)}{b\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{B\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)x\sqrt{c\,x^{2}+b}}{\sqrt{c}\,\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	72
pseudoelliptic	$-\frac{-\frac{B\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)b\,x^{2}}{2}+\frac{B\ln\left(2\right)b\,x^{2}}{2}+A\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}}{\sqrt{c}\,b\,x^{2}}$	78

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(c*x^2+b)^(1/2)*(-B*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b*x+A*(c*x^2+b)^(1/2)*c^
(1/2))/(c*x^4+b*x^2)^(1/2)/c^(1/2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.39

$$\int \frac{A+Bx^2}{x\sqrt{bx^2+cx^4}}\,dx = \left[\frac{Bb\sqrt{c}x^2\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}Ac}{2\,bcx^2}, -\frac{Bb\sqrt{-c}x^2\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}Ac}{bcx^2}\right]$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(B*b*sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2
*sqrt(c*x^4 + b*x^2)*A*c)/(b*c*x^2), -(B*b*sqrt(-c)*x^2*arctan(sqrt(c*x^4 +
 b*x^2)*sqrt(-c)/(c*x^2 + b)) + sqrt(c*x^4 + b*x^2)*A*c)/(b*c*x^2)]
```

Sympy [F]

$$\int \frac{A + Bx^2}{x\sqrt{bx^2 + cx^4}}\,dx = \int \frac{A + Bx^2}{x\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \frac{A + Bx^2}{x\sqrt{bx^2 + cx^4}}\,dx = \frac{B\log\left(2\,cx^2 + b + 2\,\sqrt{cx^4 + bx^2}\sqrt{c}\right)}{2\,\sqrt{c}} - \frac{\sqrt{cx^4 + bx^2}A}{bx^2}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*B*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/sqrt(c) - sqrt(c*x^4
 + b*x^2)*A/(b*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16

$$\int \frac{A + Bx^2}{x\sqrt{bx^2 + cx^4}}\,dx = -\frac{B\log\left(\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^2\right)}{2\,\sqrt{c}\mathrm{sgn}\left(x\right)} + \frac{2\,A\sqrt{c}}{\left(\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^2 - b\right)\mathrm{sgn}\left(x\right)}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*B*log((sqrt(c)*x - sqrt(c*x^2 + b))^2)/(sqrt(c)*sgn(x)) + 2*A*sqrt(c)/
(((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x\sqrt{bx^2+cx^4}}\,dx=\frac{B\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{2\,\sqrt{c}}-\frac{A\,\sqrt{c\,x^4+b\,x^2}}{b\,x^2}$$

```
[In] int((A + B*x^2)/(x*(b*x^2 + c*x^4)^(1/2)),x)
[Out] (B*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(2*c^(1/2)) - (A*(b*
x^2 + c*x^4)^(1/2))/(b*x^2)
```

3.135 $\int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 61

$$\int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}}\,dx = -\frac{A\sqrt{bx^2+cx^4}}{3bx^4}-\frac{(3bB-2Ac)\sqrt{bx^2+cx^4}}{3b^2x^2}$$

```
[Out] -1/3*A*(c*x^4+b*x^2)^(1/2)/b/x^4-1/3*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/b^2
/x^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2059, 806, 664}

$$\int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{bx^2+cx^4}(3bB-2Ac)}{3b^2x^2}-\frac{A\sqrt{bx^2+cx^4}}{3bx^4}$$

```
[In] Int[(A + B*x^2)/(x^3*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/3*(A*Sqrt[b*x^2 + c*x^4])/(b*x^4) - ((3*b*B - 2*A*c)*Sqrt[b*x^2 + c*x^4]
)/(3*b^2*x^2)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\
&= -\frac{A\sqrt{bx^2+cx^4}}{3bx^4} + \frac{\left(-2(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{3b}\\
&= -\frac{A\sqrt{bx^2+cx^4}}{3bx^4} - \frac{(3bB-2Ac)\sqrt{bx^2+cx^4}}{3b^2x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{x^2(b+cx^2)}\left(3bBx^2+A(b-2cx^2)\right)}{3b^2x^4}$$

```
[In] Integrate[(A + B*x^2)/(x^3*Sqrt[b*x^2 + c*x^4]),x]
[Out] -1/3*(Sqrt[x^2*(b + c*x^2)]*(3*b*B*x^2 + A*(b - 2*c*x^2)))/(b^2*x^4)
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

method	result	size
trager	$-\frac{\left(-2 A c \,x^{2}+3 b B \,x^{2}+A b \right) \sqrt{x^{4} c +b \,x^{2}}}{3 b^{2} x^{4}}$	40
gosper	$-\frac{\left(c \,x^{2}+b \right) \left(-2 A c \,x^{2}+3 b B \,x^{2}+A b \right)}{3 x^{2} b^{2} \sqrt{x^{4} c +b \,x^{2}}}$	47
default	$-\frac{\left(c \,x^{2}+b \right) \left(-2 A c \,x^{2}+3 b B \,x^{2}+A b \right)}{3 x^{2} b^{2} \sqrt{x^{4} c +b \,x^{2}}}$	47
risch	$-\frac{\left(c \,x^{2}+b \right) \left(-2 A c \,x^{2}+3 b B \,x^{2}+A b \right)}{3 x^{2} \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, b^{2}}$	47
pseudoelliptic	$-\frac{\left(\left(3 x^{2} B +A \right) b -2 A c \,x^{2}\right) \left(c \,x^{2}+b \right)}{3 \sqrt{x^{2} \left(c \,x^{2}+b \right)}\, x^{2} b^{2}}$	47

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(-2*A*c*x^2+3*B*b*x^2+A*b)/b^2/x^4*(c*x^4+b*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.62

$$\int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{cx^4+bx^2}((3\,Bb-2\,Ac)x^2+Ab)}{3\,b^2x^4}$$

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(c*x^4 + b*x^2)*((3*B*b - 2*A*c)*x^2 + A*b)/(b^2*x^4)
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^3\sqrt{x^2\,(b+cx^2)}}\,dx$$

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral((A + B*x**2)/(x**3*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}}\,dx = \frac{1}{3}\,A\left(\frac{2\,\sqrt{cx^4+bx^2}c}{b^2x^2} - \frac{\sqrt{cx^4+bx^2}}{bx^4}\right) - \frac{\sqrt{cx^4+bx^2}B}{bx^2}$$

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*A*(2*sqrt(c*x^4 + b*x^2)*c/(b^2*x^2) - sqrt(c*x^4 + b*x^2)/(b*x^4)) - s
qrt(c*x^4 + b*x^2)*B/(b*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(53) = 106.

Time = 0.39 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.03

$$\int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}}\,dx = \frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4B\sqrt{c} - 6\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2Bb\sqrt{c} + 6\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2Ac^{\frac{3}{2}} + 3\,Bb^2\sqrt{c} - 2\,A\right.}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^3\mathrm{sgn}\left(x\right)}$$

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] 2/3*(3*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*sqrt(c) - 6*(sqrt(c)*x - sqrt(c*x^
2 + b))^2*B*b*sqrt(c) + 6*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*c^(3/2) + 3*B*b
^2*sqrt(c) - 2*A*b*c^(3/2))/(((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^3*sgn(x)
)
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx^2}{x^3\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{c\,x^4+b\,x^2}\,\left(A\,b-2\,A\,c\,x^2+3\,B\,b\,x^2\right)}{3\,b^2\,x^4}$$

```
[In] int((A + B*x^2)/(x^3*(b*x^2 + c*x^4)^(1/2)),x)
[Out] -((b*x^2 + c*x^4)^(1/2)*(A*b - 2*A*c*x^2 + 3*B*b*x^2))/(3*b^2*x^4)
```

3.136 $\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}\,dx = -\frac{A\sqrt{bx^2+cx^4}}{5bx^6}-\frac{(5bB-4Ac)\sqrt{bx^2+cx^4}}{15b^2x^4}+\frac{2c(5bB-4Ac)\sqrt{bx^2+cx^4}}{15b^3x^2}$$

```
[Out] -1/5*A*(c*x^4+b*x^2)^(1/2)/b/x^6-1/15*(-4*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/b^
2/x^4+2/15*c*(-4*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2059, 806, 672, 664}

$$\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}\,dx = \frac{2c\sqrt{bx^2+cx^4}(5bB-4Ac)}{15b^3x^2}-\frac{\sqrt{bx^2+cx^4}(5bB-4Ac)}{15b^2x^4}-\frac{A\sqrt{bx^2+cx^4}}{5bx^6}$$

```
[In] Int[(A + B*x^2)/(x^5*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/5*(A*Sqrt[b*x^2 + c*x^4])/(b*x^6) - ((5*b*B - 4*A*c)*Sqrt[b*x^2 + c*x^4]
)/(15*b^2*x^4) + (2*c*(5*b*B - 4*A*c)*Sqrt[b*x^2 + c*x^4])/(15*b^3*x^2)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
```

```
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^3\sqrt{bx+cx^2}}\,dx,x,x^2\right)\\
&= -\frac{A\sqrt{bx^2+cx^4}}{5bx^6}+\frac{\left(-3(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{5b}\\
&= -\frac{A\sqrt{bx^2+cx^4}}{5bx^6}-\frac{(5bB-4Ac)\sqrt{bx^2+cx^4}}{15b^2x^4}-\frac{(c(5bB-4Ac))\text{Subst}\left(\int \frac{1}{x\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{15b^2}\\
&= -\frac{A\sqrt{bx^2+cx^4}}{5bx^6}-\frac{(5bB-4Ac)\sqrt{bx^2+cx^4}}{15b^2x^4}+\frac{2c(5bB-4Ac)\sqrt{bx^2+cx^4}}{15b^3x^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.67

$$\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}\,dx=\frac{\sqrt{x^2\left(b+cx^2\right)}(-5bBx^2(b-2cx^2)+A(-3b^2+4bcx^2-8c^2x^4))}{15b^3x^6}$$

```
[In] Integrate[(A + B*x^2)/(x^5*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-5*b*B*x^2*(b - 2*c*x^2) + A*(-3*b^2 + 4*b*c*x^2 -
8*c^2*x^4)))/(15*b^3*x^6)
```

Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

method	result	size
trager	$-\frac{\left(8A\,c^2x^4-10x^4Bbc-4Abc\,x^2+5b^2B\,x^2+3b^2A\right)\sqrt{x^4c+b\,x^2}}{15b^3x^6}$	63
pseudoelliptic	$-\frac{\left(\left(\frac{5x^2B}{3}+A\right)b^2-\frac{4x^2\left(\frac{5x^2B}{2}+A\right)cb}{3}+\frac{8A\,c^2x^4}{3}\right)\left(c\,x^2+b\right)}{5\sqrt{x^2\left(c\,x^2+b\right)}\,x^4b^3}$	66
gosper	$-\frac{\left(c\,x^2+b\right)\left(8A\,c^2x^4-10x^4Bbc-4Abc\,x^2+5b^2B\,x^2+3b^2A\right)}{15x^4b^3\sqrt{x^4c+b\,x^2}}$	70
default	$-\frac{\left(c\,x^2+b\right)\left(8A\,c^2x^4-10x^4Bbc-4Abc\,x^2+5b^2B\,x^2+3b^2A\right)}{15x^4b^3\sqrt{x^4c+b\,x^2}}$	70
risch	$-\frac{\left(c\,x^2+b\right)\left(8A\,c^2x^4-10x^4Bbc-4Abc\,x^2+5b^2B\,x^2+3b^2A\right)}{15x^4\sqrt{x^2\left(c\,x^2+b\right)}\,b^3}$	70

```
[In] int((B*x^2+A)/x^5/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(8*A*c^2*x^4-10*B*b*c*x^4-4*A*b*c*x^2+5*B*b^2*x^2+3*A*b^2)/b^3/x^6*(c
*x^4+b*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}\,dx=\frac{(2\,(5\,Bbc-4\,Ac^2)x^4-3\,Ab^2-(5\,Bb^2-4\,Abc)x^2)\sqrt{cx^4+bx^2}}{15\,b^3x^6}$$

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(5*B*b*c - 4*A*c^2)*x^4 - 3*A*b^2 - (5*B*b^2 - 4*A*b*c)*x^2)*sqrt(c
*x^4 + b*x^2)/(b^3*x^6)
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^5\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

```
[In] integrate((B*x**2+A)/x**5/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral((A + B*x**2)/(x**5*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}\,dx = \frac{1}{3}B\left(\frac{2\sqrt{cx^4+bx^2}c}{b^2x^2} - \frac{\sqrt{cx^4+bx^2}}{bx^4}\right) - \frac{1}{15}A\left(\frac{8\sqrt{cx^4+bx^2}c^2}{b^3x^2} - \frac{4\sqrt{cx^4+bx^2}c}{b^2x^4} + \frac{3\sqrt{cx^4+bx^2}}{bx^6}\right)$$

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*B*(2*sqrt(c*x^4 + b*x^2)*c/(b^2*x^2) - sqrt(c*x^4 + b*x^2)/(b*x^4)) - 1
/15*A*(8*sqrt(c*x^4 + b*x^2)*c^2/(b^3*x^2) - 4*sqrt(c*x^4 + b*x^2)*c/(b^2*x
^4) + 3*sqrt(c*x^4 + b*x^2)/(b*x^6))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(84) = 168.

Time = 0.58 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.88

$$\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}\,dx = \frac{4\left(15\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6Bc^{\frac{3}{2}} - 35\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4Bbc^{\frac{3}{2}} + 40\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4Ac^{\frac{5}{2}} + 25\left(\sqrt{c}x-\sqrt{\right.}\right.}{15\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^5\mathrm{sgn}\left(x\right.}$$

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] 4/15*(15*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*c^(3/2) - 35*(sqrt(c)*x - sqrt(c
*x^2 + b))^4*B*b*c^(3/2) + 40*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*c^(5/2) + 2
5*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^2*c^(3/2) - 20*(sqrt(c)*x - sqrt(c*x^
2 + b))^2*A*b*c^(5/2) - 5*B*b^3*c^(3/2) + 4*A*b^2*c^(5/2))/(((sqrt(c)*x - s
qrt(c*x^2 + b))^2 - b)^5*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx^2}{x^5\sqrt{bx^2+cx^4}}\,dx=-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(5\,B\,b^2\,x^2+3\,A\,b^2-10\,B\,b\,c\,x^4-4\,A\,b\,c\,x^2+8\,A\,c^2\,x^4\right)}{15\,b^3\,x^6}$$

```
[In] int((A + B*x^2)/(x^5*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*(3*A*b^2 + 5*B*b^2*x^2 + 8*A*c^2*x^4 - 4*A*b*c*x^2
- 10*B*b*c*x^4))/(15*b^3*x^6)
```

3.137 $\int \frac{A+Bx^2}{x^7\sqrt{bx^2+cx^4}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 133

$$\int \frac{A+Bx^2}{x^7\sqrt{bx^2+cx^4}} \, dx = -\frac{A\sqrt{bx^2+cx^4}}{7bx^8} - \frac{(7bB-6Ac)\sqrt{bx^2+cx^4}}{35b^2x^6} + \frac{4c(7bB-6Ac)\sqrt{bx^2+cx^4}}{105b^3x^4} - \frac{8c^2(7bB-6Ac)\sqrt{bx^2+cx^4}}{105b^4x^2}$$

```
[Out] -1/7*A*(c*x^4+b*x^2)^(1/2)/b/x^8-1/35*(-6*A*c+7*B*b)*(c*x^4+b*x^2)^(1/2)/b^
2/x^6+4/105*c*(-6*A*c+7*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^4-8/105*c^2*(-6*A*c+
7*B*b)*(c*x^4+b*x^2)^(1/2)/b^4/x^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2059, 806, 672, 664}

$$\int \frac{A+Bx^2}{x^7\sqrt{bx^2+cx^4}} \, dx = -\frac{8c^2\sqrt{bx^2+cx^4}(7bB-6Ac)}{105b^4x^2} + \frac{4c\sqrt{bx^2+cx^4}(7bB-6Ac)}{105b^3x^4} - \frac{\sqrt{bx^2+cx^4}(7bB-6Ac)}{35b^2x^6} - \frac{A\sqrt{bx^2+cx^4}}{7bx^8}$$

```
[In] Int[(A + B*x^2)/(x^7*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/7*(A*Sqrt[b*x^2 + c*x^4])/(b*x^8) - ((7*b*B - 6*A*c)*Sqrt[b*x^2 + c*x^4]
)/(35*b^2*x^6) + (4*c*(7*b*B - 6*A*c)*Sqrt[b*x^2 + c*x^4])/(105*b^3*x^4) -
(8*c^2*(7*b*B - 6*A*c)*Sqrt[b*x^2 + c*x^4])/(105*b^4*x^2)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^4\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{7bx^8} + \frac{\left(-4(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{1}{x^3\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{7b}$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{7bx^8} - \frac{(7bB-6Ac)\sqrt{bx^2+cx^4}}{35b^2x^6} - \frac{(2c(7bB-6Ac))\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{35b^2}$$

$$
\begin{aligned}
&= -\frac{A\sqrt{bx^2+cx^4}}{7bx^8} - \frac{(7bB-6Ac)\sqrt{bx^2+cx^4}}{35b^2x^6} + \frac{4c(7bB-6Ac)\sqrt{bx^2+cx^4}}{105b^3x^4} \\
&\quad + \frac{(4c^2(7bB-6Ac))\,\text{Subst}\left(\int \frac{1}{x\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{105b^3} \\
&= -\frac{A\sqrt{bx^2+cx^4}}{7bx^8} - \frac{(7bB-6Ac)\sqrt{bx^2+cx^4}}{35b^2x^6} \\
&\quad + \frac{4c(7bB-6Ac)\sqrt{bx^2+cx^4}}{105b^3x^4} - \frac{8c^2(7bB-6Ac)\sqrt{bx^2+cx^4}}{105b^4x^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.67

$$
\int \frac{A+Bx^2}{x^7\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{x^2(b+cx^2)}(7bBx^2(3b^2-4bcx^2+8c^2x^4)+3A(5b^3-6b^2cx^2+8bc^2x^4-16c^3x^6))}{105b^4x^8}
$$

```
[In] Integrate[(A + B*x^2)/(x^7*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/105*(Sqrt[x^2*(b + c*x^2)]*(7*b*B*x^2*(3*b^2 - 4*b*c*x^2 + 8*c^2*x^4) +
3*A*(5*b^3 - 6*b^2*c*x^2 + 8*b*c^2*x^4 - 16*c^3*x^6)))/(b^4*x^8)
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$-\frac{\left(\left(\frac{7x^2B}{5}+A\right)b^3-\frac{6x^2c\left(\frac{14x^2B}{9}+A\right)b^2}{5}+\frac{8x^4\left(\frac{7x^2B}{3}+A\right)c^2b}{5}-\frac{16Ac^3x^6}{5}\right)(cx^2+b)}{7\sqrt{x^2(cx^2+b)}\,x^6b^4}$	85
trager	$-\frac{(-48Ac^3x^6+56x^6Bbc^2+24Abc^2x^4-28x^4Bb^2c-18Ab^2cx^2+21b^3Bx^2+15b^3A)\sqrt{x^4c+bx^2}}{105x^8b^4}$	87
gosper	$-\frac{(cx^2+b)(-48Ac^3x^6+56x^6Bbc^2+24Abc^2x^4-28x^4Bb^2c-18Ab^2cx^2+21b^3Bx^2+15b^3A)}{105x^6b^4\sqrt{x^4c+bx^2}}$	94
default	$-\frac{(cx^2+b)(-48Ac^3x^6+56x^6Bbc^2+24Abc^2x^4-28x^4Bb^2c-18Ab^2cx^2+21b^3Bx^2+15b^3A)}{105x^6b^4\sqrt{x^4c+bx^2}}$	94
risch	$-\frac{(cx^2+b)(-48Ac^3x^6+56x^6Bbc^2+24Abc^2x^4-28x^4Bb^2c-18Ab^2cx^2+21b^3Bx^2+15b^3A)}{105x^6\sqrt{x^2(cx^2+b)}\,b^4}$	94

```
[In] int((B*x^2+A)/x^7/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*((7/5*x^2*B+A)*b^3-6/5*x^2*c*(14/9*x^2*B+A)*b^2+8/5*x^4*(7/3*x^2*B+A)*
c^2*b-16/5*A*c^3*x^6)/(x^2*(c*x^2+b))^(1/2)*(c*x^2+b)/x^6/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx^2}{x^7\sqrt{bx^2+cx^4}}\,dx = -\frac{(8\,(7\,Bbc^2-6\,Ac^3)x^6-4\,(7\,Bb^2c-6\,Abc^2)x^4+15\,Ab^3+3\,(7\,Bb^3-6\,Ab^2c)x^2)\sqrt{cx^4+bx^2}}{105\,b^4x^8}$$

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/105*(8*(7*B*b*c^2 - 6*A*c^3)*x^6 - 4*(7*B*b^2*c - 6*A*b*c^2)*x^4 + 15*A*
b^3 + 3*(7*B*b^3 - 6*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^4*x^8)
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^7\sqrt{bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^7\sqrt{x^2\,(b+cx^2)}}\,dx$$

```
[In] integrate((B*x**2+A)/x**7/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**7*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.26

$$\begin{aligned}\int \frac{A+Bx^2}{x^7\sqrt{bx^2+cx^4}}\,dx &= -\frac{1}{15}\,B\left(\frac{8\,\sqrt{cx^4+bx^2}c^2}{b^3x^2}-\frac{4\,\sqrt{cx^4+bx^2}c}{b^2x^4}+\frac{3\,\sqrt{cx^4+bx^2}}{bx^6}\right)\\ &\quad+\frac{1}{35}\,A\left(\frac{16\,\sqrt{cx^4+bx^2}c^3}{b^4x^2}-\frac{8\,\sqrt{cx^4+bx^2}c^2}{b^3x^4}+\frac{6\,\sqrt{cx^4+bx^2}c}{b^2x^6}-\frac{5\,\sqrt{cx^4+bx^2}}{bx^8}\right)\end{aligned}$$

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/15*B*(8*sqrt(c*x^4 + b*x^2)*c^2/(b^3*x^2) - 4*sqrt(c*x^4 + b*x^2)*c/(b^2
*x^4) + 3*sqrt(c*x^4 + b*x^2)/(b*x^6)) + 1/35*A*(16*sqrt(c*x^4 + b*x^2)*c^3
/(b^4*x^2) - 8*sqrt(c*x^4 + b*x^2)*c^2/(b^3*x^4) + 6*sqrt(c*x^4 + b*x^2)*c/
(b^2*x^6) - 5*sqrt(c*x^4 + b*x^2)/(b*x^8))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(117) = 234$.

Time = 0.73 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.77

$$\int \frac{A + Bx^2}{x^7\sqrt{bx^2 + cx^4}}\,dx = \frac{16\left(70\left(\sqrt{c}x - \sqrt{cx^2+b}\right)^8 Bc^{\frac{5}{2}} - 175\left(\sqrt{c}x - \sqrt{cx^2+b}\right)^6 Bbc^{\frac{5}{2}} + 210\left(\sqrt{c}x - \sqrt{cx^2+b}\right)^6 Ac^{\frac{7}{2}} + 147\left(\sqrt{c}x\right.\right.}{10}$$

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 16/105*(70*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*c^(5/2) - 175*(sqrt(c)*x - sqr
t(c*x^2 + b))^6*B*b*c^(5/2) + 210*(sqrt(c)*x - sqrt(c*x^2 + b))^6*A*c^(7/2)
 + 147*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^2*c^(5/2) - 126*(sqrt(c)*x - sqr
t(c*x^2 + b))^4*A*b*c^(7/2) - 49*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^3*c^(5
/2) + 42*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^2*c^(7/2) + 7*B*b^4*c^(5/2) -
6*A*b^3*c^(7/2))/(((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^7*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \frac{A + Bx^2}{x^7\sqrt{bx^2 + cx^4}}\,dx = {} & \frac{(6\,A\,c - 7\,B\,b)\,\sqrt{c\,x^4 + b\,x^2}}{35\,b^2\,x^6} - \frac{A\,\sqrt{c\,x^4 + b\,x^2}}{7\,b\,x^8} \\ & - \frac{(24\,A\,c^2 - 28\,B\,b\,c)\,\sqrt{c\,x^4 + b\,x^2}}{105\,b^3\,x^4} \\ & + \frac{(48\,A\,c^3 - 56\,B\,b\,c^2)\,\sqrt{c\,x^4 + b\,x^2}}{105\,b^4\,x^2}\end{aligned}$$

```
[In] int((A + B*x^2)/(x^7*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] ((6*A*c - 7*B*b)*(b*x^2 + c*x^4)^(1/2))/(35*b^2*x^6) - (A*(b*x^2 + c*x^4)^(
1/2))/(7*b*x^8) - ((24*A*c^2 - 28*B*b*c)*(b*x^2 + c*x^4)^(1/2))/(105*b^3*x^
4) + ((48*A*c^3 - 56*B*b*c^2)*(b*x^2 + c*x^4)^(1/2))/(105*b^4*x^2)
```

3.138 $\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 170

$$\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx=-\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}}-\frac{(9bB-8Ac)\sqrt{bx^2+cx^4}}{63b^2x^8}+\frac{2c(9bB-8Ac)\sqrt{bx^2+cx^4}}{105b^3x^6}-\frac{8c^2(9bB-8Ac)\sqrt{bx^2+cx^4}}{315b^4x^4}+\frac{16c^3(9bB-8Ac)\sqrt{bx^2+cx^4}}{315b^5x^2}$$

```
[Out] -1/9*A*(c*x^4+b*x^2)^(1/2)/b/x^10-1/63*(-8*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/b
^2/x^8+2/105*c*(-8*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^6-8/315*c^2*(-8*A*c
+9*B*b)*(c*x^4+b*x^2)^(1/2)/b^4/x^4+16/315*c^3*(-8*A*c+9*B*b)*(c*x^4+b*x^2)
^(1/2)/b^5/x^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2059, 806, 672, 664}

$$\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx=\frac{16c^3\sqrt{bx^2+cx^4}(9bB-8Ac)}{315b^5x^2}-\frac{8c^2\sqrt{bx^2+cx^4}(9bB-8Ac)}{315b^4x^4}+\frac{2c\sqrt{bx^2+cx^4}(9bB-8Ac)}{105b^3x^6}-\frac{\sqrt{bx^2+cx^4}(9bB-8Ac)}{63b^2x^8}-\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}}$$

```
[In] Int[(A + B*x^2)/(x^9*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/9*(A*Sqrt[b*x^2 + c*x^4])/(b*x^10) - ((9*b*B - 8*A*c)*Sqrt[b*x^2 + c*x^4
])/(63*b^2*x^8) + (2*c*(9*b*B - 8*A*c)*Sqrt[b*x^2 + c*x^4])/(105*b^3*x^6) -
 (8*c^2*(9*b*B - 8*A*c)*Sqrt[b*x^2 + c*x^4])/(315*b^4*x^4) + (16*c^3*(9*b*B
 - 8*A*c)*Sqrt[b*x^2 + c*x^4])/(315*b^5*x^2)
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^5\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}} + \frac{\left(-5(-bB+Ac)+\frac{1}{2}(-bB+2Ac)\right)\text{Subst}\left(\int \frac{1}{x^4\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{9b}$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}} - \frac{(9bB-8Ac)\sqrt{bx^2+cx^4}}{63b^2x^8} - \frac{(c(9bB-8Ac))\text{Subst}\left(\int \frac{1}{x^3\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{21b^2}$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}} - \frac{(9bB-8Ac)\sqrt{bx^2+cx^4}}{63b^2x^8} + \frac{2c(9bB-8Ac)\sqrt{bx^2+cx^4}}{105b^3x^6} + \frac{(4c^2(9bB-8Ac))\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{105b^3}$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}} - \frac{(9bB-8Ac)\sqrt{bx^2+cx^4}}{63b^2x^8} + \frac{2c(9bB-8Ac)\sqrt{bx^2+cx^4}}{105b^3x^6} - \frac{8c^2(9bB-8Ac)\sqrt{bx^2+cx^4}}{315b^4x^4} - \frac{(8c^3(9bB-8Ac))\text{Subst}\left(\int \frac{1}{x\sqrt{bx+cx^2}}\,dx,x,x^2\right)}{315b^4}$$

$$= -\frac{A\sqrt{bx^2+cx^4}}{9bx^{10}} - \frac{(9bB-8Ac)\sqrt{bx^2+cx^4}}{63b^2x^8} + \frac{2c(9bB-8Ac)\sqrt{bx^2+cx^4}}{105b^3x^6} - \frac{8c^2(9bB-8Ac)\sqrt{bx^2+cx^4}}{315b^4x^4} + \frac{16c^3(9bB-8Ac)\sqrt{bx^2+cx^4}}{315b^5x^2}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{x^2(b+cx^2)}(9bBx^2(-5b^3+6b^2cx^2-8bc^2x^4+16c^3x^6)+A(-35b^4+40b^3cx^2-48b^2c^2x^4+64bc^3x^6-12}{315b^5x^{10}}$$

```
[In] Integrate[(A + B*x^2)/(x^9*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(9*b*B*x^2*(-5*b^3 + 6*b^2*c*x^2 - 8*b*c^2*x^4 + 16*
c^3*x^6) + A*(-35*b^4 + 40*b^3*c*x^2 - 48*b^2*c^2*x^4 + 64*b*c^3*x^6 - 128*
c^4*x^8)))/(315*b^5*x^10)
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$-\frac{\left(\left(\frac{9x^2B}{7}+A\right)b^4-\frac{8x^2c\left(\frac{27x^2B}{20}+A\right)b^3}{7}+\frac{48x^4\left(\frac{3x^2B}{2}+A\right)c^2b^2}{35}-\frac{64x^6\left(\frac{9x^2B}{4}+A\right)c^3b}{35}+\frac{128Ax^8c^4}{35}\right)(c\,x^2+b)}{9\sqrt{x^2(c\,x^2+b)}\,x^8b^5}$
trager	$-\frac{(128A\,x^8c^4-144B\,x^8b\,c^3-64A\,x^6b\,c^3+72B\,x^6b^2c^2+48A\,b^2c^2x^4-54B\,b^3c\,x^4-40A\,x^2b^3c+45B\,x^2b^4+35A\,b^4)\sqrt{x^4c+b\,x^2}}{315b^5x^{10}}$
gosper	$-\frac{(c\,x^2+b)(128A\,x^8c^4-144B\,x^8b\,c^3-64A\,x^6b\,c^3+72B\,x^6b^2c^2+48A\,b^2c^2x^4-54B\,b^3c\,x^4-40A\,x^2b^3c+45B\,x^2b^4+35A\,b^4)}{315x^8b^5\sqrt{x^4c+b\,x^2}}$
default	$-\frac{(c\,x^2+b)(128A\,x^8c^4-144B\,x^8b\,c^3-64A\,x^6b\,c^3+72B\,x^6b^2c^2+48A\,b^2c^2x^4-54B\,b^3c\,x^4-40A\,x^2b^3c+45B\,x^2b^4+35A\,b^4)}{315x^8b^5\sqrt{x^4c+b\,x^2}}$
risch	$-\frac{(c\,x^2+b)(128A\,x^8c^4-144B\,x^8b\,c^3-64A\,x^6b\,c^3+72B\,x^6b^2c^2+48A\,b^2c^2x^4-54B\,b^3c\,x^4-40A\,x^2b^3c+45B\,x^2b^4+35A\,b^4)}{315x^8\sqrt{x^2(c\,x^2+b)}\,b^5}$

```
[In] int((B*x^2+A)/x^9/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*((9/7*x^2*B+A)*b^4-8/7*x^2*c*(27/20*x^2*B+A)*b^3+48/35*x^4*(3/2*x^2*B+
A)*c^2*b^2-64/35*x^6*(9/4*x^2*B+A)*c^3*b+128/35*A*x^8*c^4)/(x^2*(c*x^2+b))^
(1/2)*(c*x^2+b)/x^8/b^5
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx = \frac{(16\,(9\,Bbc^3-8\,Ac^4)x^8-8\,(9\,Bb^2c^2-8\,Abc^3)x^6-35\,Ab^4+6\,(9\,Bb^3c-8\,Ab^2c^2)x^4-5\,(9\,Bb^4-8\,Ab^3c)x}{315\,b^5x^{10}}$$

```
[In] integrate((B*x^2+A)/x^9/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(16*(9*B*b*c^3 - 8*A*c^4)*x^8 - 8*(9*B*b^2*c^2 - 8*A*b*c^3)*x^6 - 35*
A*b^4 + 6*(9*B*b^3*c - 8*A*b^2*c^2)*x^4 - 5*(9*B*b^4 - 8*A*b^3*c)*x^2)*sqrt
(c*x^4 + b*x^2)/(b^5*x^10)
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^9\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

```
[In] integrate((B*x**2+A)/x**9/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral((A + B*x**2)/(x**9*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.26

$$\begin{aligned}&\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx\\&=\frac{1}{35}\,B\left(\frac{16\sqrt{cx^4+bx^2}c^3}{b^4x^2}-\frac{8\sqrt{cx^4+bx^2}c^2}{b^3x^4}+\frac{6\sqrt{cx^4+bx^2}c}{b^2x^6}-\frac{5\sqrt{cx^4+bx^2}}{bx^8}\right)\\&\quad-\frac{1}{315}\,A\left(\frac{128\sqrt{cx^4+bx^2}c^4}{b^5x^2}-\frac{64\sqrt{cx^4+bx^2}c^3}{b^4x^4}+\frac{48\sqrt{cx^4+bx^2}c^2}{b^3x^6}-\frac{40\sqrt{cx^4+bx^2}c}{b^2x^8}+\frac{35\sqrt{cx^4+bx^2}}{bx^{10}}\right)\end{aligned}$$

```
[In] integrate((B*x^2+A)/x^9/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/35*B*(16*sqrt(c*x^4 + b*x^2)*c^3/(b^4*x^2) - 8*sqrt(c*x^4 + b*x^2)*c^2/(b
^3*x^4) + 6*sqrt(c*x^4 + b*x^2)*c/(b^2*x^6) - 5*sqrt(c*x^4 + b*x^2)/(b*x^8)
) - 1/315*A*(128*sqrt(c*x^4 + b*x^2)*c^4/(b^5*x^2) - 64*sqrt(c*x^4 + b*x^2)
*c^3/(b^4*x^4) + 48*sqrt(c*x^4 + b*x^2)*c^2/(b^3*x^6) - 40*sqrt(c*x^4 + b*x
^2)*c/(b^2*x^8) + 35*sqrt(c*x^4 + b*x^2)/(b*x^10))
```

Giac [A] (verification not implemented)

none

Time = 0.98 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.72

$$\begin{aligned}&\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx\\&=\frac{32\left(315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Bc^{\frac{7}{2}}-819\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Bbc^{\frac{7}{2}}+1008\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}Ac^{\frac{9}{2}}+756\left(\sqrt{\right.\right.}{}\end{aligned}$$

```
[In] integrate((B*x^2+A)/x^9/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 32/315*(315*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*c^(7/2) - 819*(sqrt(c)*x - s
qrt(c*x^2 + b))^8*B*b*c^(7/2) + 1008*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*c^(9
/2) + 756*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^2*c^(7/2) - 672*(sqrt(c)*x -
sqrt(c*x^2 + b))^6*A*b*c^(9/2) - 324*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^3*
c^(7/2) + 288*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^2*c^(9/2) + 81*(sqrt(c)*x
 - sqrt(c*x^2 + b))^2*B*b^4*c^(7/2) - 72*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*
b^3*c^(9/2) - 9*B*b^5*c^(7/2) + 8*A*b^4*c^(9/2))/(((sqrt(c)*x - sqrt(c*x^2
+ b))^2 - b)^9*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2}{x^9\sqrt{bx^2+cx^4}}\,dx = \frac{(8\,A\,c-9\,B\,b)\,\sqrt{c\,x^4+b\,x^2}}{63\,b^2\,x^8} - \frac{A\,\sqrt{c\,x^4+b\,x^2}}{9\,b\,x^{10}} - \frac{(16\,A\,c^2-18\,B\,b\,c)\,\sqrt{c\,x^4+b\,x^2}}{105\,b^3\,x^6} + \frac{(64\,A\,c^3-72\,B\,b\,c^2)\,\sqrt{c\,x^4+b\,x^2}}{315\,b^4\,x^4} - \frac{(128\,A\,c^4-144\,B\,b\,c^3)\,\sqrt{c\,x^4+b\,x^2}}{315\,b^5\,x^2}$$

```
[In] int((A + B*x^2)/(x^9*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] ((8*A*c - 9*B*b)*(b*x^2 + c*x^4)^(1/2))/(63*b^2*x^8) - (A*(b*x^2 + c*x^4)^(
1/2))/(9*b*x^10) - ((16*A*c^2 - 18*B*b*c)*(b*x^2 + c*x^4)^(1/2))/(105*b^3*x
^6) + ((64*A*c^3 - 72*B*b*c^2)*(b*x^2 + c*x^4)^(1/2))/(315*b^4*x^4) - ((128
*A*c^4 - 144*B*b*c^3)*(b*x^2 + c*x^4)^(1/2))/(315*b^5*x^2)
```

3.139 $\int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$

Optimal result

Integrand size = 26, antiderivative size = 131

$$\int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx = -\frac{8b^2(6bB-7Ac)\sqrt{bx^2+cx^4}}{105c^4x} + \frac{4b(6bB-7Ac)x\sqrt{bx^2+cx^4}}{105c^3} - \frac{(6bB-7Ac)x^3\sqrt{bx^2+cx^4}}{35c^2} + \frac{Bx^5\sqrt{bx^2+cx^4}}{7c}$$

```
[Out] -8/105*b^2*(-7*A*c+6*B*b)*(c*x^4+b*x^2)^(1/2)/c^4/x+4/105*b*(-7*A*c+6*B*b)*
x*(c*x^4+b*x^2)^(1/2)/c^3-1/35*(-7*A*c+6*B*b)*x^3*(c*x^4+b*x^2)^(1/2)/c^2+1
/7*B*x^5*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2064, 2041, 1602}

$$\int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx = -\frac{8b^2\sqrt{bx^2+cx^4}(6bB-7Ac)}{105c^4x} + \frac{4bx\sqrt{bx^2+cx^4}(6bB-7Ac)}{105c^3} - \frac{x^3\sqrt{bx^2+cx^4}(6bB-7Ac)}{35c^2} + \frac{Bx^5\sqrt{bx^2+cx^4}}{7c}$$

```
[In] Int[(x^6*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-8*b^2*(6*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(105*c^4*x) + (4*b*(6*b*B - 7*
A*c)*x*Sqrt[b*x^2 + c*x^4])/(105*c^3) - ((6*b*B - 7*A*c)*x^3*Sqrt[b*x^2 + c
*x^4])/(35*c^2) + (B*x^5*Sqrt[b*x^2 + c*x^4])/(7*c)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Bx^5\sqrt{bx^2+cx^4}}{7c} - \frac{(6bB-7Ac)\int \frac{x^6}{\sqrt{bx^2+cx^4}}\,dx}{7c} \\
&= -\frac{(6bB-7Ac)x^3\sqrt{bx^2+cx^4}}{35c^2} + \frac{Bx^5\sqrt{bx^2+cx^4}}{7c} + \frac{(4b(6bB-7Ac))\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx}{35c^2} \\
&= \frac{4b(6bB-7Ac)x\sqrt{bx^2+cx^4}}{105c^3} - \frac{(6bB-7Ac)x^3\sqrt{bx^2+cx^4}}{35c^2} \\
&\quad + \frac{Bx^5\sqrt{bx^2+cx^4}}{7c} - \frac{(8b^2(6bB-7Ac))\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{105c^3} \\
&= -\frac{8b^2(6bB-7Ac)\sqrt{bx^2+cx^4}}{105c^4x} + \frac{4b(6bB-7Ac)x\sqrt{bx^2+cx^4}}{105c^3} \\
&\quad - \frac{(6bB-7Ac)x^3\sqrt{bx^2+cx^4}}{35c^2} + \frac{Bx^5\sqrt{bx^2+cx^4}}{7c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.65

$$\int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{x^2\left(b+cx^2\right)}(-48b^3B+8b^2c(7A+3Bx^2)+3c^3x^4(7A+5Bx^2)-2bc^2x^2(14A+9Bx^2))}{105c^4x}$$

```
[In] Integrate[(x^6*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-48*b^3*B + 8*b^2*c*(7*A + 3*B*x^2) + 3*c^3*x^4*(7*
A + 5*B*x^2) - 2*b*c^2*x^2*(14*A + 9*B*x^2)))/(105*c^4*x)
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.64

method	result	size
trager	$\frac{\left(15B\,c^{3}x^{6}+21A\,c^{3}x^{4}-18Bb\,c^{2}x^{4}-28Ab\,c^{2}x^{2}+24B\,b^{2}c\,x^{2}+56b^{2}Ac-48B\,b^{3}\right)\sqrt{x^{4}c+b\,x^{2}}}{105c^{4}x}$	84
gosper	$\frac{\left(c\,x^{2}+b\right)\left(15B\,c^{3}x^{6}+21A\,c^{3}x^{4}-18Bb\,c^{2}x^{4}-28Ab\,c^{2}x^{2}+24B\,b^{2}c\,x^{2}+56b^{2}Ac-48B\,b^{3}\right)x}{105c^{4}\sqrt{x^{4}c+b\,x^{2}}}$	89
default	$\frac{\left(c\,x^{2}+b\right)\left(15B\,c^{3}x^{6}+21A\,c^{3}x^{4}-18Bb\,c^{2}x^{4}-28Ab\,c^{2}x^{2}+24B\,b^{2}c\,x^{2}+56b^{2}Ac-48B\,b^{3}\right)x}{105c^{4}\sqrt{x^{4}c+b\,x^{2}}}$	89
risch	$\frac{x\left(c\,x^{2}+b\right)\left(15B\,c^{3}x^{6}+21A\,c^{3}x^{4}-18Bb\,c^{2}x^{4}-28Ab\,c^{2}x^{2}+24B\,b^{2}c\,x^{2}+56b^{2}Ac-48B\,b^{3}\right)}{105\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,c^{4}}$	89

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*(15*B*c^3*x^6+21*A*c^3*x^4-18*B*b*c^2*x^4-28*A*b*c^2*x^2+24*B*b^2*c*x
^2+56*A*b^2*c-48*B*b^3)/c^4/x*(c*x^4+b*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.63

$$\int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{(15\,Bc^3x^6-3\,(6\,Bbc^2-7\,Ac^3)x^4-48\,Bb^3+56\,Ab^2c+4\,(6\,Bb^2c-7\,Abc^2)x^2)\sqrt{cx^4+bx^2}}{105\,c^4x}$$

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*c^3*x^6 - 3*(6*B*b*c^2 - 7*A*c^3)*x^4 - 48*B*b^3 + 56*A*b^2*c +
 4*(6*B*b^2*c - 7*A*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^4*x)
```

Sympy [F]

$$\int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^6(A+Bx^2)}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**6*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.81

$$\int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{(3\,c^3x^6 - bc^2x^4 + 4\,b^2cx^2 + 8\,b^3)A}{15\,\sqrt{cx^2+b}c^3} + \frac{(5\,c^4x^8 - bc^3x^6 + 2\,b^2c^2x^4 - 8\,b^3cx^2 - 16\,b^4)B}{35\,\sqrt{cx^2+b}c^4}$$

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*c^3*x^6 - b*c^2*x^4 + 4*b^2*c*x^2 + 8*b^3)*A/(sqrt(c*x^2 + b)*c^3)
+ 1/35*(5*c^4*x^8 - b*c^3*x^6 + 2*b^2*c^2*x^4 - 8*b^3*c*x^2 - 16*b^4)*B/(sq
rt(c*x^2 + b)*c^4)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.99

$$\int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{8\left(6\,Bb^{\frac{7}{2}} - 7\,Ab^{\frac{5}{2}}c\right)\mathrm{sgn}(x)}{105\,c^4} - \frac{(Bb^3 - Ab^2c)\sqrt{cx^2+b}}{c^4\mathrm{sgn}\,(x)} + \frac{15\,(cx^2+b)^{\frac{7}{2}}B - 63\,(cx^2+b)^{\frac{5}{2}}Bb + 105\,(cx^2+b)^{\frac{3}{2}}Bb^2 + 21\,(cx^2+b)^{\frac{5}{2}}Ac - 70\,(cx^2+b)^{\frac{3}{2}}Abc}{105\,c^4\mathrm{sgn}\,(x)}$$

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 8/105*(6*B*b^(7/2) - 7*A*b^(5/2)*c)*sgn(x)/c^4 - (B*b^3 - A*b^2*c)*sqrt(c*x
^2 + b)/(c^4*sgn(x)) + 1/105*(15*(c*x^2 + b)^(7/2)*B - 63*(c*x^2 + b)^(5/2)
*B*b + 105*(c*x^2 + b)^(3/2)*B*b^2 + 21*(c*x^2 + b)^(5/2)*A*c - 70*(c*x^2 +
 b)^(3/2)*A*b*c)/(c^4*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.66

$$\int \frac{x^6(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{48\,B\,b^3-56\,A\,b^2\,c}{105\,c^4}-\frac{B\,x^6}{7\,c}-\frac{x^4\,\left(21\,A\,c^3-18\,B\,b\,c^2\right)}{105\,c^4}+\frac{4\,b\,x^2\,\left(7\,A\,c-6\,B\,b\right)}{105\,c^3}\right)}{x}$$

```
[In] int((x^6*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*((48*B*b^3 - 56*A*b^2*c)/(105*c^4) - (B*x^6)/(7*c)
- (x^4*(21*A*c^3 - 18*B*b*c^2))/(105*c^4) + (4*b*x^2*(7*A*c - 6*B*b))/(105*
c^3)))/x
```

3.140 $\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$

Optimal result

Integrand size = 26, antiderivative size = 94

$$\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx = \frac{2b(4bB-5Ac)\sqrt{bx^2+cx^4}}{15c^3x} - \frac{(4bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^2} + \frac{Bx^3\sqrt{bx^2+cx^4}}{5c}$$

```
[Out] 2/15*b*(-5*A*c+4*B*b)*(c*x^4+b*x^2)^(1/2)/c^3/x-1/15*(-5*A*c+4*B*b)*x*(c*x^
4+b*x^2)^(1/2)/c^2+1/5*B*x^3*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2064, 2041, 1602}

$$\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx = \frac{2b\sqrt{bx^2+cx^4}(4bB-5Ac)}{15c^3x} - \frac{x\sqrt{bx^2+cx^4}(4bB-5Ac)}{15c^2} + \frac{Bx^3\sqrt{bx^2+cx^4}}{5c}$$

```
[In] Int[(x^4*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*b*(4*b*B - 5*A*c)*Sqrt[b*x^2 + c*x^4])/(15*c^3*x) - ((4*b*B - 5*A*c)*x*S
qrt[b*x^2 + c*x^4])/(15*c^2) + (B*x^3*Sqrt[b*x^2 + c*x^4])/(5*c)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
```

```
x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Bx^3\sqrt{bx^2+cx^4}}{5c} - \frac{(4bB-5Ac)\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx}{5c} \\
&= -\frac{(4bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^2} + \frac{Bx^3\sqrt{bx^2+cx^4}}{5c} + \frac{(2b(4bB-5Ac))\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{15c^2} \\
&= \frac{2b(4bB-5Ac)\sqrt{bx^2+cx^4}}{15c^3x} - \frac{(4bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^2} + \frac{Bx^3\sqrt{bx^2+cx^4}}{5c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.67

$$\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{x^2(b+cx^2)}(8b^2B-2bc(5A+2Bx^2)+c^2x^2(5A+3Bx^2))}{15c^3x}$$

```
[In] Integrate[(x^4*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
[Out] (Sqrt[x^2*(b + c*x^2)]*(8*b^2*B - 2*b*c*(5*A + 2*B*x^2) + c^2*x^2*(5*A + 3*
B*x^2)))/(15*c^3*x)
```

Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.64

method	result	size
trager	$-\frac{\left(-3B\,c^{2}x^{4}-5A\,c^{2}x^{2}+4Bbc\,x^{2}+10Abc-8B\,b^{2}\right)\sqrt{x^{4}c+b\,x^{2}}}{15c^{3}x}$	60
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(-3B\,c^{2}x^{4}-5A\,c^{2}x^{2}+4Bbc\,x^{2}+10Abc-8B\,b^{2}\right)x}{15c^{3}\sqrt{x^{4}c+b\,x^{2}}}$	65
default	$-\frac{\left(c\,x^{2}+b\right)\left(-3B\,c^{2}x^{4}-5A\,c^{2}x^{2}+4Bbc\,x^{2}+10Abc-8B\,b^{2}\right)x}{15c^{3}\sqrt{x^{4}c+b\,x^{2}}}$	65
risch	$-\frac{x\left(c\,x^{2}+b\right)\left(-3B\,c^{2}x^{4}-5A\,c^{2}x^{2}+4Bbc\,x^{2}+10Abc-8B\,b^{2}\right)}{15\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,c^{3}}$	65

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/15*(-3*B*c^2*x^4-5*A*c^2*x^2+4*B*b*c*x^2+10*A*b*c-8*B*b^2)/c^3/x*(c*x^4+
b*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.63

$$\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{(3\,Bc^2x^4+8\,Bb^2-10\,Abc-(4\,Bbc-5\,Ac^2)x^2)\sqrt{cx^4+bx^2}}{15\,c^3x}$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/15*(3*B*c^2*x^4 + 8*B*b^2 - 10*A*b*c - (4*B*b*c - 5*A*c^2)*x^2)*sqrt(c*x^
4 + b*x^2)/(c^3*x)
```

Sympy [F]

$$\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^4(A+Bx^2)}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x**4*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

$$\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{(c^2x^4-bcx^2-2\,b^2)A}{3\,\sqrt{cx^2+b}c^2} + \frac{(3\,c^3x^6-bc^2x^4+4\,b^2cx^2+8\,b^3)B}{15\,\sqrt{cx^2+b}c^3}$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*(c^2*x^4 - b*c*x^2 - 2*b^2)*A/(sqrt(c*x^2 + b)*c^2) + 1/15*(3*c^3*x^6 -
 b*c^2*x^4 + 4*b^2*c*x^2 + 8*b^3)*B/(sqrt(c*x^2 + b)*c^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04

$$\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{2\left(4\,Bb^{\frac{5}{2}}-5\,Ab^{\frac{3}{2}}c\right)\mathrm{sgn}(x)}{15\,c^3} + \frac{(Bb^2-Abc)\sqrt{cx^2+b}}{c^3\mathrm{sgn}\,(x)} + \frac{3\,(cx^2+b)^{\frac{5}{2}}B-10\,(cx^2+b)^{\frac{3}{2}}Bb+5\,(cx^2+b)^{\frac{3}{2}}Ac}{15\,c^3\mathrm{sgn}\,(x)}$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] -2/15*(4*B*b^(5/2) - 5*A*b^(3/2)*c)*sgn(x)/c^3 + (B*b^2 - A*b*c)*sqrt(c*x^2
 + b)/(c^3*sgn(x)) + 1/15*(3*(c*x^2 + b)^(5/2)*B - 10*(c*x^2 + b)^(3/2)*B*b
 + 5*(c*x^2 + b)^(3/2)*A*c)/(c^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.68

$$\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{c\,x^4+b\,x^2}\,\left(\frac{8\,B\,b^2-10\,A\,b\,c}{15\,c^3}+\frac{x^2\,\left(5\,A\,c^2-4\,B\,b\,c\right)}{15\,c^3}+\frac{B\,x^4}{5\,c}\right)}{x}$$

```
[In] int((x^4*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
[Out] ((b*x^2 + c*x^4)^(1/2)*((8*B*b^2 - 10*A*b*c)/(15*c^3) + (x^2*(5*A*c^2 - 4*B
*b*c))/(15*c^3) + (B*x^4)/(5*c)))/x
```

3.141 $\int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 59

$$\int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{(2bB-3Ac)\sqrt{bx^2+cx^4}}{3c^2x}+\frac{Bx\sqrt{bx^2+cx^4}}{3c}$$

```
[Out] -1/3*(-3*A*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/c^2/x+1/3*B*x*(c*x^4+b*x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2064, 1602}

$$\int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{Bx\sqrt{bx^2+cx^4}}{3c}-\frac{\sqrt{bx^2+cx^4}(2bB-3Ac)}{3c^2x}$$

```
[In] Int[(x^2*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -1/3*((2*b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(c^2*x) + (B*x*Sqrt[b*x^2 + c*x^
4])/(3*c)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{Bx\sqrt{bx^2+cx^4}}{3c} - \frac{(2bB-3Ac)\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{3c} \\ &= -\frac{(2bB-3Ac)\sqrt{bx^2+cx^4}}{3c^2x} + \frac{Bx\sqrt{bx^2+cx^4}}{3c}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{x^2(b+cx^2)}(-2bB+3Ac+Bcx^2)}{3c^2x}$$

```
[In] Integrate[(x^2*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
[Out] (Sqrt[x^2*(b + c*x^2)]*(-2*b*B + 3*A*c + B*c*x^2))/(3*c^2*x)
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.63

method	result	size
trager	$\frac{(Bc\,x^2+3Ac-2Bb)\sqrt{x^4c+b\,x^2}}{3c^2x}$	37
gosper	$\frac{(c\,x^2+b)\,(Bc\,x^2+3Ac-2Bb)x}{3c^2\sqrt{x^4c+b\,x^2}}$	42
default	$\frac{(c\,x^2+b)\,(Bc\,x^2+3Ac-2Bb)x}{3c^2\sqrt{x^4c+b\,x^2}}$	42
risch	$\frac{x(c\,x^2+b)\,(Bc\,x^2+3Ac-2Bb)}{3\sqrt{x^2(c\,x^2+b)}\,c^2}$	42

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(B*c*x^2+3*A*c-2*B*b)/c^2/x*(c*x^4+b*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

$$\int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{cx^4+bx^2}(Bcx^2-2\,Bb+3\,Ac)}{3\,c^2x}$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(B*c*x^2 - 2*B*b + 3*A*c)/(c^2*x)
```

Sympy [F]

$$\int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^2(A+Bx^2)}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x**2*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\sqrt{cx^2+b}A}{c} + \frac{(c^2x^4-bcx^2-2\,b^2)B}{3\,\sqrt{cx^2+b}c^2}$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] sqrt(c*x^2 + b)*A/c + 1/3*(c^2*x^4 - b*c*x^2 - 2*b^2)*B/(sqrt(c*x^2 + b)*c^
2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

$$\int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\left(2\,Bb^{\frac{3}{2}}-3\,A\sqrt{b}c\right)\mathrm{sgn}(x)}{3\,c^2} + \frac{(cx^2+b)^{\frac{3}{2}}B}{3\,c^2\mathrm{sgn}\,(x)} - \frac{\sqrt{cx^2+b}(Bb-Ac)}{c^2\mathrm{sgn}\,(x)}$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] 1/3*(2*B*b^(3/2) - 3*A*sqrt(b)*c)*sgn(x)/c^2 + 1/3*(c*x^2 + b)^(3/2)*B/(c^2
*sgn(x)) - sqrt(c*x^2 + b)*(B*b - A*c)/(c^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

$$\int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{\left(\frac{3\,A\,c-2\,B\,b}{3\,c^2}+\frac{B\,x^2}{3\,c}\right)\,\sqrt{c\,x^4+b\,x^2}}{x}$$

```
[In] int((x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
[Out] (((3*A*c - 2*B*b)/(3*c^2) + (B*x^2)/(3*c))*(b*x^2 + c*x^4)^(1/2))/x
```

3.142 $\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}} dx$

Optimal result

Integrand size = 23, antiderivative size = 55

$$\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}} dx = \frac{B\sqrt{bx^2+cx^4}}{cx} - \frac{A\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

```
[Out] -A*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(1/2)+B*(c*x^4+b*x^2)^(1/2)/c/x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1159, 2033, 212}

$$\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}} dx = \frac{B\sqrt{bx^2+cx^4}}{cx} - \frac{A\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

```
[In] Int[(A + B*x^2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (B*Sqrt[b*x^2 + c*x^4])/(c*x) - (A*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
)/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
Simp[e*((b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 3)*x)), x] - Dist[(b*e*(2*p + 1)
```

```
- c*d*(4*p + 3))/(c*(4*p + 3)), Int[(b*x^2 + c*x^4)^p, x], x] /; FreeQ[{b,
c, d, e, p}, x] &&  !IntegerQ[p] && NeQ[4*p + 3, 0] && NeQ[b*e*(2*p + 1) -
c*d*(4*p + 3), 0]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{B\sqrt{bx^2+cx^4}}{cx} + A\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx \\ &= \frac{B\sqrt{bx^2+cx^4}}{cx} - A\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right) \\ &= \frac{B\sqrt{bx^2+cx^4}}{cx} - \frac{A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}}\,dx = \frac{x\left(\sqrt{b}B(b+cx^2) - Ac\sqrt{b+cx^2}\,\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{\sqrt{b}c\sqrt{x^2(b+cx^2)}}$$

```
[In] Integrate[(A + B*x^2)/Sqrt[b*x^2 + c*x^4],x]
[Out] (x*(Sqrt[b]*B*(b + c*x^2) - A*c*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqr
t[b]]))/(Sqrt[b]*c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.31

method	result	size
default	$-\frac{x\sqrt{c\,x^2+b}\left(A\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)c-B\sqrt{c\,x^2+b}\,\sqrt{b}\right)}{\sqrt{x^4c+b\,x^2}\,c\sqrt{b}}$	72

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x*(c*x^2+b)^(1/2)*(A*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*c-B*(c*x^2+b)^(1/
2)*b^(1/2))/(c*x^4+b*x^2)^(1/2)/c/b^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.51

$$\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}}\,dx = \left[\frac{A\sqrt{b}cx\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}Bb}{2\,bcx}, \frac{A\sqrt{-b}cx\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}B}{bcx}\right.$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(A*sqrt(b)*c*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^
3) + 2*sqrt(c*x^4 + b*x^2)*B*b)/(b*c*x), (A*sqrt(-b)*c*x*arctan(sqrt(c*x^4
+ b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*B*b)/(b*c*x)]
```

Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}}\,dx$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

$$\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}}\,dx = -\frac{\left(Ac\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+B\sqrt{-b}\sqrt{b}\right)\operatorname{sgn}(x)}{\sqrt{-b}c}+\frac{\frac{A\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+\frac{\sqrt{cx^2+b}B}{c}}{\operatorname{sgn}(x)}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] -(A*c*arctan(sqrt(b)/sqrt(-b)) + B*sqrt(-b)*sqrt(b))*sgn(x)/(sqrt(-b)*c) +
(A*arctan(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b) + sqrt(c*x^2 + b)*B/c)/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{B\,x^2+A}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

```
[In] int((A + B*x^2)/(b*x^2 + c*x^4)^(1/2),x)
[Out] int((A + B*x^2)/(b*x^2 + c*x^4)^(1/2), x)
```

3.143 $\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 68

$$\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}\,dx=-\frac{A\sqrt{bx^2+cx^4}}{2bx^3}-\frac{(2bB-Ac)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}$$

```
[Out] -1/2*(-A*c+2*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(3/2)-1/2*A*(c*x
^4+b*x^2)^(1/2)/b/x^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2063, 2033, 212}

$$\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}\,dx=-\frac{(2bB-Ac)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{A\sqrt{bx^2+cx^4}}{2bx^3}$$

```
[In] Int[(A + B*x^2)/(x^2*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*(A*Sqrt[b*x^2 + c*x^4])/(b*x^3) - ((2*b*B - A*c)*ArcTanh[(Sqrt[b]*x)/S
qrt[b*x^2 + c*x^4]])/(2*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{A\sqrt{bx^2+cx^4}}{2bx^3} - \frac{(-2bB+Ac)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{2b} \\
&= -\frac{A\sqrt{bx^2+cx^4}}{2bx^3} + \frac{(-2bB+Ac)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)}{2b} \\
&= -\frac{A\sqrt{bx^2+cx^4}}{2bx^3} - \frac{(2bB-Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.26

$$
\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}\,dx = \frac{-A\sqrt{b}(b+cx^2)-(2bB-Ac)x^2\sqrt{b+cx^2}\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{2b^{3/2}x\sqrt{x^2(b+cx^2)}}
$$

```
[In] Integrate[(A + B*x^2)/(x^2*Sqrt[b*x^2 + c*x^4]),x]
[Out] (-(A*Sqrt[b]*(b + c*x^2)) - (2*b*B - A*c)*x^2*Sqrt[b + c*x^2]*ArcTanh[Sqrt[
b + c*x^2]/Sqrt[b]])/(2*b^(3/2)*x*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.32

method	result	size
risch	$-\frac{A(c\,x^2+b)}{2bx\sqrt{x^2(c\,x^2+b)}}+\frac{(Ac-2Bb)\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)x\sqrt{c\,x^2+b}}{2b^{\frac{3}{2}}\sqrt{x^2(c\,x^2+b)}}$	90
default	$-\frac{\sqrt{c\,x^2+b}\left(2B\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)b^2x^2-A\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)bc\,x^2+A\sqrt{c\,x^2+b}\,b^{\frac{3}{2}}\right)}{2x\sqrt{x^4c+b\,x^2}\,b^{\frac{5}{2}}}$	105

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2/b*A*(c*x^2+b)/x/(x^2*(c*x^2+b))^(1/2)+1/2*(A*c-2*B*b)/b^(3/2)*ln((2*b+
2*b^(1/2)*(c*x^2+b)^(1/2))/x)*x/(x^2*(c*x^2+b))^(1/2)*(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.24

$$\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}\,dx = \left[-\frac{(2\,Bb-Ac)\sqrt{b}x^3\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}Ab}{4\,b^2x^3},\ \frac{(2\,Bb-Ac)\sqrt{-b}x^3\arctan\left(\frac{\sqrt{cx^4+bx^2}}{cx^3+b}\right.}{2\,b^2x^3}\right.$$

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] [-1/4*((2*B*b - A*c)*sqrt(b)*x^3*log(-(c*x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2
)*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*A*b)/(b^2*x^3), 1/2*((2*B*b - A*c)*
sqrt(-b)*x^3*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) - sqrt(c*x^
4 + b*x^2)*A*b)/(b^2*x^3)]
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^2\sqrt{x^2\,(b+cx^2)}}\,dx$$

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral((A + B*x**2)/(x**2*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^2}\,dx$$

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}\,dx = \frac{\frac{(2\,Bbc-Ac^2)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b} - \frac{\sqrt{cx^2+b}Ac}{bx^2}}{2\,c\mathrm{sgn}\,(x)}$$

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] 1/2*((2*B*b*c - A*c^2)*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b) - sqrt
(c*x^2 + b)*A*c/(b*x^2))/(c*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^2\sqrt{bx^2+cx^4}}\,dx = \int \frac{B\,x^2+A}{x^2\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

```
[In] int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(1/2)),x)
[Out] int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(1/2)), x)
```

3.144 $\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 103

$$\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}\,dx = -\frac{A\sqrt{bx^2+cx^4}}{4bx^5} - \frac{(4bB-3Ac)\sqrt{bx^2+cx^4}}{8b^2x^3} + \frac{c(4bB-3Ac)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}$$

```
[Out] 1/8*c*(-3*A*c+4*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(5/2)-1/4*A*(
c*x^4+b*x^2)^(1/2)/b/x^5-1/8*(-3*A*c+4*B*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2063, 2050, 2033, 212}

$$\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}\,dx = \frac{c(4bB-3Ac)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}} - \frac{\sqrt{bx^2+cx^4}(4bB-3Ac)}{8b^2x^3} - \frac{A\sqrt{bx^2+cx^4}}{4bx^5}$$

```
[In] Int[(A + B*x^2)/(x^4*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -1/4*(A*Sqrt[b*x^2 + c*x^4])/(b*x^5) - ((4*b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4]
)/(8*b^2*x^3) + (c*(4*b*B - 3*A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
)/(8*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{A\sqrt{bx^2+cx^4}}{4bx^5} - \frac{(-4bB+3Ac)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{4b} \\
&= -\frac{A\sqrt{bx^2+cx^4}}{4bx^5} - \frac{(4bB-3Ac)\sqrt{bx^2+cx^4}}{8b^2x^3} - \frac{(c(4bB-3Ac))\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b^2} \\
&= -\frac{A\sqrt{bx^2+cx^4}}{4bx^5} - \frac{(4bB-3Ac)\sqrt{bx^2+cx^4}}{8b^2x^3} + \frac{(c(4bB-3Ac))\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)}{8b^2} \\
&= -\frac{A\sqrt{bx^2+cx^4}}{4bx^5} - \frac{(4bB-3Ac)\sqrt{bx^2+cx^4}}{8b^2x^3} + \frac{c(4bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{-\sqrt{b}(b+cx^2)\left(2Ab+4bBx^2-3Acx^2\right)+c(4bB-3Ac)x^4\sqrt{b+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{8b^{5/2}x^3\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(A + B*x^2)/(x^4*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-(Sqrt[b]*(b + c*x^2)*(2*A*b + 4*b*B*x^2 - 3*A*c*x^2)) + c*(4*b*B - 3*A*c)
*x^4*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])/(8*b^(5/2)*x^3*Sqrt[
x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{(c x^2+b)(-3Ac x^2+4bB x^2+2Ab)}{8b^2x^3\sqrt{x^2(c x^2+b)}}-\frac{(3Ac-4Bb)c\ln\left(\frac{2b+2\sqrt{b}\sqrt{c x^2+b}}{x}\right)x\sqrt{c x^2+b}}{8b^{\frac{5}{2}}\sqrt{x^2(c x^2+b)}}$
default	$-\frac{\sqrt{c x^2+b}\left(3A\ln\left(\frac{2b+2\sqrt{b}\sqrt{c x^2+b}}{x}\right)b c^2x^4-4B\ln\left(\frac{2b+2\sqrt{b}\sqrt{c x^2+b}}{x}\right)b^2c x^4-3A\sqrt{c x^2+b}\,b^{\frac{3}{2}}c x^2+4B\sqrt{c x^2+b}\,b^{\frac{5}{2}}x^2+2A\sqrt{c x^2+b}\,b\right.}{8x^3\sqrt{x^4c+b x^2}\,b^{\frac{7}{2}}}$

```
[In] int((B*x^2+A)/x^4/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(c*x^2+b)*(-3*A*c*x^2+4*B*b*x^2+2*A*b)/b^2/x^3/(x^2*(c*x^2+b))^(1/2)-1
/8*(3*A*c-4*B*b)*c/b^(5/2)*ln((2*b+2*b^(1/2)*(c*x^2+b)^(1/2))/x)*x/(x^2*(c*
x^2+b))^(1/2)*(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.93

$$\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}\,dx$$
$$=\left[-\frac{(4\,Bbc-3\,Ac^2)\sqrt{b}x^5\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}(2\,Ab^2+(4\,Bb^2-3\,Abc)x^2)}{16\,b^3x^5},\right.$$
$$\left.-\frac{(4\,Bbc-3\,Ac^2)\sqrt{-b}x^5\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)+\sqrt{cx^4+bx^2}(2\,Ab^2+(4\,Bb^2-3\,Abc)x^2)}{8\,b^3x^5}\right]$$

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((4*B*b*c - 3*A*c^2)*sqrt(b)*x^5*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4
+ b*x^2)*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(2*A*b^2 + (4*B*b^2 - 3*A*b*
c)*x^2))/(b^3*x^5), -1/8*((4*B*b*c - 3*A*c^2)*sqrt(-b)*x^5*arctan(sqrt(c*x^
4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x)) + sqrt(c*x^4 + b*x^2)*(2*A*b^2 + (4*B*b^
2 - 3*A*b*c)*x^2))/(b^3*x^5)]
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}\,dx=\int \frac{A+Bx^2}{x^4\sqrt{x^2\,(b+cx^2)}}\,dx$$

```
[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**4*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^4}\,dx$$

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}\,dx = -\frac{\frac{\left(4\,Bbc^2-3\,Ac^3\right)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2}+\frac{4\,\left(cx^2+b\right)^{\frac{3}{2}}Bbc^2-4\,\sqrt{cx^2+b}Bb^2c^2-3\,\left(cx^2+b\right)^{\frac{3}{2}}Ac^3+5\,\sqrt{cx^2+b}Abc^3}{b^2c^2x^4}}{8\,c\mathrm{sgn}\left(x\right)}$$

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*((4*B*b*c^2 - 3*A*c^3)*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^2)
 + (4*(c*x^2 + b)^(3/2)*B*b*c^2 - 4*sqrt(c*x^2 + b)*B*b^2*c^2 - 3*(c*x^2 +
b)^(3/2)*A*c^3 + 5*sqrt(c*x^2 + b)*A*b*c^3)/(b^2*c^2*x^4))/(c*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^4\sqrt{bx^2+cx^4}}\,dx = \int \frac{B\,x^2+A}{x^4\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

```
[In] int((A + B*x^2)/(x^4*(b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((A + B*x^2)/(x^4*(b*x^2 + c*x^4)^(1/2)), x)
```

3.145 $\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 184

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=-\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}}+\frac{5b(7bB-6Ac)\sqrt{bx^2+cx^4}}{16c^4}-\frac{5(7bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^3}+\frac{(7bB-6Ac)x^4\sqrt{bx^2+cx^4}}{6bc^2}-\frac{5b^2(7bB-6Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{9/2}}$$

```
[Out] -5/16*b^2*(-6*A*c+7*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(9/2)-(
-A*c+B*b)*x^8/b/c/(c*x^4+b*x^2)^(1/2)+5/16*b*(-6*A*c+7*B*b)*(c*x^4+b*x^2)^(
1/2)/c^4-5/24*(-6*A*c+7*B*b)*x^2*(c*x^4+b*x^2)^(1/2)/c^3+1/6*(-6*A*c+7*B*b)
*x^4*(c*x^4+b*x^2)^(1/2)/b/c^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2059, 802, 684, 654, 634, 212}

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=-\frac{5b^2(7bB-6Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{9/2}}+\frac{5b\sqrt{bx^2+cx^4}(7bB-6Ac)}{16c^4}-\frac{5x^2\sqrt{bx^2+cx^4}(7bB-6Ac)}{24c^3}+\frac{x^4\sqrt{bx^2+cx^4}(7bB-6Ac)}{6bc^2}-\frac{x^8(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[In] Int[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^8)/(b*c*Sqrt[b*x^2 + c*x^4])) + (5*b*(7*b*B - 6*A*c)*Sqrt[
b*x^2 + c*x^4])/(16*c^4) - (5*(7*b*B - 6*A*c)*x^2*Sqrt[b*x^2 + c*x^4])/(24*
c^3) + ((7*b*B - 6*A*c)*x^4*Sqrt[b*x^2 + c*x^4])/(6*b*c^2) - (5*b^2*(7*b*B
- 6*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 802

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Dist[e*((m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e))), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
```

```
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^4(A+Bx)}{(bx+cx^2)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} + \frac{1}{2}\left(-\frac{6A}{b}+\frac{7B}{c}\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} + \frac{(7bB-6Ac)x^4\sqrt{bx^2+cx^4}}{6bc^2} - \frac{(5(7bB-6Ac))\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{12c^2}$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} - \frac{5(7bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^3} + \frac{(7bB-6Ac)x^4\sqrt{bx^2+cx^4}}{6bc^2} + \frac{(5b(7bB-6Ac))\text{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{16c^3}$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} + \frac{5b(7bB-6Ac)\sqrt{bx^2+cx^4}}{16c^4} - \frac{5(7bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^3} + \frac{(7bB-6Ac)x^4\sqrt{bx^2+cx^4}}{6bc^2} - \frac{(5b^2(7bB-6Ac))\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{32c^4}$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} + \frac{5b(7bB-6Ac)\sqrt{bx^2+cx^4}}{16c^4} - \frac{5(7bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^3} + \frac{(7bB-6Ac)x^4\sqrt{bx^2+cx^4}}{6bc^2} - \frac{(5b^2(7bB-6Ac))\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^4}$$

$$= -\frac{(bB-Ac)x^8}{bc\sqrt{bx^2+cx^4}} + \frac{5b(7bB-6Ac)\sqrt{bx^2+cx^4}}{16c^4} - \frac{5(7bB-6Ac)x^2\sqrt{bx^2+cx^4}}{24c^3} + \frac{(7bB-6Ac)x^4\sqrt{bx^2+cx^4}}{6bc^2} - \frac{5b^2(7bB-6Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{9/2}}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.85

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x^3\Big(\sqrt{c}x(b+cx^2)\,(105b^3B+4c^3x^4(3A+2Bx^2)-2bc^2x^2(15A+7Bx^2)+b^2(-90Ac}{48c^{9/2}\,(x^2\,(b+cx^2))}$$

[In] Integrate[(x^9*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]

[Out] (x^3*(Sqrt[c]*x*(b + c*x^2)*(105*b^3*B + 4*c^3*x^4*(3*A + 2*B*x^2) - 2*b*c^2*x^2*(15*A + 7*B*x^2) + b^2*(-90*A*c + 35*B*c*x^2)) - 30*b^2*(7*b*B - 6*A*c)*(b + c*x^2)^(3/2)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[b] + Sqrt[b + c*x^2])]))/(48*c^(9/2)*(x^2*(b + c*x^2))^(3/2))

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{-\frac{15x^2\left(-\frac{7x^2B}{18}+A\right)b^2c^{\frac{3}{2}}}{8}-\frac{5x^4\left(\frac{7x^2B}{15}+A\right)b\,c^{\frac{5}{2}}}{8}+\frac{x^6\left(\frac{2x^2B}{3}+A\right)c^{\frac{7}{2}}}{4}+\frac{15\left(\frac{7Bb\,x^2\sqrt{c}}{3}+\left(Ac-\frac{7Bb}{6}\right)\sqrt{x^2\left(c\,x^2+b\right)}\,(-\ln(2)+\ln(\frac{2c\,x^2}{}\cdots\right.}{16}}{c^{\frac{9}{2}}\sqrt{x^2(c\,x^2+b)}}$
default	$\frac{x^3\left(c\,x^2+b\right)(8x^7B\,c^{\frac{9}{2}}+12A\,c^{\frac{9}{2}}x^5-14x^5Bb\,c^{\frac{7}{2}}-30A\,c^{\frac{7}{2}}b\,x^3+35b^2x^3B\,c^{\frac{5}{2}}-90A\,c^{\frac{5}{2}}b^2x+105Bx\,b^3c^{\frac{3}{2}}+90A\sqrt{c\,x^2+b}\,\ln(\sqrt{c}\,x\cdots}{48(x^4c+b\,x^2)^{\frac{3}{2}}c^{\frac{11}{2}}}$
risch	$-\frac{x^2\left(-8B\,c^2x^4-12A\,c^2x^2+22Bbc\,x^2+42Abc-57B\,b^2\right)\left(c\,x^2+b\right)}{48c^4\sqrt{x^2(c\,x^2+b)}}+\frac{b^2(-\frac{19Bbx}{\sqrt{c\,x^2+b}}+\frac{14Acx}{\sqrt{c\,x^2+b}}+(30A\,c^2-35Bbc)(-\frac{x}{c\sqrt{c\,x^2}\cdots}}{16c^4\sqrt{x^2(c\,x^2+b)}}$

[In] int(x^9*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 15/16*(-2*x^2*(-7/18*x^2*B+A)*b^2*c^(3/2)-2/3*x^4*(7/15*x^2*B+A)*b*c^(5/2)+4/15*x^6*(2/3*x^2*B+A)*c^(7/2)+(7/3*B*b*x^2*c^(1/2)+(A*c-7/6*B*b)*(x^2*(c*x^2+b))^(1/2)*(-ln(2)+ln((2*c*x^2+2*(x^2*(c*x^2+b))^(1/2)*c^(1/2)+b)/c^(1/2))))*b^2)/c^(9/2)/(x^2*(c*x^2+b))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.85

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \left[-\frac{15\,(7\,Bb^4-6\,Ab^3c+(7\,Bb^3c-6\,Ab^2c^2)x^2)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{}\right.$$

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*(7*B*b^4 - 6*A*b^3*c + (7*B*b^3*c - 6*A*b^2*c^2)*x^2)*sqrt(c)*lo
g(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*(8*B*c^4*x^6 + 105*B*b^
3*c - 90*A*b^2*c^2 - 2*(7*B*b*c^3 - 6*A*c^4)*x^4 + 5*(7*B*b^2*c^2 - 6*A*b*c
^3)*x^2)*sqrt(c*x^4 + b*x^2))/(c^6*x^2 + b*c^5), 1/48*(15*(7*B*b^4 - 6*A*b^
3*c + (7*B*b^3*c - 6*A*b^2*c^2)*x^2)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sq
rt(-c)/(c*x^2 + b)) + (8*B*c^4*x^6 + 105*B*b^3*c - 90*A*b^2*c^2 - 2*(7*B*b*
c^3 - 6*A*c^4)*x^4 + 5*(7*B*b^2*c^2 - 6*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/
(c^6*x^2 + b*c^5)]
```

Sympy [F]

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^9(A+Bx^2)}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**9*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.29

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{1}{16}\left(\frac{4\,x^6}{\sqrt{cx^4+bx^2}c}-\frac{10\,bx^4}{\sqrt{cx^4+bx^2}c^2}-\frac{30\,b^2x^2}{\sqrt{cx^4+bx^2}c^3}+\frac{15\,b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+\right.}{c^{\frac{7}{2}}}\right.$$
$$+\frac{1}{96}\left(\frac{16\,x^8}{\sqrt{cx^4+bx^2}c}-\frac{28\,bx^6}{\sqrt{cx^4+bx^2}c^2}+\frac{70\,b^2x^4}{\sqrt{cx^4+bx^2}c^3}+\frac{210\,b^3x^2}{\sqrt{cx^4+bx^2}c^4}-\frac{105\,b^3\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2\right.}{c^{\frac{9}{2}}}\right.$$

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*x^6/(sqrt(c*x^4 + b*x^2)*c) - 10*b*x^4/(sqrt(c*x^4 + b*x^2)*c^2) -
30*b^2*x^2/(sqrt(c*x^4 + b*x^2)*c^3) + 15*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^
4 + b*x^2)*sqrt(c))/c^(7/2))*A + 1/96*(16*x^8/(sqrt(c*x^4 + b*x^2)*c) - 28*
b*x^6/(sqrt(c*x^4 + b*x^2)*c^2) + 70*b^2*x^4/(sqrt(c*x^4 + b*x^2)*c^3) + 21
0*b^3*x^2/(sqrt(c*x^4 + b*x^2)*c^4) - 105*b^3*log(2*c*x^2 + b + 2*sqrt(c*x^
4 + b*x^2)*sqrt(c))/c^(9/2))*B
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.99

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\left(\left(2\,x^2\left(\frac{4\,Bx^2}{c\mathrm{sgn}(x)} - \frac{7\,Bbc^5\mathrm{sgn}(x) - 6\,Ac^6\mathrm{sgn}(x)}{c^7}\right) + \frac{5\,(7\,Bb^2c^4\mathrm{sgn}(x) - 6\,Abc^5\mathrm{sgn}(x))}{c^7}\right)x^2 + \frac{15\,(7\,Bb^3c^3\mathrm{sgn}(\cdots}{\cdots}\right)\cdots}{48\,\sqrt{cx^2+b}} - \frac{5\,(7\,Bb^3\log(|b|) - 6\,Ab^2c\log(|b|))\mathrm{sgn}(x)}{32\,c^{\frac{9}{2}}} + \frac{5\,(7\,Bb^3 - 6\,Ab^2c)\log\left(\left|-\sqrt{c}x + \sqrt{cx^2+b}\right|\right)}{16\,c^{\frac{9}{2}}\mathrm{sgn}\,(x)}$$

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*((2*x^2*(4*B*x^2/(c*sgn(x)) - (7*B*b*c^5*sgn(x) - 6*A*c^6*sgn(x))/c^7)
 + 5*(7*B*b^2*c^4*sgn(x) - 6*A*b*c^5*sgn(x))/c^7)*x^2 + 15*(7*B*b^3*c^3*sgn
(x) - 6*A*b^2*c^4*sgn(x))/c^7)*x/sqrt(c*x^2 + b) - 5/32*(7*B*b^3*log(abs(b)
) - 6*A*b^2*c*log(abs(b)))*sgn(x)/c^(9/2) + 5/16*(7*B*b^3 - 6*A*b^2*c)*log(
abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(9/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^9\,(B\,x^2+A)}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

```
[In] int((x^9*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((x^9*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.146 $\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^6}{bc\sqrt{bx^2+cx^4}}-\frac{3(5bB-4Ac)\sqrt{bx^2+cx^4}}{8c^3}+\frac{(5bB-4Ac)x^2\sqrt{bx^2+cx^4}}{4bc^2}+\frac{3b(5bB-4Ac)\,\mathrm{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}$$

```
[Out] 3/8*b*(-4*A*c+5*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(7/2)-(-A*c
+B*b)*x^6/b/c/(c*x^4+b*x^2)^(1/2)-3/8*(-4*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/c^
3+1/4*(-4*A*c+5*B*b)*x^2*(c*x^4+b*x^2)^(1/2)/b/c^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2059, 802, 684, 654, 634, 212}

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{3b(5bB-4Ac)\,\mathrm{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}-\frac{3\sqrt{bx^2+cx^4}(5bB-4Ac)}{8c^3}+\frac{x^2\sqrt{bx^2+cx^4}(5bB-4Ac)}{4bc^2}-\frac{x^6(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[In] Int[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^6)/(b*c*Sqrt[b*x^2 + c*x^4])) - (3*(5*b*B - 4*A*c)*Sqrt[b*
x^2 + c*x^4])/(8*c^3) + ((5*b*B - 4*A*c)*x^2*Sqrt[b*x^2 + c*x^4])/(4*b*c^2)
```

```
 + (3*b*(5*b*B - 4*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(7
/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 802

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Dist[e*((m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e))), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
```

```
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(A+Bx)}{(bx+cx^2)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{(bB-Ac)x^6}{bc\sqrt{bx^2+cx^4}} + \frac{1}{2}\left(-\frac{4A}{b}+\frac{5B}{c}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)\\
&= -\frac{(bB-Ac)x^6}{bc\sqrt{bx^2+cx^4}} + \frac{(5bB-4Ac)x^2\sqrt{bx^2+cx^4}}{4bc^2} - \frac{(3(5bB-4Ac))\text{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{8c^2}\\
&= -\frac{(bB-Ac)x^6}{bc\sqrt{bx^2+cx^4}} - \frac{3(5bB-4Ac)\sqrt{bx^2+cx^4}}{8c^3} + \frac{(5bB-4Ac)x^2\sqrt{bx^2+cx^4}}{4bc^2}\\
&\quad + \frac{(3b(5bB-4Ac))\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{16c^3}\\
&= -\frac{(bB-Ac)x^6}{bc\sqrt{bx^2+cx^4}} - \frac{3(5bB-4Ac)\sqrt{bx^2+cx^4}}{8c^3} + \frac{(5bB-4Ac)x^2\sqrt{bx^2+cx^4}}{4bc^2}\\
&\quad + \frac{(3b(5bB-4Ac))\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^3}\\
&= -\frac{(bB-Ac)x^6}{bc\sqrt{bx^2+cx^4}} - \frac{3(5bB-4Ac)\sqrt{bx^2+cx^4}}{8c^3}\\
&\quad + \frac{(5bB-4Ac)x^2\sqrt{bx^2+cx^4}}{4bc^2} + \frac{3b(5bB-4Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90

$$
\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x^3\left(\sqrt{c}(b+cx^2)\left(-15b^2Bx+bcx(12A-5Bx^2)+2c^2x^3(2A+Bx^2)\right)+6b(5bB-4Ac\right.}{8c^{7/2}\left(x^2\left(b+cx^2\right)\right)^{3/2}}
$$

```
[In] Integrate[(x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^3*(Sqrt[c]*(b + c*x^2)*(-15*b^2*B*x + b*c*x*(12*A - 5*B*x^2) + 2*c^2*x^3
*(2*A + B*x^2)) + 6*b*(5*b*B - 4*A*c)*(b + c*x^2)^(3/2)*ArcTanh[(Sqrt[c]*x)
/(-Sqrt[b] + Sqrt[b + c*x^2])]))/(8*c^(7/2)*(x^2*(b + c*x^2))^(3/2))
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.95

method	result
default	$-\frac{x^{3}\left(c\,x^{2}+b\right)\left(-2x^{5}B\,c^{\frac{7}{2}}-4A\,c^{\frac{7}{2}}x^{3}+5x^{3}Bb\,c^{\frac{5}{2}}-12A\,c^{\frac{5}{2}}bx+15b^{2}Bx\,c^{\frac{3}{2}}+12A\sqrt{c\,x^{2}+b}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)b\,c^{2}-15B\sqrt{c\,x^{2}}\right.}{8\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}c^{\frac{9}{2}}}$
risch	$\frac{x^{2}\left(2Bc\,x^{2}+4Ac-7Bb\right)\left(c\,x^{2}+b\right)}{8c^{3}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{b\left(-\frac{7Bbx}{\sqrt{c\,x^{2}+b}}+\frac{4Acx}{\sqrt{c\,x^{2}+b}}+\left(12A\,c^{2}-15Bbc\right)\left(-\frac{x}{c\sqrt{c\,x^{2}+b}}+\frac{\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+b}\right)}{c^{\frac{3}{2}}}\right)\right)x\sqrt{c\,x^{2}}}{8c^{3}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$
pseudoelliptic	$\frac{4B\,c^{\frac{5}{2}}x^{6}+8A\,c^{\frac{5}{2}}x^{4}-10Bb\,c^{\frac{3}{2}}x^{4}+24Ab\,c^{\frac{3}{2}}x^{2}-30B\,b^{2}x^{2}\sqrt{c}+12A\ln(2)bc\sqrt{x^{2}\left(c\,x^{2}+b\right)}-12A\ln\left(\frac{2c\,x^{2}+2\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,\sqrt{c}+b}{\sqrt{c}}\right)}{16c^{\frac{7}{2}}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*x^3*(c*x^2+b)*(-2*x^5*B*c^(7/2)-4*A*c^(7/2)*x^3+5*x^3*B*b*c^(5/2)-12*A
*c^(5/2)*b*x+15*b^2*B*x*c^(3/2)+12*A*(c*x^2+b)^(1/2)*ln(c^(1/2)*x+(c*x^2+b)
^(1/2))*b*c^2-15*B*(c*x^2+b)^(1/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*b^2*c)/(c*
x^4+b*x^2)^(3/2)/c^(9/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.97

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=\left[-\frac{3\,(5\,Bb^3-4\,Ab^2c+(5\,Bb^2c-4\,Abc^2)x^2)\sqrt{c}\log\left(-2\,cx^2-b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{16\,(c^5x^2+bc^4)}\right.$$

$$-\frac{3\,(5\,Bb^3-4\,Ab^2c+(5\,Bb^2c-4\,Abc^2)x^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)-(2\,Bc^3x^4-15\,Bb^2c+12\,Abc^2-(5}{8\,(c^5x^2+bc^4)}$$

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*(5*B*b^3 - 4*A*b^2*c + (5*B*b^2*c - 4*A*b*c^2)*x^2)*sqrt(c)*log(-
2*c*x^2 - b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*(2*B*c^3*x^4 - 15*B*b^2*c
+ 12*A*b*c^2 - (5*B*b*c^2 - 4*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/(c^5*x^2 + b
*c^4), -1/8*(3*(5*B*b^3 - 4*A*b^2*c + (5*B*b^2*c - 4*A*b*c^2)*x^2)*sqrt(-c)
*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) - (2*B*c^3*x^4 - 15*B*b^2
*c + 12*A*b*c^2 - (5*B*b*c^2 - 4*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2))/(c^5*x^2
+ b*c^4)]
```

Sympy [F]

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^7(A+Bx^2)}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(x**7*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.27

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{1}{4}\left(\frac{2\,x^4}{\sqrt{cx^4+bx^2}c}+\frac{6\,bx^2}{\sqrt{cx^4+bx^2}c^2}-\frac{3\,b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{5}{2}}}\right)A + \frac{1}{16}\left(\frac{4\,x^6}{\sqrt{cx^4+bx^2}c}-\frac{10\,bx^4}{\sqrt{cx^4+bx^2}c^2}-\frac{30\,b^2x^2}{\sqrt{cx^4+bx^2}c^3}+\frac{15\,b^2\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{7}{2}}}\right)B$$

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] 1/4*(2*x^4/(sqrt(c*x^4 + b*x^2)*c) + 6*b*x^2/(sqrt(c*x^4 + b*x^2)*c^2) - 3*
b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2))*A + 1/16*(4*x^6
/(sqrt(c*x^4 + b*x^2)*c) - 10*b*x^4/(sqrt(c*x^4 + b*x^2)*c^2) - 30*b^2*x^2/
(sqrt(c*x^4 + b*x^2)*c^3) + 15*b^2*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*
sqrt(c))/c^(7/2))*B
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\left(x^2\left(\frac{2\,Bx^2}{c\mathrm{sgn}(x)}-\frac{5\,Bbc^3\mathrm{sgn}(x)-4\,Ac^4\mathrm{sgn}(x)}{c^5}\right)-\frac{3\,(5\,Bb^2c^2\mathrm{sgn}(x)-4\,Abc^3\mathrm{sgn}(x))}{c^5}\right)x}{8\,\sqrt{cx^2+b}} + \frac{3\,(5\,Bb^2\log(|b|)-4\,Abc\log(|b|))\mathrm{sgn}(x)}{16\,c^{\frac{7}{2}}} - \frac{3\,(5\,Bb^2-4\,Abc)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{8\,c^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] 1/8*(x^2*(2*B*x^2/(c*sgn(x)) - (5*B*b*c^3*sgn(x) - 4*A*c^4*sgn(x))/c^5) - 3
*(5*B*b^2*c^2*sgn(x) - 4*A*b*c^3*sgn(x))/c^5)*x/sqrt(c*x^2 + b) + 3/16*(5*B
*b^2*log(abs(b)) - 4*A*b*c*log(abs(b)))*sgn(x)/c^(7/2) - 3/8*(5*B*b^2 - 4*A
*b*c)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(7/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^7\,(B\,x^2+A)}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

```
[In] int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
[Out] int((x^7*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.147 $\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 112

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^4}{bc\sqrt{bx^2+cx^4}} + \frac{(3bB-2Ac)\sqrt{bx^2+cx^4}}{2bc^2} - \frac{(3bB-2Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}$$

```
[Out] -1/2*(-2*A*c+3*B*b)*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(5/2)-(-A*c+
B*b)*x^4/b/c/(c*x^4+b*x^2)^(1/2)+1/2*(-2*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/b/c
^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2059, 802, 654, 634, 212}

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(3bB-2Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}} + \frac{\sqrt{bx^2+cx^4}(3bB-2Ac)}{2bc^2} - \frac{x^4(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[In] Int[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^4)/(b*c*Sqrt[b*x^2 + c*x^4])) + ((3*b*B - 2*A*c)*Sqrt[b*x^
2 + c*x^4])/(2*b*c^2) - ((3*b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
 c*x^4]])/(2*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 802

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((a
 + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Dist[e*((m*(g*(c*d
 - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e))), Int
[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d,
e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ
[p, -1] && GtQ[m, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(bx+cx^2)^{3/2}}\,dx, x, x^2\right) \\
&= -\frac{(bB-Ac)x^4}{bc\sqrt{bx^2+cx^4}} + \frac{1}{2}\left(-\frac{2A}{b}+\frac{3B}{c}\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+cx^2}}\,dx, x, x^2\right) \\
&= -\frac{(bB-Ac)x^4}{bc\sqrt{bx^2+cx^4}} + \frac{(3bB-2Ac)\sqrt{bx^2+cx^4}}{2bc^2} - \frac{(3bB-2Ac)\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{4c^2}
\end{aligned}$$

$$= -\frac{(bB - Ac)x^4}{bc\sqrt{bx^2 + cx^4}} + \frac{(3bB - 2Ac)\sqrt{bx^2 + cx^4}}{2bc^2} - \frac{(3bB - 2Ac)\text{Subst}\left(\int \frac{1}{1-cx^2} dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^2}$$

$$= -\frac{(bB - Ac)x^4}{bc\sqrt{bx^2 + cx^4}} + \frac{(3bB - 2Ac)\sqrt{bx^2 + cx^4}}{2bc^2} - \frac{(3bB - 2Ac)\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96

$$\int \frac{x^5(A + Bx^2)}{(bx^2 + cx^4)^{3/2}} dx = \frac{x^3\left(\sqrt{c}x(b + cx^2)(3bB - 2Ac + Bcx^2) + 2(-3bB + 2Ac)(b + cx^2)^{3/2} \text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{b}+\sqrt{b+cx^2}}\right)\right)}{2c^{5/2}(x^2(b + cx^2))^{3/2}}$$

```
[In] Integrate[(x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^3*(Sqrt[c]*x*(b + c*x^2)*(3*b*B - 2*A*c + B*c*x^2) + 2*(-3*b*B + 2*A*c)*
(b + c*x^2)^(3/2)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[b] + Sqrt[b + c*x^2])]))/(2*c^
(5/2)*(x^2*(b + c*x^2))^(3/2))
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02

method	result
default	$\frac{x^3(c\,x^2+b)\left(x^3 B\,c^{\frac{5}{2}}-2A\,c^{\frac{5}{2}}x+3bBx\,c^{\frac{3}{2}}+2A\sqrt{c\,x^2+b}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)c^2-3B\sqrt{c\,x^2+b}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)bc\right)}{2(x^4c+b\,x^2)^{\frac{3}{2}}c^{\frac{7}{2}}}$
risch	$\frac{B\,x^2(c\,x^2+b)}{2c^2\sqrt{x^2(c\,x^2+b)}}+\frac{\left(-\frac{Bbx}{\sqrt{c\,x^2+b}}+(2A\,c^2-3Bbc)\left(-\frac{x}{c\sqrt{c\,x^2+b}}+\frac{\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+b}\right)}{c^{\frac{3}{2}}}\right)\right)x\sqrt{c\,x^2+b}}{2c^2\sqrt{x^2(c\,x^2+b)}}$
pseudoelliptic	$\frac{2B\,c^{\frac{3}{2}}x^4-4A\,c^{\frac{3}{2}}x^2+6Bb\,x^2\sqrt{c}-2A\ln(2)c\sqrt{x^2(c\,x^2+b)}+2A\ln\left(\frac{2c\,x^2+2\sqrt{x^2(c\,x^2+b)}\,\sqrt{c}+b}{\sqrt{c}}\right)c\sqrt{x^2(c\,x^2+b)}+3B\ln(2)b\sqrt{x^2(c}}{4c^{\frac{5}{2}}\sqrt{x^2(c\,x^2+b)}}$

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^3*(c*x^2+b)*(x^3*B*c^(5/2)-2*A*c^(5/2)*x+3*b*B*x*c^(3/2)+2*A*(c*x^2+b
)^(1/2)*ln(c^(1/2)*x+(c*x^2+b)^(1/2))*c^2-3*B*(c*x^2+b)^(1/2)*ln(c^(1/2)*x+
(c*x^2+b)^(1/2))*b*c)/(c*x^4+b*x^2)^(3/2)/c^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.05

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \left[-\frac{(3\,Bb^2-2\,Abc+(3\,Bbc-2\,Ac^2)x^2)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,(}{4\,(c^4x^2+bc^3)}\right.$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((3*B*b^2 - 2*A*b*c + (3*B*b*c - 2*A*c^2)*x^2)*sqrt(c)*log(-2*c*x^2 -
 b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c)) - 2*(B*c^2*x^2 + 3*B*b*c - 2*A*c^2)*sqr
t(c*x^4 + b*x^2))/(c^4*x^2 + b*c^3), 1/2*((3*B*b^2 - 2*A*b*c + (3*B*b*c - 2
*A*c^2)*x^2)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b)) + (B
*c^2*x^2 + 3*B*b*c - 2*A*c^2)*sqrt(c*x^4 + b*x^2))/(c^4*x^2 + b*c^3)]
```

Sympy [F]

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^5(A+Bx^2)}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**5*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.23

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{1}{4}\left(\frac{2\,x^4}{\sqrt{cx^4+bx^2}c}+\frac{6\,bx^2}{\sqrt{cx^4+bx^2}c^2}-\frac{3\,b\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{5}{2}}}\right)B - \frac{1}{2}\,A\left(\frac{2\,x^2}{\sqrt{cx^4+bx^2}c}-\frac{\log\left(2\,cx^2+b+2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{3}{2}}}\right)$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*x^4/(sqrt(c*x^4 + b*x^2)*c) + 6*b*x^2/(sqrt(c*x^4 + b*x^2)*c^2) - 3*
b*log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c))/c^(5/2))*B - 1/2*A*(2*x^
2/(sqrt(c*x^4 + b*x^2)*c) - log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*x^2)*sqrt(c)
)/c^(3/2))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.93

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x\left(\frac{Bx^2}{c\mathrm{sgn}(x)}+\frac{3\,Bbc\mathrm{sgn}(x)-2\,Ac^2\mathrm{sgn}(x)}{c^3}\right)}{2\,\sqrt{cx^2+b}} - \frac{(3\,Bb\log\left(|b|\right)-2\,Ac\log\left(|b|\right))\mathrm{sgn}(x)}{4\,c^{\frac{5}{2}}} + \frac{(3\,Bb-2\,Ac)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{2\,c^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*x*(B*x^2/(c*sgn(x)) + (3*B*b*c*sgn(x) - 2*A*c^2*sgn(x))/c^3)/sqrt(c*x^2
 + b) - 1/4*(3*B*b*log(abs(b)) - 2*A*c*log(abs(b)))*sgn(x)/c^(5/2) + 1/2*(3
*B*b - 2*A*c)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(5/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^5\,\left(B\,x^2+A\right)}{{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

```
[In] int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((x^5*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.148 $\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 67

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^2}{bc\sqrt{bx^2+cx^4}}+\frac{B\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}$$

```
[Out] B*arctanh(x^2*c^(1/2)/(c*x^4+b*x^2)^(1/2))/c^(3/2)-(-A*c+B*b)*x^2/b/c/(c*x^
4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2059, 791, 634, 212}

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{B\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}-\frac{x^2(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[In] Int[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^2)/(b*c*Sqrt[b*x^2 + c*x^4])) + (B*ArcTanh[(Sqrt[c]*x^2)/S
qrt[b*x^2 + c*x^4]])/c^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 791

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b
*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[p, -1]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{(bx+cx^2)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{(bB-Ac)x^2}{bc\sqrt{bx^2+cx^4}}+\frac{B\text{Subst}\left(\int \frac{1}{\sqrt{bx+cx^2}}\,dx, x, x^2\right)}{2c}\\
&= -\frac{(bB-Ac)x^2}{bc\sqrt{bx^2+cx^4}}+\frac{B\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x^2}{\sqrt{bx^2+cx^4}}\right)}{c}\\
&= -\frac{(bB-Ac)x^2}{bc\sqrt{bx^2+cx^4}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{x\left(\sqrt{c}(bB-Ac)x+bB\sqrt{b+cx^2}\log\left(-\sqrt{c}x+\sqrt{b+cx^2}\right)\right)}{bc^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(x^3*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
[Out] -((x*(Sqrt[c]*(b*B - A*c)*x + b*B*Sqrt[b + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[b
 + c*x^2]]))/(b*c^(3/2)*Sqrt[x^2*(b + c*x^2)]))
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{x^3(cx^2+b)\left(Ac^{\frac{5}{2}}x-bBxc^{\frac{3}{2}}+B\sqrt{cx^2+b}\,\ln\left(\sqrt{c}x+\sqrt{cx^2+b}\right)bc\right)}{(x^4c+bx^2)^{\frac{3}{2}}bc^{\frac{5}{2}}}$	75
pseudoelliptic	$\frac{2Ac^{\frac{3}{2}}x^2-2Bbx^2\sqrt{c}+B\ln\left(\frac{2cx^2+2\sqrt{x^2(cx^2+b)}\,\sqrt{c}+b}{\sqrt{c}}\right)b\sqrt{x^2(cx^2+b)}-B\ln(2)b\sqrt{x^2(cx^2+b)}}{2c^{\frac{3}{2}}\sqrt{x^2(cx^2+b)}\,b}$	108

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] x^3*(c*x^2+b)*(A*c^(5/2)*x-b*B*x*c^(3/2)+B*(c*x^2+b)^(1/2)*ln(c^(1/2)*x+(c*
x^2+b)^(1/2))*b*c)/(c*x^4+b*x^2)^(3/2)/b/c^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.81

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \left[\frac{(Bbcx^2+Bb^2)\sqrt{c}\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}(Bbc-Ac^2)}{2\,(bc^3x^2+b^2c^2)}, -\frac{(Bbcx^2+Bb^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-c}}{cx^2+b}\right)+\sqrt{cx^4+bx^2}(Bbc-Ac^2)}{bc^3x^2+b^2c^2}\right]$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
[Out] [1/2*((B*b*c*x^2 + B*b^2)*sqrt(c)*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*
sqrt(c)) - 2*sqrt(c*x^4 + b*x^2)*(B*b*c - A*c^2))/(b*c^3*x^2 + b^2*c^2), -(
```

```
(B*b*c*x^2 + B*b^2)*sqrt(-c)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-c)/(c*x^2 + b
)) + sqrt(c*x^4 + b*x^2)*(B*b*c - A*c^2))/(b*c^3*x^2 + b^2*c^2)]
```

Sympy [F]

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^3(A+Bx^2)}{(x^2(b+cx^2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(x**3*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.18

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{1}{2}B\left(\frac{2x^2}{\sqrt{cx^4+bx^2}c} - \frac{\log\left(2cx^2+b+2\sqrt{cx^4+bx^2}\sqrt{c}\right)}{c^{\frac{3}{2}}}\right) + \frac{Ax^2}{\sqrt{cx^4+bx^2}b}$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/2*B*(2*x^2/(sqrt(c*x^4 + b*x^2)*c) - log(2*c*x^2 + b + 2*sqrt(c*x^4 + b*
x^2)*sqrt(c))/c^(3/2)) + A*x^2/(sqrt(c*x^4 + b*x^2)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{B\log(|b|)\,\mathrm{sgn}(x)}{2c^{\frac{3}{2}}} - \frac{(Bb\mathrm{sgn}(x) - Ac\mathrm{sgn}(x))x}{\sqrt{cx^2+b}bc} - \frac{B\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)}{c^{\frac{3}{2}}\mathrm{sgn}(x)}$$

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] 1/2*B*log(abs(b))*sgn(x)/c^(3/2) - (B*b*sgn(x) - A*c*sgn(x))*x/(sqrt(c*x^2
+ b)*b*c) - B*log(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/(c^(3/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16

$$\int \frac{x^3(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{B\,\ln\left(\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2}\right)}{2\,c^{3/2}}+\frac{A\,x^2}{b\,\sqrt{c\,x^4+b\,x^2}}-\frac{B\,x^2}{c\,\sqrt{c\,x^4+b\,x^2}}$$

```
[In] int((x^3*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] (B*log((b/2 + c*x^2)/c^(1/2) + (b*x^2 + c*x^4)^(1/2)))/(2*c^(3/2)) + (A*x^2
)/(b*(b*x^2 + c*x^4)^(1/2)) - (B*x^2)/(c*(b*x^2 + c*x^4)^(1/2))
```

3.149 $\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 37

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{Ab-(bB-2Ac)x^2}{b^2\sqrt{bx^2+cx^4}}$$

```
[Out] (-A*b+(-2*A*c+B*b)*x^2)/b^2/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2059, 650}

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{Ab-x^2(bB-2Ac)}{b^2\sqrt{bx^2+cx^4}}$$

```
[In] Int[(x*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((A*b - (b*B - 2*A*c)*x^2)/(b^2*Sqrt[b*x^2 + c*x^4]))
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
```

```
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{(bx+cx^2)^{3/2}}\,dx, x, x^2\right)$$
$$= -\frac{Ab-(bB-2Ac)x^2}{b^2\sqrt{bx^2+cx^4}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{bBx^2-A(b+2cx^2)}{b^2\sqrt{x^2(b+cx^2)}}$$

```
[In] Integrate[(x*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
[Out] (b*B*x^2 - A*(b + 2*c*x^2))/(b^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

method	result	size
pseudoelliptic	$\frac{(-2c\,x^2-b)A+bB\,x^2}{\sqrt{x^2(c\,x^2+b)}\,b^2}$	37
gosper	$-\frac{x^2(c\,x^2+b)(2Ac\,x^2-bB\,x^2+Ab)}{b^2(x^4c+b\,x^2)^{\frac{3}{2}}}$	47
default	$-\frac{x^2(c\,x^2+b)(2Ac\,x^2-bB\,x^2+Ab)}{b^2(x^4c+b\,x^2)^{\frac{3}{2}}}$	47
trager	$-\frac{(2Ac\,x^2-bB\,x^2+Ab)\sqrt{x^4c+b\,x^2}}{(c\,x^2+b)b^2x^2}$	49
risch	$-\frac{A(c\,x^2+b)}{b^2\sqrt{x^2(c\,x^2+b)}}-\frac{x^2(Ac-Bb)}{b^2\sqrt{x^2(c\,x^2+b)}}$	57

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] ((-2*c*x^2-b)*A+b*B*x^2)/(x^2*(c*x^2+b))^(1/2)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{cx^4+bx^2}((Bb-2\,Ac)x^2-Ab)}{b^2cx^4+b^3x^2}$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
[Out] sqrt(c*x^4 + b*x^2)*((B*b - 2*A*c)*x^2 - A*b)/(b^2*c*x^4 + b^3*x^2)
```

Sympy [F]

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x(A+Bx^2)}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(x*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.76

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -A\left(\frac{2\,cx^2}{\sqrt{cx^4+bx^2}b^2}+\frac{1}{\sqrt{cx^4+bx^2}b}\right)+\frac{Bx^2}{\sqrt{cx^4+bx^2}b}$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] -A*(2*c*x^2/(sqrt(c*x^4 + b*x^2)*b^2) + 1/(sqrt(c*x^4 + b*x^2)*b)) + B*x^2/
(sqrt(c*x^4 + b*x^2)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.76

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{2\,A\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)b\mathrm{sgn}\,(x)}+\frac{(Bb-Ac)x}{\sqrt{cx^2+b}b^2\mathrm{sgn}\,(x)}$$

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] 2*A*sqrt(c)/(((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)*b*sgn(x)) + (B*b - A*c)*
x/(sqrt(c*x^2 + b)*b^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

$$\int \frac{x(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{\left(\frac{A}{b}-x^2\left(\frac{B}{b}-\frac{2\,A\,c}{b^2}\right)\right)\sqrt{c\,x^4+b\,x^2}}{x\,\left(c\,x^3+b\,x\right)}$$

```
[In] int((x*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
[Out] -((A/b - x^2*(B/b - (2*A*c)/b^2))*(b*x^2 + c*x^4)^(1/2))/(x*(b*x + c*x^3))
```

3.150 $\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 66

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^{3/2}}\,dx=-\frac{A}{3bx^2\sqrt{bx^2+cx^4}}-\frac{(3bB-4Ac)\left(b+2cx^2\right)}{3b^3\sqrt{bx^2+cx^4}}$$

```
[Out] -1/3*A/b/x^2/(c*x^4+b*x^2)^(1/2)-1/3*(-4*A*c+3*B*b)*(2*c*x^2+b)/b^3/(c*x^4+
b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2059, 806, 627}

$$\int \frac{A+Bx^2}{x\left(bx^2+cx^4\right)^{3/2}}\,dx=-\frac{\left(b+2cx^2\right)(3bB-4Ac)}{3b^3\sqrt{bx^2+cx^4}}-\frac{A}{3bx^2\sqrt{bx^2+cx^4}}$$

```
[In] Int[(A + B*x^2)/(x*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -1/3*A/(b*x^2*Sqrt[b*x^2 + c*x^4]) - ((3*b*B - 4*A*c)*(b + 2*c*x^2))/(3*b^3
*Sqrt[b*x^2 + c*x^4])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x\,(bx+cx^2)^{3/2}}\,dx, x, x^2\right) \\
&= -\frac{A}{3bx^2\sqrt{bx^2+cx^4}} + \frac{\left(bB-Ac+\frac{1}{2}(bB-2Ac)\right)\text{Subst}\left(\int \frac{1}{(bx+cx^2)^{3/2}}\,dx, x, x^2\right)}{3b} \\
&= -\frac{A}{3bx^2\sqrt{bx^2+cx^4}} - \frac{(3bB-4Ac)\,(b+2cx^2)}{3b^3\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^2}{x\,(bx^2+cx^4)^{3/2}}\,dx = \frac{-3bBx^2(b+2cx^2)+A(-b^2+4bcx^2+8c^2x^4)}{3b^3x^2\sqrt{x^2\,(b+cx^2)}}$$

```
[In] Integrate[(A + B*x^2)/(x*(b*x^2 + c*x^4)^(3/2)),x]
[Out] (-3*b*B*x^2*(b + 2*c*x^2) + A*(-b^2 + 4*b*c*x^2 + 8*c^2*x^4))/(3*b^3*x^2*Sq
rt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

method	result	size
pseudoelliptic	$-\frac{(3x^2B+A)b^2-4\left(-\frac{3x^2B}{2}+A\right)x^2cb-8A\,c^2x^4}{3\sqrt{x^2(c\,x^2+b)}\,b^3x^2}$	59
gosper	$-\frac{(c\,x^2+b)\left(-8A\,c^2x^4+6x^4Bbc-4Abc\,x^2+3b^2B\,x^2+b^2A\right)}{3b^3(x^4c+b\,x^2)^{\frac{3}{2}}}$	66
default	$-\frac{(c\,x^2+b)\left(-8A\,c^2x^4+6x^4Bbc-4Abc\,x^2+3b^2B\,x^2+b^2A\right)}{3b^3(x^4c+b\,x^2)^{\frac{3}{2}}}$	66
trager	$-\frac{\left(-8A\,c^2x^4+6x^4Bbc-4Abc\,x^2+3b^2B\,x^2+b^2A\right)\sqrt{x^4c+b\,x^2}}{3(c\,x^2+b)b^3x^4}$	71
risch	$-\frac{(c\,x^2+b)\left(-5Ac\,x^2+3bB\,x^2+Ab\right)}{3b^3x^2\sqrt{x^2(c\,x^2+b)}}+\frac{x^2(Ac-Bb)c}{b^3\sqrt{x^2(c\,x^2+b)}}$	77

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*((3*B*x^2+A)*b^2-4*(-3/2*x^2*B+A)*x^2*c*b-8*A*c^2*x^4)/(x^2*(c*x^2+b))
^(1/2)/b^3/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^2}{x\,(bx^2+cx^4)^{3/2}}\,dx = -\frac{(2\,(3\,Bbc-4\,Ac^2)x^4+Ab^2+(3\,Bb^2-4\,Abc)x^2)\sqrt{cx^4+bx^2}}{3\,(b^3cx^6+b^4x^4)}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(3*B*b*c - 4*A*c^2)*x^4 + A*b^2 + (3*B*b^2 - 4*A*b*c)*x^2)*sqrt(c*x
^4 + b*x^2)/(b^3*c*x^6 + b^4*x^4)
```

Sympy [F]

$$\int \frac{A+Bx^2}{x\,(bx^2+cx^4)^{3/2}}\,dx = \int \frac{A+Bx^2}{x\,(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)/(x*(x**2*(b + c*x**2))**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.70

$$\int \frac{A+Bx^2}{x\,(bx^2+cx^4)^{3/2}}\,dx = -B\left(\frac{2\,cx^2}{\sqrt{cx^4+bx^2}b^2}+\frac{1}{\sqrt{cx^4+bx^2}b}\right) + \frac{1}{3}\,A\left(\frac{8\,c^2x^2}{\sqrt{cx^4+bx^2}b^3}+\frac{4\,c}{\sqrt{cx^4+bx^2}b^2}-\frac{1}{\sqrt{cx^4+bx^2}bx^2}\right)$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -B*(2*c*x^2/(sqrt(c*x^4 + b*x^2)*b^2) + 1/(sqrt(c*x^4 + b*x^2)*b)) + 1/3*A*
(8*c^2*x^2/(sqrt(c*x^4 + b*x^2)*b^3) + 4*c/(sqrt(c*x^4 + b*x^2)*b^2) - 1/(s
qrt(c*x^4 + b*x^2)*b*x^2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(58) = 116.

Time = 0.50 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.86

$$\int \frac{A+Bx^2}{x\,(bx^2+cx^4)^{3/2}}\,dx = -\frac{(Bbc-Ac^2)x}{\sqrt{cx^2+b}b^3\mathrm{sgn}\,(x)} + \frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4Bb\sqrt{c}-3\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4Ac^{\frac{3}{2}}-6\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2Bb^2\sqrt{c}+12\left(\sqrt{c}x-\sqrt{}\right.\right.}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^3b^2\mathrm{sgn}\,(x)}$$

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(B*b*c - A*c^2)*x/(sqrt(c*x^2 + b)*b^3*sgn(x)) + 2/3*(3*(sqrt(c)*x - sqrt(
c*x^2 + b))^4*B*b*sqrt(c) - 3*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*c^(3/2) - 6
*(sqrt(c)*x - sqrt(c*x^2 + b))^2*B*b^2*sqrt(c) + 12*(sqrt(c)*x - sqrt(c*x^2
 + b))^2*A*b*c^(3/2) + 3*B*b^3*sqrt(c) - 5*A*b^2*c^(3/2))/(((sqrt(c)*x - sq
rt(c*x^2 + b))^2 - b)^3*b^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2}{x\,(bx^2+cx^4)^{3/2}}\,dx=-\frac{\sqrt{c\,x^4+b\,x^2}\,\left(3\,B\,b^2\,x^2+A\,b^2+6\,B\,b\,c\,x^4-4\,A\,b\,c\,x^2-8\,A\,c^2\,x^4\right)}{3\,b^3\,x^4\,\left(c\,x^2+b\right)}$$

```
[In] int((A + B*x^2)/(x*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*(A*b^2 + 3*B*b^2*x^2 - 8*A*c^2*x^4 - 4*A*b*c*x^2 +
6*B*b*c*x^4))/(3*b^3*x^4*(b + c*x^2))
```

3.151 $\int \frac{A+Bx^2}{x^3(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 101

$$\int \frac{A+Bx^2}{x^3\,(bx^2+cx^4)^{3/2}}\,dx = -\frac{A}{5bx^4\sqrt{bx^2+cx^4}}-\frac{5bB-6Ac}{15b^2x^2\sqrt{bx^2+cx^4}}+\frac{4c(5bB-6Ac)\,(b+2cx^2)}{15b^4\sqrt{bx^2+cx^4}}$$

```
[Out] -1/5*A/b/x^4/(c*x^4+b*x^2)^(1/2)+1/15*(6*A*c-5*B*b)/b^2/x^2/(c*x^4+b*x^2)^(
1/2)+4/15*c*(-6*A*c+5*B*b)*(2*c*x^2+b)/b^4/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2059, 806, 672, 627}

$$\int \frac{A+Bx^2}{x^3\,(bx^2+cx^4)^{3/2}}\,dx = \frac{4c(b+2cx^2)\,(5bB-6Ac)}{15b^4\sqrt{bx^2+cx^4}}-\frac{5bB-6Ac}{15b^2x^2\sqrt{bx^2+cx^4}}-\frac{A}{5bx^4\sqrt{bx^2+cx^4}}$$

```
[In] Int[(A + B*x^2)/(x^3*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -1/5*A/(b*x^4*Sqrt[b*x^2 + c*x^4]) - (5*b*B - 6*A*c)/(15*b^2*x^2*Sqrt[b*x^2
 + c*x^4]) + (4*c*(5*b*B - 6*A*c)*(b + 2*c*x^2))/(15*b^4*Sqrt[b*x^2 + c*x^4
])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2\left(bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= -\frac{A}{5bx^4\sqrt{bx^2+cx^4}}+\frac{\left(\frac{1}{2}(bB-2Ac)-2(-bB+Ac)\right)\text{Subst}\left(\int \frac{1}{x\left(bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)}{5b}\\
&= -\frac{A}{5bx^4\sqrt{bx^2+cx^4}}-\frac{5bB-6Ac}{15b^2x^2\sqrt{bx^2+cx^4}}-\frac{(2c(5bB-6Ac))\text{Subst}\left(\int \frac{1}{\left(bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)}{15b^2}\\
&= -\frac{A}{5bx^4\sqrt{bx^2+cx^4}}-\frac{5bB-6Ac}{15b^2x^2\sqrt{bx^2+cx^4}}+\frac{4c(5bB-6Ac)\left(b+2cx^2\right)}{15b^4\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^2}{x^3 \left(bx^2+cx^4\right)^{3/2}}\, dx = \frac{-5bBx^2(b^2-4bcx^2-8c^2x^4)-3A(b^3-2b^2cx^2+8bc^2x^4+16c^3x^6)}{15b^4x^4\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(A + B*x^2)/(x^3*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-5*b*B*x^2*(b^2 - 4*b*c*x^2 - 8*c^2*x^4) - 3*A*(b^3 - 2*b^2*c*x^2 + 8*b*c^
2*x^4 + 16*c^3*x^6))/(15*b^4*x^4*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

method	result	size
pseudoelliptic	$\frac{\left(-5x^{2}B-3A\right)b^{3}+6x^{2}\left(\frac{10x^{2}B}{3}+A\right)c\,b^{2}-24x^{4}c^{2}\left(-\frac{5x^{2}B}{3}+A\right)b-48A\,c^{3}x^{6}}{15\sqrt{x^{2}\left(c\,x^{2}+b\right)}\,x^{4}b^{4}}$	80
gosper	$-\frac{\left(c\,x^{2}+b\right)\left(48A\,c^{3}x^{6}-40x^{6}Bb\,c^{2}+24Ab\,c^{2}x^{4}-20x^{4}B\,b^{2}c-6A\,b^{2}c\,x^{2}+5b^{3}B\,x^{2}+3b^{3}A\right)}{15x^{2}b^{4}\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}}$	94
default	$-\frac{\left(c\,x^{2}+b\right)\left(48A\,c^{3}x^{6}-40x^{6}Bb\,c^{2}+24Ab\,c^{2}x^{4}-20x^{4}B\,b^{2}c-6A\,b^{2}c\,x^{2}+5b^{3}B\,x^{2}+3b^{3}A\right)}{15x^{2}b^{4}\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}}$	94
trager	$-\frac{\left(48A\,c^{3}x^{6}-40x^{6}Bb\,c^{2}+24Ab\,c^{2}x^{4}-20x^{4}B\,b^{2}c-6A\,b^{2}c\,x^{2}+5b^{3}B\,x^{2}+3b^{3}A\right)\sqrt{x^{4}c+b\,x^{2}}}{15\left(c\,x^{2}+b\right)b^{4}x^{6}}$	96
risch	$-\frac{\left(c\,x^{2}+b\right)\left(33A\,c^{2}x^{4}-25x^{4}Bbc-9Abc\,x^{2}+5b^{2}B\,x^{2}+3b^{2}A\right)}{15b^{4}x^{4}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{x^{2}c^{2}\left(Ac-Bb\right)}{b^{4}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$	103

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*((-5*B*x^2-3*A)*b^3+6*x^2*(10/3*x^2*B+A)*c*b^2-24*x^4*c^2*(-5/3*x^2*B+
A)*b-48*A*c^3*x^6)/(x^2*(c*x^2+b))^(1/2)/x^4/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^2}{x^3 \left(bx^2+cx^4\right)^{3/2}}\, dx = \frac{(8\,(5\,Bbc^2-6\,Ac^3)x^6+4\,(5\,Bb^2c-6\,Abc^2)x^4-3\,Ab^3-(5\,Bb^3-6\,Ab^2c)x^2)\sqrt{cx^4}}{15\,(b^4cx^8+b^5x^6)}$$

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*(5*B*b*c^2 - 6*A*c^3)*x^6 + 4*(5*B*b^2*c - 6*A*b*c^2)*x^4 - 3*A*b^3
 - (5*B*b^3 - 6*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2)/(b^4*c*x^8 + b^5*x^6)
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{A+Bx^2}{x^3\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral((A + B*x**2)/(x**3*(x**2*(b + c*x**2))**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.58

$$\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)^{3/2}}\,dx=\frac{1}{3}\,B\left(\frac{8\,c^2x^2}{\sqrt{cx^4+bx^2}b^3}+\frac{4\,c}{\sqrt{cx^4+bx^2}b^2}-\frac{1}{\sqrt{cx^4+bx^2}bx^2}\right)-\frac{1}{5}\,A\left(\frac{16\,c^3x^2}{\sqrt{cx^4+bx^2}b^4}+\frac{8\,c^2}{\sqrt{cx^4+bx^2}b^3}-\frac{2\,c}{\sqrt{cx^4+bx^2}b^2x^2}+\frac{1}{\sqrt{cx^4+bx^2}bx^4}\right)$$

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] 1/3*B*(8*c^2*x^2/(sqrt(c*x^4 + b*x^2)*b^3) + 4*c/(sqrt(c*x^4 + b*x^2)*b^2)
- 1/(sqrt(c*x^4 + b*x^2)*b*x^2)) - 1/5*A*(16*c^3*x^2/(sqrt(c*x^4 + b*x^2)*b
^4) + 8*c^2/(sqrt(c*x^4 + b*x^2)*b^3) - 2*c/(sqrt(c*x^4 + b*x^2)*b^2*x^2) +
 1/(sqrt(c*x^4 + b*x^2)*b*x^4))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(89) = 178.

Time = 0.92 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.99

$$\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)^{3/2}}\,dx=\frac{(Bbc^2-Ac^3)x}{\sqrt{cx^2+b}b^4\mathrm{sgn}\left(x\right)}-\frac{2\left(15\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^8Bbc^{\frac{3}{2}}-15\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^8Ac^{\frac{5}{2}}-90\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6Bb^2c^{\frac{3}{2}}+90\left(\sqrt{c}x-\right.}{}$$

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] (B*b*c^2 - A*c^3)*x/(sqrt(c*x^2 + b)*b^4*sgn(x)) - 2/15*(15*(sqrt(c)*x - sq
rt(c*x^2 + b))^8*B*b*c^(3/2) - 15*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*c^(5/2)
 - 90*(sqrt(c)*x - sqrt(c*x^2 + b))^6*B*b^2*c^(3/2) + 90*(sqrt(c)*x - sqrt(
```

```
c*x^2 + b))^6*A*b*c^(5/2) + 160*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^3*c^(3/
2) - 240*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^2*c^(5/2) - 110*(sqrt(c)*x - s
qrt(c*x^2 + b))^2*B*b^4*c^(3/2) + 150*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^3
*c^(5/2) + 25*B*b^5*c^(3/2) - 33*A*b^4*c^(5/2))/(((sqrt(c)*x - sqrt(c*x^2 +
 b))^2 - b)^5*b^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{x^3\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{\sqrt{c\,x^4+b\,x^2}\,\left(5\,B\,b^3\,x^2+3\,A\,b^3-20\,B\,b^2\,c\,x^4-6\,A\,b^2\,c\,x^2-40\,B\,b\,c^2\,x^6+24\,A\,b\,c^2\,x^4+48\,A\,c^3\,x^6\right)}{15\,b^4\,x^6\,\left(c\,x^2+b\right)}$$

```
[In] int((A + B*x^2)/(x^3*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] -((b*x^2 + c*x^4)^(1/2)*(3*A*b^3 + 5*B*b^3*x^2 + 48*A*c^3*x^6 - 6*A*b^2*c*x
^2 + 24*A*b*c^2*x^4 - 20*B*b^2*c*x^4 - 40*B*b*c^2*x^6))/(15*b^4*x^6*(b + c*
x^2))
```

3.152 $\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 138

$$\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{A}{7bx^6\sqrt{bx^2+cx^4}}-\frac{7bB-8Ac}{35b^2x^4\sqrt{bx^2+cx^4}}+\frac{2c(7bB-8Ac)}{35b^3x^2\sqrt{bx^2+cx^4}}-\frac{8c^2(7bB-8Ac)\left(b+2cx^2\right)}{35b^5\sqrt{bx^2+cx^4}}$$

```
[Out] -1/7*A/b/x^6/(c*x^4+b*x^2)^(1/2)+1/35*(8*A*c-7*B*b)/b^2/x^4/(c*x^4+b*x^2)^(
1/2)+2/35*c*(-8*A*c+7*B*b)/b^3/x^2/(c*x^4+b*x^2)^(1/2)-8/35*c^2*(-8*A*c+7*B
*b)*(2*c*x^2+b)/b^5/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2059, 806, 672, 627}

$$\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{8c^2\left(b+2cx^2\right)(7bB-8Ac)}{35b^5\sqrt{bx^2+cx^4}}+\frac{2c(7bB-8Ac)}{35b^3x^2\sqrt{bx^2+cx^4}}-\frac{7bB-8Ac}{35b^2x^4\sqrt{bx^2+cx^4}}-\frac{A}{7bx^6\sqrt{bx^2+cx^4}}$$

```
[In] Int[(A + B*x^2)/(x^5*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -1/7*A/(b*x^6*Sqrt[b*x^2 + c*x^4]) - (7*b*B - 8*A*c)/(35*b^2*x^4*Sqrt[b*x^2
 + c*x^4]) + (2*c*(7*b*B - 8*A*c))/(35*b^3*x^2*Sqrt[b*x^2 + c*x^4]) - (8*c^
2*(7*b*B - 8*A*c)*(b + 2*c*x^2))/(35*b^5*Sqrt[b*x^2 + c*x^4])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 672

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Dist[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d,
 e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^3\,(bx+cx^2)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{A}{7bx^6\sqrt{bx^2+cx^4}} + \frac{\left(\frac{1}{2}(bB-2Ac)-3(-bB+Ac)\right)\text{Subst}\left(\int \frac{1}{x^2(bx+cx^2)^{3/2}}\,dx, x, x^2\right)}{7b}$$

$$= -\frac{A}{7bx^6\sqrt{bx^2+cx^4}} - \frac{7bB-8Ac}{35b^2x^4\sqrt{bx^2+cx^4}} - \frac{(3c(7bB-8Ac))\text{Subst}\left(\int \frac{1}{x(bx+cx^2)^{3/2}}\,dx, x, x^2\right)}{35b^2}$$

$$= -\frac{A}{7bx^6\sqrt{bx^2+cx^4}} - \frac{7bB-8Ac}{35b^2x^4\sqrt{bx^2+cx^4}} + \frac{2c(7bB-8Ac)}{35b^3x^2\sqrt{bx^2+cx^4}} + \frac{(4c^2(7bB-8Ac))\,\text{Subst}\left(\int \frac{1}{(bx+cx^2)^{3/2}}\,dx, x, x^2\right)}{35b^3}$$

$$= -\frac{A}{7bx^6\sqrt{bx^2+cx^4}} - \frac{7bB-8Ac}{35b^2x^4\sqrt{bx^2+cx^4}} + \frac{2c(7bB-8Ac)}{35b^3x^2\sqrt{bx^2+cx^4}} - \frac{8c^2(7bB-8Ac)\,(b+2cx^2)}{35b^5\sqrt{bx^2+cx^4}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx^2}{x^5\,(bx^2+cx^4)^{3/2}}\,dx = \frac{-7bBx^2(b^3-2b^2cx^2+8bc^2x^4+16c^3x^6)+A(-5b^4+8b^3cx^2-16b^2c^2x^4+64bc^3x^6}{35b^5x^6\sqrt{x^2\,(b+cx^2)}}$$

```
[In] Integrate[(A + B*x^2)/(x^5*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-7*b*B*x^2*(b^3 - 2*b^2*c*x^2 + 8*b*c^2*x^4 + 16*c^3*x^6) + A*(-5*b^4 + 8*
b^3*c*x^2 - 16*b^2*c^2*x^4 + 64*b*c^3*x^6 + 128*c^4*x^8))/(35*b^5*x^6*Sqrt[
x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$-\frac{\left(\frac{7x^2B}{5}+A\right)b^4-\frac{8x^2\left(\frac{7x^2B}{4}+A\right)c\,b^3}{5}+\frac{16x^4\left(\frac{7x^2B}{2}+A\right)c^2b^2}{5}-\frac{64\left(-\frac{7x^2B}{4}+A\right)x^6c^3b}{5}-\frac{128A\,x^8c^4}{5}}{7\sqrt{x^2(c\,x^2+b)}\,x^6b^5}$
gosper	$-\frac{(c\,x^2+b)\left(-128A\,x^8c^4+112B\,x^8b\,c^3-64A\,x^6b\,c^3+56B\,x^6b^2c^2+16A\,b^2c^2x^4-14B\,b^3c\,x^4-8A\,x^2b^3c+7B\,x^2b^4+5A\,b^4\right)}{35x^4b^5(x^4c+b\,x^2)^{\frac{3}{2}}}$
default	$-\frac{(c\,x^2+b)\left(-128A\,x^8c^4+112B\,x^8b\,c^3-64A\,x^6b\,c^3+56B\,x^6b^2c^2+16A\,b^2c^2x^4-14B\,b^3c\,x^4-8A\,x^2b^3c+7B\,x^2b^4+5A\,b^4\right)}{35x^4b^5(x^4c+b\,x^2)^{\frac{3}{2}}}$
trager	$-\frac{\left(-128A\,x^8c^4+112B\,x^8b\,c^3-64A\,x^6b\,c^3+56B\,x^6b^2c^2+16A\,b^2c^2x^4-14B\,b^3c\,x^4-8A\,x^2b^3c+7B\,x^2b^4+5A\,b^4\right)\sqrt{x^4c+b\,x^2}}{35(c\,x^2+b)b^5x^8}$
risch	$-\frac{(c\,x^2+b)\left(-93A\,c^3x^6+77x^6Bb\,c^2+29Ab\,c^2x^4-21x^4B\,b^2c-13A\,b^2c\,x^2+7b^3B\,x^2+5b^3A\right)}{35b^5x^6\sqrt{x^2(c\,x^2+b)}}+\frac{x^2c^3(Ac-Bb)}{b^5\sqrt{x^2(c\,x^2+b)}}$

```
[In] int((B*x^2+A)/x^5/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/(x^2*(c*x^2+b))^(1/2)*((7/5*x^2*B+A)*b^4-8/5*x^2*(7/4*x^2*B+A)*c*b^3+1
6/5*x^4*(7/2*x^2*B+A)*c^2*b^2-64/5*(-7/4*x^2*B+A)*x^6*c^3*b-128/5*A*x^8*c^4
)/x^6/b^5
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)^{3/2}}\,dx =$$
$$-\frac{\left(16\left(7\,Bbc^3-8\,Ac^4\right)x^8+8\left(7\,Bb^2c^2-8\,Abc^3\right)x^6+5\,Ab^4-2\left(7\,Bb^3c-8\,Ab^2c^2\right)x^4+\left(7\,Bb^4-8\,Ab^3c\right)x^2\right)\sqrt{}}{35\left(b^5cx^{10}+b^6x^8\right)}$$

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/35*(16*(7*B*b*c^3 - 8*A*c^4)*x^8 + 8*(7*B*b^2*c^2 - 8*A*b*c^3)*x^6 + 5*A
*b^4 - 2*(7*B*b^3*c - 8*A*b^2*c^2)*x^4 + (7*B*b^4 - 8*A*b^3*c)*x^2)*sqrt(c*
x^4 + b*x^2)/(b^5*c*x^10 + b^6*x^8)
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{A+Bx^2}{x^5\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)/x**5/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral((A + B*x**2)/(x**5*(x**2*(b + c*x**2))**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.51

$$\int \frac{A+Bx^2}{x^5\left(bx^2+cx^4\right)^{3/2}}\,dx =$$
$$-\frac{1}{5}\,B\left(\frac{16\,c^3x^2}{\sqrt{cx^4+bx^2}b^4}+\frac{8\,c^2}{\sqrt{cx^4+bx^2}b^3}-\frac{2\,c}{\sqrt{cx^4+bx^2}b^2x^2}+\frac{1}{\sqrt{cx^4+bx^2}bx^4}\right)$$
$$+\frac{1}{35}\,A\left(\frac{128\,c^4x^2}{\sqrt{cx^4+bx^2}b^5}+\frac{64\,c^3}{\sqrt{cx^4+bx^2}b^4}-\frac{16\,c^2}{\sqrt{cx^4+bx^2}b^3x^2}+\frac{8\,c}{\sqrt{cx^4+bx^2}b^2x^4}-\frac{5}{\sqrt{cx^4+bx^2}bx^6}\right)$$

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/5*B*(16*c^3*x^2/(sqrt(c*x^4 + b*x^2)*b^4) + 8*c^2/(sqrt(c*x^4 + b*x^2)*b
^3) - 2*c/(sqrt(c*x^4 + b*x^2)*b^2*x^2) + 1/(sqrt(c*x^4 + b*x^2)*b*x^4)) +
1/35*A*(128*c^4*x^2/(sqrt(c*x^4 + b*x^2)*b^5) + 64*c^3/(sqrt(c*x^4 + b*x^2)
*b^4) - 16*c^2/(sqrt(c*x^4 + b*x^2)*b^3*x^2) + 8*c/(sqrt(c*x^4 + b*x^2)*b^2
*x^4) - 5/(sqrt(c*x^4 + b*x^2)*b*x^6))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(122) = 244.

Time = 1.28 (sec) , antiderivative size = 415, normalized size of antiderivative = 3.01

$$\int \frac{A+Bx^2}{x^5 (bx^2+cx^4)^{3/2}} dx = -\frac{(Bbc^3 - Ac^4)x}{\sqrt{cx^2+b}b^5\mathrm{sgn}(x)} + \frac{2\left(35\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Bbc^{\frac{5}{2}} - 35\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}Ac^{\frac{7}{2}} - 280\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}Bb^2c^{\frac{5}{2}} + 280\left(\sqrt{c}x\right.\right.}{}$$

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(B*b*c^3 - A*c^4)*x/(sqrt(c*x^2 + b)*b^5*sgn(x)) + 2/35*(35*(sqrt(c)*x - s
qrt(c*x^2 + b))^12*B*b*c^(5/2) - 35*(sqrt(c)*x - sqrt(c*x^2 + b))^12*A*c^(7
/2) - 280*(sqrt(c)*x - sqrt(c*x^2 + b))^10*B*b^2*c^(5/2) + 280*(sqrt(c)*x -
 sqrt(c*x^2 + b))^10*A*b*c^(7/2) + 1015*(sqrt(c)*x - sqrt(c*x^2 + b))^8*B*b
^3*c^(5/2) - 1015*(sqrt(c)*x - sqrt(c*x^2 + b))^8*A*b^2*c^(7/2) - 1680*(sqr
t(c)*x - sqrt(c*x^2 + b))^6*B*b^4*c^(5/2) + 2240*(sqrt(c)*x - sqrt(c*x^2 +
b))^6*A*b^3*c^(7/2) + 1337*(sqrt(c)*x - sqrt(c*x^2 + b))^4*B*b^5*c^(5/2) -
1673*(sqrt(c)*x - sqrt(c*x^2 + b))^4*A*b^4*c^(7/2) - 504*(sqrt(c)*x - sqrt(
c*x^2 + b))^2*B*b^6*c^(5/2) + 616*(sqrt(c)*x - sqrt(c*x^2 + b))^2*A*b^5*c^(
7/2) + 77*B*b^7*c^(5/2) - 93*A*b^6*c^(7/2))/(((sqrt(c)*x - sqrt(c*x^2 + b))
^2 - b)^7*b^4*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx^2}{x^5 (bx^2+cx^4)^{3/2}} dx = -\frac{(7\,B\,b^2-13\,A\,b\,c)\,\sqrt{c\,x^4+b\,x^2}}{35\,b^4\,x^6} - \frac{\left(x^2\left(\frac{58\,A\,c^4-42\,B\,b\,c^3}{35\,b^5}-\frac{2\,c^3\,(93\,A\,c-77\,B\,b)}{35\,b^5}\right)-\frac{c^2\,(93\,A\,c-77\,B\,b)}{35\,b^4}\right)\sqrt{c\,x^4+b\,x^2}}{x^2\,\left(c\,x^2+b\right)} - \frac{A\,\sqrt{c\,x^4+b\,x^2}}{7\,b^2\,x^8} - \frac{c\,(29\,A\,c-21\,B\,b)\,\sqrt{c\,x^4+b\,x^2}}{35\,b^4\,x^4}$$

```
[In] int((A + B*x^2)/(x^5*(b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] - ((7*B*b^2 - 13*A*b*c)*(b*x^2 + c*x^4)^(1/2))/(35*b^4*x^6) - ((x^2*((58*A*
c^4 - 42*B*b*c^3)/(35*b^5) - (2*c^3*(93*A*c - 77*B*b))/(35*b^5)) - (c^2*(93
*A*c - 77*B*b))/(35*b^4))*(b*x^2 + c*x^4)^(1/2))/(x^2*(b + c*x^2)) - (A*(b*
x^2 + c*x^4)^(1/2))/(7*b^2*x^8) - (c*(29*A*c - 21*B*b)*(b*x^2 + c*x^4)^(1/2
))/(35*b^4*x^4)
```

3.153 $\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$

Optimal result

Integrand size = 26, antiderivative size = 139

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx = -\frac{(bB-Ac)x^7}{bc\sqrt{bx^2+cx^4}} + \frac{8b(6bB-5Ac)\sqrt{bx^2+cx^4}}{15c^4x} - \frac{4(6bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^3} + \frac{(6bB-5Ac)x^3\sqrt{bx^2+cx^4}}{5bc^2}$$

```
[Out] -(-A*c+B*b)*x^7/b/c/(c*x^4+b*x^2)^(1/2)+8/15*b*(-5*A*c+6*B*b)*(c*x^4+b*x^2)
^(1/2)/c^4/x-4/15*(-5*A*c+6*B*b)*x*(c*x^4+b*x^2)^(1/2)/c^3+1/5*(-5*A*c+6*B*
b)*x^3*(c*x^4+b*x^2)^(1/2)/b/c^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2062, 2041, 1602}

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx = \frac{8b\sqrt{bx^2+cx^4}(6bB-5Ac)}{15c^4x} - \frac{4x\sqrt{bx^2+cx^4}(6bB-5Ac)}{15c^3} + \frac{x^3\sqrt{bx^2+cx^4}(6bB-5Ac)}{5bc^2} - \frac{x^7(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[In] Int[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^7)/(b*c*Sqrt[b*x^2 + c*x^4])) + (8*b*(6*b*B - 5*A*c)*Sqrt[
b*x^2 + c*x^4])/(15*c^4*x) - (4*(6*b*B - 5*A*c)*x*Sqrt[b*x^2 + c*x^4])/(15*
c^3) + ((6*b*B - 5*A*c)*x^3*Sqrt[b*x^2 + c*x^4])/(5*b*c^2)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2062

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Dist[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1))), Int[(e*x)^(m
- j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n
}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(bB-Ac)x^7}{bc\sqrt{bx^2+cx^4}}+\frac{(6bB-5Ac)\int\frac{x^6}{\sqrt{bx^2+cx^4}}\,dx}{bc}\\
&= -\frac{(bB-Ac)x^7}{bc\sqrt{bx^2+cx^4}}+\frac{(6bB-5Ac)x^3\sqrt{bx^2+cx^4}}{5bc^2}-\frac{(4(6bB-5Ac))\int\frac{x^4}{\sqrt{bx^2+cx^4}}\,dx}{5c^2}\\
&= -\frac{(bB-Ac)x^7}{bc\sqrt{bx^2+cx^4}}-\frac{4(6bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^3}\\
&\quad+\frac{(6bB-5Ac)x^3\sqrt{bx^2+cx^4}}{5bc^2}+\frac{(8b(6bB-5Ac))\int\frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{15c^3}\\
&= -\frac{(bB-Ac)x^7}{bc\sqrt{bx^2+cx^4}}+\frac{8b(6bB-5Ac)\sqrt{bx^2+cx^4}}{15c^4x}\\
&\quad-\frac{4(6bB-5Ac)x\sqrt{bx^2+cx^4}}{15c^3}+\frac{(6bB-5Ac)x^3\sqrt{bx^2+cx^4}}{5bc^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.59

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x(48b^3B-8b^2c(5A-3Bx^2)+c^3x^4(5A+3Bx^2)-2bc^2x^2(10A+3Bx^2))}{15c^4\sqrt{x^2\,(b+cx^2)}}$$

```
[In] Integrate[(x^8*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(48*b^3*B - 8*b^2*c*(5*A - 3*B*x^2) + c^3*x^4*(5*A + 3*B*x^2) - 2*b*c^2*
x^2*(10*A + 3*B*x^2)))/(15*c^4*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.65

method	result	size
gosper	$-\frac{(c\,x^2+b)\left(-3B\,c^3x^6-5A\,c^3x^4+6Bb\,c^2x^4+20Ab\,c^2x^2-24B\,b^2c\,x^2+40b^2Ac-48B\,b^3\right)x^3}{15c^4\left(x^4c+b\,x^2\right)^{\frac{3}{2}}}$	91
default	$-\frac{(c\,x^2+b)\left(-3B\,c^3x^6-5A\,c^3x^4+6Bb\,c^2x^4+20Ab\,c^2x^2-24B\,b^2c\,x^2+40b^2Ac-48B\,b^3\right)x^3}{15c^4\left(x^4c+b\,x^2\right)^{\frac{3}{2}}}$	91
trager	$-\frac{\left(-3B\,c^3x^6-5A\,c^3x^4+6Bb\,c^2x^4+20Ab\,c^2x^2-24B\,b^2c\,x^2+40b^2Ac-48B\,b^3\right)\sqrt{x^4c+b\,x^2}}{15(c\,x^2+b)c^4x}$	93
risch	$-\frac{\left(-3B\,c^2x^4-5A\,c^2x^2+9Bbc\,x^2+25Abc-33B\,b^2\right)(c\,x^2+b)x}{15c^4\sqrt{x^2(c\,x^2+b)}}-\frac{b^2(Ac-Bb)x}{c^4\sqrt{x^2(c\,x^2+b)}}$	96

```
[In] int(x^8*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(c*x^2+b)*(-3*B*c^3*x^6-5*A*c^3*x^4+6*B*b*c^2*x^4+20*A*b*c^2*x^2-24*B
*b^2*c*x^2+40*A*b^2*c-48*B*b^3)*x^3/c^4/(c*x^4+b*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.67

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{(3\,Bc^3x^6-(6\,Bbc^2-5\,Ac^3)x^4+48\,Bb^3-40\,Ab^2c+4\,(6\,Bb^2c-5\,Abc^2)x^2)\sqrt{cx^4+b}}{15\,(c^5x^3+bc^4x)}$$

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*c^3*x^6 - (6*B*b*c^2 - 5*A*c^3)*x^4 + 48*B*b^3 - 40*A*b^2*c + 4*(
6*B*b^2*c - 5*A*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^5*x^3 + b*c^4*x)
```

Sympy [F]

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^8(A+Bx^2)}{(x^2(b+cx^2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(x**8*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.59

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{(c^2x^4-4\,bcx^2-8\,b^2)A}{3\,\sqrt{cx^2+b}c^3} + \frac{(c^3x^6-2\,bc^2x^4+8\,b^2cx^2+16\,b^3)B}{5\,\sqrt{cx^2+b}c^4}$$

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] 1/3*(c^2*x^4 - 4*b*c*x^2 - 8*b^2)*A/(sqrt(c*x^2 + b)*c^3) + 1/5*(c^3*x^6 -
2*b*c^2*x^4 + 8*b^2*c*x^2 + 16*b^3)*B/(sqrt(c*x^2 + b)*c^4)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.04

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{8\,(6\,Bb^3-5\,Ab^2c)\mathrm{sgn}(x)}{15\,\sqrt{b}c^4} + \frac{Bb^3-Ab^2c}{\sqrt{cx^2+b}c^4\mathrm{sgn}\,(x)} + \frac{3\,(cx^2+b)^{\frac{5}{2}}Bc^{16}-15\,(cx^2+b)^{\frac{3}{2}}Bbc^{16}+45\,\sqrt{cx^2+b}Bb^2c^{16}+5\,(cx^2+b)^{\frac{3}{2}}Ac^{17}-30\,\sqrt{cx^2+b}Abc^{17}}{15\,c^{20}\mathrm{sgn}\,(x)}$$

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] -8/15*(6*B*b^3 - 5*A*b^2*c)*sgn(x)/(sqrt(b)*c^4) + (B*b^3 - A*b^2*c)/(sqrt(
c*x^2 + b)*c^4*sgn(x)) + 1/15*(3*(c*x^2 + b)^(5/2)*B*c^16 - 15*(c*x^2 + b)^
(3/2)*B*b*c^16 + 45*sqrt(c*x^2 + b)*B*b^2*c^16 + 5*(c*x^2 + b)^(3/2)*A*c^17
 - 30*sqrt(c*x^2 + b)*A*b*c^17)/(c^20*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.66

$$\int \frac{x^8(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{c\,x^4+b\,x^2}\,(48\,B\,b^3+24\,B\,b^2\,c\,x^2-40\,A\,b^2\,c-6\,B\,b\,c^2\,x^4-20\,A\,b\,c^2\,x^2+3\,B\,c^3\,x^6}{15\,c^4\,x\,\left(c\,x^2+b\right)}$$

```
[In] int((x^8*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*(48*B*b^3 + 5*A*c^3*x^4 + 3*B*c^3*x^6 - 40*A*b^2*c -
 20*A*b*c^2*x^2 + 24*B*b^2*c*x^2 - 6*B*b*c^2*x^4))/(15*c^4*x*(b + c*x^2))
```

3.154 $\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 104

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^5}{bc\sqrt{bx^2+cx^4}} - \frac{2(4bB-3Ac)\sqrt{bx^2+cx^4}}{3c^3x} + \frac{(4bB-3Ac)x\sqrt{bx^2+cx^4}}{3bc^2}$$

```
[Out] -(-A*c+B*b)*x^5/b/c/(c*x^4+b*x^2)^(1/2)-2/3*(-3*A*c+4*B*b)*(c*x^4+b*x^2)^(1
/2)/c^3/x+1/3*(-3*A*c+4*B*b)*x*(c*x^4+b*x^2)^(1/2)/b/c^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2062, 2041, 1602}

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{2\sqrt{bx^2+cx^4}(4bB-3Ac)}{3c^3x} + \frac{x\sqrt{bx^2+cx^4}(4bB-3Ac)}{3bc^2} - \frac{x^5(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[In] Int[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^5)/(b*c*Sqrt[b*x^2 + c*x^4])) - (2*(4*b*B - 3*A*c)*Sqrt[b*
x^2 + c*x^4])/(3*c^3*x) + ((4*b*B - 3*A*c)*x*Sqrt[b*x^2 + c*x^4])/(3*b*c^2)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
```

```
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2062

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Dist[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1))), Int[(e*x)^(m
- j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n
}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(bB-Ac)x^5}{bc\sqrt{bx^2+cx^4}}+\frac{(4bB-3Ac)\int\frac{x^4}{\sqrt{bx^2+cx^4}}\,dx}{bc}\\
&= -\frac{(bB-Ac)x^5}{bc\sqrt{bx^2+cx^4}}+\frac{(4bB-3Ac)x\sqrt{bx^2+cx^4}}{3bc^2}-\frac{(2(4bB-3Ac))\int\frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{3c^2}\\
&= -\frac{(bB-Ac)x^5}{bc\sqrt{bx^2+cx^4}}-\frac{2(4bB-3Ac)\sqrt{bx^2+cx^4}}{3c^3x}+\frac{(4bB-3Ac)x\sqrt{bx^2+cx^4}}{3bc^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.58

$$\int\frac{x^6(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=\frac{x(-8b^2B+c^2x^2(3A+Bx^2)+b(6Ac-4Bcx^2))}{3c^3\sqrt{x^2\,(b+cx^2)}}$$

```
[In] Integrate[(x^6*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
[Out] (x*(-8*b^2*B + c^2*x^2*(3*A + B*x^2) + b*(6*A*c - 4*B*c*x^2)))/(3*c^3*Sqrt[
x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{\left(c\,x^{2}+b\right)\left(B\,c^{2} x^{4}+3 A\,c^{2} x^{2}-4 B b c\,x^{2}+6 A b c-8 B\,b^{2}\right) x^{3}}{3 c^{3} \left(x^{4} c+b\,x^{2}\right)^{\frac{3}{2}}}$	66
default	$\frac{\left(c\,x^{2}+b\right)\left(B\,c^{2} x^{4}+3 A\,c^{2} x^{2}-4 B b c\,x^{2}+6 A b c-8 B\,b^{2}\right) x^{3}}{3 c^{3} \left(x^{4} c+b\,x^{2}\right)^{\frac{3}{2}}}$	66
trager	$\frac{\left(B\,c^{2} x^{4}+3 A\,c^{2} x^{2}-4 B b c\,x^{2}+6 A b c-8 B\,b^{2}\right) \sqrt{x^{4} c+b\,x^{2}}}{3 \left(c\,x^{2}+b\right) c^{3} x}$	68
risch	$\frac{\left(B c\,x^{2}+3 A c-5 B b\right)\left(c\,x^{2}+b\right) x}{3 c^{3} \sqrt{x^{2} \left(c\,x^{2}+b\right)}}+\frac{b \left(A c-B b\right) x}{c^{3} \sqrt{x^{2} \left(c\,x^{2}+b\right)}}$	70

[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/3*(c*x^2+b)*(B*c^2*x^4+3*A*c^2*x^2-4*B*b*c*x^2+6*A*b*c-8*B*b^2)*x^3/c^3/(c*x^4+b*x^2)^(3/2)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.65

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{(Bc^2x^4 - 8\,Bb^2 + 6\,Abc - (4\,Bbc - 3\,Ac^2)x^2)\sqrt{cx^4+bx^2}}{3\,(c^4x^3 + bc^3x)}$$

[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")

[Out] 1/3*(B*c^2*x^4 - 8*B*b^2 + 6*A*b*c - (4*B*b*c - 3*A*c^2)*x^2)*sqrt(c*x^4 + b*x^2)/(c^4*x^3 + b*c^3*x)

Sympy [F]

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^6(A+Bx^2)}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)

[Out] Integral(x**6*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.57

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{(cx^2+2\,b)A}{\sqrt{cx^2+b}c^2} + \frac{(c^2x^4-4\,bcx^2-8\,b^2)B}{3\,\sqrt{cx^2+b}c^3}$$

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (c*x^2 + 2*b)*A/(sqrt(c*x^2 + b)*c^2) + 1/3*(c^2*x^4 - 4*b*c*x^2 - 8*b^2)*B
/(sqrt(c*x^2 + b)*c^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{2\,(4\,Bb^2-3\,Abc)\mathrm{sgn}(x)}{3\,\sqrt{b}c^3} - \frac{Bb^2-Abc}{\sqrt{cx^2+b}c^3\mathrm{sgn}\,(x)} + \frac{(cx^2+b)^{\frac{3}{2}}Bc^6-6\,\sqrt{cx^2+b}Bbc^6+3\,\sqrt{cx^2+b}Ac^7}{3\,c^9\mathrm{sgn}\,(x)}$$

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(4*B*b^2 - 3*A*b*c)*sgn(x)/(sqrt(b)*c^3) - (B*b^2 - A*b*c)/(sqrt(c*x^2
+ b)*c^3*sgn(x)) + 1/3*((c*x^2 + b)^(3/2)*B*c^6 - 6*sqrt(c*x^2 + b)*B*b*c^6
 + 3*sqrt(c*x^2 + b)*A*c^7)/(c^9*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

$$\int \frac{x^6(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{c\,x^4+b\,x^2}\,\left(-8\,B\,b^2-4\,B\,b\,c\,x^2+6\,A\,b\,c+B\,c^2\,x^4+3\,A\,c^2\,x^2\right)}{3\,c^3\,x\,\left(c\,x^2+b\right)}$$

```
[In] int((x^6*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] ((b*x^2 + c*x^4)^(1/2)*(3*A*c^2*x^2 - 8*B*b^2 + B*c^2*x^4 + 6*A*b*c - 4*B*b
*c*x^2))/(3*c^3*x*(b + c*x^2))
```

3.155 $\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 69

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=-\frac{(bB-Ac)x^3}{bc\sqrt{bx^2+cx^4}}+\frac{(2bB-Ac)\sqrt{bx^2+cx^4}}{bc^2x}$$

```
[Out] -(-A*c+B*b)*x^3/b/c/(c*x^4+b*x^2)^(1/2)+(-A*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/b/
c^2/x
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2062, 1602}

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=\frac{\sqrt{bx^2+cx^4}(2bB-Ac)}{bc^2x}-\frac{x^3(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[In] Int[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^3)/(b*c*Sqrt[b*x^2 + c*x^4])) + ((2*b*B - A*c)*Sqrt[b*x^2
+ c*x^4])/(b*c^2*x)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2062

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Dist[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1))), Int[(e*x)^(m
- j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n
}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(bB-Ac)x^3}{bc\sqrt{bx^2+cx^4}}+\frac{(2bB-Ac)\int\frac{x^2}{\sqrt{bx^2+cx^4}}\,dx}{bc}\\ &= -\frac{(bB-Ac)x^3}{bc\sqrt{bx^2+cx^4}}+\frac{(2bB-Ac)\sqrt{bx^2+cx^4}}{bc^2x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.51

$$\int\frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=\frac{x(2bB-Ac+Bcx^2)}{c^2\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(x^4*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
[Out] (x*(2*b*B - A*c + B*c*x^2))/(c^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{(c\,x^2+b)\,(-B c\,x^2+Ac-2Bb)x^3}{c^2(x^4c+b\,x^2)^{\frac{3}{2}}}$	44
default	$-\frac{(c\,x^2+b)\,(-B c\,x^2+Ac-2Bb)x^3}{c^2(x^4c+b\,x^2)^{\frac{3}{2}}}$	44
trager	$-\frac{(-B c\,x^2+Ac-2Bb)\sqrt{x^4c+b\,x^2}}{(c\,x^2+b)c^2x}$	46
risch	$\frac{B(c\,x^2+b)x}{c^2\sqrt{x^2(c\,x^2+b)}}-\frac{(Ac-Bb)x}{c^2\sqrt{x^2(c\,x^2+b)}}$	55

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -(c*x^2+b)*(-B*c*x^2+A*c-2*B*b)*x^3/c^2/(c*x^4+b*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{cx^4+bx^2}(Bcx^2+2\,Bb-Ac)}{c^3x^3+bc^2x}$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
[Out] sqrt(c*x^4 + b*x^2)*(B*c*x^2 + 2*B*b - A*c)/(c^3*x^3 + b*c^2*x)
```

Sympy [F]

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^4(A+Bx^2)}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(x**4*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{(cx^2+2\,b)B}{\sqrt{cx^2+b}c^2} - \frac{A}{\sqrt{cx^2+b}c}$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] (c*x^2 + 2*b)*B/(sqrt(c*x^2 + b)*c^2) - A/(sqrt(c*x^2 + b)*c)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(2\,Bb-Ac)\mathrm{sgn}(x)}{\sqrt{b}c^2} + \frac{\sqrt{cx^2+b}B}{c^2\mathrm{sgn}\,(x)} + \frac{Bb-Ac}{\sqrt{cx^2+b}c^2\mathrm{sgn}\,(x)}$$

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] -(2*B*b - A*c)*sgn(x)/(sqrt(b)*c^2) + sqrt(c*x^2 + b)*B/(c^2*sgn(x)) + (B*b
 - A*c)/(sqrt(c*x^2 + b)*c^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

$$\int \frac{x^4(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=\frac{\sqrt{c\,x^4+b\,x^2}\,\left(B\,c\,x^2-A\,c+2\,B\,b\right)}{c^2\,x\,\left(c\,x^2+b\right)}$$

```
[In] int((x^4*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
[Out] ((b*x^2 + c*x^4)^(1/2)*(2*B*b - A*c + B*c*x^2))/(c^2*x*(b + c*x^2))
```

3.156 $\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 64

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x}{bc\sqrt{bx^2+cx^4}} - \frac{A\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}$$

```
[Out] -A*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(3/2)-(-A*c+B*b)*x/b/c/(c*x^4+b
*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2062, 2033, 212}

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{A\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}} - \frac{x(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[In] Int[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x)/(b*c*Sqrt[b*x^2 + c*x^4])) - (A*ArcTanh[(Sqrt[b]*x)/Sqrt[
b*x^2 + c*x^4]])/b^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2062

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Dist[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1))), Int[(e*x)^(m
- j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n
}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(bB-Ac)x}{bc\sqrt{bx^2+cx^4}}+\frac{A\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{b}\\
&= -\frac{(bB-Ac)x}{bc\sqrt{bx^2+cx^4}}-\frac{A\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{bx^2+cx^4}}\right)}{b}\\
&= -\frac{(bB-Ac)x}{bc\sqrt{bx^2+cx^4}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{x\left(\sqrt{b}(bB-Ac)+Ac\sqrt{b+cx^2}\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)\right)}{b^{3/2}c\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
[Out] -((x*(Sqrt[b]*(b*B - A*c) + A*c*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqr
t[b]]))/(b^(3/2)*c*Sqrt[x^2*(b + c*x^2)]))
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.23

method	result	size
default	$-\frac{x^{3}\left(c\,x^{2}+b\right)\left(A\sqrt{c\,x^{2}+b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^{2}+b}}{x}\right)bc-A\,b^{\frac{3}{2}}c+B\,b^{\frac{5}{2}}\right)}{\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}c\,b^{\frac{5}{2}}}$	79

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -x^3*(c*x^2+b)*(A*(c*x^2+b)^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b*c-A
*b^(3/2)*c+B*b^(5/2))/(c*x^4+b*x^2)^(3/2)/c/b^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 199, normalized size of antiderivative = 3.11

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \left[\frac{(Ac^2x^3+Abcx)\sqrt{b}\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)-2\,\sqrt{cx^4+bx^2}(Bb^2-Abc)}{2\,(b^2c^2x^3+b^3cx)}, \frac{(Ac^2x^3}{}\right.$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
[Out] [1/2*((A*c^2*x^3 + A*b*c*x)*sqrt(b)*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*
x^2)*sqrt(b))/x^3) - 2*sqrt(c*x^4 + b*x^2)*(B*b^2 - A*b*c))/(b^2*c^2*x^3 +
b^3*c*x), ((A*c^2*x^3 + A*b*c*x)*sqrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-
b)/(c*x^3 + b*x)) - sqrt(c*x^4 + b*x^2)*(B*b^2 - A*b*c))/(b^2*c^2*x^3 + b^3
*c*x)]
```

Sympy [F]

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^2(A+Bx^2)}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(x**2*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^2}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*x^2/(c*x^4 + b*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.70

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{A\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b\mathrm{sgn}\left(x\right)} - \frac{\left(A\sqrt{b}c\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)-B\sqrt{-b}b+A\sqrt{-b}c\right)\mathrm{sgn}(x)}{\sqrt{-b}b^{\frac{3}{2}}c} - \frac{Bb-Ac}{\sqrt{cx^2+b}bc\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] A*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b*sgn(x)) - (A*sqrt(b)*c*arcta
n(sqrt(b)/sqrt(-b)) - B*sqrt(-b)*b + A*sqrt(-b)*c)*sgn(x)/(sqrt(-b)*b^(3/2)
*c) - (B*b - A*c)/(sqrt(c*x^2 + b)*b*c*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^2\left(B\,x^2+A\right)}{{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

```
[In] int((x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
[Out] int((x^2*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.157 $\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 142

$$\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^{3/2}}\,dx=-\frac{B}{3cx\sqrt{bx^2+cx^4}}-\frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}}+\frac{(2bB-3Ac)\sqrt{bx^2+cx^4}}{2b^2cx^3}-\frac{(2bB-3Ac)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}$$

```
[Out] -1/2*(-3*A*c+2*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(5/2)-1/3*B/c/
x/(c*x^4+b*x^2)^(1/2)+1/3*(3*A*c-2*B*b)/b/c/x/(c*x^4+b*x^2)^(1/2)+1/2*(-3*A
*c+2*B*b)*(c*x^4+b*x^2)^(1/2)/b^2/c/x^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1159, 2031, 2050, 2033, 212}

$$\int \frac{A+Bx^2}{\left(bx^2+cx^4\right)^{3/2}}\,dx=-\frac{(2bB-3Ac)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}+\frac{\sqrt{bx^2+cx^4}(2bB-3Ac)}{2b^2cx^3}-\frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}}-\frac{B}{3cx\sqrt{bx^2+cx^4}}$$

```
[In] Int[(A + B*x^2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -1/3*B/(c*x*Sqrt[b*x^2 + c*x^4]) - (2*b*B - 3*A*c)/(3*b*c*x*Sqrt[b*x^2 + c*
x^4]) + ((2*b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(2*b^2*c*x^3) - ((2*b*B - 3*A
*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
Simp[e*((b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 3)*x)), x] - Dist[(b*e*(2*p + 1)
- c*d*(4*p + 3))/(c*(4*p + 3)), Int[(b*x^2 + c*x^4)^p, x], x] /; FreeQ[{b,
c, d, e, p}, x] &&  !IntegerQ[p] && NeQ[4*p + 3, 0] && NeQ[b*e*(2*p + 1) -
c*d*(4*p + 3), 0]
```

Rule 2031

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{B}{3cx\sqrt{bx^2+cx^4}} + \frac{(-2bB+3Ac)\int \frac{1}{(bx^2+cx^4)^{3/2}}\,dx}{3c} \\
&= -\frac{B}{3cx\sqrt{bx^2+cx^4}} - \frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}} + \frac{(-2bB+3Ac)\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{bc} \\
&= -\frac{B}{3cx\sqrt{bx^2+cx^4}} - \frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}} + \frac{(2bB-3Ac)\sqrt{bx^2+cx^4}}{2b^2cx^3} + \frac{(2bB-3Ac)\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx}{2b^2}
\end{aligned}
$$

$$= -\frac{B}{3cx\sqrt{bx^2+cx^4}} - \frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}} + \frac{(2bB-3Ac)\sqrt{bx^2+cx^4}}{2b^2cx^3} - \frac{(2bB-3Ac)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)}{2b^2}$$

$$= -\frac{B}{3cx\sqrt{bx^2+cx^4}} - \frac{2bB-3Ac}{3bcx\sqrt{bx^2+cx^4}} + \frac{(2bB-3Ac)\sqrt{bx^2+cx^4}}{2b^2cx^3} - \frac{(2bB-3Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{b}(2bBx^2-A(b+3cx^2))-(2bB-3Ac)x^2\sqrt{b+cx^2}\text{arctanh}\left(\frac{\sqrt{b+cx^2}}{\sqrt{b}}\right)}{2b^{5/2}x\sqrt{x^2(b+cx^2)}}$$

```
[In] Integrate[(A + B*x^2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[b]*(2*b*B*x^2 - A*(b + 3*c*x^2)) - (2*b*B - 3*A*c)*x^2*Sqrt[b + c*x^2
]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])/(2*b^(5/2)*x*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{A(c\,x^2+b)}{2b^2x\sqrt{x^2(c\,x^2+b)}}-\frac{\left(-\frac{Ac}{\sqrt{c\,x^2+b}}+b(3Ac-2Bb)\left(\frac{1}{b\sqrt{c\,x^2+b}}-\frac{\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)}{b^{\frac{3}{2}}}\right)\right)x\sqrt{c\,x^2+b}}{2b^2\sqrt{x^2(c\,x^2+b)}}$	126
default	$\frac{x(c\,x^2+b)\left(3A\sqrt{c\,x^2+b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)bc\,x^2-3A\,b^{\frac{3}{2}}c\,x^2-2B\sqrt{c\,x^2+b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{c\,x^2+b}}{x}\right)b^2x^2+2B\,b^{\frac{5}{2}}x^2-A\,b^{\frac{5}{2}}\right)}{2(x^4c+b\,x^2)^{\frac{3}{2}}b^{\frac{7}{2}}}$	130

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b^2*A*(c*x^2+b)/x/(x^2*(c*x^2+b))^(1/2)-1/2/b^2*(-A*c/(c*x^2+b)^(1/2)+
b*(3*A*c-2*B*b)*(1/b/(c*x^2+b)^(1/2)-1/b^(3/2)*ln((2*b+2*b^(1/2)*(c*x^2+b)^
(1/2))/x)))*x/(x^2*(c*x^2+b))^(1/2)*(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.83

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^{3/2}}\,dx = \left[-\frac{((2\,Bbc-3\,Ac^2)x^5+(2\,Bb^2-3\,Abc)x^3)\sqrt{b}\log\left(-\frac{cx^3+2\,bx+2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx}}{4\,(b^3cx^5+b^4x^3)}\right.$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*B*b*c - 3*A*c^2)*x^5 + (2*B*b^2 - 3*A*b*c)*x^3)*sqrt(b)*log(-(c*
x^3 + 2*b*x + 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(
A*b^2 - (2*B*b^2 - 3*A*b*c)*x^2))/(b^3*c*x^5 + b^4*x^3), 1/2*(((2*B*b*c - 3
*A*c^2)*x^5 + (2*B*b^2 - 3*A*b*c)*x^3)*sqrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*
sqrt(-b)/(c*x^3 + b*x)) - sqrt(c*x^4 + b*x^2)*(A*b^2 - (2*B*b^2 - 3*A*b*c)*
x^2))/(b^3*c*x^5 + b^4*x^3)]
```

Sympy [F]

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{A+Bx^2}{\left(x^2\,(b+cx^2)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(c*x^4 + b*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^{3/2}}\,dx = \frac{(2\,Bb-3\,Ac)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{2\,\sqrt{-b}b^2\mathrm{sgn}\,(x)} + \frac{2\,(cx^2+b)Bb-2\,Bb^2-3\,(cx^2+b)Ac+2\,Abc}{2\left((cx^2+b)^{\frac{3}{2}}-\sqrt{cx^2+b}b\right)b^2\mathrm{sgn}\,(x)}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*B*b - 3*A*c)*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^2*sgn(x))
+ 1/2*(2*(c*x^2 + b)*B*b - 2*B*b^2 - 3*(c*x^2 + b)*A*c + 2*A*b*c)/(((c*x^2
+ b)^(3/2) - sqrt(c*x^2 + b)*b)*b^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{B\,x^2+A}{{\left(c\,x^4+b\,x^2\right)}^{3/2}}\,dx$$

```
[In] int((A + B*x^2)/(b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((A + B*x^2)/(b*x^2 + c*x^4)^(3/2), x)
```

3.158 $\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{A}{4bx^3\sqrt{bx^2+cx^4}}+\frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}} - \frac{3(4bB-5Ac)\sqrt{bx^2+cx^4}}{8b^3x^3}+\frac{3c(4bB-5Ac)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}$$

```
[Out] 3/8*c*(-5*A*c+4*B*b)*arctanh(x*b^(1/2)/(c*x^4+b*x^2)^(1/2))/b^(7/2)-1/4*A/b
/x^3/(c*x^4+b*x^2)^(1/2)+1/4*(-5*A*c+4*B*b)/b^2/x/(c*x^4+b*x^2)^(1/2)-3/8*(
-5*A*c+4*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2063, 2031, 2050, 2033, 212}

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{3c(4bB-5Ac)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}} - \frac{3\sqrt{bx^2+cx^4}(4bB-5Ac)}{8b^3x^3}+\frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}}-\frac{A}{4bx^3\sqrt{bx^2+cx^4}}$$

```
[In] Int[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -1/4*A/(b*x^3*Sqrt[b*x^2 + c*x^4]) + (4*b*B - 5*A*c)/(4*b^2*x*Sqrt[b*x^2 +
c*x^4]) - (3*(4*b*B - 5*A*c)*Sqrt[b*x^2 + c*x^4])/(8*b^3*x^3) + (3*c*(4*b*B
 - 5*A*c)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2031

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{A}{4bx^3\sqrt{bx^2+cx^4}} - \frac{(-4bB+5Ac)\int \frac{1}{(bx^2+cx^4)^{3/2}}\,dx}{4b}\\ &= -\frac{A}{4bx^3\sqrt{bx^2+cx^4}} + \frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}} + \frac{(3(4bB-5Ac))\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx}{4b^2}\end{aligned}$$

$$= -\frac{A}{4bx^3\sqrt{bx^2+cx^4}} + \frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}} - \frac{3(4bB-5Ac)\sqrt{bx^2+cx^4}}{8b^3x^3} - \frac{(3c(4bB-5Ac))\int\frac{1}{\sqrt{bx^2+cx^4}}\,dx}{8b^3}$$

$$= -\frac{A}{4bx^3\sqrt{bx^2+cx^4}} + \frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}} - \frac{3(4bB-5Ac)\sqrt{bx^2+cx^4}}{8b^3x^3} + \frac{(3c(4bB-5Ac))\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+cx^4}}\right)}{8b^3}$$

$$= -\frac{A}{4bx^3\sqrt{bx^2+cx^4}} + \frac{4bB-5Ac}{4b^2x\sqrt{bx^2+cx^4}} - \frac{3(4bB-5Ac)\sqrt{bx^2+cx^4}}{8b^3x^3} + \frac{3c(4bB-5Ac)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int\frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{\sqrt{b}(-4bBx^2(b+3cx^2)+A(-2b^2+5bcx^2+15c^2x^4))+3c(4bB-5Ac)x^4\sqrt{b+cx^2}}{8b^{7/2}x^3\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[b]*(-4*b*B*x^2*(b + 3*c*x^2) + A*(-2*b^2 + 5*b*c*x^2 + 15*c^2*x^4)) +
 3*c*(4*b*B - 5*A*c)*x^4*Sqrt[b + c*x^2]*ArcTanh[Sqrt[b + c*x^2]/Sqrt[b]])/
(8*b^(7/2)*x^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{(cx^2+b)(-7Acx^2+4bBx^2+2Ab)}{8b^3x^3\sqrt{x^2(cx^2+b)}} + \frac{c\left(-\frac{7Ac-4Bb}{\sqrt{cx^2+b}}+3b(5Ac-4Bb)\left(\frac{1}{b\sqrt{cx^2+b}}-\frac{\ln\left(\frac{2b+2\sqrt{b}\sqrt{cx^2+b}}{x}\right)}{b^{\frac{3}{2}}}\right)\right)x\sqrt{cx^2+b}}{8b^3\sqrt{x^2(cx^2+b)}}$
default	$-\frac{(cx^2+b)\left(15A\sqrt{cx^2+b}\,\ln\left(\frac{2b+2\sqrt{b}\sqrt{cx^2+b}}{x}\right)bc^2x^4-15Ab^{\frac{3}{2}}c^2x^4-12B\sqrt{cx^2+b}\,\ln\left(\frac{2b+2\sqrt{b}\sqrt{cx^2+b}}{x}\right)b^2cx^4+12Bb^{\frac{5}{2}}cx^4-5Ab^{\frac{5}{2}}\right.}{8x(x^4c+bx^2)^{\frac{3}{2}}b^{\frac{9}{2}}}$

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(c*x^2+b)*(-7*A*c*x^2+4*B*b*x^2+2*A*b)/b^3/x^3/(x^2*(c*x^2+b))^(1/2)+1
/8*c/b^3*(-(7*A*c-4*B*b)/(c*x^2+b)^(1/2)+3*b*(5*A*c-4*B*b)*(1/b/(c*x^2+b)^(
```

```
1/2)-1/b^(3/2)*ln((2*b+2*b^(1/2)*(c*x^2+b)^(1/2))/x)))*x/(x^2*(c*x^2+b))^(1
/2)*(c*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.30

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx = \left[-\frac{3\left((4\,Bbc^2-5\,Ac^3)x^7+(4\,Bb^2c-5\,Abc^2)x^5\right)\sqrt{b}\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2}{16\left(b^4cx^7+b^5x\right.}\right.$$

$$-\frac{3\left((4\,Bbc^2-5\,Ac^3)x^7+(4\,Bb^2c-5\,Abc^2)x^5\right)\sqrt{-b}\arctan\left(\frac{\sqrt{cx^4+bx^2}\sqrt{-b}}{cx^3+bx}\right)+\left(3\,(4\,Bb^2c-5\,Abc^2)x^4+2\,Ab^3-\right.}{8\left(b^4cx^7+b^5x^5\right)}$$

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((4*B*b*c^2 - 5*A*c^3)*x^7 + (4*B*b^2*c - 5*A*b*c^2)*x^5)*sqrt(b)
*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(b))/x^3) + 2*(3*(4*B*b^2*
c - 5*A*b*c^2)*x^4 + 2*A*b^3 + (4*B*b^3 - 5*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^
2))/(b^4*c*x^7 + b^5*x^5), -1/8*(3*((4*B*b*c^2 - 5*A*c^3)*x^7 + (4*B*b^2*c
- 5*A*b*c^2)*x^5)*sqrt(-b)*arctan(sqrt(c*x^4 + b*x^2)*sqrt(-b)/(c*x^3 + b*x
)) + (3*(4*B*b^2*c - 5*A*b*c^2)*x^4 + 2*A*b^3 + (4*B*b^3 - 5*A*b^2*c)*x^2)*
sqrt(c*x^4 + b*x^2))/(b^4*c*x^7 + b^5*x^5)]
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{A+Bx^2}{x^2\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**2*(x**2*(b + c*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{Bx^2+A}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx=-\frac{3\left(4\,Bbc-5\,Ac^2\right)\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{8\,\sqrt{-b}b^3\mathrm{sgn}\left(x\right)}-\frac{Bbc-Ac^2}{\sqrt{cx^2+b}b^3\mathrm{sgn}\left(x\right)}-\frac{4\left(cx^2+b\right)^{\frac{3}{2}}Bbc-4\sqrt{cx^2+b}Bb^2c-7\left(cx^2+b\right)^{\frac{3}{2}}Ac^2+9\sqrt{cx^2+b}Abc^2}{8\,b^3c^2x^4\mathrm{sgn}\left(x\right)}$$

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] -3/8*(4*B*b*c - 5*A*c^2)*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^3*sgn
(x)) - (B*b*c - A*c^2)/(sqrt(c*x^2 + b)*b^3*sgn(x)) - 1/8*(4*(c*x^2 + b)^(3
/2)*B*b*c - 4*sqrt(c*x^2 + b)*B*b^2*c - 7*(c*x^2 + b)^(3/2)*A*c^2 + 9*sqrt(
c*x^2 + b)*A*b*c^2)/(b^3*c^2*x^4*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 10.23 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx^2}{x^2\left(bx^2+cx^4\right)^{3/2}}\,dx=-\frac{A\,{\left(\frac{b}{c\,x^2}+1\right)}^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{b}{c\,x^2}\right)}{7\,x\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}-\frac{B\,x\,{\left(\frac{b}{c\,x^2}+1\right)}^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{b}{c\,x^2}\right)}{5\,{\left(c\,x^4+b\,x^2\right)}^{3/2}}$$

```
[In] int((A + B*x^2)/(x^2*(b*x^2 + c*x^4)^(3/2)),x)
[Out] - (A*(b/(c*x^2) + 1)^(3/2)*hypergeom([3/2, 7/2], 9/2, -b/(c*x^2)))/(7*x*(b*
x^2 + c*x^4)^(3/2)) - (B*x*(b/(c*x^2) + 1)^(3/2)*hypergeom([3/2, 5/2], 7/2,
 -b/(c*x^2)))/(5*(b*x^2 + c*x^4)^(3/2))
```

3.159 $\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 39

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{13}Abx^{13/2}+\frac{2}{17}(bB+Ac)x^{17/2}+\frac{2}{21}Bcx^{21/2}$$

```
[Out] 2/13*A*b*x^(13/2)+2/17*(A*c+B*b)*x^(17/2)+2/21*B*c*x^(21/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{17}x^{17/2}(Ac+bB)+\frac{2}{13}Abx^{13/2}+\frac{2}{21}Bcx^{21/2}$$

```
[In] Int[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*A*b*x^(13/2))/13 + (2*(b*B + A*c)*x^(17/2))/17 + (2*B*c*x^(21/2))/21
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{11/2}\left(A+Bx^2\right)\left(b+cx^2\right)\,dx\\ &= \int \left(Abx^{11/2}+(bB+Ac)x^{15/2}+Bcx^{19/2}\right)\,dx\\ &= \frac{2}{13}Abx^{13/2}+\frac{2}{17}(bB+Ac)x^{17/2}+\frac{2}{21}Bcx^{21/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx=\frac{2x^{13/2}(357Ab+273bBx^2+273Acx^2+221Bcx^4)}{4641}$$

```
[In] Integrate[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
[Out] (2*x^(13/2)*(357*A*b + 273*b*B*x^2 + 273*A*c*x^2 + 221*B*c*x^4))/4641
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2Ab\,x^{\frac{13}{2}}}{13}+\frac{2(Ac+Bb)x^{\frac{17}{2}}}{17}+\frac{2Bc\,x^{\frac{21}{2}}}{21}$	28
default	$\frac{2Ab\,x^{\frac{13}{2}}}{13}+\frac{2(Ac+Bb)x^{\frac{17}{2}}}{17}+\frac{2Bc\,x^{\frac{21}{2}}}{21}$	28
gosper	$\frac{2x^{\frac{13}{2}}\left(221Bc\,x^{4}+273Ac\,x^{2}+273bB\,x^{2}+357Ab\right)}{4641}$	32
trager	$\frac{2x^{\frac{13}{2}}\left(221Bc\,x^{4}+273Ac\,x^{2}+273bB\,x^{2}+357Ab\right)}{4641}$	32
risch	$\frac{2x^{\frac{13}{2}}\left(221Bc\,x^{4}+273Ac\,x^{2}+273bB\,x^{2}+357Ab\right)}{4641}$	32

```
[In] int(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] 2/13*A*b*x^(13/2)+2/17*(A*c+B*b)*x^(17/2)+2/21*B*c*x^(21/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{4641}\left(221\,Bcx^{10}+273\,(Bb+Ac)x^8+357\,Abx^6\right)\sqrt{x}$$

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 2/4641*(221*B*c*x^10 + 273*(B*b + A*c)*x^8 + 357*A*b*x^6)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2Abx^{\frac{13}{2}}}{13}+\frac{2Acx^{\frac{17}{2}}}{17}+\frac{2Bbx^{\frac{17}{2}}}{17}+\frac{2Bcx^{\frac{21}{2}}}{21}$$

```
[In] integrate(x**(7/2)*(B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] 2*A*b*x**(13/2)/13 + 2*A*c*x**(17/2)/17 + 2*B*b*x**(17/2)/17 + 2*B*c*x**(21
/2)/21
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{21}\,Bcx^{\frac{21}{2}}+\frac{2}{17}\,(Bb+Ac)x^{\frac{17}{2}}+\frac{2}{13}\,Abx^{\frac{13}{2}}$$

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2/21*B*c*x^(21/2) + 2/17*(B*b + A*c)*x^(17/2) + 2/13*A*b*x^(13/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{21}\,Bcx^{\frac{21}{2}}+\frac{2}{17}\,Bbx^{\frac{17}{2}}+\frac{2}{17}\,Acx^{\frac{17}{2}}+\frac{2}{13}\,Abx^{\frac{13}{2}}$$

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/21*B*c*x^(21/2) + 2/17*B*b*x^(17/2) + 2/17*A*c*x^(17/2) + 2/13*A*b*x^(13/
2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx=\frac{2\,x^{13/2}\,\left(357\,A\,b+273\,A\,c\,x^2+273\,B\,b\,x^2+221\,B\,c\,x^4\right)}{4641}$$

```
[In] int(x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4),x)
[Out] (2*x^(13/2)*(357*A*b + 273*A*c*x^2 + 273*B*b*x^2 + 221*B*c*x^4))/4641
```

3.160 $\int x^{5/2}(A+Bx^2)(bx^2+cx^4)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 39

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{11}Abx^{11/2}+\frac{2}{15}(bB+Ac)x^{15/2}+\frac{2}{19}Bcx^{19/2}$$

```
[Out] 2/11*A*b*x^(11/2)+2/15*(A*c+B*b)*x^(15/2)+2/19*B*c*x^(19/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{15}x^{15/2}(Ac+bB)+\frac{2}{11}Abx^{11/2}+\frac{2}{19}Bcx^{19/2}$$

```
[In] Int[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*A*b*x^(11/2))/11 + (2*(b*B + A*c)*x^(15/2))/15 + (2*B*c*x^(19/2))/19
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{9/2}\left(A+Bx^2\right)\left(b+cx^2\right)\,dx\\ &= \int \left(Abx^{9/2}+(bB+Ac)x^{13/2}+Bcx^{17/2}\right)\,dx\\ &= \frac{2}{11}Abx^{11/2}+\frac{2}{15}(bB+Ac)x^{15/2}+\frac{2}{19}Bcx^{19/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx=\frac{2x^{11/2}(285Ab+209bBx^2+209Acx^2+165Bcx^4)}{3135}$$

```
[In] Integrate[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
[Out] (2*x^(11/2)*(285*A*b + 209*b*B*x^2 + 209*A*c*x^2 + 165*B*c*x^4))/3135
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2Ab\,x^{\frac{11}{2}}}{11}+\frac{2(Ac+Bb)x^{\frac{15}{2}}}{15}+\frac{2Bc\,x^{\frac{19}{2}}}{19}$	28
default	$\frac{2Ab\,x^{\frac{11}{2}}}{11}+\frac{2(Ac+Bb)x^{\frac{15}{2}}}{15}+\frac{2Bc\,x^{\frac{19}{2}}}{19}$	28
gosper	$\frac{2x^{\frac{11}{2}}\left(165Bc\,x^4+209Ac\,x^2+209bB\,x^2+285Ab\right)}{3135}$	32
trager	$\frac{2x^{\frac{11}{2}}\left(165Bc\,x^4+209Ac\,x^2+209bB\,x^2+285Ab\right)}{3135}$	32
risch	$\frac{2x^{\frac{11}{2}}\left(165Bc\,x^4+209Ac\,x^2+209bB\,x^2+285Ab\right)}{3135}$	32

```
[In] int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] 2/11*A*b*x^(11/2)+2/15*(A*c+B*b)*x^(15/2)+2/19*B*c*x^(19/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{3135}\left(165\,Bcx^9+209\,(Bb+Ac)x^7+285\,Abx^5\right)\sqrt{x}$$

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 2/3135*(165*B*c*x^9 + 209*(B*b + A*c)*x^7 + 285*A*b*x^5)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2Abx^{\frac{11}{2}}}{11}+\frac{2Acx^{\frac{15}{2}}}{15}+\frac{2Bbx^{\frac{15}{2}}}{15}+\frac{2Bcx^{\frac{19}{2}}}{19}$$

```
[In] integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] 2*A*b*x**(11/2)/11 + 2*A*c*x**(15/2)/15 + 2*B*b*x**(15/2)/15 + 2*B*c*x**(19
/2)/19
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{19}\,Bcx^{\frac{19}{2}}+\frac{2}{15}\,(Bb+Ac)x^{\frac{15}{2}}+\frac{2}{11}\,Abx^{\frac{11}{2}}$$

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2/19*B*c*x^(19/2) + 2/15*(B*b + A*c)*x^(15/2) + 2/11*A*b*x^(11/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{19}\,Bcx^{\frac{19}{2}}+\frac{2}{15}\,Bbx^{\frac{15}{2}}+\frac{2}{15}\,Acx^{\frac{15}{2}}+\frac{2}{11}\,Abx^{\frac{11}{2}}$$

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/19*B*c*x^(19/2) + 2/15*B*b*x^(15/2) + 2/15*A*c*x^(15/2) + 2/11*A*b*x^(11/
2)
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx=\frac{2\,x^{11/2}\,\left(285\,A\,b+209\,A\,c\,x^2+209\,B\,b\,x^2+165\,B\,c\,x^4\right)}{3135}$$

```
[In] int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4),x)
[Out] (2*x^(11/2)*(285*A*b + 209*A*c*x^2 + 209*B*b*x^2 + 165*B*c*x^4))/3135
```

3.161 $\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 39

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{9}Abx^{9/2}+\frac{2}{13}(bB+Ac)x^{13/2}+\frac{2}{17}Bcx^{17/2}$$

```
[Out] 2/9*A*b*x^(9/2)+2/13*(A*c+B*b)*x^(13/2)+2/17*B*c*x^(17/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{13}x^{13/2}(Ac+bB)+\frac{2}{9}Abx^{9/2}+\frac{2}{17}Bcx^{17/2}$$

```
[In] Int[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*A*b*x^(9/2))/9 + (2*(b*B + A*c)*x^(13/2))/13 + (2*B*c*x^(17/2))/17
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^{7/2}\left(A+Bx^2\right)\left(b+cx^2\right)\,dx \\
&= \int \left(Abx^{7/2}+(bB+Ac)x^{11/2}+Bcx^{15/2}\right)\,dx \\
&= \frac{2}{9}Abx^{9/2}+\frac{2}{13}(bB+Ac)x^{13/2}+\frac{2}{17}Bcx^{17/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx=\frac{2\left(221Abx^{9/2}+153bBx^{13/2}+153Acx^{13/2}+117Bcx^{17/2}\right)}{1989}$$

```
[In] Integrate[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4),x]
[Out] (2*(221*A*b*x^(9/2) + 153*b*B*x^(13/2) + 153*A*c*x^(13/2) + 117*B*c*x^(17/2
)))/1989
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2Ab\,x^{\frac{9}{2}}}{9}+\frac{2(Ac+Bb)x^{\frac{13}{2}}}{13}+\frac{2Bc\,x^{\frac{17}{2}}}{17}$	28
default	$\frac{2Ab\,x^{\frac{9}{2}}}{9}+\frac{2(Ac+Bb)x^{\frac{13}{2}}}{13}+\frac{2Bc\,x^{\frac{17}{2}}}{17}$	28
gosper	$\frac{2x^{\frac{9}{2}}\left(117Bc\,x^4+153Ac\,x^2+153bB\,x^2+221Ab\right)}{1989}$	32
trager	$\frac{2x^{\frac{9}{2}}\left(117Bc\,x^4+153Ac\,x^2+153bB\,x^2+221Ab\right)}{1989}$	32
risch	$\frac{2x^{\frac{9}{2}}\left(117Bc\,x^4+153Ac\,x^2+153bB\,x^2+221Ab\right)}{1989}$	32

```
[In] int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] 2/9*A*b*x^(9/2)+2/13*(A*c+B*b)*x^(13/2)+2/17*B*c*x^(17/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{1989}\left(117\,Bcx^8+153\,(Bb+Ac)x^6+221\,Abx^4\right)\sqrt{x}$$

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 2/1989*(117*B*c*x^8 + 153*(B*b + A*c)*x^6 + 221*A*b*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2Abx^{\frac{9}{2}}}{9}+\frac{2Acx^{\frac{13}{2}}}{13}+\frac{2Bbx^{\frac{13}{2}}}{13}+\frac{2Bcx^{\frac{17}{2}}}{17}$$

```
[In] integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2),x)
[Out] 2*A*b*x**(9/2)/9 + 2*A*c*x**(13/2)/13 + 2*B*b*x**(13/2)/13 + 2*B*c*x**(17/2
)/17
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{17}\,Bcx^{\frac{17}{2}}+\frac{2}{13}\,(Bb+Ac)x^{\frac{13}{2}}+\frac{2}{9}\,Abx^{\frac{9}{2}}$$

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 2/17*B*c*x^(17/2) + 2/13*(B*b + A*c)*x^(13/2) + 2/9*A*b*x^(9/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{17}\,Bcx^{\frac{17}{2}}+\frac{2}{13}\,Bbx^{\frac{13}{2}}+\frac{2}{13}\,Acx^{\frac{13}{2}}+\frac{2}{9}\,Abx^{\frac{9}{2}}$$

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
[Out] 2/17*B*c*x^(17/2) + 2/13*B*b*x^(13/2) + 2/13*A*c*x^(13/2) + 2/9*A*b*x^(9/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2\,x^{9/2}\,\left(221\,A\,b+153\,A\,c\,x^2+153\,B\,b\,x^2+117\,B\,c\,x^4\right)}{1989}$$

```
[In] int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4),x)
[Out] (2*x^(9/2)*(221*A*b + 153*A*c*x^2 + 153*B*b*x^2 + 117*B*c*x^4))/1989
```

3.162 $\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 39

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{7}Abx^{7/2}+\frac{2}{11}(bB+Ac)x^{11/2}+\frac{2}{15}Bcx^{15/2}$$

```
[Out] 2/7*A*b*x^(7/2)+2/11*(A*c+B*b)*x^(11/2)+2/15*B*c*x^(15/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{11}x^{11/2}(Ac+bB)+\frac{2}{7}Abx^{7/2}+\frac{2}{15}Bcx^{15/2}$$

```
[In] Int[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*A*b*x^(7/2))/7 + (2*(b*B + A*c)*x^(11/2))/11 + (2*B*c*x^(15/2))/15
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{5/2}\left(A+Bx^2\right)\left(b+cx^2\right)\,dx\\ &= \int \left(Abx^{5/2}+(bB+Ac)x^{9/2}+Bcx^{13/2}\right)\,dx\\ &= \frac{2}{7}Abx^{7/2}+\frac{2}{11}(bB+Ac)x^{11/2}+\frac{2}{15}Bcx^{15/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx = \frac{2x^{7/2}(165Ab+105bBx^2+105Acx^2+77Bcx^4)}{1155}$$

```
[In] Integrate[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4),x]
[Out] (2*x^(7/2)*(165*A*b + 105*b*B*x^2 + 105*A*c*x^2 + 77*B*c*x^4))/1155
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2Ab\,x^{\frac{7}{2}}}{7}+\frac{2(Ac+Bb)x^{\frac{11}{2}}}{11}+\frac{2Bc\,x^{\frac{15}{2}}}{15}$	28
default	$\frac{2Ab\,x^{\frac{7}{2}}}{7}+\frac{2(Ac+Bb)x^{\frac{11}{2}}}{11}+\frac{2Bc\,x^{\frac{15}{2}}}{15}$	28
gosper	$\frac{2x^{\frac{7}{2}}\left(77Bc\,x^4+105Ac\,x^2+105bB\,x^2+165Ab\right)}{1155}$	32
trager	$\frac{2x^{\frac{7}{2}}\left(77Bc\,x^4+105Ac\,x^2+105bB\,x^2+165Ab\right)}{1155}$	32
risch	$\frac{2x^{\frac{7}{2}}\left(77Bc\,x^4+105Ac\,x^2+105bB\,x^2+165Ab\right)}{1155}$	32

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)*x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/7*A*b*x^(7/2)+2/11*(A*c+B*b)*x^(11/2)+2/15*B*c*x^(15/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{1155}\left(77\,Bcx^7+105\,(Bb+Ac)x^5+165\,Abx^3\right)\sqrt{x}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)*x^(1/2),x, algorithm="fricas")
[Out] 2/1155*(77*B*c*x^7 + 105*(B*b + A*c)*x^5 + 165*A*b*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2Abx^{\frac{7}{2}}}{7}+\frac{2Bcx^{\frac{15}{2}}}{15}+\frac{2x^{\frac{11}{2}}(Ac+Bb)}{11}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)*x**(1/2),x)
[Out] 2*A*b*x**(7/2)/7 + 2*B*c*x**(15/2)/15 + 2*x**(11/2)*(A*c + B*b)/11
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{15}\,Bcx^{\frac{15}{2}}+\frac{2}{11}\,(Bb+Ac)x^{\frac{11}{2}}+\frac{2}{7}\,Abx^{\frac{7}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)*x^(1/2),x, algorithm="maxima")
[Out] 2/15*B*c*x^(15/2) + 2/11*(B*b + A*c)*x^(11/2) + 2/7*A*b*x^(7/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2}{15}\,Bcx^{\frac{15}{2}}+\frac{2}{11}\,Bbx^{\frac{11}{2}}+\frac{2}{11}\,Acx^{\frac{11}{2}}+\frac{2}{7}\,Abx^{\frac{7}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)*x^(1/2),x, algorithm="giac")
[Out] 2/15*B*c*x^(15/2) + 2/11*B*b*x^(11/2) + 2/11*A*c*x^(11/2) + 2/7*A*b*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)\,dx=\frac{2\,x^{7/2}\,\left(165\,A\,b+105\,A\,c\,x^2+105\,B\,b\,x^2+77\,B\,c\,x^4\right)}{1155}$$

```
[In] int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4),x)
[Out] (2*x^(7/2)*(165*A*b + 105*A*c*x^2 + 105*B*b*x^2 + 77*B*c*x^4))/1155
```

3.163 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 39

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}} \, dx = \frac{2}{5}Abx^{5/2} + \frac{2}{9}(bB+Ac)x^{9/2} + \frac{2}{13}Bcx^{13/2}$$

```
[Out] 2/5*A*b*x^(5/2)+2/9*(A*c+B*b)*x^(9/2)+2/13*B*c*x^(13/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}} \, dx = \frac{2}{9}x^{9/2}(Ac+bB) + \frac{2}{5}Abx^{5/2} + \frac{2}{13}Bcx^{13/2}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/Sqrt[x],x]
```

```
[Out] (2*A*b*x^(5/2))/5 + (2*(b*B + A*c)*x^(9/2))/9 + (2*B*c*x^(13/2))/13
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{3/2}\left(A+Bx^2\right)\left(b+cx^2\right)\,dx\\ &= \int \left(Abx^{3/2}+(bB+Ac)x^{7/2}+Bcx^{11/2}\right)\,dx\\ &= \frac{2}{5}Abx^{5/2}+\frac{2}{9}(bB+Ac)x^{9/2}+\frac{2}{13}Bcx^{13/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}}\,dx = \frac{2}{585}x^{5/2}\left(117Ab+65bBx^2+65Acx^2+45Bcx^4\right)$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/Sqrt[x],x]
[Out] (2*x^(5/2)*(117*A*b + 65*b*B*x^2 + 65*A*c*x^2 + 45*B*c*x^4))/585
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2Ab\,x^{\frac{5}{2}}}{5}+\frac{2(Ac+Bb)x^{\frac{9}{2}}}{9}+\frac{2Bc\,x^{\frac{13}{2}}}{13}$	28
default	$\frac{2Ab\,x^{\frac{5}{2}}}{5}+\frac{2(Ac+Bb)x^{\frac{9}{2}}}{9}+\frac{2Bc\,x^{\frac{13}{2}}}{13}$	28
gosper	$\frac{2x^{\frac{5}{2}}\left(45Bc\,x^{4}+65Ac\,x^{2}+65bB\,x^{2}+117Ab\right)}{585}$	32
trager	$\frac{2x^{\frac{5}{2}}\left(45Bc\,x^{4}+65Ac\,x^{2}+65bB\,x^{2}+117Ab\right)}{585}$	32
risch	$\frac{2x^{\frac{5}{2}}\left(45Bc\,x^{4}+65Ac\,x^{2}+65bB\,x^{2}+117Ab\right)}{585}$	32

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5*A*b*x^(5/2)+2/9*(A*c+B*b)*x^(9/2)+2/13*B*c*x^(13/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}}\,dx = \frac{2}{585}\left(45\,Bcx^6+65\,(Bb+Ac)x^4+117\,Abx^2\right)\sqrt{x}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(1/2),x, algorithm="fricas")
[Out] 2/585*(45*B*c*x^6 + 65*(B*b + A*c)*x^4 + 117*A*b*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}}\,dx = \frac{2Abx^{\frac{5}{2}}}{5}+\frac{2Acx^{\frac{9}{2}}}{9}+\frac{2Bbx^{\frac{9}{2}}}{9}+\frac{2Bcx^{\frac{13}{2}}}{13}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**(1/2),x)
[Out] 2*A*b*x**(5/2)/5 + 2*A*c*x**(9/2)/9 + 2*B*b*x**(9/2)/9 + 2*B*c*x**(13/2)/13
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}}\,dx = \frac{2}{13}\,Bcx^{\frac{13}{2}}+\frac{2}{9}\,(Bb+Ac)x^{\frac{9}{2}}+\frac{2}{5}\,Abx^{\frac{5}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(1/2),x, algorithm="maxima")
[Out] 2/13*B*c*x^(13/2) + 2/9*(B*b + A*c)*x^(9/2) + 2/5*A*b*x^(5/2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{\sqrt{x}}\,dx = \frac{2}{13}\,Bcx^{\frac{13}{2}}+\frac{2}{9}\,Bbx^{\frac{9}{2}}+\frac{2}{9}\,Acx^{\frac{9}{2}}+\frac{2}{5}\,Abx^{\frac{5}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(1/2),x, algorithm="giac")
[Out] 2/13*B*c*x^(13/2) + 2/9*B*b*x^(9/2) + 2/9*A*c*x^(9/2) + 2/5*A*b*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)}{\sqrt{x}}\,dx = \frac{2\,x^{5/2}\,(117\,A\,b+65\,A\,c\,x^2+65\,B\,b\,x^2+45\,B\,c\,x^4)}{585}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^(1/2),x)
[Out] (2*x^(5/2)*(117*A*b + 65*A*c*x^2 + 65*B*b*x^2 + 45*B*c*x^4))/585
```

3.164 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 39

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}} \, dx = \frac{2}{3}Abx^{3/2} + \frac{2}{7}(bB+Ac)x^{7/2} + \frac{2}{11}Bcx^{11/2}$$

```
[Out] 2/3*A*b*x^(3/2)+2/7*(A*c+B*b)*x^(7/2)+2/11*B*c*x^(11/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}} \, dx = \frac{2}{7}x^{7/2}(Ac+bB) + \frac{2}{3}Abx^{3/2} + \frac{2}{11}Bcx^{11/2}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^(3/2),x]
```

```
[Out] (2*A*b*x^(3/2))/3 + (2*(b*B + A*c)*x^(7/2))/7 + (2*B*c*x^(11/2))/11
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \sqrt{x}\left(A+Bx^2\right)\left(b+cx^2\right)\,dx\\ &= \int \left(Ab\sqrt{x}+(bB+Ac)x^{5/2}+Bcx^{9/2}\right)\,dx\\ &= \frac{2}{3}Abx^{3/2}+\frac{2}{7}(bB+Ac)x^{7/2}+\frac{2}{11}Bcx^{11/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)}{x^{3/2}}\,dx=\frac{2}{231}x^{3/2}\left(77Ab+33bBx^2+33Acx^2+21Bcx^4\right)$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^(3/2),x]
[Out] (2*x^(3/2)*(77*A*b + 33*b*B*x^2 + 33*A*c*x^2 + 21*B*c*x^4))/231
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2Ab\,x^{\frac{3}{2}}}{3}+\frac{2(Ac+Bb)x^{\frac{7}{2}}}{7}+\frac{2Bc\,x^{\frac{11}{2}}}{11}$	28
default	$\frac{2Ab\,x^{\frac{3}{2}}}{3}+\frac{2(Ac+Bb)x^{\frac{7}{2}}}{7}+\frac{2Bc\,x^{\frac{11}{2}}}{11}$	28
gosper	$\frac{2x^{\frac{3}{2}}\left(21Bc\,x^4+33Ac\,x^2+33bB\,x^2+77Ab\right)}{231}$	32
trager	$\frac{2x^{\frac{3}{2}}\left(21Bc\,x^4+33Ac\,x^2+33bB\,x^2+77Ab\right)}{231}$	32
risch	$\frac{2x^{\frac{3}{2}}\left(21Bc\,x^4+33Ac\,x^2+33bB\,x^2+77Ab\right)}{231}$	32

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/3*A*b*x^(3/2)+2/7*(A*c+B*b)*x^(7/2)+2/11*B*c*x^(11/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}}\,dx = \frac{2}{231}\left(21\,Bcx^5+33\,(Bb+Ac)x^3+77\,Abx\right)\sqrt{x}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(3/2),x, algorithm="fricas")
[Out] 2/231*(21*B*c*x^5 + 33*(B*b + A*c)*x^3 + 77*A*b*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}}\,dx = \frac{2Abx^{\frac{3}{2}}}{3}+\frac{2Acx^{\frac{7}{2}}}{7}+\frac{2Bbx^{\frac{7}{2}}}{7}+\frac{2Bcx^{\frac{11}{2}}}{11}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**(3/2),x)
[Out] 2*A*b*x**(3/2)/3 + 2*A*c*x**(7/2)/7 + 2*B*b*x**(7/2)/7 + 2*B*c*x**(11/2)/11
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}}\,dx = \frac{2}{11}\,Bcx^{\frac{11}{2}}+\frac{2}{7}\,(Bb+Ac)x^{\frac{7}{2}}+\frac{2}{3}\,Abx^{\frac{3}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(3/2),x, algorithm="maxima")
[Out] 2/11*B*c*x^(11/2) + 2/7*(B*b + A*c)*x^(7/2) + 2/3*A*b*x^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}}\,dx = \frac{2}{11}\,Bcx^{\frac{11}{2}}+\frac{2}{7}\,Bbx^{\frac{7}{2}}+\frac{2}{7}\,Acx^{\frac{7}{2}}+\frac{2}{3}\,Abx^{\frac{3}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(3/2),x, algorithm="giac")
[Out] 2/11*B*c*x^(11/2) + 2/7*B*b*x^(7/2) + 2/7*A*c*x^(7/2) + 2/3*A*b*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{3/2}}\,dx = \frac{2\,x^{3/2}\,(77\,A\,b+33\,A\,c\,x^2+33\,B\,b\,x^2+21\,B\,c\,x^4)}{231}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^(3/2),x)
[Out] (2*x^(3/2)*(77*A*b + 33*A*c*x^2 + 33*B*b*x^2 + 21*B*c*x^4))/231
```

3.165 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 37

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}}\,dx = 2Ab\sqrt{x}+\frac{2}{5}(bB+Ac)x^{5/2}+\frac{2}{9}Bcx^{9/2}$$

```
[Out] 2/5*(A*c+B*b)*x^(5/2)+2/9*B*c*x^(9/2)+2*A*b*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}}\,dx = \frac{2}{5}x^{5/2}(Ac+bB)+2Ab\sqrt{x}+\frac{2}{9}Bcx^{9/2}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^(5/2),x]
```

```
[Out] 2*A*b*Sqrt[x] + (2*(b*B + A*c)*x^(5/2))/5 + (2*B*c*x^(9/2))/9
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(A+Bx^2)(b+cx^2)}{\sqrt{x}}\,dx\\ &= \int \left(\frac{Ab}{\sqrt{x}}+(bB+Ac)x^{3/2}+Bcx^{7/2}\right)dx\\ &= 2Ab\sqrt{x}+\frac{2}{5}(bB+Ac)x^{5/2}+\frac{2}{9}Bcx^{9/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}}\,dx = \frac{2}{45}\sqrt{x}\left(45Ab+9bBx^2+9Acx^2+5Bcx^4\right)$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^(5/2),x]
[Out] (2*Sqrt[x]*(45*A*b + 9*b*B*x^2 + 9*A*c*x^2 + 5*B*c*x^4))/45
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{2(Ac+Bb)x^{\frac{5}{2}}}{5}+\frac{2Bc\,x^{\frac{9}{2}}}{9}+2Ab\sqrt{x}$	28
default	$\frac{2(Ac+Bb)x^{\frac{5}{2}}}{5}+\frac{2Bc\,x^{\frac{9}{2}}}{9}+2Ab\sqrt{x}$	28
trager	$\left(\frac{2}{9}Bc\,x^4+\frac{2}{5}Ac\,x^2+\frac{2}{5}bB\,x^2+2Ab\right)\sqrt{x}$	31
gosper	$\frac{2\sqrt{x}\,(5Bc\,x^4+9Ac\,x^2+9bB\,x^2+45Ab)}{45}$	32
risch	$\frac{2\sqrt{x}\,(5Bc\,x^4+9Ac\,x^2+9bB\,x^2+45Ab)}{45}$	32

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/5*(A*c+B*b)*x^(5/2)+2/9*B*c*x^(9/2)+2*A*b*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}}\,dx = \frac{2}{45}\left(5\,Bcx^4+9\,(Bb+Ac)x^2+45\,Ab\right)\sqrt{x}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(5/2),x, algorithm="fricas")
[Out] 2/45*(5*B*c*x^4 + 9*(B*b + A*c)*x^2 + 45*A*b)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}}\,dx = 2Ab\sqrt{x}+\frac{2Acx^{\frac{5}{2}}}{5}+\frac{2Bbx^{\frac{5}{2}}}{5}+\frac{2Bcx^{\frac{9}{2}}}{9}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**(5/2),x)
[Out] 2*A*b*sqrt(x) + 2*A*c*x**(5/2)/5 + 2*B*b*x**(5/2)/5 + 2*B*c*x**(9/2)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}}\,dx = \frac{2}{9}\,Bcx^{\frac{9}{2}}+\frac{2}{5}\,(Bb+Ac)x^{\frac{5}{2}}+2\,Ab\sqrt{x}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(5/2),x, algorithm="maxima")
[Out] 2/9*B*c*x^(9/2) + 2/5*(B*b + A*c)*x^(5/2) + 2*A*b*sqrt(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}}\,dx = \frac{2}{9}\,Bcx^{\frac{9}{2}}+\frac{2}{5}\,Bbx^{\frac{5}{2}}+\frac{2}{5}\,Acx^{\frac{5}{2}}+2\,Ab\sqrt{x}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(5/2),x, algorithm="giac")
[Out] 2/9*B*c*x^(9/2) + 2/5*B*b*x^(5/2) + 2/5*A*c*x^(5/2) + 2*A*b*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{5/2}}\,dx = \frac{2\sqrt{x}\,(45\,A\,b+9\,A\,c\,x^2+9\,B\,b\,x^2+5\,B\,c\,x^4)}{45}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^(5/2),x)
[Out] (2*x^(1/2)*(45*A*b + 9*A*c*x^2 + 9*B*b*x^2 + 5*B*c*x^4))/45
```

3.166 $\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 37

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}} dx = -\frac{2Ab}{\sqrt{x}} + \frac{2}{3}(bB+Ac)x^{3/2} + \frac{2}{7}Bcx^{7/2}$$

```
[Out] 2/3*(A*c+B*b)*x^(3/2)+2/7*B*c*x^(7/2)-2*A*b/x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}} dx = \frac{2}{3}x^{3/2}(Ac+bB) - \frac{2Ab}{\sqrt{x}} + \frac{2}{7}Bcx^{7/2}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4))/x^(7/2),x]
```

```
[Out] (-2*A*b)/Sqrt[x] + (2*(b*B + A*c)*x^(3/2))/3 + (2*B*c*x^(7/2))/7
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{(A+Bx^2)(b+cx^2)}{x^{3/2}}\,dx$$
$$= \int \left(\frac{Ab}{x^{3/2}} + (bB+Ac)\sqrt{x} + Bcx^{5/2}\right) dx$$
$$= -\frac{2Ab}{\sqrt{x}} + \frac{2}{3}(bB+Ac)x^{3/2} + \frac{2}{7}Bcx^{7/2}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}}\,dx = \frac{2(-21Ab+7bBx^2+7Acx^2+3Bcx^4)}{21\sqrt{x}}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4))/x^(7/2),x]
[Out] (2*(-21*A*b + 7*b*B*x^2 + 7*A*c*x^2 + 3*B*c*x^4))/(21*Sqrt[x])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{2Bc\,x^{\frac{7}{2}}}{7}+\frac{2Ac\,x^{\frac{3}{2}}}{3}+\frac{2bB\,x^{\frac{3}{2}}}{3}-\frac{2Ab}{\sqrt{x}}$	30
default	$\frac{2Bc\,x^{\frac{7}{2}}}{7}+\frac{2Ac\,x^{\frac{3}{2}}}{3}+\frac{2bB\,x^{\frac{3}{2}}}{3}-\frac{2Ab}{\sqrt{x}}$	30
gosper	$-\frac{2(-3Bc\,x^4-7Ac\,x^2-7bB\,x^2+21Ab)}{21\sqrt{x}}$	32
trager	$-\frac{2(-3Bc\,x^4-7Ac\,x^2-7bB\,x^2+21Ab)}{21\sqrt{x}}$	32
risch	$-\frac{2(-3Bc\,x^4-7Ac\,x^2-7bB\,x^2+21Ab)}{21\sqrt{x}}$	32

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)/x^(7/2),x,method=_RETURNVERBOSE)
[Out] 2/7*B*c*x^(7/2)+2/3*A*c*x^(3/2)+2/3*b*B*x^(3/2)-2*A*b/x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}}\,dx = \frac{2\,(3\,Bcx^4+7\,(Bb+Ac)x^2-21\,Ab)}{21\,\sqrt{x}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(7/2),x, algorithm="fricas")
[Out] 2/21*(3*B*c*x^4 + 7*(B*b + A*c)*x^2 - 21*A*b)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}}\,dx = -\frac{2Ab}{\sqrt{x}}+\frac{2Acx^{\frac{3}{2}}}{3}+\frac{2Bbx^{\frac{3}{2}}}{3}+\frac{2Bcx^{\frac{7}{2}}}{7}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)/x**(7/2),x)
[Out] -2*A*b/sqrt(x) + 2*A*c*x**(3/2)/3 + 2*B*b*x**(3/2)/3 + 2*B*c*x**(7/2)/7
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}}\,dx = \frac{2}{7}\,Bcx^{\frac{7}{2}}+\frac{2}{3}\,(Bb+Ac)x^{\frac{3}{2}}-\frac{2\,Ab}{\sqrt{x}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(7/2),x, algorithm="maxima")
[Out] 2/7*B*c*x^(7/2) + 2/3*(B*b + A*c)*x^(3/2) - 2*A*b/sqrt(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}}\,dx = \frac{2}{7}\,Bcx^{\frac{7}{2}}+\frac{2}{3}\,Bbx^{\frac{3}{2}}+\frac{2}{3}\,Acx^{\frac{3}{2}}-\frac{2\,Ab}{\sqrt{x}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)/x^(7/2),x, algorithm="giac")
[Out] 2/7*B*c*x^(7/2) + 2/3*B*b*x^(3/2) + 2/3*A*c*x^(3/2) - 2*A*b/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx^2)(bx^2+cx^4)}{x^{7/2}}\,dx = \frac{14\,A\,c\,x^2-42\,A\,b+14\,B\,b\,x^2+6\,B\,c\,x^4}{21\,\sqrt{x}}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4))/x^(7/2),x)
[Out] (14*A*c*x^2 - 42*A*b + 14*B*b*x^2 + 6*B*c*x^4)/(21*x^(1/2))
```

3.167 $\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 63

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{17}Ab^2x^{17/2}+\frac{2}{21}b(bB+2Ac)x^{21/2}+\frac{2}{25}c(2bB+Ac)x^{25/2}+\frac{2}{29}Bc^2x^{29/2}$$

```
[Out] 2/17*A*b^2*x^(17/2)+2/21*b*(2*A*c+B*b)*x^(21/2)+2/25*c*(A*c+2*B*b)*x^(25/2)
+2/29*B*c^2*x^(29/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1598, 459}

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{17}Ab^2x^{17/2}+\frac{2}{25}cx^{25/2}(Ac+2bB)+\frac{2}{21}bx^{21/2}(2Ac+bB)+\frac{2}{29}Bc^2x^{29/2}$$

```
[In] Int[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*A*b^2*x^(17/2))/17 + (2*b*(b*B + 2*A*c)*x^(21/2))/21 + (2*c*(2*b*B + A*c
)*x^(25/2))/25 + (2*B*c^2*x^(29/2))/29
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{15/2}\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx\\ &= \int \left(Ab^2x^{15/2}+b(bB+2Ac)x^{19/2}+c(2bB+Ac)x^{23/2}+Bc^2x^{27/2}\right)\,dx\\ &= \frac{2}{17}Ab^2x^{17/2}+\frac{2}{21}b(bB+2Ac)x^{21/2}+\frac{2}{25}c(2bB+Ac)x^{25/2}+\frac{2}{29}Bc^2x^{29/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx=\frac{2x^{17/2}(29A(525b^2+850bcx^2+357c^2x^4)+17Bx^2(725b^2+1218bcx^2+525c^2x^4))}{258825}$$

```
[In] Integrate[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
[Out] (2*x^(17/2)*(29*A*(525*b^2 + 850*b*c*x^2 + 357*c^2*x^4) + 17*B*x^2*(725*b^2
 + 1218*b*c*x^2 + 525*c^2*x^4)))/258825
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2B\,c^{2}x^{\frac{29}{2}}}{29}+\frac{2(A\,c^{2}+2Bbc)\,x^{\frac{25}{2}}}{25}+\frac{2(2Abc+B\,b^{2})\,x^{\frac{21}{2}}}{21}+\frac{2A\,b^{2}x^{\frac{17}{2}}}{17}$	52
default	$\frac{2B\,c^{2}x^{\frac{29}{2}}}{29}+\frac{2(A\,c^{2}+2Bbc)\,x^{\frac{25}{2}}}{25}+\frac{2(2Abc+B\,b^{2})\,x^{\frac{21}{2}}}{21}+\frac{2A\,b^{2}x^{\frac{17}{2}}}{17}$	52
gosper	$\frac{2x^{\frac{17}{2}}(8925B\,c^{2}x^{6}+10353A\,c^{2}x^{4}+20706x^{4}Bbc+24650Abc\,x^{2}+12325b^{2}B\,x^{2}+15225b^{2}A)}{258825}$	56
trager	$\frac{2x^{\frac{17}{2}}(8925B\,c^{2}x^{6}+10353A\,c^{2}x^{4}+20706x^{4}Bbc+24650Abc\,x^{2}+12325b^{2}B\,x^{2}+15225b^{2}A)}{258825}$	56
risch	$\frac{2x^{\frac{17}{2}}(8925B\,c^{2}x^{6}+10353A\,c^{2}x^{4}+20706x^{4}Bbc+24650Abc\,x^{2}+12325b^{2}B\,x^{2}+15225b^{2}A)}{258825}$	56

```
[In] int(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/29*B*c^2*x^(29/2)+2/25*(A*c^2+2*B*b*c)*x^(25/2)+2/21*(2*A*b*c+B*b^2)*x^(2
1/2)+2/17*A*b^2*x^(17/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{258825}\left(8925\,Bc^2x^{14}+10353\left(2\,Bbc+Ac^2\right)x^{12}+15225\,Ab^2x^8+12325\left(Bb^2+2\,Abc\right)x^{10}\right)\sqrt{x}$$

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 2/258825*(8925*B*c^2*x^14 + 10353*(2*B*b*c + A*c^2)*x^12 + 15225*A*b^2*x^8
+ 12325*(B*b^2 + 2*A*b*c)*x^10)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2Ab^2x^{\frac{17}{2}}}{17}+\frac{4Abcx^{\frac{21}{2}}}{21}+\frac{2Ac^2x^{\frac{25}{2}}}{25}+\frac{2Bb^2x^{\frac{21}{2}}}{21}+\frac{4Bbcx^{\frac{25}{2}}}{25}+\frac{2Bc^2x^{\frac{29}{2}}}{29}$$

```
[In] integrate(x**(7/2)*(B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*A*b**2*x**(17/2)/17 + 4*A*b*c*x**(21/2)/21 + 2*A*c**2*x**(25/2)/25 + 2*B*
b**2*x**(21/2)/21 + 4*B*b*c*x**(25/2)/25 + 2*B*c**2*x**(29/2)/29
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{29}\,Bc^2x^{\frac{29}{2}}+\frac{2}{25}\,(2\,Bbc+Ac^2)x^{\frac{25}{2}}+\frac{2}{17}\,Ab^2x^{\frac{17}{2}}+\frac{2}{21}\,(Bb^2+2\,Abc)x^{\frac{21}{2}}$$

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 2/29*B*c^2*x^(29/2) + 2/25*(2*B*b*c + A*c^2)*x^(25/2) + 2/17*A*b^2*x^(17/2)
 + 2/21*(B*b^2 + 2*A*b*c)*x^(21/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{29}\,Bc^2x^{\frac{29}{2}}+\frac{4}{25}\,Bbcx^{\frac{25}{2}}+\frac{2}{25}\,Ac^2x^{\frac{25}{2}}+\frac{2}{21}\,Bb^2x^{\frac{21}{2}}+\frac{4}{21}\,Abcx^{\frac{21}{2}}+\frac{2}{17}\,Ab^2x^{\frac{17}{2}}$$

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2/29*B*c^2*x^(29/2) + 4/25*B*b*c*x^(25/2) + 2/25*A*c^2*x^(25/2) + 2/21*B*b^
2*x^(21/2) + 4/21*A*b*c*x^(21/2) + 2/17*A*b^2*x^(17/2)
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=x^{21/2}\left(\frac{2\,B\,b^2}{21}+\frac{4\,A\,c\,b}{21}\right)+x^{25/2}\left(\frac{2\,A\,c^2}{25}+\frac{4\,B\,b\,c}{25}\right)+\frac{2\,A\,b^2\,x^{17/2}}{17}+\frac{2\,B\,c^2\,x^{29/2}}{29}$$

```
[In] int(x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(21/2)*((2*B*b^2)/21 + (4*A*b*c)/21) + x^(25/2)*((2*A*c^2)/25 + (4*B*b*c)
/25) + (2*A*b^2*x^(17/2))/17 + (2*B*c^2*x^(29/2))/29
```

3.168 $\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 63

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{15}Ab^2x^{15/2}+\frac{2}{19}b(bB+2Ac)x^{19/2}+\frac{2}{23}c(2bB+Ac)x^{23/2}+\frac{2}{27}Bc^2x^{27/2}$$

```
[Out] 2/15*A*b^2*x^(15/2)+2/19*b*(2*A*c+B*b)*x^(19/2)+2/23*c*(A*c+2*B*b)*x^(23/2)
+2/27*B*c^2*x^(27/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1598, 459}

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{15}Ab^2x^{15/2}+\frac{2}{23}cx^{23/2}(Ac+2bB)+\frac{2}{19}bx^{19/2}(2Ac+bB)+\frac{2}{27}Bc^2x^{27/2}$$

```
[In] Int[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*A*b^2*x^(15/2))/15 + (2*b*(b*B + 2*A*c)*x^(19/2))/19 + (2*c*(2*b*B + A*c
)*x^(23/2))/23 + (2*B*c^2*x^(27/2))/27
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{13/2}\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx\\ &= \int \left(Ab^2x^{13/2}+b(bB+2Ac)x^{17/2}+c(2bB+Ac)x^{21/2}+Bc^2x^{25/2}\right)\,dx\\ &= \frac{2}{15}Ab^2x^{15/2}+\frac{2}{19}b(bB+2Ac)x^{19/2}+\frac{2}{23}c(2bB+Ac)x^{23/2}+\frac{2}{27}Bc^2x^{27/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx=\frac{2x^{15/2}(9A(437b^2+690bcx^2+285c^2x^4)+5Bx^2(621b^2+1026bcx^2+437c^2x^4))}{58995}$$

```
[In] Integrate[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
[Out] (2*x^(15/2)*(9*A*(437*b^2 + 690*b*c*x^2 + 285*c^2*x^4) + 5*B*x^2*(621*b^2 +
 1026*b*c*x^2 + 437*c^2*x^4)))/58995
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2B\,c^2x^{\frac{27}{2}}}{27}+\frac{2(A\,c^2+2Bbc)\,x^{\frac{23}{2}}}{23}+\frac{2(2Abc+B\,b^2)\,x^{\frac{19}{2}}}{19}+\frac{2A\,b^2x^{\frac{15}{2}}}{15}$	52
default	$\frac{2B\,c^2x^{\frac{27}{2}}}{27}+\frac{2(A\,c^2+2Bbc)\,x^{\frac{23}{2}}}{23}+\frac{2(2Abc+B\,b^2)\,x^{\frac{19}{2}}}{19}+\frac{2A\,b^2x^{\frac{15}{2}}}{15}$	52
gosper	$\frac{2x^{\frac{15}{2}}\left(2185B\,c^2x^6+2565A\,c^2x^4+5130x^4Bbc+6210Abc\,x^2+3105b^2B\,x^2+3933b^2A\right)}{58995}$	56
trager	$\frac{2x^{\frac{15}{2}}\left(2185B\,c^2x^6+2565A\,c^2x^4+5130x^4Bbc+6210Abc\,x^2+3105b^2B\,x^2+3933b^2A\right)}{58995}$	56
risch	$\frac{2x^{\frac{15}{2}}\left(2185B\,c^2x^6+2565A\,c^2x^4+5130x^4Bbc+6210Abc\,x^2+3105b^2B\,x^2+3933b^2A\right)}{58995}$	56

```
[In] int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
[Out] 2/27*B*c^2*x^(27/2)+2/23*(A*c^2+2*B*b*c)*x^(23/2)+2/19*(2*A*b*c+B*b^2)*x^(1
9/2)+2/15*A*b^2*x^(15/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{58995}\left(2185\,Bc^2x^{13}+2565\left(2\,Bbc+Ac^2\right)x^{11}+3933\,Ab^2x^7+3105\left(Bb^2+2\,Abc\right)x^9\right)\sqrt{x}$$

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
[Out] 2/58995*(2185*B*c^2*x^13 + 2565*(2*B*b*c + A*c^2)*x^11 + 3933*A*b^2*x^7 + 3
105*(B*b^2 + 2*A*b*c)*x^9)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2Ab^2x^{\frac{15}{2}}}{15}+\frac{4Abcx^{\frac{19}{2}}}{19}+\frac{2Ac^2x^{\frac{23}{2}}}{23}+\frac{2Bb^2x^{\frac{19}{2}}}{19}+\frac{4Bbcx^{\frac{23}{2}}}{23}+\frac{2Bc^2x^{\frac{27}{2}}}{27}$$

```
[In] integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2)**2,x)
[Out] 2*A*b**2*x**(15/2)/15 + 4*A*b*c*x**(19/2)/19 + 2*A*c**2*x**(23/2)/23 + 2*B*
b**2*x**(19/2)/19 + 4*B*b*c*x**(23/2)/23 + 2*B*c**2*x**(27/2)/27
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{27}\,Bc^2x^{\frac{27}{2}}+\frac{2}{23}\,\left(2\,Bbc+Ac^2\right)x^{\frac{23}{2}}+\frac{2}{15}\,Ab^2x^{\frac{15}{2}}+\frac{2}{19}\,\left(Bb^2+2\,Abc\right)x^{\frac{19}{2}}$$

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 2/27*B*c^2*x^(27/2) + 2/23*(2*B*b*c + A*c^2)*x^(23/2) + 2/15*A*b^2*x^(15/2)
 + 2/19*(B*b^2 + 2*A*b*c)*x^(19/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{27}\,Bc^2x^{\frac{27}{2}}+\frac{4}{23}\,Bbcx^{\frac{23}{2}}+\frac{2}{23}\,Ac^2x^{\frac{23}{2}}+\frac{2}{19}\,Bb^2x^{\frac{19}{2}}+\frac{4}{19}\,Abcx^{\frac{19}{2}}+\frac{2}{15}\,Ab^2x^{\frac{15}{2}}$$

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2/27*B*c^2*x^(27/2) + 4/23*B*b*c*x^(23/2) + 2/23*A*c^2*x^(23/2) + 2/19*B*b^
2*x^(19/2) + 4/19*A*b*c*x^(19/2) + 2/15*A*b^2*x^(15/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx=x^{19/2}\left(\frac{2\,B\,b^2}{19}+\frac{4\,A\,c\,b}{19}\right)+x^{23/2}\left(\frac{2\,A\,c^2}{23}+\frac{4\,B\,b\,c}{23}\right)+\frac{2\,A\,b^2\,x^{15/2}}{15}+\frac{2\,B\,c^2\,x^{27/2}}{27}$$

```
[In] int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(19/2)*((2*B*b^2)/19 + (4*A*b*c)/19) + x^(23/2)*((2*A*c^2)/23 + (4*B*b*c)
/23) + (2*A*b^2*x^(15/2))/15 + (2*B*c^2*x^(27/2))/27
```

3.169 $\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 63

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{13}Ab^2x^{13/2}+\frac{2}{17}b(bB+2Ac)x^{17/2}+\frac{2}{21}c(2bB+Ac)x^{21/2}+\frac{2}{25}Bc^2x^{25/2}$$

```
[Out] 2/13*A*b^2*x^(13/2)+2/17*b*(2*A*c+B*b)*x^(17/2)+2/21*c*(A*c+2*B*b)*x^(21/2)
+2/25*B*c^2*x^(25/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1598, 459}

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{13}Ab^2x^{13/2}+\frac{2}{21}cx^{21/2}(Ac+2bB)+\frac{2}{17}bx^{17/2}(2Ac+bB)+\frac{2}{25}Bc^2x^{25/2}$$

```
[In] Int[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*A*b^2*x^(13/2))/13 + (2*b*(b*B + 2*A*c)*x^(17/2))/17 + (2*c*(2*b*B + A*c
)*x^(21/2))/21 + (2*B*c^2*x^(25/2))/25
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{11/2}(A+Bx^2)\left(b+cx^2\right)^2\,dx\\ &= \int \left(Ab^2x^{11/2}+b(bB+2Ac)x^{15/2}+c(2bB+Ac)x^{19/2}+Bc^2x^{23/2}\right)\,dx\\ &= \frac{2}{13}Ab^2x^{13/2}+\frac{2}{17}b(bB+2Ac)x^{17/2}+\frac{2}{21}c(2bB+Ac)x^{21/2}+\frac{2}{25}Bc^2x^{25/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2x^{13/2}(25A(357b^2+546bcx^2+221c^2x^4)+13Bx^2(525b^2+850bcx^2+357c^2x^4))}{116025}$$

```
[In] Integrate[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
[Out] (2*x^(13/2)*(25*A*(357*b^2 + 546*b*c*x^2 + 221*c^2*x^4) + 13*B*x^2*(525*b^2
 + 850*b*c*x^2 + 357*c^2*x^4)))/116025
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2B\,c^{2}x^{\frac{25}{2}}}{25}+\frac{2\left(A\,c^{2}+2Bbc\right)x^{\frac{21}{2}}}{21}+\frac{2\left(2Abc+B\,b^{2}\right)x^{\frac{17}{2}}}{17}+\frac{2A\,b^{2}x^{\frac{13}{2}}}{13}$	52
default	$\frac{2B\,c^{2}x^{\frac{25}{2}}}{25}+\frac{2\left(A\,c^{2}+2Bbc\right)x^{\frac{21}{2}}}{21}+\frac{2\left(2Abc+B\,b^{2}\right)x^{\frac{17}{2}}}{17}+\frac{2A\,b^{2}x^{\frac{13}{2}}}{13}$	52
gosper	$\frac{2x^{\frac{13}{2}}\left(4641B\,c^{2}x^{6}+5525A\,c^{2}x^{4}+11050x^{4}Bbc+13650Abc\,x^{2}+6825b^{2}B\,x^{2}+8925b^{2}A\right)}{116025}$	56
trager	$\frac{2x^{\frac{13}{2}}\left(4641B\,c^{2}x^{6}+5525A\,c^{2}x^{4}+11050x^{4}Bbc+13650Abc\,x^{2}+6825b^{2}B\,x^{2}+8925b^{2}A\right)}{116025}$	56
risch	$\frac{2x^{\frac{13}{2}}\left(4641B\,c^{2}x^{6}+5525A\,c^{2}x^{4}+11050x^{4}Bbc+13650Abc\,x^{2}+6825b^{2}B\,x^{2}+8925b^{2}A\right)}{116025}$	56

```
[In] int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/25*B*c^2*x^(25/2)+2/21*(A*c^2+2*B*b*c)*x^(21/2)+2/17*(2*A*b*c+B*b^2)*x^(1
7/2)+2/13*A*b^2*x^(13/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{116025}\left(4641\,Bc^2x^{12}+5525\left(2\,Bbc+Ac^2\right)x^{10}+8925\,Ab^2x^6+6825\left(Bb^2+2\,Abc\right)x^8\right)\sqrt{x}$$

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 2/116025*(4641*B*c^2*x^12 + 5525*(2*B*b*c + A*c^2)*x^10 + 8925*A*b^2*x^6 +
6825*(B*b^2 + 2*A*b*c)*x^8)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2Ab^2x^{\frac{13}{2}}}{13}+\frac{4Abcx^{\frac{17}{2}}}{17}+\frac{2Ac^2x^{\frac{21}{2}}}{21}+\frac{2Bb^2x^{\frac{17}{2}}}{17}+\frac{4Bbcx^{\frac{21}{2}}}{21}+\frac{2Bc^2x^{\frac{25}{2}}}{25}$$

```
[In] integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*A*b**2*x**(13/2)/13 + 4*A*b*c*x**(17/2)/17 + 2*A*c**2*x**(21/2)/21 + 2*B*
b**2*x**(17/2)/17 + 4*B*b*c*x**(21/2)/21 + 2*B*c**2*x**(25/2)/25
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{25}\,Bc^2x^{\frac{25}{2}}+\frac{2}{21}\,(2\,Bbc+Ac^2)x^{\frac{21}{2}}+\frac{2}{13}\,Ab^2x^{\frac{13}{2}}+\frac{2}{17}\,(Bb^2+2\,Abc)x^{\frac{17}{2}}$$

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
[Out] 2/25*B*c^2*x^(25/2) + 2/21*(2*B*b*c + A*c^2)*x^(21/2) + 2/13*A*b^2*x^(13/2)
 + 2/17*(B*b^2 + 2*A*b*c)*x^(17/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{25}\,Bc^2x^{\frac{25}{2}}+\frac{4}{21}\,Bbcx^{\frac{21}{2}}+\frac{2}{21}\,Ac^2x^{\frac{21}{2}}+\frac{2}{17}\,Bb^2x^{\frac{17}{2}}+\frac{4}{17}\,Abcx^{\frac{17}{2}}+\frac{2}{13}\,Ab^2x^{\frac{13}{2}}$$

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")
[Out] 2/25*B*c^2*x^(25/2) + 4/21*B*b*c*x^(21/2) + 2/21*A*c^2*x^(21/2) + 2/17*B*b^
2*x^(17/2) + 4/17*A*b*c*x^(17/2) + 2/13*A*b^2*x^(13/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=x^{17/2}\left(\frac{2\,B\,b^2}{17}+\frac{4\,A\,c\,b}{17}\right)+x^{21/2}\left(\frac{2\,A\,c^2}{21}+\frac{4\,B\,b\,c}{21}\right)+\frac{2\,A\,b^2\,x^{13/2}}{13}+\frac{2\,B\,c^2\,x^{25/2}}{25}$$

```
[In] int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x)
[Out] x^(17/2)*((2*B*b^2)/17 + (4*A*b*c)/17) + x^(21/2)*((2*A*c^2)/21 + (4*B*b*c)
/21) + (2*A*b^2*x^(13/2))/13 + (2*B*c^2*x^(25/2))/25
```

3.170 $\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{11}Ab^2x^{11/2}+\frac{2}{15}b(bB+2Ac)x^{15/2}+\frac{2}{19}c(2bB+Ac)x^{19/2}+\frac{2}{23}Bc^2x^{23/2}$$

```
[Out] 2/11*A*b^2*x^(11/2)+2/15*b*(2*A*c+B*b)*x^(15/2)+2/19*c*(A*c+2*B*b)*x^(19/2)
+2/23*B*c^2*x^(23/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1598, 459}

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx=\frac{2}{11}Ab^2x^{11/2}+\frac{2}{19}cx^{19/2}(Ac+2bB)+\frac{2}{15}bx^{15/2}(2Ac+bB)+\frac{2}{23}Bc^2x^{23/2}$$

```
[In] Int[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*A*b^2*x^(11/2))/11 + (2*b*(b*B + 2*A*c)*x^(15/2))/15 + (2*c*(2*b*B + A*c
)*x^(19/2))/19 + (2*B*c^2*x^(23/2))/23
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{9/2}\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx\\ &= \int \left(Ab^2x^{9/2}+b(bB+2Ac)x^{13/2}+c(2bB+Ac)x^{17/2}+Bc^2x^{21/2}\right)\,dx\\ &= \frac{2}{11}Ab^2x^{11/2}+\frac{2}{15}b(bB+2Ac)x^{15/2}+\frac{2}{19}c(2bB+Ac)x^{19/2}+\frac{2}{23}Bc^2x^{23/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx = \frac{2x^{11/2}(23A(285b^2+418bcx^2+165c^2x^4)+11Bx^2(437b^2+690bcx^2+285c^2x^4))}{72105}$$

```
[In] Integrate[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
[Out] (2*x^(11/2)*(23*A*(285*b^2 + 418*b*c*x^2 + 165*c^2*x^4) + 11*B*x^2*(437*b^2
 + 690*b*c*x^2 + 285*c^2*x^4)))/72105
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2B\,c^{2}x^{\frac{23}{2}}}{23}+\frac{2(A\,c^{2}+2Bbc)\,x^{\frac{19}{2}}}{19}+\frac{2(2Abc+B\,b^{2})\,x^{\frac{15}{2}}}{15}+\frac{2A\,b^{2}x^{\frac{11}{2}}}{11}$	52
default	$\frac{2B\,c^{2}x^{\frac{23}{2}}}{23}+\frac{2(A\,c^{2}+2Bbc)\,x^{\frac{19}{2}}}{19}+\frac{2(2Abc+B\,b^{2})\,x^{\frac{15}{2}}}{15}+\frac{2A\,b^{2}x^{\frac{11}{2}}}{11}$	52
gosper	$\frac{2x^{\frac{11}{2}}\left(3135B\,c^{2}x^{6}+3795A\,c^{2}x^{4}+7590x^{4}Bbc+9614Abc\,x^{2}+4807b^{2}B\,x^{2}+6555b^{2}A\right)}{72105}$	56
trager	$\frac{2x^{\frac{11}{2}}\left(3135B\,c^{2}x^{6}+3795A\,c^{2}x^{4}+7590x^{4}Bbc+9614Abc\,x^{2}+4807b^{2}B\,x^{2}+6555b^{2}A\right)}{72105}$	56
risch	$\frac{2x^{\frac{11}{2}}\left(3135B\,c^{2}x^{6}+3795A\,c^{2}x^{4}+7590x^{4}Bbc+9614Abc\,x^{2}+4807b^{2}B\,x^{2}+6555b^{2}A\right)}{72105}$	56

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/23*B*c^2*x^(23/2)+2/19*(A*c^2+2*B*b*c)*x^(19/2)+2/15*(2*A*b*c+B*b^2)*x^(1
5/2)+2/11*A*b^2*x^(11/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx = \frac{2}{72105}\left(3135\,Bc^2x^{11}+3795\left(2\,Bbc+Ac^2\right)x^9+6555\,Ab^2x^5+4807\left(Bb^2+2\,Abc\right)x^7\right)\sqrt{x}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/72105*(3135*B*c^2*x^11 + 3795*(2*B*b*c + A*c^2)*x^9 + 6555*A*b^2*x^5 + 48
07*(B*b^2 + 2*A*b*c)*x^7)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx = \frac{2Ab^2x^{\frac{11}{2}}}{11}+\frac{2Bc^2x^{\frac{23}{2}}}{23}+\frac{2x^{\frac{19}{2}}(Ac^2+2Bbc)}{19}+\frac{2x^{\frac{15}{2}}\cdot(2Abc+Bb^2)}{15}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2*x**(1/2),x)
```

```
[Out] 2*A*b**2*x**(11/2)/11 + 2*B*c**2*x**(23/2)/23 + 2*x**(19/2)*(A*c**2 + 2*B*b
*c)/19 + 2*x**(15/2)*(2*A*b*c + B*b**2)/15
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx = \frac{2}{23}\,Bc^2x^{\frac{23}{2}} + \frac{2}{19}\left(2\,Bbc+Ac^2\right)x^{\frac{19}{2}} + \frac{2}{11}\,Ab^2x^{\frac{11}{2}} + \frac{2}{15}\left(Bb^2+2\,Abc\right)x^{\frac{15}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2*x^(1/2),x, algorithm="maxima")
[Out] 2/23*B*c^2*x^(23/2) + 2/19*(2*B*b*c + A*c^2)*x^(19/2) + 2/11*A*b^2*x^(11/2)
 + 2/15*(B*b^2 + 2*A*b*c)*x^(15/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx = \frac{2}{23}\,Bc^2x^{\frac{23}{2}} + \frac{4}{19}\,Bbcx^{\frac{19}{2}} + \frac{2}{19}\,Ac^2x^{\frac{19}{2}} + \frac{2}{15}\,Bb^2x^{\frac{15}{2}} + \frac{4}{15}\,Abcx^{\frac{15}{2}} + \frac{2}{11}\,Ab^2x^{\frac{11}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2*x^(1/2),x, algorithm="giac")
[Out] 2/23*B*c^2*x^(23/2) + 4/19*B*b*c*x^(19/2) + 2/19*A*c^2*x^(19/2) + 2/15*B*b^
2*x^(15/2) + 4/15*A*b*c*x^(15/2) + 2/11*A*b^2*x^(11/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx = x^{15/2}\left(\frac{2\,B\,b^2}{15}+\frac{4\,A\,c\,b}{15}\right) + x^{19/2}\left(\frac{2\,A\,c^2}{19}+\frac{4\,B\,b\,c}{19}\right) + \frac{2\,A\,b^2\,x^{11/2}}{11} + \frac{2\,B\,c^2\,x^{23/2}}{23}$$

```
[In] int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^2,x)
[Out] x^(15/2)*((2*B*b^2)/15 + (4*A*b*c)/15) + x^(19/2)*((2*A*c^2)/19 + (4*B*b*c)
/19) + (2*A*b^2*x^(11/2))/11 + (2*B*c^2*x^(23/2))/23
```

3.171 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{\sqrt{x}} \, dx = \frac{2}{9}Ab^2x^{9/2} + \frac{2}{13}b(bB+2Ac)x^{13/2} + \frac{2}{17}c(2bB+Ac)x^{17/2} + \frac{2}{21}Bc^2x^{21/2}$$

```
[Out] 2/9*A*b^2*x^(9/2)+2/13*b*(2*A*c+B*b)*x^(13/2)+2/17*c*(A*c+2*B*b)*x^(17/2)+2
/21*B*c^2*x^(21/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{\sqrt{x}} \, dx = \frac{2}{9}Ab^2x^{9/2} + \frac{2}{17}cx^{17/2}(Ac+2bB) + \frac{2}{13}bx^{13/2}(2Ac+bB) + \frac{2}{21}Bc^2x^{21/2}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/Sqrt[x],x]
```

```
[Out] (2*A*b^2*x^(9/2))/9 + (2*b*(b*B + 2*A*c)*x^(13/2))/13 + (2*c*(2*b*B + A*c)*
x^(17/2))/17 + (2*B*c^2*x^(21/2))/21
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
```

```
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^{7/2}\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx\\
&= \int \left(Ab^2x^{7/2}+b(bB+2Ac)x^{11/2}+c(2bB+Ac)x^{15/2}+Bc^2x^{19/2}\right)\,dx\\
&= \frac{2}{9}Ab^2x^{9/2}+\frac{2}{13}b(bB+2Ac)x^{13/2}+\frac{2}{17}c(2bB+Ac)x^{17/2}+\frac{2}{21}Bc^2x^{21/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{\sqrt{x}}\,dx = \frac{2x^{9/2}(7A(221b^2+306bcx^2+117c^2x^4)+3Bx^2(357b^2+546bcx^2+221c^2x^4))}{13923}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/Sqrt[x],x]
[Out] (2*x^(9/2)*(7*A*(221*b^2 + 306*b*c*x^2 + 117*c^2*x^4) + 3*B*x^2*(357*b^2 +
546*b*c*x^2 + 221*c^2*x^4)))/13923
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2B\,c^{2}x^{\frac{21}{2}}}{21}+\frac{2\left(A\,c^{2}+2Bbc\right)x^{\frac{17}{2}}}{17}+\frac{2\left(2Abc+B\,b^{2}\right)x^{\frac{13}{2}}}{13}+\frac{2A\,b^{2}x^{\frac{9}{2}}}{9}$	52
default	$\frac{2B\,c^{2}x^{\frac{21}{2}}}{21}+\frac{2\left(A\,c^{2}+2Bbc\right)x^{\frac{17}{2}}}{17}+\frac{2\left(2Abc+B\,b^{2}\right)x^{\frac{13}{2}}}{13}+\frac{2A\,b^{2}x^{\frac{9}{2}}}{9}$	52
gosper	$\frac{2x^{\frac{9}{2}}\left(663B\,c^{2}x^{6}+819A\,c^{2}x^{4}+1638x^{4}Bbc+2142Abc\,x^{2}+1071b^{2}B\,x^{2}+1547b^{2}A\right)}{13923}$	56
trager	$\frac{2x^{\frac{9}{2}}\left(663B\,c^{2}x^{6}+819A\,c^{2}x^{4}+1638x^{4}Bbc+2142Abc\,x^{2}+1071b^{2}B\,x^{2}+1547b^{2}A\right)}{13923}$	56
risch	$\frac{2x^{\frac{9}{2}}\left(663B\,c^{2}x^{6}+819A\,c^{2}x^{4}+1638x^{4}Bbc+2142Abc\,x^{2}+1071b^{2}B\,x^{2}+1547b^{2}A\right)}{13923}$	56

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/21*B*c^2*x^(21/2)+2/17*(A*c^2+2*B*b*c)*x^(17/2)+2/13*(2*A*b*c+B*b^2)*x^(1
3/2)+2/9*A*b^2*x^(9/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{\sqrt{x}}\,dx = \frac{2}{13923}\left(663\,Bc^2x^{10}+819\,\left(2\,Bbc+Ac^2\right)x^8+1547\,Ab^2x^4+1071\,\left(Bb^2+2\,Abc\right)x^6\right)\sqrt{x}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="fricas")
[Out] 2/13923*(663*B*c^2*x^10 + 819*(2*B*b*c + A*c^2)*x^8 + 1547*A*b^2*x^4 + 1071
*(B*b^2 + 2*A*b*c)*x^6)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{\sqrt{x}}\,dx = \frac{2Ab^2x^{\frac{9}{2}}}{9}+\frac{4Abcx^{\frac{13}{2}}}{13}+\frac{2Ac^2x^{\frac{17}{2}}}{17}+\frac{2Bb^2x^{\frac{13}{2}}}{13}+\frac{4Bbcx^{\frac{17}{2}}}{17}+\frac{2Bc^2x^{\frac{21}{2}}}{21}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**(1/2),x)
[Out] 2*A*b**2*x**(9/2)/9 + 4*A*b*c*x**(13/2)/13 + 2*A*c**2*x**(17/2)/17 + 2*B*b*
*2*x**(13/2)/13 + 4*B*b*c*x**(17/2)/17 + 2*B*c**2*x**(21/2)/21
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{\sqrt{x}}\,dx = \frac{2}{21}\,Bc^2x^{\frac{21}{2}} + \frac{2}{17}\,(2\,Bbc + Ac^2)x^{\frac{17}{2}} + \frac{2}{9}\,Ab^2x^{\frac{9}{2}} + \frac{2}{13}\,(Bb^2 + 2\,Abc)x^{\frac{13}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*B*c^2*x^(21/2) + 2/17*(2*B*b*c + A*c^2)*x^(17/2) + 2/9*A*b^2*x^(9/2) +
 2/13*(B*b^2 + 2*A*b*c)*x^(13/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{\sqrt{x}}\,dx = \frac{2}{21}\,Bc^2x^{\frac{21}{2}} + \frac{4}{17}\,Bbcx^{\frac{17}{2}} + \frac{2}{17}\,Ac^2x^{\frac{17}{2}} + \frac{2}{13}\,Bb^2x^{\frac{13}{2}} + \frac{4}{13}\,Abcx^{\frac{13}{2}} + \frac{2}{9}\,Ab^2x^{\frac{9}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/21*B*c^2*x^(21/2) + 4/17*B*b*c*x^(17/2) + 2/17*A*c^2*x^(17/2) + 2/13*B*b^
2*x^(13/2) + 4/13*A*b*c*x^(13/2) + 2/9*A*b^2*x^(9/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{\sqrt{x}}\,dx = x^{13/2}\left(\frac{2\,B\,b^2}{13} + \frac{4\,A\,c\,b}{13}\right) + x^{17/2}\left(\frac{2\,A\,c^2}{17} + \frac{4\,B\,b\,c}{17}\right) + \frac{2\,A\,b^2\,x^{9/2}}{9} + \frac{2\,B\,c^2\,x^{21/2}}{21}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(1/2),x)
```

```
[Out] x^(13/2)*((2*B*b^2)/13 + (4*A*b*c)/13) + x^(17/2)*((2*A*c^2)/17 + (4*B*b*c)
/17) + (2*A*b^2*x^(9/2))/9 + (2*B*c^2*x^(21/2))/21
```

3.172 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{3/2}} dx$

Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{3/2}} dx = \frac{2}{7}Ab^2x^{7/2} + \frac{2}{11}b(bB+2Ac)x^{11/2} + \frac{2}{15}c(2bB+Ac)x^{15/2} + \frac{2}{19}Bc^2x^{19/2}$$

```
[Out] 2/7*A*b^2*x^(7/2)+2/11*b*(2*A*c+B*b)*x^(11/2)+2/15*c*(A*c+2*B*b)*x^(15/2)+2
/19*B*c^2*x^(19/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{3/2}} dx = \frac{2}{7}Ab^2x^{7/2} + \frac{2}{15}cx^{15/2}(Ac+2bB) + \frac{2}{11}bx^{11/2}(2Ac+bB) + \frac{2}{19}Bc^2x^{19/2}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(3/2),x]
```

```
[Out] (2*A*b^2*x^(7/2))/7 + (2*b*(b*B + 2*A*c)*x^(11/2))/11 + (2*c*(2*b*B + A*c)*
x^(15/2))/15 + (2*B*c^2*x^(19/2))/19
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{5/2}\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx\\&= \int \left(Ab^2x^{5/2}+b(bB+2Ac)x^{9/2}+c(2bB+Ac)x^{13/2}+Bc^2x^{17/2}\right)\,dx\\&= \frac{2}{7}Ab^2x^{7/2}+\frac{2}{11}b(bB+2Ac)x^{11/2}+\frac{2}{15}c(2bB+Ac)x^{15/2}+\frac{2}{19}Bc^2x^{19/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2}{x^{3/2}}\,dx = \frac{2x^{7/2}(19A(165b^2+210bcx^2+77c^2x^4)+7Bx^2(285b^2+418bcx^2+165c^2x^4)}{21945}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(3/2),x]
[Out] (2*x^(7/2)*(19*A*(165*b^2 + 210*b*c*x^2 + 77*c^2*x^4) + 7*B*x^2*(285*b^2 +
418*b*c*x^2 + 165*c^2*x^4)))/21945
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2Bc^2x^{\frac{19}{2}}}{19}+\frac{2(Ac^2+2Bbc)x^{\frac{15}{2}}}{15}+\frac{2(2Abc+Bb^2)x^{\frac{11}{2}}}{11}+\frac{2Ab^2x^{\frac{7}{2}}}{7}$	52
default	$\frac{2Bc^2x^{\frac{19}{2}}}{19}+\frac{2(Ac^2+2Bbc)x^{\frac{15}{2}}}{15}+\frac{2(2Abc+Bb^2)x^{\frac{11}{2}}}{11}+\frac{2Ab^2x^{\frac{7}{2}}}{7}$	52
gosper	$\frac{2x^{\frac{7}{2}}(1155Bc^2x^6+1463Ac^2x^4+2926x^4Bbc+3990Abcx^2+1995b^2Bx^2+3135b^2A)}{21945}$	56
trager	$\frac{2x^{\frac{7}{2}}(1155Bc^2x^6+1463Ac^2x^4+2926x^4Bbc+3990Abcx^2+1995b^2Bx^2+3135b^2A)}{21945}$	56
risch	$\frac{2x^{\frac{7}{2}}(1155Bc^2x^6+1463Ac^2x^4+2926x^4Bbc+3990Abcx^2+1995b^2Bx^2+3135b^2A)}{21945}$	56

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/19*B*c^2*x^(19/2)+2/15*(A*c^2+2*B*b*c)*x^(15/2)+2/11*(2*A*b*c+B*b^2)*x^(1
1/2)+2/7*A*b^2*x^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^{3/2}}\,dx = \frac{2}{21945}\,\big(1155\,Bc^2x^9 + 1463\,\big(2\,Bbc + Ac^2\big)x^7 + 3135\,Ab^2x^3 + 1995\,\big(Bb^2 + 2\,A$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*B*c^2*x^9 + 1463*(2*B*b*c + A*c^2)*x^7 + 3135*A*b^2*x^3 + 199
5*(B*b^2 + 2*A*b*c)*x^5)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^{3/2}}\,dx = \frac{2Ab^2x^{\frac{7}{2}}}{7} + \frac{4Abcx^{\frac{11}{2}}}{11} + \frac{2Ac^2x^{\frac{15}{2}}}{15} + \frac{2Bb^2x^{\frac{11}{2}}}{11} + \frac{4Bbcx^{\frac{15}{2}}}{15} + \frac{2Bc^2x^{\frac{19}{2}}}{19}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**(3/2),x)
```

```
[Out] 2*A*b**2*x**(7/2)/7 + 4*A*b*c*x**(11/2)/11 + 2*A*c**2*x**(15/2)/15 + 2*B*b*
*2*x**(11/2)/11 + 4*B*b*c*x**(15/2)/15 + 2*B*c**2*x**(19/2)/19
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^{3/2}}\,dx = \frac{2}{19}\,Bc^2x^{\frac{19}{2}} + \frac{2}{15}\,\big(2\,Bbc + Ac^2\big)x^{\frac{15}{2}} + \frac{2}{7}\,Ab^2x^{\frac{7}{2}} + \frac{2}{11}\,\big(Bb^2 + 2\,Abc\big)x^{\frac{11}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/19*B*c^2*x^(19/2) + 2/15*(2*B*b*c + A*c^2)*x^(15/2) + 2/7*A*b^2*x^(7/2) +
 2/11*(B*b^2 + 2*A*b*c)*x^(11/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{3/2}}\,dx = \frac{2}{19}\,Bc^2x^{\frac{19}{2}} + \frac{4}{15}\,Bbcx^{\frac{15}{2}} + \frac{2}{15}\,Ac^2x^{\frac{15}{2}} + \frac{2}{11}\,Bb^2x^{\frac{11}{2}} + \frac{4}{11}\,Abcx^{\frac{11}{2}} + \frac{2}{7}\,Ab^2x^{\frac{7}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/19*B*c^2*x^(19/2) + 4/15*B*b*c*x^(15/2) + 2/15*A*c^2*x^(15/2) + 2/11*B*b^
2*x^(11/2) + 4/11*A*b*c*x^(11/2) + 2/7*A*b^2*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{3/2}}\,dx = x^{11/2}\left(\frac{2\,B\,b^2}{11} + \frac{4\,A\,c\,b}{11}\right) + x^{15/2}\left(\frac{2\,A\,c^2}{15} + \frac{4\,B\,b\,c}{15}\right) + \frac{2\,A\,b^2\,x^{7/2}}{7} + \frac{2\,B\,c^2\,x^{19/2}}{19}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(3/2),x)
```

```
[Out] x^(11/2)*((2*B*b^2)/11 + (4*A*b*c)/11) + x^(15/2)*((2*A*c^2)/15 + (4*B*b*c)
/15) + (2*A*b^2*x^(7/2))/7 + (2*B*c^2*x^(19/2))/19
```

3.173 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{5/2}}\,dx = \frac{2}{5}Ab^2x^{5/2} + \frac{2}{9}b(bB+2Ac)x^{9/2} + \frac{2}{13}c(2bB+Ac)x^{13/2} + \frac{2}{17}Bc^2x^{17/2}$$

```
[Out] 2/5*A*b^2*x^(5/2)+2/9*b*(2*A*c+B*b)*x^(9/2)+2/13*c*(A*c+2*B*b)*x^(13/2)+2/1
7*B*c^2*x^(17/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{5/2}}\,dx = \frac{2}{5}Ab^2x^{5/2} + \frac{2}{13}cx^{13/2}(Ac+2bB) + \frac{2}{9}bx^{9/2}(2Ac+bB) + \frac{2}{17}Bc^2x^{17/2}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(5/2),x]
```

```
[Out] (2*A*b^2*x^(5/2))/5 + (2*b*(b*B + 2*A*c)*x^(9/2))/9 + (2*c*(2*b*B + A*c)*x^
(13/2))/13 + (2*B*c^2*x^(17/2))/17
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{3/2}\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx\\ &= \int \left(Ab^2x^{3/2}+b(bB+2Ac)x^{7/2}+c(2bB+Ac)x^{11/2}+Bc^2x^{15/2}\right)\,dx\\ &= \frac{2}{5}Ab^2x^{5/2}+\frac{2}{9}b(bB+2Ac)x^{9/2}+\frac{2}{13}c(2bB+Ac)x^{13/2}+\frac{2}{17}Bc^2x^{17/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^{5/2}}\,dx = \frac{2x^{5/2}(17A(117b^2+130bcx^2+45c^2x^4)+5Bx^2(221b^2+306bcx^2+117c^2x^4)}{9945}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(5/2),x]
[Out] (2*x^(5/2)*(17*A*(117*b^2 + 130*b*c*x^2 + 45*c^2*x^4) + 5*B*x^2*(221*b^2 +
306*b*c*x^2 + 117*c^2*x^4)))/9945
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2Bc^2x^{\frac{17}{2}}}{17}+\frac{2(Ac^2+2Bbc)x^{\frac{13}{2}}}{13}+\frac{2(2Abc+Bb^2)x^{\frac{9}{2}}}{9}+\frac{2Ab^2x^{\frac{5}{2}}}{5}$	52
default	$\frac{2Bc^2x^{\frac{17}{2}}}{17}+\frac{2(Ac^2+2Bbc)x^{\frac{13}{2}}}{13}+\frac{2(2Abc+Bb^2)x^{\frac{9}{2}}}{9}+\frac{2Ab^2x^{\frac{5}{2}}}{5}$	52
gosper	$\frac{2x^{\frac{5}{2}}(585Bc^2x^6+765Ac^2x^4+1530x^4Bbc+2210Abcx^2+1105b^2Bx^2+1989b^2A)}{9945}$	56
trager	$\frac{2x^{\frac{5}{2}}(585Bc^2x^6+765Ac^2x^4+1530x^4Bbc+2210Abcx^2+1105b^2Bx^2+1989b^2A)}{9945}$	56
risch	$\frac{2x^{\frac{5}{2}}(585Bc^2x^6+765Ac^2x^4+1530x^4Bbc+2210Abcx^2+1105b^2Bx^2+1989b^2A)}{9945}$	56

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/17*B*c^2*x^(17/2)+2/13*(A*c^2+2*B*b*c)*x^(13/2)+2/9*(2*A*b*c+B*b^2)*x^(9/
2)+2/5*A*b^2*x^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{5/2}}\,dx = \frac{2}{9945}\left(585\,Bc^2x^8 + 765\left(2\,Bbc + Ac^2\right)x^6 + 1989\,Ab^2x^2 + 1105\left(Bb^2 + 2\,Abc\right)\right.$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*B*c^2*x^8 + 765*(2*B*b*c + A*c^2)*x^6 + 1989*A*b^2*x^2 + 1105*(
B*b^2 + 2*A*b*c)*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{5/2}}\,dx = \frac{2Ab^2x^{\frac{5}{2}}}{5} + \frac{4Abcx^{\frac{9}{2}}}{9} + \frac{2Ac^2x^{\frac{13}{2}}}{13} + \frac{2Bb^2x^{\frac{9}{2}}}{9} + \frac{4Bbcx^{\frac{13}{2}}}{13} + \frac{2Bc^2x^{\frac{17}{2}}}{17}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**(5/2),x)
```

```
[Out] 2*A*b**2*x**(5/2)/5 + 4*A*b*c*x**(9/2)/9 + 2*A*c**2*x**(13/2)/13 + 2*B*b**2
*x**(9/2)/9 + 4*B*b*c*x**(13/2)/13 + 2*B*c**2*x**(17/2)/17
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{5/2}}\,dx = \frac{2}{17}\,Bc^2x^{\frac{17}{2}} + \frac{2}{13}\left(2\,Bbc + Ac^2\right)x^{\frac{13}{2}} + \frac{2}{5}\,Ab^2x^{\frac{5}{2}} + \frac{2}{9}\left(Bb^2 + 2\,Abc\right)x^{\frac{9}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/17*B*c^2*x^(17/2) + 2/13*(2*B*b*c + A*c^2)*x^(13/2) + 2/5*A*b^2*x^(5/2) +
 2/9*(B*b^2 + 2*A*b*c)*x^(9/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^{5/2}}\,dx = \frac{2}{17}\,Bc^2x^{\frac{17}{2}}+\frac{4}{13}\,Bbcx^{\frac{13}{2}}+\frac{2}{13}\,Ac^2x^{\frac{13}{2}}+\frac{2}{9}\,Bb^2x^{\frac{9}{2}}+\frac{4}{9}\,Abcx^{\frac{9}{2}}+\frac{2}{5}\,Ab^2x^{\frac{5}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/17*B*c^2*x^(17/2) + 4/13*B*b*c*x^(13/2) + 2/13*A*c^2*x^(13/2) + 2/9*B*b^2
*x^(9/2) + 4/9*A*b*c*x^(9/2) + 2/5*A*b^2*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^2}{x^{5/2}}\,dx = x^{9/2}\left(\frac{2\,B\,b^2}{9}+\frac{4\,A\,c\,b}{9}\right)+x^{13/2}\left(\frac{2\,A\,c^2}{13}+\frac{4\,B\,b\,c}{13}\right)+\frac{2\,A\,b^2\,x^{5/2}}{5}+\frac{2\,B\,c^2\,x^{17/2}}{17}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(5/2),x)
```

```
[Out] x^(9/2)*((2*B*b^2)/9 + (4*A*b*c)/9) + x^(13/2)*((2*A*c^2)/13 + (4*B*b*c)/13
) + (2*A*b^2*x^(5/2))/5 + (2*B*c^2*x^(17/2))/17
```

3.174 $\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{7/2}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 63

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{7/2}} \, dx = \frac{2}{3}Ab^2x^{3/2} + \frac{2}{7}b(bB+2Ac)x^{7/2} + \frac{2}{11}c(2bB+Ac)x^{11/2} + \frac{2}{15}Bc^2x^{15/2}$$

```
[Out] 2/3*A*b^2*x^(3/2)+2/7*b*(2*A*c+B*b)*x^(7/2)+2/11*c*(A*c+2*B*b)*x^(11/2)+2/1
5*B*c^2*x^(15/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{7/2}} \, dx = \frac{2}{3}Ab^2x^{3/2} + \frac{2}{11}cx^{11/2}(Ac+2bB) + \frac{2}{7}bx^{7/2}(2Ac+bB) + \frac{2}{15}Bc^2x^{15/2}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(7/2),x]
```

```
[Out] (2*A*b^2*x^(3/2))/3 + (2*b*(b*B + 2*A*c)*x^(7/2))/7 + (2*c*(2*b*B + A*c)*x^
(11/2))/11 + (2*B*c^2*x^(15/2))/15
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \sqrt{x}\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx\\ &= \int \left(Ab^2\sqrt{x}+b(bB+2Ac)x^{5/2}+c(2bB+Ac)x^{9/2}+Bc^2x^{13/2}\right)\,dx\\ &= \frac{2}{3}Ab^2x^{3/2}+\frac{2}{7}b(bB+2Ac)x^{7/2}+\frac{2}{11}c(2bB+Ac)x^{11/2}+\frac{2}{15}Bc^2x^{15/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^{7/2}}\,dx = \frac{2x^{3/2}(5A(77b^2+66bcx^2+21c^2x^4)+Bx^2(165b^2+210bcx^2+77c^2x^4))}{1155}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(7/2),x]
[Out] (2*x^(3/2)*(5*A*(77*b^2 + 66*b*c*x^2 + 21*c^2*x^4) + B*x^2*(165*b^2 + 210*b
*c*x^2 + 77*c^2*x^4)))/1155
```

Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2Bc^2x^{\frac{15}{2}}}{15}+\frac{2(Ac^2+2Bbc)x^{\frac{11}{2}}}{11}+\frac{2(2Abc+Bb^2)x^{\frac{7}{2}}}{7}+\frac{2Ab^2x^{\frac{3}{2}}}{3}$	52
default	$\frac{2Bc^2x^{\frac{15}{2}}}{15}+\frac{2(Ac^2+2Bbc)x^{\frac{11}{2}}}{11}+\frac{2(2Abc+Bb^2)x^{\frac{7}{2}}}{7}+\frac{2Ab^2x^{\frac{3}{2}}}{3}$	52
gosper	$\frac{2x^{\frac{3}{2}}(77Bc^2x^6+105Ac^2x^4+210x^4Bbc+330Abcx^2+165b^2Bx^2+385b^2A)}{1155}$	56
trager	$\frac{2x^{\frac{3}{2}}(77Bc^2x^6+105Ac^2x^4+210x^4Bbc+330Abcx^2+165b^2Bx^2+385b^2A)}{1155}$	56
risch	$\frac{2x^{\frac{3}{2}}(77Bc^2x^6+105Ac^2x^4+210x^4Bbc+330Abcx^2+165b^2Bx^2+385b^2A)}{1155}$	56

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^2/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*B*c^2*x^(15/2)+2/11*(A*c^2+2*B*b*c)*x^(11/2)+2/7*(2*A*b*c+B*b^2)*x^(7/
2)+2/3*A*b^2*x^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{7/2}}\,dx = \frac{2}{1155}\left(77\,Bc^2x^7+105\left(2\,Bbc+Ac^2\right)x^5+385\,Ab^2x+165\left(Bb^2+2\,Abc\right)x^3\right)\sqrt{x}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*B*c^2*x^7 + 105*(2*B*b*c + A*c^2)*x^5 + 385*A*b^2*x + 165*(B*b^2
 + 2*A*b*c)*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{7/2}}\,dx = \frac{2Ab^2x^{\frac{3}{2}}}{3}+\frac{4Abcx^{\frac{7}{2}}}{7}+\frac{2Ac^2x^{\frac{11}{2}}}{11}+\frac{2Bb^2x^{\frac{7}{2}}}{7}+\frac{4Bbcx^{\frac{11}{2}}}{11}+\frac{2Bc^2x^{\frac{15}{2}}}{15}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**2/x**(7/2),x)
```

```
[Out] 2*A*b**2*x**(3/2)/3 + 4*A*b*c*x**(7/2)/7 + 2*A*c**2*x**(11/2)/11 + 2*B*b**2
*x**(7/2)/7 + 4*B*b*c*x**(11/2)/11 + 2*B*c**2*x**(15/2)/15
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^2}{x^{7/2}}\,dx = \frac{2}{15}\,Bc^2x^{\frac{15}{2}}+\frac{2}{11}\left(2\,Bbc+Ac^2\right)x^{\frac{11}{2}}+\frac{2}{3}\,Ab^2x^{\frac{3}{2}}+\frac{2}{7}\left(Bb^2+2\,Abc\right)x^{\frac{7}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*c^2*x^(15/2) + 2/11*(2*B*b*c + A*c^2)*x^(11/2) + 2/3*A*b^2*x^(3/2) +
 2/7*(B*b^2 + 2*A*b*c)*x^(7/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^{7/2}}\,dx = \frac{2}{15}\,Bc^2x^{\frac{15}{2}} + \frac{4}{11}\,Bbcx^{\frac{11}{2}} + \frac{2}{11}\,Ac^2x^{\frac{11}{2}} + \frac{2}{7}\,Bb^2x^{\frac{7}{2}} + \frac{4}{7}\,Abcx^{\frac{7}{2}} + \frac{2}{3}\,Ab^2x^{\frac{3}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^2/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*B*c^2*x^(15/2) + 4/11*B*b*c*x^(11/2) + 2/11*A*c^2*x^(11/2) + 2/7*B*b^2
*x^(7/2) + 4/7*A*b*c*x^(7/2) + 2/3*A*b^2*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^2}{x^{7/2}}\,dx = x^{7/2}\left(\frac{2\,B\,b^2}{7}+\frac{4\,A\,c\,b}{7}\right) + x^{11/2}\left(\frac{2\,A\,c^2}{11}+\frac{4\,B\,b\,c}{11}\right) + \frac{2\,A\,b^2\,x^{3/2}}{3} + \frac{2\,B\,c^2\,x^{15/2}}{15}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^2)/x^(7/2),x)
```

```
[Out] x^(7/2)*((2*B*b^2)/7 + (4*A*b*c)/7) + x^(11/2)*((2*A*c^2)/11 + (4*B*b*c)/11
) + (2*A*b^2*x^(3/2))/3 + (2*B*c^2*x^(15/2))/15
```

3.175 $\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx$

Optimal result

Integrand size = 26, antiderivative size = 85

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{21}Ab^3x^{21/2}+\frac{2}{25}b^2(bB+3Ac)x^{25/2}+\frac{6}{29}bc(bB+Ac)x^{29/2}+\frac{2}{33}c^2(3bB+Ac)x^{33/2}+\frac{2}{37}Bc^3x^{37/2}$$

```
[Out] 2/21*A*b^3*x^(21/2)+2/25*b^2*(3*A*c+B*b)*x^(25/2)+6/29*b*c*(A*c+B*b)*x^(29/
2)+2/33*c^2*(A*c+3*B*b)*x^(33/2)+2/37*B*c^3*x^(37/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1598, 459}

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{21}Ab^3x^{21/2}+\frac{2}{25}b^2x^{25/2}(3Ac+bB)+\frac{2}{33}c^2x^{33/2}(Ac+3bB)+\frac{6}{29}bcx^{29/2}(Ac+bB)+\frac{2}{37}Bc^3x^{37/2}$$

```
[In] Int[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(21/2))/21 + (2*b^2*(b*B + 3*A*c)*x^(25/2))/25 + (6*b*c*(b*B + A
*c)*x^(29/2))/29 + (2*c^2*(3*b*B + A*c)*x^(33/2))/33 + (2*B*c^3*x^(37/2))/3
7
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
```

```
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{19/2}\left(A+Bx^2\right)\left(b+cx^2\right)^3\,dx\\ &= \int \left(Ab^3x^{19/2}+b^2(bB+3Ac)x^{23/2}+3bc(bB+Ac)x^{27/2}+c^2(3bB+Ac)x^{31/2}+Bc^3x^{35/2}\right)\,dx\\ &= \frac{2}{21}Ab^3x^{21/2}\\ &\quad+\frac{2}{25}b^2(bB+3Ac)x^{25/2}+\frac{6}{29}bc(bB+Ac)x^{29/2}+\frac{2}{33}c^2(3bB+Ac)x^{33/2}+\frac{2}{37}Bc^3x^{37/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{21}Ab^3x^{21/2}+\frac{2}{25}b^2(bB+3Ac)x^{25/2}+\frac{6}{29}bc(bB+Ac)x^{29/2}+\frac{2}{33}c^2(3bB+Ac)x^{33/2}+\frac{2}{37}Bc^3x^{37/2}$$

```
[In] Integrate[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
[Out] (2*A*b^3*x^(21/2))/21 + (2*b^2*(b*B + 3*A*c)*x^(25/2))/25 + (6*b*c*(b*B + A
*c)*x^(29/2))/29 + (2*c^2*(3*b*B + A*c)*x^(33/2))/33 + (2*B*c^3*x^(37/2))/3
7
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2B\,c^3x^{\frac{37}{2}}}{37}+\frac{2(A\,c^3+3Bb\,c^2)x^{\frac{33}{2}}}{33}+\frac{2(3Ab\,c^2+3B\,b^2c)x^{\frac{29}{2}}}{29}+\frac{2(3b^2Ac+B\,b^3)x^{\frac{25}{2}}}{25}+\frac{2A\,b^3x^{\frac{21}{2}}}{21}$
default	$\frac{2B\,c^3x^{\frac{37}{2}}}{37}+\frac{2(A\,c^3+3Bb\,c^2)x^{\frac{33}{2}}}{33}+\frac{2(3Ab\,c^2+3B\,b^2c)x^{\frac{29}{2}}}{29}+\frac{2(3b^2Ac+B\,b^3)x^{\frac{25}{2}}}{25}+\frac{2A\,b^3x^{\frac{21}{2}}}{21}$
gosper	$\frac{2x^{\frac{21}{2}}(167475B\,c^3x^8+187775A\,c^3x^6+563325x^6Bb\,c^2+641025Ab\,c^2x^4+641025x^4B\,b^2c+743589A\,b^2c\,x^2+247863b^3B\,x^2+}{6196575}$
trager	$\frac{2x^{\frac{21}{2}}(167475B\,c^3x^8+187775A\,c^3x^6+563325x^6Bb\,c^2+641025Ab\,c^2x^4+641025x^4B\,b^2c+743589A\,b^2c\,x^2+247863b^3B\,x^2+}{6196575}$
risch	$\frac{2x^{\frac{21}{2}}(167475B\,c^3x^8+187775A\,c^3x^6+563325x^6Bb\,c^2+641025Ab\,c^2x^4+641025x^4B\,b^2c+743589A\,b^2c\,x^2+247863b^3B\,x^2+}{6196575}$

```
[In] int(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/37*B*c^3*x^(37/2)+2/33*(A*c^3+3*B*b*c^2)*x^(33/2)+2/29*(3*A*b*c^2+3*B*b^2
*c)*x^(29/2)+2/25*(3*A*b^2*c+B*b^3)*x^(25/2)+2/21*A*b^3*x^(21/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{6196575}\left(167475\,Bc^3x^{18}+187775\left(3\,Bbc^2+Ac^3\right)x^{16}+641025\left(Bb^2c+Abc^2\right)x^{14}+295075\,Ab^3\right.$$

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/6196575*(167475*B*c^3*x^18 + 187775*(3*B*b*c^2 + A*c^3)*x^16 + 641025*(B*
b^2*c + A*b*c^2)*x^14 + 295075*A*b^3*x^10 + 247863*(B*b^3 + 3*A*b^2*c)*x^12
)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 3.36 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx=\frac{2Ab^3x^{\frac{21}{2}}}{21}+\frac{6Ab^2cx^{\frac{25}{2}}}{25}+\frac{6Abc^2x^{\frac{29}{2}}}{29}+\frac{2Ac^3x^{\frac{33}{2}}}{33}+\frac{2Bb^3x^{\frac{25}{2}}}{25}+\frac{6Bb^2cx^{\frac{29}{2}}}{29}+\frac{2Bbc^2x^{\frac{33}{2}}}{11}+\frac{2Bc^3x^{\frac{37}{2}}}{37}$$

```
[In] integrate(x**(7/2)*(B*x**2+A)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*A*b**3*x**(21/2)/21 + 6*A*b**2*c*x**(25/2)/25 + 6*A*b*c**2*x**(29/2)/29 +
 2*A*c**3*x**(33/2)/33 + 2*B*b**3*x**(25/2)/25 + 6*B*b**2*c*x**(29/2)/29 +
2*B*b*c**2*x**(33/2)/11 + 2*B*c**3*x**(37/2)/37
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{37}\,Bc^3x^{\frac{37}{2}}+\frac{2}{33}\,(3\,Bbc^2+Ac^3)x^{\frac{33}{2}}+\frac{6}{29}\,(Bb^2c+Abc^2)x^{\frac{29}{2}}+\frac{2}{21}\,Ab^3x^{\frac{21}{2}}+\frac{2}{25}\,(Bb^3+3\,Ab^2c)x^{\frac{25}{2}}$$

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/37*B*c^3*x^(37/2) + 2/33*(3*B*b*c^2 + A*c^3)*x^(33/2) + 6/29*(B*b^2*c + A
*b*c^2)*x^(29/2) + 2/21*A*b^3*x^(21/2) + 2/25*(B*b^3 + 3*A*b^2*c)*x^(25/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{37}\,Bc^3x^{\frac{37}{2}}+\frac{2}{11}\,Bbc^2x^{\frac{33}{2}}+\frac{2}{33}\,Ac^3x^{\frac{33}{2}}+\frac{6}{29}\,Bb^2cx^{\frac{29}{2}}+\frac{6}{29}\,Abc^2x^{\frac{29}{2}}+\frac{2}{25}\,Bb^3x^{\frac{25}{2}}+\frac{6}{25}\,Ab^2cx^{\frac{25}{2}}+\frac{2}{21}\,Ab^3x^{\frac{21}{2}}$$

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/37*B*c^3*x^(37/2) + 2/11*B*b*c^2*x^(33/2) + 2/33*A*c^3*x^(33/2) + 6/29*B*
b^2*c*x^(29/2) + 6/29*A*b*c^2*x^(29/2) + 2/25*B*b^3*x^(25/2) + 6/25*A*b^2*c
*x^(25/2) + 2/21*A*b^3*x^(21/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=x^{25/2}\left(\frac{2\,B\,b^3}{25}+\frac{6\,A\,c\,b^2}{25}\right)+x^{33/2}\left(\frac{2\,A\,c^3}{33}+\frac{2\,B\,b\,c^2}{11}\right)+\frac{2\,A\,b^3\,x^{21/2}}{21}+\frac{2\,B\,c^3\,x^{37/2}}{37}+\frac{6\,b\,c\,x^{29/2}\,(A\,c+B\,b)}{29}$$

```
[In] int(x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] x^(25/2)*((2*B*b^3)/25 + (6*A*b^2*c)/25) + x^(33/2)*((2*A*c^3)/33 + (2*B*b*
c^2)/11) + (2*A*b^3*x^(21/2))/21 + (2*B*c^3*x^(37/2))/37 + (6*b*c*x^(29/2)*
(A*c + B*b))/29
```

3.176 $\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx$

Optimal result

Integrand size = 26, antiderivative size = 85

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{19}Ab^3x^{19/2}+\frac{2}{23}b^2(bB+3Ac)x^{23/2}+\frac{2}{9}bc(bB+Ac)x^{27/2}+\frac{2}{31}c^2(3bB+Ac)x^{31/2}+\frac{2}{35}Bc^3x^{35/2}$$

```
[Out] 2/19*A*b^3*x^(19/2)+2/23*b^2*(3*A*c+B*b)*x^(23/2)+2/9*b*c*(A*c+B*b)*x^(27/2
)+2/31*c^2*(A*c+3*B*b)*x^(31/2)+2/35*B*c^3*x^(35/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1598, 459}

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{19}Ab^3x^{19/2}+\frac{2}{23}b^2x^{23/2}(3Ac+bB)+\frac{2}{31}c^2x^{31/2}(Ac+3bB)+\frac{2}{9}bcx^{27/2}(Ac+bB)+\frac{2}{35}Bc^3x^{35/2}$$

```
[In] Int[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(19/2))/19 + (2*b^2*(b*B + 3*A*c)*x^(23/2))/23 + (2*b*c*(b*B + A
*c)*x^(27/2))/9 + (2*c^2*(3*b*B + A*c)*x^(31/2))/31 + (2*B*c^3*x^(35/2))/35
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{17/2}(A+Bx^2)\left(b+cx^2\right)^3\,dx\\ &= \int \left(Ab^3x^{17/2}+b^2(bB+3Ac)x^{21/2}+3bc(bB+Ac)x^{25/2}+c^2(3bB+Ac)x^{29/2}+Bc^3x^{33/2}\right)\,dx\\ &= \frac{2}{19}Ab^3x^{19/2}+\frac{2}{23}b^2(bB+3Ac)x^{23/2}+\frac{2}{9}bc(bB+Ac)x^{27/2}+\frac{2}{31}c^2(3bB+Ac)x^{31/2}+\frac{2}{35}Bc^3x^{35/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx = \frac{2}{19}Ab^3x^{19/2} + \frac{2}{23}b^2(bB+3Ac)x^{23/2}+\frac{2}{9}bc(bB+Ac)x^{27/2}+\frac{2}{31}c^2(3bB+Ac)x^{31/2}+\frac{2}{35}Bc^3x^{35/2}$$

```
[In] Integrate[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
[Out] (2*A*b^3*x^(19/2))/19 + (2*b^2*(b*B + 3*A*c)*x^(23/2))/23 + (2*b*c*(b*B + A
*c)*x^(27/2))/9 + (2*c^2*(3*b*B + A*c)*x^(31/2))/31 + (2*B*c^3*x^(35/2))/35
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2B\,c^{3}x^{\frac{35}{2}}}{35}+\frac{2(A\,c^{3}+3Bb\,c^{2})x^{\frac{31}{2}}}{31}+\frac{2(3Ab\,c^{2}+3B\,b^{2}c)x^{\frac{27}{2}}}{27}+\frac{2(3b^{2}Ac+B\,b^{3})x^{\frac{23}{2}}}{23}+\frac{2A\,b^{3}x^{\frac{19}{2}}}{19}$
default	$\frac{2B\,c^{3}x^{\frac{35}{2}}}{35}+\frac{2(A\,c^{3}+3Bb\,c^{2})x^{\frac{31}{2}}}{31}+\frac{2(3Ab\,c^{2}+3B\,b^{2}c)x^{\frac{27}{2}}}{27}+\frac{2(3b^{2}Ac+B\,b^{3})x^{\frac{23}{2}}}{23}+\frac{2A\,b^{3}x^{\frac{19}{2}}}{19}$
gosper	$\frac{2x^{\frac{19}{2}}(121923B\,c^{3}x^{8}+137655A\,c^{3}x^{6}+412965x^{6}Bb\,c^{2}+474145Ab\,c^{2}x^{4}+474145x^{4}B\,b^{2}c+556605A\,b^{2}c\,x^{2}+185535b^{3}B\,x^{2}+}{4267305}$
trager	$\frac{2x^{\frac{19}{2}}(121923B\,c^{3}x^{8}+137655A\,c^{3}x^{6}+412965x^{6}Bb\,c^{2}+474145Ab\,c^{2}x^{4}+474145x^{4}B\,b^{2}c+556605A\,b^{2}c\,x^{2}+185535b^{3}B\,x^{2}+}{4267305}$
risch	$\frac{2x^{\frac{19}{2}}(121923B\,c^{3}x^{8}+137655A\,c^{3}x^{6}+412965x^{6}Bb\,c^{2}+474145Ab\,c^{2}x^{4}+474145x^{4}B\,b^{2}c+556605A\,b^{2}c\,x^{2}+185535b^{3}B\,x^{2}+}{4267305}$

```
[In] int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/35*B*c^3*x^(35/2)+2/31*(A*c^3+3*B*b*c^2)*x^(31/2)+2/27*(3*A*b*c^2+3*B*b^2
*c)*x^(27/2)+2/23*(3*A*b^2*c+B*b^3)*x^(23/2)+2/19*A*b^3*x^(19/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{5/2}\left(A+Bx^{2}\right)\left(bx^{2}+cx^{4}\right)^{3}\,dx=\frac{2}{4267305}\left(121923\,Bc^{3}x^{17}+137655\left(3\,Bbc^{2}+Ac^{3}\right)x^{15}+474145\left(Bb^{2}c+Abc^{2}\right)x^{13}+224595\,Ab^{3}\right.$$

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 2/4267305*(121923*B*c^3*x^17 + 137655*(3*B*b*c^2 + A*c^3)*x^15 + 474145*(B*
b^2*c + A*b*c^2)*x^13 + 224595*A*b^3*x^9 + 185535*(B*b^3 + 3*A*b^2*c)*x^11)
*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int x^{5/2}\left(A+Bx^{2}\right)\left(bx^{2}+cx^{4}\right)^{3}\,dx=\frac{2Ab^{3}x^{\frac{19}{2}}}{19}+\frac{6Ab^{2}cx^{\frac{23}{2}}}{23}+\frac{2Abc^{2}x^{\frac{27}{2}}}{9}+\frac{2Ac^{3}x^{\frac{31}{2}}}{31}+\frac{2Bb^{3}x^{\frac{23}{2}}}{23}+\frac{2Bb^{2}cx^{\frac{27}{2}}}{9}+\frac{6Bbc^{2}x^{\frac{31}{2}}}{31}+\frac{2Bc^{3}x^{\frac{35}{2}}}{35}$$

```
[In] integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*A*b**3*x**(19/2)/19 + 6*A*b**2*c*x**(23/2)/23 + 2*A*b*c**2*x**(27/2)/9 +
2*A*c**3*x**(31/2)/31 + 2*B*b**3*x**(23/2)/23 + 2*B*b**2*c*x**(27/2)/9 + 6*
B*b*c**2*x**(31/2)/31 + 2*B*c**3*x**(35/2)/35
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{35}\,Bc^3x^{\frac{35}{2}}+\frac{2}{31}\left(3\,Bbc^2+Ac^3\right)x^{\frac{31}{2}}+\frac{2}{9}\left(Bb^2c+Abc^2\right)x^{\frac{27}{2}}+\frac{2}{19}\,Ab^3x^{\frac{19}{2}}+\frac{2}{23}\left(Bb^3+3\,Ab^2c\right)x^{\frac{23}{2}}$$

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/35*B*c^3*x^(35/2) + 2/31*(3*B*b*c^2 + A*c^3)*x^(31/2) + 2/9*(B*b^2*c + A*
b*c^2)*x^(27/2) + 2/19*A*b^3*x^(19/2) + 2/23*(B*b^3 + 3*A*b^2*c)*x^(23/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{35}\,Bc^3x^{\frac{35}{2}}+\frac{6}{31}\,Bbc^2x^{\frac{31}{2}}+\frac{2}{31}\,Ac^3x^{\frac{31}{2}}+\frac{2}{9}\,Bb^2cx^{\frac{27}{2}}+\frac{2}{9}\,Abc^2x^{\frac{27}{2}}+\frac{2}{23}\,Bb^3x^{\frac{23}{2}}+\frac{6}{23}\,Ab^2cx^{\frac{23}{2}}+\frac{2}{19}\,Ab^3x^{\frac{19}{2}}$$

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/35*B*c^3*x^(35/2) + 6/31*B*b*c^2*x^(31/2) + 2/31*A*c^3*x^(31/2) + 2/9*B*b
^2*c*x^(27/2) + 2/9*A*b*c^2*x^(27/2) + 2/23*B*b^3*x^(23/2) + 6/23*A*b^2*c*x
^(23/2) + 2/19*A*b^3*x^(19/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx=x^{23/2}\left(\frac{2\,B\,b^3}{23}+\frac{6\,A\,c\,b^2}{23}\right)+x^{31/2}\left(\frac{2\,A\,c^3}{31}+\frac{6\,B\,b\,c^2}{31}\right)+\frac{2\,A\,b^3\,x^{19/2}}{19}+\frac{2\,B\,c^3\,x^{35/2}}{35}+\frac{2\,b\,c\,x^{27/2}\,\left(A\,c+B\,b\right)}{9}$$

```
[In] int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] x^(23/2)*((2*B*b^3)/23 + (6*A*b^2*c)/23) + x^(31/2)*((2*A*c^3)/31 + (6*B*b*
c^2)/31) + (2*A*b^3*x^(19/2))/19 + (2*B*c^3*x^(35/2))/35 + (2*b*c*x^(27/2)*
(A*c + B*b))/9
```

3.177 $\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx$

Optimal result

Integrand size = 26, antiderivative size = 85

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{17}Ab^3x^{17/2}+\frac{2}{21}b^2(bB+3Ac)x^{21/2}+\frac{6}{25}bc(bB+Ac)x^{25/2}+\frac{2}{29}c^2(3bB+Ac)x^{29/2}+\frac{2}{33}Bc^3x^{33/2}$$

```
[Out] 2/17*A*b^3*x^(17/2)+2/21*b^2*(3*A*c+B*b)*x^(21/2)+6/25*b*c*(A*c+B*b)*x^(25/
2)+2/29*c^2*(A*c+3*B*b)*x^(29/2)+2/33*B*c^3*x^(33/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1598, 459}

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{17}Ab^3x^{17/2}+\frac{2}{21}b^2x^{21/2}(3Ac+bB)+\frac{2}{29}c^2x^{29/2}(Ac+3bB)+\frac{6}{25}bcx^{25/2}(Ac+bB)+\frac{2}{33}Bc^3x^{33/2}$$

```
[In] Int[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(17/2))/17 + (2*b^2*(b*B + 3*A*c)*x^(21/2))/21 + (6*b*c*(b*B + A
*c)*x^(25/2))/25 + (2*c^2*(3*b*B + A*c)*x^(29/2))/29 + (2*B*c^3*x^(33/2))/3
3
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
```

```
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^{15/2}\left(A+Bx^2\right)\left(b+cx^2\right)^3\,dx\\
&= \int \left(Ab^3x^{15/2}+b^2(bB+3Ac)x^{19/2}+3bc(bB+Ac)x^{23/2}+c^2(3bB+Ac)x^{27/2}+Bc^3x^{31/2}\right)\,dx\\
&= \frac{2}{17}Ab^3x^{17/2}\\
&\quad+\frac{2}{21}b^2(bB+3Ac)x^{21/2}+\frac{6}{25}bc(bB+Ac)x^{25/2}+\frac{2}{29}c^2(3bB+Ac)x^{29/2}+\frac{2}{33}Bc^3x^{33/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.14

$$\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx=\frac{2\left(167475Ab^3x^{17/2}+135575b^3Bx^{21/2}+406725Ab^2cx^{21/2}+341649b^2Bcx^{25/2}+341649Abc^2x^{25/}\right.}{2847075}$$

```
[In] Integrate[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*(167475*A*b^3*x^(17/2) + 135575*b^3*B*x^(21/2) + 406725*A*b^2*c*x^(21/2)
 + 341649*b^2*B*c*x^(25/2) + 341649*A*b*c^2*x^(25/2) + 294525*b*B*c^2*x^(29
/2) + 98175*A*c^3*x^(29/2) + 86275*B*c^3*x^(33/2)))/2847075
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2B\,c^{3}x^{\frac{33}{2}}}{33}+\frac{2(A\,c^{3}+3Bb\,c^{2})x^{\frac{29}{2}}}{29}+\frac{2(3Ab\,c^{2}+3B\,b^{2}c)x^{\frac{25}{2}}}{25}+\frac{2(3b^{2}Ac+B\,b^{3})x^{\frac{21}{2}}}{21}+\frac{2A\,b^{3}x^{\frac{17}{2}}}{17}$
default	$\frac{2B\,c^{3}x^{\frac{33}{2}}}{33}+\frac{2(A\,c^{3}+3Bb\,c^{2})x^{\frac{29}{2}}}{29}+\frac{2(3Ab\,c^{2}+3B\,b^{2}c)x^{\frac{25}{2}}}{25}+\frac{2(3b^{2}Ac+B\,b^{3})x^{\frac{21}{2}}}{21}+\frac{2A\,b^{3}x^{\frac{17}{2}}}{17}$
gosper	$\frac{2x^{\frac{17}{2}}(86275B\,c^{3}x^{8}+98175A\,c^{3}x^{6}+294525x^{6}Bb\,c^{2}+341649Ab\,c^{2}x^{4}+341649x^{4}B\,b^{2}c+406725A\,b^{2}c\,x^{2}+135575b^{3}B\,x^{2}+16}{2847075}$
trager	$\frac{2x^{\frac{17}{2}}(86275B\,c^{3}x^{8}+98175A\,c^{3}x^{6}+294525x^{6}Bb\,c^{2}+341649Ab\,c^{2}x^{4}+341649x^{4}B\,b^{2}c+406725A\,b^{2}c\,x^{2}+135575b^{3}B\,x^{2}+16}{2847075}$
risch	$\frac{2x^{\frac{17}{2}}(86275B\,c^{3}x^{8}+98175A\,c^{3}x^{6}+294525x^{6}Bb\,c^{2}+341649Ab\,c^{2}x^{4}+341649x^{4}B\,b^{2}c+406725A\,b^{2}c\,x^{2}+135575b^{3}B\,x^{2}+16}{2847075}$

```
[In] int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
[Out] 2/33*B*c^3*x^(33/2)+2/29*(A*c^3+3*B*b*c^2)*x^(29/2)+2/25*(3*A*b*c^2+3*B*b^2
*c)*x^(25/2)+2/21*(3*A*b^2*c+B*b^3)*x^(21/2)+2/17*A*b^3*x^(17/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{2847075}\left(86275\,Bc^3x^{16}+98175\left(3\,Bbc^2+Ac^3\right)x^{14}+341649\left(Bb^2c+Abc^2\right)x^{12}+167475\,Ab^3x^8\right.$$

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="fricas")
[Out] 2/2847075*(86275*B*c^3*x^16 + 98175*(3*B*b*c^2 + A*c^3)*x^14 + 341649*(B*b^
2*c + A*b*c^2)*x^12 + 167475*A*b^3*x^8 + 135575*(B*b^3 + 3*A*b^2*c)*x^10)*s
qrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2Ab^3x^{\frac{17}{2}}}{17}+\frac{2Ab^2cx^{\frac{21}{2}}}{7}+\frac{6Abc^2x^{\frac{25}{2}}}{25}+\frac{2Ac^3x^{\frac{29}{2}}}{29}+\frac{2Bb^3x^{\frac{21}{2}}}{21}+\frac{6Bb^2cx^{\frac{25}{2}}}{25}+\frac{6Bbc^2x^{\frac{29}{2}}}{29}+\frac{2Bc^3x^{\frac{33}{2}}}{33}$$

```
[In] integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2)**3,x)
[Out] 2*A*b**3*x**(17/2)/17 + 2*A*b**2*c*x**(21/2)/7 + 6*A*b*c**2*x**(25/2)/25 +
2*A*c**3*x**(29/2)/29 + 2*B*b**3*x**(21/2)/21 + 6*B*b**2*c*x**(25/2)/25 + 6
*B*b*c**2*x**(29/2)/29 + 2*B*c**3*x**(33/2)/33
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{33}\,Bc^3x^{\frac{33}{2}}+\frac{2}{29}\left(3\,Bbc^2+Ac^3\right)x^{\frac{29}{2}}+\frac{6}{25}\left(Bb^2c+Abc^2\right)x^{\frac{25}{2}}+\frac{2}{17}\,Ab^3x^{\frac{17}{2}}+\frac{2}{21}\left(Bb^3+3\,Ab^2c\right)x^{\frac{21}{2}}$$

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 2/33*B*c^3*x^(33/2) + 2/29*(3*B*b*c^2 + A*c^3)*x^(29/2) + 6/25*(B*b^2*c + A
*b*c^2)*x^(25/2) + 2/17*A*b^3*x^(17/2) + 2/21*(B*b^3 + 3*A*b^2*c)*x^(21/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2}{33}\,Bc^3x^{\frac{33}{2}}+\frac{6}{29}\,Bbc^2x^{\frac{29}{2}}+\frac{2}{29}\,Ac^3x^{\frac{29}{2}}+\frac{6}{25}\,Bb^2cx^{\frac{25}{2}}+\frac{6}{25}\,Abc^2x^{\frac{25}{2}}+\frac{2}{21}\,Bb^3x^{\frac{21}{2}}+\frac{2}{7}\,Ab^2cx^{\frac{21}{2}}+\frac{2}{17}\,Ab^3x^{\frac{17}{2}}$$

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/33*B*c^3*x^(33/2) + 6/29*B*b*c^2*x^(29/2) + 2/29*A*c^3*x^(29/2) + 6/25*B*
b^2*c*x^(25/2) + 6/25*A*b*c^2*x^(25/2) + 2/21*B*b^3*x^(21/2) + 2/7*A*b^2*c*
x^(21/2) + 2/17*A*b^3*x^(17/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=x^{21/2}\left(\frac{2\,B\,b^3}{21}+\frac{2\,A\,c\,b^2}{7}\right)+x^{29/2}\left(\frac{2\,A\,c^3}{29}+\frac{6\,B\,b\,c^2}{29}\right)+\frac{2\,A\,b^3\,x^{17/2}}{17}+\frac{2\,B\,c^3\,x^{33/2}}{33}+\frac{6\,b\,c\,x^{25/2}\,(A\,c+B\,b)}{25}$$

```
[In] int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] x^(21/2)*((2*B*b^3)/21 + (2*A*b^2*c)/7) + x^(29/2)*((2*A*c^3)/29 + (6*B*b*c
^2)/29) + (2*A*b^3*x^(17/2))/17 + (2*B*c^3*x^(33/2))/33 + (6*b*c*x^(25/2)*(
A*c + B*b))/25
```

3.178 $\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^3 \, dx$

Optimal result

Integrand size = 26, antiderivative size = 85

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^3 \, dx = \frac{2}{15}Ab^3x^{15/2}+\frac{2}{19}b^2(bB+3Ac)x^{19/2}+\frac{6}{23}bc(bB+Ac)x^{23/2} + \frac{2}{27}c^2(3bB+Ac)x^{27/2}+\frac{2}{31}Bc^3x^{31/2}$$

```
[Out] 2/15*A*b^3*x^(15/2)+2/19*b^2*(3*A*c+B*b)*x^(19/2)+6/23*b*c*(A*c+B*b)*x^(23/
2)+2/27*c^2*(A*c+3*B*b)*x^(27/2)+2/31*B*c^3*x^(31/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1598, 459}

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^3 \, dx$$
$$= \frac{2}{15}Ab^3x^{15/2} + \frac{2}{19}b^2x^{19/2}(3Ac+bB)+\frac{2}{27}c^2x^{27/2}(Ac+3bB)+\frac{6}{23}bcx^{23/2}(Ac+bB)+\frac{2}{31}Bc^3x^{31/2}$$

```
[In] Int[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*A*b^3*x^(15/2))/15 + (2*b^2*(b*B + 3*A*c)*x^(19/2))/19 + (6*b*c*(b*B + A
*c)*x^(23/2))/23 + (2*c^2*(3*b*B + A*c)*x^(27/2))/27 + (2*B*c^3*x^(31/2))/3
1
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{13/2}\left(A+Bx^2\right)\left(b+cx^2\right)^3\,dx\\
&= \int \left(Ab^3x^{13/2}+b^2(bB+3Ac)x^{17/2}+3bc(bB+Ac)x^{21/2}+c^2(3bB+Ac)x^{25/2}+Bc^3x^{29/2}\right)\,dx\\
&= \frac{2}{15}Ab^3x^{15/2}\\
&\quad+\frac{2}{19}b^2(bB+3Ac)x^{19/2}+\frac{6}{23}bc(bB+Ac)x^{23/2}+\frac{2}{27}c^2(3bB+Ac)x^{27/2}+\frac{2}{31}Bc^3x^{31/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx = \frac{2x^{15/2}(31A(3933b^3+9315b^2cx^2+7695bc^2x^4+2185c^3x^6)+15Bx^2(6417b^3+15903b^2cx^2+13547bc^2x^4+}{1828845}$$

```
[In] Integrate[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*x^(15/2)*(31*A*(3933*b^3 + 9315*b^2*c*x^2 + 7695*b*c^2*x^4 + 2185*c^3*x^
6) + 15*B*x^2*(6417*b^3 + 15903*b^2*c*x^2 + 13547*b*c^2*x^4 + 3933*c^3*x^6)
))/1828845
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2B\,c^3x^{\frac{31}{2}}}{31}+\frac{2(A\,c^3+3Bb\,c^2)x^{\frac{27}{2}}}{27}+\frac{2(3Ab\,c^2+3B\,b^2c)x^{\frac{23}{2}}}{23}+\frac{2(3b^2Ac+B\,b^3)x^{\frac{19}{2}}}{19}+\frac{2A\,b^3x^{\frac{15}{2}}}{15}$
default	$\frac{2B\,c^3x^{\frac{31}{2}}}{31}+\frac{2(A\,c^3+3Bb\,c^2)x^{\frac{27}{2}}}{27}+\frac{2(3Ab\,c^2+3B\,b^2c)x^{\frac{23}{2}}}{23}+\frac{2(3b^2Ac+B\,b^3)x^{\frac{19}{2}}}{19}+\frac{2A\,b^3x^{\frac{15}{2}}}{15}$
gosper	$\frac{2x^{\frac{15}{2}}(58995B\,c^3x^8+67735A\,c^3x^6+203205x^6Bb\,c^2+238545Ab\,c^2x^4+238545x^4B\,b^2c+288765A\,b^2c\,x^2+96255b^3B\,x^2+121}{1828845}$
trager	$\frac{2x^{\frac{15}{2}}(58995B\,c^3x^8+67735A\,c^3x^6+203205x^6Bb\,c^2+238545Ab\,c^2x^4+238545x^4B\,b^2c+288765A\,b^2c\,x^2+96255b^3B\,x^2+121}{1828845}$
risch	$\frac{2x^{\frac{15}{2}}(58995B\,c^3x^8+67735A\,c^3x^6+203205x^6Bb\,c^2+238545Ab\,c^2x^4+238545x^4B\,b^2c+288765A\,b^2c\,x^2+96255b^3B\,x^2+121}{1828845}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/31*B*c^3*x^(31/2)+2/27*(A*c^3+3*B*b*c^2)*x^(27/2)+2/23*(3*A*b*c^2+3*B*b^2
*c)*x^(23/2)+2/19*(3*A*b^2*c+B*b^3)*x^(19/2)+2/15*A*b^3*x^(15/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx$$
$$=\frac{2}{1828845}\left(58995\,Bc^3x^{15}+67735\left(3\,Bbc^2+Ac^3\right)x^{13}+238545\left(Bb^2c+Abc^2\right)x^{11}+121923\,Ab^3x^7+96255\,(\right.$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1828845*(58995*B*c^3*x^15 + 67735*(3*B*b*c^2 + A*c^3)*x^13 + 238545*(B*b^
2*c + A*b*c^2)*x^11 + 121923*A*b^3*x^7 + 96255*(B*b^3 + 3*A*b^2*c)*x^9)*sqr
t(x)
```

Sympy [A] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{2Ab^3x^{\frac{15}{2}}}{15}+\frac{2Bc^3x^{\frac{31}{2}}}{31}+\frac{2x^{\frac{27}{2}}(Ac^3+3Bbc^2)}{27}+\frac{2x^{\frac{23}{2}}\cdot(3Abc^2+3Bb^2c)}{23}+\frac{2x^{\frac{19}{2}}\cdot(3Ab^2c+Bb^3)}{19}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3*x**(1/2),x)
```

```
[Out] 2*A*b**3*x**(15/2)/15 + 2*B*c**3*x**(31/2)/31 + 2*x**(27/2)*(A*c**3 + 3*B*b
*c**2)/27 + 2*x**(23/2)*(3*A*b*c**2 + 3*B*b**2*c)/23 + 2*x**(19/2)*(3*A*b**
2*c + B*b**3)/19
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx = \frac{2}{31}\,Bc^3x^{\frac{31}{2}}+\frac{2}{27}\left(3\,Bbc^2+Ac^3\right)x^{\frac{27}{2}}+\frac{6}{23}\left(Bb^2c+Abc^2\right)x^{\frac{23}{2}}+\frac{2}{15}\,Ab^3x^{\frac{15}{2}}+\frac{2}{19}\left(Bb^3+3\,Ab^2c\right)x^{\frac{19}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/31*B*c^3*x^(31/2) + 2/27*(3*B*b*c^2 + A*c^3)*x^(27/2) + 6/23*(B*b^2*c + A
*b*c^2)*x^(23/2) + 2/15*A*b^3*x^(15/2) + 2/19*(B*b^3 + 3*A*b^2*c)*x^(19/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx = \frac{2}{31}\,Bc^3x^{\frac{31}{2}}+\frac{2}{9}\,Bbc^2x^{\frac{27}{2}}+\frac{2}{27}\,Ac^3x^{\frac{27}{2}}+\frac{6}{23}\,Bb^2cx^{\frac{23}{2}}+\frac{6}{23}\,Abc^2x^{\frac{23}{2}}+\frac{2}{19}\,Bb^3x^{\frac{19}{2}}+\frac{6}{19}\,Ab^2cx^{\frac{19}{2}}+\frac{2}{15}\,Ab^3x^{\frac{15}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/31*B*c^3*x^(31/2) + 2/9*B*b*c^2*x^(27/2) + 2/27*A*c^3*x^(27/2) + 6/23*B*b
^2*c*x^(23/2) + 6/23*A*b*c^2*x^(23/2) + 2/19*B*b^3*x^(19/2) + 6/19*A*b^2*c*
x^(19/2) + 2/15*A*b^3*x^(15/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \sqrt{x}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx=x^{19/2}\left(\frac{2\,B\,b^3}{19}+\frac{6\,A\,c\,b^2}{19}\right)+x^{27/2}\left(\frac{2\,A\,c^3}{27}+\frac{2\,B\,b\,c^2}{9}\right)+\frac{2\,A\,b^3\,x^{15/2}}{15}+\frac{2\,B\,c^3\,x^{31/2}}{31}+\frac{6\,b\,c\,x^{23/2}\,\left(A\,c+B\,b\right)}{23}$$

```
[In] int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] x^(19/2)*((2*B*b^3)/19 + (6*A*b^2*c)/19) + x^(27/2)*((2*A*c^3)/27 + (2*B*b*
c^2)/9) + (2*A*b^3*x^(15/2))/15 + (2*B*c^3*x^(31/2))/31 + (6*b*c*x^(23/2)*(
A*c + B*b))/23
```

3.179 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 85

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{\sqrt{x}}\,dx = \frac{2}{13}Ab^3x^{13/2}+\frac{2}{17}b^2(bB+3Ac)x^{17/2} +\frac{2}{7}bc(bB+Ac)x^{21/2}+\frac{2}{25}c^2(3bB+Ac)x^{25/2}+\frac{2}{29}Bc^3x^{29/2}$$

```
[Out] 2/13*A*b^3*x^(13/2)+2/17*b^2*(3*A*c+B*b)*x^(17/2)+2/7*b*c*(A*c+B*b)*x^(21/2
)+2/25*c^2*(A*c+3*B*b)*x^(25/2)+2/29*B*c^3*x^(29/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{\sqrt{x}}\,dx = \frac{2}{13}Ab^3x^{13/2}+\frac{2}{17}b^2x^{17/2}(3Ac+bB) +\frac{2}{25}c^2x^{25/2}(Ac+3bB)+\frac{2}{7}bcx^{21/2}(Ac+bB)+\frac{2}{29}Bc^3x^{29/2}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/Sqrt[x],x]
```

```
[Out] (2*A*b^3*x^(13/2))/13 + (2*b^2*(b*B + 3*A*c)*x^(17/2))/17 + (2*b*c*(b*B + A
*c)*x^(21/2))/7 + (2*c^2*(3*b*B + A*c)*x^(25/2))/25 + (2*B*c^3*x^(29/2))/29
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
```

```
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int x^{11/2}\left(A+Bx^2\right)\left(b+cx^2\right)^3\,dx$$

$$= \int \left(Ab^3x^{11/2}+b^2(bB+3Ac)x^{15/2}+3bc(bB+Ac)x^{19/2}+c^2(3bB+Ac)x^{23/2}+Bc^3x^{27/2}\right)\,dx$$

$$= \frac{2}{13}Ab^3x^{13/2}+\frac{2}{17}b^2(bB+3Ac)x^{17/2}+\frac{2}{7}bc(bB+Ac)x^{21/2}+\frac{2}{25}c^2(3bB+Ac)x^{25/2}+\frac{2}{29}Bc^3x^{29/2}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^3}{\sqrt{x}}\,dx = \frac{2x^{13/2}(29A(2975b^3+6825b^2cx^2+5525bc^2x^4+1547c^3x^6)+13Bx^2(5075b^3+12325b^2cx^2+10353bc^2x^4+2}{1121575}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/Sqrt[x],x]
```

```
[Out] (2*x^(13/2)*(29*A*(2975*b^3 + 6825*b^2*c*x^2 + 5525*b*c^2*x^4 + 1547*c^3*x^
6) + 13*B*x^2*(5075*b^3 + 12325*b^2*c*x^2 + 10353*b*c^2*x^4 + 2975*c^3*x^6)
))/1121575
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2B\,c^{3}x^{\frac{29}{2}}}{29}+\frac{2(A\,c^{3}+3Bb\,c^{2})x^{\frac{25}{2}}}{25}+\frac{2(3Ab\,c^{2}+3B\,b^{2}c)x^{\frac{21}{2}}}{21}+\frac{2(3b^{2}Ac+B\,b^{3})x^{\frac{17}{2}}}{17}+\frac{2A\,b^{3}x^{\frac{13}{2}}}{13}$
default	$\frac{2B\,c^{3}x^{\frac{29}{2}}}{29}+\frac{2(A\,c^{3}+3Bb\,c^{2})x^{\frac{25}{2}}}{25}+\frac{2(3Ab\,c^{2}+3B\,b^{2}c)x^{\frac{21}{2}}}{21}+\frac{2(3b^{2}Ac+B\,b^{3})x^{\frac{17}{2}}}{17}+\frac{2A\,b^{3}x^{\frac{13}{2}}}{13}$
gosper	$\frac{2x^{\frac{13}{2}}(38675B\,c^{3}x^{8}+44863A\,c^{3}x^{6}+134589x^{6}Bb\,c^{2}+160225Ab\,c^{2}x^{4}+160225x^{4}B\,b^{2}c+197925A\,b^{2}c\,x^{2}+65975b^{3}B\,x^{2}+8}{1121575}$
trager	$\frac{2x^{\frac{13}{2}}(38675B\,c^{3}x^{8}+44863A\,c^{3}x^{6}+134589x^{6}Bb\,c^{2}+160225Ab\,c^{2}x^{4}+160225x^{4}B\,b^{2}c+197925A\,b^{2}c\,x^{2}+65975b^{3}B\,x^{2}+8}{1121575}$
risch	$\frac{2x^{\frac{13}{2}}(38675B\,c^{3}x^{8}+44863A\,c^{3}x^{6}+134589x^{6}Bb\,c^{2}+160225Ab\,c^{2}x^{4}+160225x^{4}B\,b^{2}c+197925A\,b^{2}c\,x^{2}+65975b^{3}B\,x^{2}+8}{1121575}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/29*B*c^3*x^(29/2)+2/25*(A*c^3+3*B*b*c^2)*x^(25/2)+2/21*(3*A*b*c^2+3*B*b^2
*c)*x^(21/2)+2/17*(3*A*b^2*c+B*b^3)*x^(17/2)+2/13*A*b^3*x^(13/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{\sqrt{x}}\,dx$$
$$=\frac{2}{1121575}\left(38675\,Bc^3x^{14}+44863\left(3\,Bbc^2+Ac^3\right)x^{12}+160225\left(Bb^2c+Abc^2\right)x^{10}+86275\,Ab^3x^6+65975\,(\right.$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="fricas")
[Out] 2/1121575*(38675*B*c^3*x^14 + 44863*(3*B*b*c^2 + A*c^3)*x^12 + 160225*(B*b^
2*c + A*b*c^2)*x^10 + 86275*A*b^3*x^6 + 65975*(B*b^3 + 3*A*b^2*c)*x^8)*sqrt
(x)
```

Sympy [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{\sqrt{x}}\,dx=\frac{2Ab^3x^{\frac{13}{2}}}{13}+\frac{6Ab^2cx^{\frac{17}{2}}}{17}+\frac{2Abc^2x^{\frac{21}{2}}}{7}+\frac{2Ac^3x^{\frac{25}{2}}}{25}+\frac{2Bb^3x^{\frac{17}{2}}}{17}+\frac{2Bb^2cx^{\frac{21}{2}}}{7}+\frac{6Bbc^2x^{\frac{25}{2}}}{25}+\frac{2Bc^3x^{\frac{29}{2}}}{29}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**(1/2),x)
[Out] 2*A*b**3*x**(13/2)/13 + 6*A*b**2*c*x**(17/2)/17 + 2*A*b*c**2*x**(21/2)/7 +
2*A*c**3*x**(25/2)/25 + 2*B*b**3*x**(17/2)/17 + 2*B*b**2*c*x**(21/2)/7 + 6*
B*b*c**2*x**(25/2)/25 + 2*B*c**3*x**(29/2)/29
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{\sqrt{x}}\,dx = \frac{2}{29}\,Bc^3x^{\frac{29}{2}} + \frac{2}{25}\,\big(3\,Bbc^2+Ac^3\big)x^{\frac{25}{2}} + \frac{2}{7}\,\big(Bb^2c+Abc^2\big)x^{\frac{21}{2}} + \frac{2}{13}\,Ab^3x^{\frac{13}{2}} + \frac{2}{17}\,\big(Bb^3+3\,Ab^2c\big)x^{\frac{17}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/29*B*c^3*x^(29/2) + 2/25*(3*B*b*c^2 + A*c^3)*x^(25/2) + 2/7*(B*b^2*c + A*
b*c^2)*x^(21/2) + 2/13*A*b^3*x^(13/2) + 2/17*(B*b^3 + 3*A*b^2*c)*x^(17/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{\sqrt{x}}\,dx = \frac{2}{29}\,Bc^3x^{\frac{29}{2}} + \frac{6}{25}\,Bbc^2x^{\frac{25}{2}} + \frac{2}{25}\,Ac^3x^{\frac{25}{2}} + \frac{2}{7}\,Bb^2cx^{\frac{21}{2}} + \frac{2}{7}\,Abc^2x^{\frac{21}{2}} + \frac{2}{17}\,Bb^3x^{\frac{17}{2}} + \frac{6}{17}\,Ab^2cx^{\frac{17}{2}} + \frac{2}{13}\,Ab^3x^{\frac{13}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/29*B*c^3*x^(29/2) + 6/25*B*b*c^2*x^(25/2) + 2/25*A*c^3*x^(25/2) + 2/7*B*b
^2*c*x^(21/2) + 2/7*A*b*c^2*x^(21/2) + 2/17*B*b^3*x^(17/2) + 6/17*A*b^2*c*x
^(17/2) + 2/13*A*b^3*x^(13/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{\sqrt{x}}\,dx = x^{17/2}\left(\frac{2\,B\,b^3}{17}+\frac{6\,A\,c\,b^2}{17}\right) + x^{25/2}\left(\frac{2\,A\,c^3}{25}+\frac{6\,B\,b\,c^2}{25}\right) + \frac{2\,A\,b^3\,x^{13/2}}{13} + \frac{2\,B\,c^3\,x^{29/2}}{29} + \frac{2\,b\,c\,x^{21/2}\,(A\,c+B\,b)}{7}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(1/2),x)
```

```
[Out] x^(17/2)*((2*B*b^3)/17 + (6*A*b^2*c)/17) + x^(25/2)*((2*A*c^3)/25 + (6*B*b*
c^2)/25) + (2*A*b^3*x^(13/2))/13 + (2*B*c^3*x^(29/2))/29 + (2*b*c*x^(21/2)*
(A*c + B*b))/7
```

3.180 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 85

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{3/2}}\,dx = \frac{2}{11}Ab^3x^{11/2} + \frac{2}{15}b^2(bB+3Ac)x^{15/2} + \frac{6}{19}bc(bB+Ac)x^{19/2} + \frac{2}{23}c^2(3bB+Ac)x^{23/2} + \frac{2}{27}Bc^3x^{27/2}$$

```
[Out] 2/11*A*b^3*x^(11/2)+2/15*b^2*(3*A*c+B*b)*x^(15/2)+6/19*b*c*(A*c+B*b)*x^(19/
2)+2/23*c^2*(A*c+3*B*b)*x^(23/2)+2/27*B*c^3*x^(27/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{3/2}}\,dx = \frac{2}{11}Ab^3x^{11/2} + \frac{2}{15}b^2x^{15/2}(3Ac+bB) + \frac{2}{23}c^2x^{23/2}(Ac+3bB) + \frac{6}{19}bcx^{19/2}(Ac+bB) + \frac{2}{27}Bc^3x^{27/2}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(3/2),x]
```

```
[Out] (2*A*b^3*x^(11/2))/11 + (2*b^2*(b*B + 3*A*c)*x^(15/2))/15 + (6*b*c*(b*B + A
*c)*x^(19/2))/19 + (2*c^2*(3*b*B + A*c)*x^(23/2))/23 + (2*B*c^3*x^(27/2))/2
7
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
```

```
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{9/2}(A+Bx^2)\left(b+cx^2\right)^3\,dx\\ &= \int \left(Ab^3x^{9/2}+b^2(bB+3Ac)x^{13/2}+3bc(bB+Ac)x^{17/2}+c^2(3bB+Ac)x^{21/2}+Bc^3x^{25/2}\right)\,dx\\ &= \frac{2}{11}Ab^3x^{11/2}\\ &\quad+\frac{2}{15}b^2(bB+3Ac)x^{15/2}+\frac{6}{19}bc(bB+Ac)x^{19/2}+\frac{2}{23}c^2(3bB+Ac)x^{23/2}+\frac{2}{27}Bc^3x^{27/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^3}{x^{3/2}}\,dx = \frac{2x^{11/2}(27A(2185b^3+4807b^2cx^2+3795bc^2x^4+1045c^3x^6)+11Bx^2(3933b^3+}{648945}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(3/2),x]
[Out] (2*x^(11/2)*(27*A*(2185*b^3 + 4807*b^2*c*x^2 + 3795*b*c^2*x^4 + 1045*c^3*x^
6) + 11*B*x^2*(3933*b^3 + 9315*b^2*c*x^2 + 7695*b*c^2*x^4 + 2185*c^3*x^6)))
/648945
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2B\,c^3 x^{\frac{27}{2}}}{27}+\frac{2(A\,c^3+3Bb\,c^2)\,x^{\frac{23}{2}}}{23}+\frac{2(3Ab\,c^2+3B\,b^2c)\,x^{\frac{19}{2}}}{19}+\frac{2(3b^2Ac+B\,b^3)\,x^{\frac{15}{2}}}{15}+\frac{2A\,b^3x^{\frac{11}{2}}}{11}$
default	$\frac{2B\,c^3 x^{\frac{27}{2}}}{27}+\frac{2(A\,c^3+3Bb\,c^2)\,x^{\frac{23}{2}}}{23}+\frac{2(3Ab\,c^2+3B\,b^2c)\,x^{\frac{19}{2}}}{19}+\frac{2(3b^2Ac+B\,b^3)\,x^{\frac{15}{2}}}{15}+\frac{2A\,b^3x^{\frac{11}{2}}}{11}$
gosper	$\frac{2x^{\frac{11}{2}}\left(24035B\,c^3x^8+28215A\,c^3x^6+84645x^6Bb\,c^2+102465Ab\,c^2x^4+102465x^4B\,b^2c+129789A\,b^2c\,x^2+43263b^3B\,x^2+58\right.}{648945}$
trager	$\frac{2x^{\frac{11}{2}}\left(24035B\,c^3x^8+28215A\,c^3x^6+84645x^6Bb\,c^2+102465Ab\,c^2x^4+102465x^4B\,b^2c+129789A\,b^2c\,x^2+43263b^3B\,x^2+58\right.}{648945}$
risch	$\frac{2x^{\frac{11}{2}}\left(24035B\,c^3x^8+28215A\,c^3x^6+84645x^6Bb\,c^2+102465Ab\,c^2x^4+102465x^4B\,b^2c+129789A\,b^2c\,x^2+43263b^3B\,x^2+58\right.}{648945}$

[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/27*B*c^3*x^(27/2)+2/23*(A*c^3+3*B*b*c^2)*x^(23/2)+2/19*(3*A*b*c^2+3*B*b^2
*c)*x^(19/2)+2/15*(3*A*b^2*c+B*b^3)*x^(15/2)+2/11*A*b^3*x^(11/2)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^3}{x^{3/2}}\,dx=\frac{2}{648945}\left(24035\,Bc^3x^{13}+28215\left(3\,Bbc^2+Ac^3\right)x^{11}+102465\left(Bb^2c+Abc^2\right)\right.$$

[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(3/2),x, algorithm="fricas")

[Out] 2/648945*(24035*B*c^3*x^13 + 28215*(3*B*b*c^2 + A*c^3)*x^11 + 102465*(B*b^2
*c + A*b*c^2)*x^9 + 58995*A*b^3*x^5 + 43263*(B*b^3 + 3*A*b^2*c)*x^7)*sqrt(x
)

Sympy [A] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^3}{x^{3/2}}\,dx=\frac{2Ab^3x^{\frac{11}{2}}}{11}+\frac{2Ab^2cx^{\frac{15}{2}}}{5}+\frac{6Abc^2x^{\frac{19}{2}}}{19}+\frac{2Ac^3x^{\frac{23}{2}}}{23}+\frac{2Bb^3x^{\frac{15}{2}}}{15}+\frac{6Bb^2cx^{\frac{19}{2}}}{19}+\frac{6Bbc^2x^{\frac{23}{2}}}{23}+\frac{2Bc^3x^{\frac{27}{2}}}{27}$$

[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**(3/2),x)

[Out] 2*A*b**3*x**(11/2)/11 + 2*A*b**2*c*x**(15/2)/5 + 6*A*b*c**2*x**(19/2)/19 +
2*A*c**3*x**(23/2)/23 + 2*B*b**3*x**(15/2)/15 + 6*B*b**2*c*x**(19/2)/19 + 6
*B*b*c**2*x**(23/2)/23 + 2*B*c**3*x**(27/2)/27

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{3/2}}\,dx = \frac{2}{27}\,Bc^3x^{\frac{27}{2}} + \frac{2}{23}\,(3\,Bbc^2+Ac^3)x^{\frac{23}{2}} + \frac{6}{19}\,(Bb^2c+Abc^2)x^{\frac{19}{2}} + \frac{2}{11}\,Ab^3x^{\frac{11}{2}} + \frac{2}{15}\,(Bb^3+3\,Ab^2c)x^{\frac{15}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/27*B*c^3*x^(27/2) + 2/23*(3*B*b*c^2 + A*c^3)*x^(23/2) + 6/19*(B*b^2*c + A
*b*c^2)*x^(19/2) + 2/11*A*b^3*x^(11/2) + 2/15*(B*b^3 + 3*A*b^2*c)*x^(15/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{3/2}}\,dx = \frac{2}{27}\,Bc^3x^{\frac{27}{2}} + \frac{6}{23}\,Bbc^2x^{\frac{23}{2}} + \frac{2}{23}\,Ac^3x^{\frac{23}{2}} + \frac{6}{19}\,Bb^2cx^{\frac{19}{2}} + \frac{6}{19}\,Abc^2x^{\frac{19}{2}} + \frac{2}{15}\,Bb^3x^{\frac{15}{2}} + \frac{2}{5}\,Ab^2cx^{\frac{15}{2}} + \frac{2}{11}\,Ab^3x^{\frac{11}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/27*B*c^3*x^(27/2) + 6/23*B*b*c^2*x^(23/2) + 2/23*A*c^3*x^(23/2) + 6/19*B*
b^2*c*x^(19/2) + 6/19*A*b*c^2*x^(19/2) + 2/15*B*b^3*x^(15/2) + 2/5*A*b^2*c*
x^(15/2) + 2/11*A*b^3*x^(11/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{3/2}}\,dx = x^{15/2}\left(\frac{2\,B\,b^3}{15}+\frac{2\,A\,c\,b^2}{5}\right) + x^{23/2}\left(\frac{2\,A\,c^3}{23}+\frac{6\,B\,b\,c^2}{23}\right) + \frac{2\,A\,b^3\,x^{11/2}}{11} + \frac{2\,B\,c^3\,x^{27/2}}{27} + \frac{6\,b\,c\,x^{19/2}\,(A\,c+B\,b)}{19}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(3/2),x)
```

```
[Out] x^(15/2)*((2*B*b^3)/15 + (2*A*b^2*c)/5) + x^(23/2)*((2*A*c^3)/23 + (6*B*b*c
^2)/23) + (2*A*b^3*x^(11/2))/11 + (2*B*c^3*x^(27/2))/27 + (6*b*c*x^(19/2)*(
A*c + B*b))/19
```

3.181 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{5/2}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 85

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{5/2}} \, dx = \frac{2}{9}Ab^3x^{9/2} + \frac{2}{13}b^2(bB+3Ac)x^{13/2} + \frac{6}{17}bc(bB+Ac)x^{17/2} + \frac{2}{21}c^2(3bB+Ac)x^{21/2} + \frac{2}{25}Bc^3x^{25/2}$$

```
[Out] 2/9*A*b^3*x^(9/2)+2/13*b^2*(3*A*c+B*b)*x^(13/2)+6/17*b*c*(A*c+B*b)*x^(17/2)
+2/21*c^2*(A*c+3*B*b)*x^(21/2)+2/25*B*c^3*x^(25/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{5/2}} \, dx = \frac{2}{9}Ab^3x^{9/2} + \frac{2}{13}b^2x^{13/2}(3Ac+bB) + \frac{2}{21}c^2x^{21/2}(Ac+3bB) + \frac{6}{17}bcx^{17/2}(Ac+bB) + \frac{2}{25}Bc^3x^{25/2}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(5/2),x]
```

```
[Out] (2*A*b^3*x^(9/2))/9 + (2*b^2*(b*B + 3*A*c)*x^(13/2))/13 + (6*b*c*(b*B + A*c
)*x^(17/2))/17 + (2*c^2*(3*b*B + A*c)*x^(21/2))/21 + (2*B*c^3*x^(25/2))/25
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
```

```
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int x^{7/2}\left(A+Bx^2\right)\left(b+cx^2\right)^3\,dx$$

$$= \int \left(Ab^3x^{7/2}+b^2(bB+3Ac)x^{11/2}+3bc(bB+Ac)x^{15/2}+c^2(3bB+Ac)x^{19/2}+Bc^3x^{23/2}\right)\,dx$$

$$= \frac{2}{9}Ab^3x^{9/2}+\frac{2}{13}b^2(bB+3Ac)x^{13/2}+\frac{6}{17}bc(bB+Ac)x^{17/2}+\frac{2}{21}c^2(3bB+Ac)x^{21/2}+\frac{2}{25}Bc^3x^{25/2}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3}{x^{5/2}}\,dx = \frac{2x^{9/2}(25A(1547b^3+3213b^2cx^2+2457bc^2x^4+663c^3x^6)+9Bx^2(2975b^3+68$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(5/2),x]
[Out] (2*x^(9/2)*(25*A*(1547*b^3 + 3213*b^2*c*x^2 + 2457*b*c^2*x^4 + 663*c^3*x^6)
 + 9*B*x^2*(2975*b^3 + 6825*b^2*c*x^2 + 5525*b*c^2*x^4 + 1547*c^3*x^6)))/34
8075
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2B\,c^3x^{\frac{25}{2}}}{25}+\frac{2(A\,c^3+3Bb\,c^2)x^{\frac{21}{2}}}{21}+\frac{2(3Ab\,c^2+3B\,b^2c)x^{\frac{17}{2}}}{17}+\frac{2(3b^2Ac+B\,b^3)x^{\frac{13}{2}}}{13}+\frac{2A\,b^3x^{\frac{9}{2}}}{9}$
default	$\frac{2B\,c^3x^{\frac{25}{2}}}{25}+\frac{2(A\,c^3+3Bb\,c^2)x^{\frac{21}{2}}}{21}+\frac{2(3Ab\,c^2+3B\,b^2c)x^{\frac{17}{2}}}{17}+\frac{2(3b^2Ac+B\,b^3)x^{\frac{13}{2}}}{13}+\frac{2A\,b^3x^{\frac{9}{2}}}{9}$
gosper	$\frac{2x^{\frac{9}{2}}(13923B\,c^3x^8+16575A\,c^3x^6+49725x^6Bb\,c^2+61425Ab\,c^2x^4+61425x^4B\,b^2c+80325A\,b^2c\,x^2+26775b^3B\,x^2+38675b}{348075}$
trager	$\frac{2x^{\frac{9}{2}}(13923B\,c^3x^8+16575A\,c^3x^6+49725x^6Bb\,c^2+61425Ab\,c^2x^4+61425x^4B\,b^2c+80325A\,b^2c\,x^2+26775b^3B\,x^2+38675b}{348075}$
risch	$\frac{2x^{\frac{9}{2}}(13923B\,c^3x^8+16575A\,c^3x^6+49725x^6Bb\,c^2+61425Ab\,c^2x^4+61425x^4B\,b^2c+80325A\,b^2c\,x^2+26775b^3B\,x^2+38675b}{348075}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/25*B*c^3*x^(25/2)+2/21*(A*c^3+3*B*b*c^2)*x^(21/2)+2/17*(3*A*b*c^2+3*B*b^2
*c)*x^(17/2)+2/13*(3*A*b^2*c+B*b^3)*x^(13/2)+2/9*A*b^3*x^(9/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^{5/2}}\,dx = \frac{2}{348075}\left(13923\,Bc^3x^{12}+16575\left(3\,Bbc^2+Ac^3\right)x^{10}+61425\left(Bb^2c+Abc^2\right)x\right.$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/348075*(13923*B*c^3*x^12 + 16575*(3*B*b*c^2 + A*c^3)*x^10 + 61425*(B*b^2*
c + A*b*c^2)*x^8 + 38675*A*b^3*x^4 + 26775*(B*b^3 + 3*A*b^2*c)*x^6)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^{5/2}}\,dx = \frac{2Ab^3x^{\frac{9}{2}}}{9}+\frac{6Ab^2cx^{\frac{13}{2}}}{13}+\frac{6Abc^2x^{\frac{17}{2}}}{17}+\frac{2Ac^3x^{\frac{21}{2}}}{21}+\frac{2Bb^3x^{\frac{13}{2}}}{13}+\frac{6Bb^2cx^{\frac{17}{2}}}{17}+\frac{2Bbc^2x^{\frac{21}{2}}}{7}+\frac{2Bc^3x^{\frac{25}{2}}}{25}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**(5/2),x)
```

```
[Out] 2*A*b**3*x**(9/2)/9 + 6*A*b**2*c*x**(13/2)/13 + 6*A*b*c**2*x**(17/2)/17 + 2
*A*c**3*x**(21/2)/21 + 2*B*b**3*x**(13/2)/13 + 6*B*b**2*c*x**(17/2)/17 + 2*
B*b*c**2*x**(21/2)/7 + 2*B*c**3*x**(25/2)/25
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^{5/2}}\,dx = \frac{2}{25}\,Bc^3x^{\frac{25}{2}} + \frac{2}{21}\,\big(3\,Bbc^2+Ac^3\big)x^{\frac{21}{2}} + \frac{6}{17}\,\big(Bb^2c+Abc^2\big)x^{\frac{17}{2}} + \frac{2}{9}\,Ab^3x^{\frac{9}{2}} + \frac{2}{13}\,\big(Bb^3+3\,Ab^2c\big)x^{\frac{13}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/25*B*c^3*x^(25/2) + 2/21*(3*B*b*c^2 + A*c^3)*x^(21/2) + 6/17*(B*b^2*c + A
*b*c^2)*x^(17/2) + 2/9*A*b^3*x^(9/2) + 2/13*(B*b^3 + 3*A*b^2*c)*x^(13/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^{5/2}}\,dx = \frac{2}{25}\,Bc^3x^{\frac{25}{2}} + \frac{2}{7}\,Bbc^2x^{\frac{21}{2}} + \frac{2}{21}\,Ac^3x^{\frac{21}{2}} + \frac{6}{17}\,Bb^2cx^{\frac{17}{2}} + \frac{6}{17}\,Abc^2x^{\frac{17}{2}} + \frac{2}{13}\,Bb^3x^{\frac{13}{2}} + \frac{6}{13}\,Ab^2cx^{\frac{13}{2}} + \frac{2}{9}\,Ab^3x^{\frac{9}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/25*B*c^3*x^(25/2) + 2/7*B*b*c^2*x^(21/2) + 2/21*A*c^3*x^(21/2) + 6/17*B*b
^2*c*x^(17/2) + 6/17*A*b*c^2*x^(17/2) + 2/13*B*b^3*x^(13/2) + 6/13*A*b^2*c*
x^(13/2) + 2/9*A*b^3*x^(9/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^{5/2}}\,dx = x^{13/2}\left(\frac{2\,B\,b^3}{13}+\frac{6\,A\,c\,b^2}{13}\right) + x^{21/2}\left(\frac{2\,A\,c^3}{21}+\frac{2\,B\,b\,c^2}{7}\right) + \frac{2\,A\,b^3\,x^{9/2}}{9} + \frac{2\,B\,c^3\,x^{25/2}}{25} + \frac{6\,b\,c\,x^{17/2}\,(A\,c+B\,b)}{17}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(5/2),x)
```

```
[Out] x^(13/2)*((2*B*b^3)/13 + (6*A*b^2*c)/13) + x^(21/2)*((2*A*c^3)/21 + (2*B*b*
c^2)/7) + (2*A*b^3*x^(9/2))/9 + (2*B*c^3*x^(25/2))/25 + (6*b*c*x^(17/2)*(A*
c + B*b))/17
```

3.182 $\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 85

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{7/2}}\,dx = \frac{2}{7}Ab^3x^{7/2} + \frac{2}{11}b^2(bB+3Ac)x^{11/2} + \frac{2}{5}bc(bB+Ac)x^{15/2} + \frac{2}{19}c^2(3bB+Ac)x^{19/2} + \frac{2}{23}Bc^3x^{23/2}$$

```
[Out] 2/7*A*b^3*x^(7/2)+2/11*b^2*(3*A*c+B*b)*x^(11/2)+2/5*b*c*(A*c+B*b)*x^(15/2)+
2/19*c^2*(A*c+3*B*b)*x^(19/2)+2/23*B*c^3*x^(23/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1598, 459}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{7/2}}\,dx = \frac{2}{7}Ab^3x^{7/2} + \frac{2}{11}b^2x^{11/2}(3Ac+bB) + \frac{2}{19}c^2x^{19/2}(Ac+3bB) + \frac{2}{5}bcx^{15/2}(Ac+bB) + \frac{2}{23}Bc^3x^{23/2}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(7/2),x]
```

```
[Out] (2*A*b^3*x^(7/2))/7 + (2*b^2*(b*B + 3*A*c)*x^(11/2))/11 + (2*b*c*(b*B + A*c
)*x^(15/2))/5 + (2*c^2*(3*b*B + A*c)*x^(19/2))/19 + (2*B*c^3*x^(23/2))/23
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
```

```
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int x^{5/2}(A+Bx^2)\left(b+cx^2\right)^3\,dx$$
$$= \int \left(Ab^3x^{5/2}+b^2(bB+3Ac)x^{9/2}+3bc(bB+Ac)x^{13/2}+c^2(3bB+Ac)x^{17/2}+Bc^3x^{21/2}\right)\,dx$$
$$= \frac{2}{7}Ab^3x^{7/2}+\frac{2}{11}b^2(bB+3Ac)x^{11/2}+\frac{2}{5}bc(bB+Ac)x^{15/2}+\frac{2}{19}c^2(3bB+Ac)x^{19/2}+\frac{2}{23}Bc^3x^{23/2}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^3}{x^{7/2}}\,dx = \frac{2x^{7/2}(23A(1045b^3+1995b^2cx^2+1463bc^2x^4+385c^3x^6)+7Bx^2(2185b^3+48$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(7/2),x]
[Out] (2*x^(7/2)*(23*A*(1045*b^3 + 1995*b^2*c*x^2 + 1463*b*c^2*x^4 + 385*c^3*x^6)
 + 7*B*x^2*(2185*b^3 + 4807*b^2*c*x^2 + 3795*b*c^2*x^4 + 1045*c^3*x^6)))/16
8245
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2B\,c^3x^{\frac{23}{2}}}{23}+\frac{2(A\,c^3+3Bb\,c^2)x^{\frac{19}{2}}}{19}+\frac{2(3Ab\,c^2+3B\,b^2c)x^{\frac{15}{2}}}{15}+\frac{2(3b^2Ac+B\,b^3)x^{\frac{11}{2}}}{11}+\frac{2A\,b^3x^{\frac{7}{2}}}{7}$
default	$\frac{2B\,c^3x^{\frac{23}{2}}}{23}+\frac{2(A\,c^3+3Bb\,c^2)x^{\frac{19}{2}}}{19}+\frac{2(3Ab\,c^2+3B\,b^2c)x^{\frac{15}{2}}}{15}+\frac{2(3b^2Ac+B\,b^3)x^{\frac{11}{2}}}{11}+\frac{2A\,b^3x^{\frac{7}{2}}}{7}$
gosper	$\frac{2x^{\frac{7}{2}}(7315B\,c^3x^8+8855A\,c^3x^6+26565x^6Bb\,c^2+33649Ab\,c^2x^4+33649x^4B\,b^2c+45885A\,b^2c\,x^2+15295b^3B\,x^2+24035b^3A}{168245}$
trager	$\frac{2x^{\frac{7}{2}}(7315B\,c^3x^8+8855A\,c^3x^6+26565x^6Bb\,c^2+33649Ab\,c^2x^4+33649x^4B\,b^2c+45885A\,b^2c\,x^2+15295b^3B\,x^2+24035b^3A}{168245}$
risch	$\frac{2x^{\frac{7}{2}}(7315B\,c^3x^8+8855A\,c^3x^6+26565x^6Bb\,c^2+33649Ab\,c^2x^4+33649x^4B\,b^2c+45885A\,b^2c\,x^2+15295b^3B\,x^2+24035b^3A}{168245}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^3/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/23*B*c^3*x^(23/2)+2/19*(A*c^3+3*B*b*c^2)*x^(19/2)+2/15*(3*A*b*c^2+3*B*b^2
*c)*x^(15/2)+2/11*(3*A*b^2*c+B*b^3)*x^(11/2)+2/7*A*b^3*x^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^{7/2}}\,dx = \frac{2}{168245}\left(7315\,Bc^3x^{11}+8855\left(3\,Bbc^2+Ac^3\right)x^9+33649\left(Bb^2c+Abc^2\right)x^7+\right.$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/168245*(7315*B*c^3*x^11 + 8855*(3*B*b*c^2 + A*c^3)*x^9 + 33649*(B*b^2*c +
 A*b*c^2)*x^7 + 24035*A*b^3*x^3 + 15295*(B*b^3 + 3*A*b^2*c)*x^5)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^3}{x^{7/2}}\,dx = \frac{2Ab^3x^{\frac{7}{2}}}{7}+\frac{6Ab^2cx^{\frac{11}{2}}}{11}+\frac{2Abc^2x^{\frac{15}{2}}}{5}+\frac{2Ac^3x^{\frac{19}{2}}}{19}+\frac{2Bb^3x^{\frac{11}{2}}}{11}+\frac{2Bb^2cx^{\frac{15}{2}}}{5}+\frac{6Bbc^2x^{\frac{19}{2}}}{19}+\frac{2Bc^3x^{\frac{23}{2}}}{23}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**3/x**(7/2),x)
```

```
[Out] 2*A*b**3*x**(7/2)/7 + 6*A*b**2*c*x**(11/2)/11 + 2*A*b*c**2*x**(15/2)/5 + 2*
A*c**3*x**(19/2)/19 + 2*B*b**3*x**(11/2)/11 + 2*B*b**2*c*x**(15/2)/5 + 6*B*
b*c**2*x**(19/2)/19 + 2*B*c**3*x**(23/2)/23
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{7/2}}\,dx = \frac{2}{23}\,Bc^3x^{\frac{23}{2}} + \frac{2}{19}\,(3\,Bbc^2+Ac^3)x^{\frac{19}{2}} + \frac{2}{5}\,(Bb^2c+Abc^2)x^{\frac{15}{2}} + \frac{2}{7}\,Ab^3x^{\frac{7}{2}} + \frac{2}{11}\,(Bb^3+3\,Ab^2c)x^{\frac{11}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/23*B*c^3*x^(23/2) + 2/19*(3*B*b*c^2 + A*c^3)*x^(19/2) + 2/5*(B*b^2*c + A*
b*c^2)*x^(15/2) + 2/7*A*b^3*x^(7/2) + 2/11*(B*b^3 + 3*A*b^2*c)*x^(11/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{7/2}}\,dx = \frac{2}{23}\,Bc^3x^{\frac{23}{2}} + \frac{6}{19}\,Bbc^2x^{\frac{19}{2}} + \frac{2}{19}\,Ac^3x^{\frac{19}{2}} + \frac{2}{5}\,Bb^2cx^{\frac{15}{2}} + \frac{2}{5}\,Abc^2x^{\frac{15}{2}} + \frac{2}{11}\,Bb^3x^{\frac{11}{2}} + \frac{6}{11}\,Ab^2cx^{\frac{11}{2}} + \frac{2}{7}\,Ab^3x^{\frac{7}{2}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^3/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/23*B*c^3*x^(23/2) + 6/19*B*b*c^2*x^(19/2) + 2/19*A*c^3*x^(19/2) + 2/5*B*b
^2*c*x^(15/2) + 2/5*A*b*c^2*x^(15/2) + 2/11*B*b^3*x^(11/2) + 6/11*A*b^2*c*x
^(11/2) + 2/7*A*b^3*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^3}{x^{7/2}}\,dx = x^{11/2}\left(\frac{2\,B\,b^3}{11}+\frac{6\,A\,c\,b^2}{11}\right) + x^{19/2}\left(\frac{2\,A\,c^3}{19}+\frac{6\,B\,b\,c^2}{19}\right) + \frac{2\,A\,b^3\,x^{7/2}}{7} + \frac{2\,B\,c^3\,x^{23/2}}{23} + \frac{2\,b\,c\,x^{15/2}\,(A\,c+B\,b)}{5}$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^3)/x^(7/2),x)
```

```
[Out] x^(11/2)*((2*B*b^3)/11 + (6*A*b^2*c)/11) + x^(19/2)*((2*A*c^3)/19 + (6*B*b*
c^2)/19) + (2*A*b^3*x^(7/2))/7 + (2*B*c^3*x^(23/2))/23 + (2*b*c*x^(15/2)*(A
*c + B*b))/5
```

3.183 $\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 278

$$\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2b(bB-Ac)x^{3/2}}{3c^3}-\frac{2(bB-Ac)x^{7/2}}{7c^2}+\frac{2Bx^{11/2}}{11c}+\frac{b^{7/4}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{15/4}}-\frac{b^{7/4}(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{15/4}}-\frac{b^{7/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{15/4}}+\frac{b^{7/4}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{15/4}}$$

```
[Out] 2/3*b*(-A*c+B*b)*x^(3/2)/c^3-2/7*(-A*c+B*b)*x^(7/2)/c^2+2/11*B*x^(11/2)/c+1
/2*b^(7/4)*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(15/4)*2^
(1/2)-1/2*b^(7/4)*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(1
5/4)*2^(1/2)-1/4*b^(7/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^
(1/2)*x^(1/2))/c^(15/4)*2^(1/2)+1/4*b^(7/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)
+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(15/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1598, 470, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{b^{7/4}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{15/4}} - \frac{b^{7/4}(bB-Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}c^{15/4}} - \frac{b^{7/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{15/4}} + \frac{b^{7/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{15/4}} + \frac{2bx^{3/2}(bB-Ac)}{3c^3} - \frac{2x^{7/2}(bB-Ac)}{7c^2} + \frac{2Bx^{11/2}}{11c}$$

```
[In] Int[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*(b*B - A*c)*x^(3/2))/(3*c^3) - (2*(b*B - A*c)*x^(7/2))/(7*c^2) + (2*B*
x^(11/2))/(11*c) + (b^(7/4)*(b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/(Sqrt[2]*c^(15/4)) - (b^(7/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(15/4)) - (b^(7/4)*(b*B - A*c)*Log[Sqr
t[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(15/4)) +
 (b^(7/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[
c]*x])/(2*Sqrt[2]*c^(15/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^{9/2}(A+Bx^2)}{b+cx^2}\,dx \\
&= \frac{2Bx^{11/2}}{11c} - \frac{\left(2\left(\frac{11bB}{2}-\frac{11Ac}{2}\right)\right)\int \frac{x^{9/2}}{b+cx^2}\,dx}{11c} \\
&= -\frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} + \frac{(b(bB-Ac))\int \frac{x^{5/2}}{b+cx^2}\,dx}{c^2} \\
&= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} - \frac{(b^2(bB-Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{c^3} \\
&= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} - \frac{(2b^2(bB-Ac))\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^3} \\
&= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c} \\
&\quad + \frac{(b^2(bB-Ac))\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{7/2}} \\
&\quad - \frac{(b^2(bB-Ac))\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{7/2}}
\end{aligned}
$$

$$= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c}$$

$$- \frac{\left(b^2(bB-Ac)\right)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{2c^4}$$

$$- \frac{\left(b^2(bB-Ac)\right)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{2c^4}$$

$$- \frac{\left(b^{7/4}(bB-Ac)\right)\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} - x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}c^{15/4}}$$

$$- \frac{\left(b^{7/4}(bB-Ac)\right)\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}} - 2x}{-\frac{\sqrt{b}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} - x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}c^{15/4}}$$

$$= \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{7/2}}{7c^2} + \frac{2Bx^{11/2}}{11c}$$

$$- \frac{b^{7/4}(bB-Ac)\log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{2\sqrt{2}c^{15/4}}$$

$$+ \frac{b^{7/4}(bB-Ac)\log\left(\sqrt{b} + \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{2\sqrt{2}c^{15/4}}$$

$$- \frac{\left(b^{7/4}(bB-Ac)\right)\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{15/4}}$$

$$+ \frac{\left(b^{7/4}(bB-Ac)\right)\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{15/4}}$$

$$
\begin{aligned}
&= \frac{2b(bB-Ac)x^{3/2}}{3c^3}-\frac{2(bB-Ac)x^{7/2}}{7c^2}+\frac{2Bx^{11/2}}{11c} \\
&+\frac{b^{7/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{15/4}}-\frac{b^{7/4}(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{15/4}} \\
&-\frac{b^{7/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{15/4}} \\
&+\frac{b^{7/4}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{15/4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.62

$$
\begin{aligned}
\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx &= \frac{2x^{3/2}(77b^2B-77Abc-33bBcx^2+33Ac^2x^2+21Bc^2x^4)}{231c^3} \\
&+\frac{b^{7/4}(bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}c^{15/4}}+\frac{b^{7/4}(bB-Ac)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}c^{15/4}}
\end{aligned}
$$

```
[In] Integrate[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(3/2)*(77*b^2*B - 77*A*b*c - 33*b*B*c*x^2 + 33*A*c^2*x^2 + 21*B*c^2*x^
4))/(231*c^3) + (b^(7/4)*(b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*
b^(1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*c^(15/4)) + (b^(7/4)*(b*B - A*c)*ArcTan
h[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*c^(15/
4))
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.59

method	result
risch	$-\frac{2x^{\frac{3}{2}}\left(-21Bc^{2}x^{4}-33Ac^{2}x^{2}+33Bbcx^{2}+77Abc-77Bb^{2}\right)}{231c^{3}}+\frac{b^{2}(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\right.\right.}{4c^{4}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
derivativedivides	$-\frac{2\left(-\frac{Bx^{\frac{11}{2}}c^{2}}{11}+\frac{\left(-Ac^{2}+Bbc\right)x^{\frac{7}{2}}}{7}+\frac{\left(Abc-Bb^{2}\right)x^{\frac{3}{2}}}{3}\right)}{c^{3}}+\frac{b^{2}(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{4c^{4}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$-\frac{2\left(-\frac{Bx^{\frac{11}{2}}c^{2}}{11}+\frac{\left(-Ac^{2}+Bbc\right)x^{\frac{7}{2}}}{7}+\frac{\left(Abc-Bb^{2}\right)x^{\frac{3}{2}}}{3}\right)}{c^{3}}+\frac{b^{2}(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{4c^{4}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

```
[In] int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/231*x^(3/2)*(-21*B*c^2*x^4-33*A*c^2*x^2+33*B*b*c*x^2+77*A*b*c-77*B*b^2)/
c^3+1/4*b^2*(A*c-B*b)/c^4/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2
)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2
*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(
1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 770, normalized size of antiderivative = 2.77

$$\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{231\,c^3\left(-\frac{B^4b^{11}-4\,AB^3b^{10}c+6\,A^2B^2b^9c^2-4\,A^3Bb^8c^3+A^4b^7c^4}{c^{15}}\right)^{\frac{1}{4}}\log\left(c^{11}\left(-\frac{B^4b^{11}-4\,AB^3b^{10}c+6\,A^2B}{c}\right.\right.}{}$$

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/462*(231*c^3*(-(B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b
^8*c^3 + A^4*b^7*c^4)/c^15)^(1/4)*log(c^11*(-(B^4*b^11 - 4*A*B^3*b^10*c + 6
*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^15)^(3/4) - (B^3*b^8 -
3*A*B^2*b^7*c + 3*A^2*B*b^6*c^2 - A^3*b^5*c^3)*sqrt(x)) - 231*I*c^3*(-(B^4*
b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/
c^15)^(1/4)*log(I*c^11*(-(B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4
*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^15)^(3/4) - (B^3*b^8 - 3*A*B^2*b^7*c + 3*A^
2*B*b^6*c^2 - A^3*b^5*c^3)*sqrt(x)) + 231*I*c^3*(-(B^4*b^11 - 4*A*B^3*b^10*
c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^15)^(1/4)*log(-I*c
```

```
^11*(-(B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^
4*b^7*c^4)/c^15)^(3/4) - (B^3*b^8 - 3*A*B^2*b^7*c + 3*A^2*B*b^6*c^2 - A^3*b
^5*c^3)*sqrt(x)) - 231*c^3*(-(B^4*b^11 - 4*A*B^3*b^10*c + 6*A^2*B^2*b^9*c^2
 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^15)^(1/4)*log(-c^11*(-(B^4*b^11 - 4*A*B
^3*b^10*c + 6*A^2*B^2*b^9*c^2 - 4*A^3*B*b^8*c^3 + A^4*b^7*c^4)/c^15)^(3/4)
- (B^3*b^8 - 3*A*B^2*b^7*c + 3*A^2*B*b^6*c^2 - A^3*b^5*c^3)*sqrt(x)) + 4*(2
1*B*c^2*x^5 - 33*(B*b*c - A*c^2)*x^3 + 77*(B*b^2 - A*b*c)*x)*sqrt(x))/c^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx=\text{Timed out}$$

```
[In] integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.85

$$\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx= -\frac{(Bb^3-Ab^2c)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac14}c^{\frac14}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac14}c^{\frac14}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac14}c^{\frac14}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac14}c^{\frac34}}+\cdots\right)}{4c^3} + \frac{2\left(21\,Bc^2x^{\frac{11}{2}}-33\,(Bbc-Ac^2)x^{\frac72}+77\,(Bb^2-Abc)x^{\frac32}\right)}{231\,c^3}$$

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] -1/4*(B*b^3 - A*b^2*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/
4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(
c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sq
rt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log
(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) +
 sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/
4)*c^(3/4)))/c^3 + 2/231*(21*B*c^2*x^(11/2) - 33*(B*b*c - A*c^2)*x^(7/2) +
77*(B*b^2 - A*b*c)*x^(3/2))/c^3
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.07

$$\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx =$$

$$-\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb^2-\left(bc^3\right)^{\frac{3}{4}}Abc\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^6}$$
$$-\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb^2-\left(bc^3\right)^{\frac{3}{4}}Abc\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^6}$$
$$+\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb^2-\left(bc^3\right)^{\frac{3}{4}}Abc\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^6}$$
$$-\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb^2-\left(bc^3\right)^{\frac{3}{4}}Abc\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^6}$$
$$+\frac{2\left(21\,Bc^{10}x^{\frac{11}{2}}-33\,Bbc^9x^{\frac{7}{2}}+33\,Ac^{10}x^{\frac{7}{2}}+77\,Bb^2c^8x^{\frac{3}{2}}-77\,Abc^9x^{\frac{3}{2}}\right)}{231\,c^{11}}$$

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((b*c^3)^(3/4)*B*b^2 - (b*c^3)^(3/4)*A*b*c)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^6 - 1/2*sqrt(2)*((b*c^3)^
(3/4)*B*b^2 - (b*c^3)^(3/4)*A*b*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
 - 2*sqrt(x))/(b/c)^(1/4))/c^6 + 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b^2 - (b*c^3)
^(3/4)*A*b*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^6 - 1/4*sq
rt(2)*((b*c^3)^(3/4)*B*b^2 - (b*c^3)^(3/4)*A*b*c)*log(-sqrt(2)*sqrt(x)*(b/c
)^(1/4) + x + sqrt(b/c))/c^6 + 2/231*(21*B*c^10*x^(11/2) - 33*B*b*c^9*x^(7/
2) + 33*A*c^10*x^(7/2) + 77*B*b^2*c^8*x^(3/2) - 77*A*b*c^9*x^(3/2))/c^11
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.41

$$\int \frac{x^{13/2}(A+Bx^2)}{bx^2+cx^4}\,dx = x^{7/2}\left(\frac{2\,A}{7\,c}-\frac{2\,B\,b}{7\,c^2}\right) + \frac{2\,B\,x^{11/2}}{11\,c} + \frac{(-b)^{7/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(A\,c-B\,b)}{c^{15/4}} - \frac{b\,x^{3/2}\,\left(\frac{2\,A}{c}-\frac{2\,B\,b}{c^2}\right)}{3\,c} + \frac{(-b)^{7/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1\mathrm{i}}{(-b)^{1/4}}\right)\,(A\,c-B\,b)\,1\mathrm{i}}{c^{15/4}}$$

```
[In] int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^(7/2)*((2*A)/(7*c) - (2*B*b)/(7*c^2)) + (2*B*x^(11/2))/(11*c) + ((-b)^(7/
4)*atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/c^(15/4) + ((-b)^(7/4)*a
tan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*(A*c - B*b)*1i)/c^(15/4) - (b*x^(3/2)*
((2*A)/c - (2*B*b)/c^2))/(3*c)
```

3.184 $\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 276

$$\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4} \, dx = \frac{2b(bB-Ac)\sqrt{x}}{c^3} - \frac{2(bB-Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} + \frac{b^{5/4}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{13/4}} - \frac{b^{5/4}(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{13/4}} + \frac{b^{5/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{13/4}} - \frac{b^{5/4}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{13/4}}$$

```
[Out] -2/5*(-A*c+B*b)*x^(5/2)/c^2+2/9*B*x^(9/2)/c+1/2*b^(5/4)*(-A*c+B*b)*arctan(1
-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(13/4)*2^(1/2)-1/2*b^(5/4)*(-A*c+B*b)*a
rctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(13/4)*2^(1/2)+1/4*b^(5/4)*(-A*c
+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)*2^(1/2
)-1/4*b^(5/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/
2))/c^(13/4)*2^(1/2)+2*b*(-A*c+B*b)*x^(1/2)/c^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1598, 470, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{b^{5/4}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{13/4}} - \frac{b^{5/4}(bB-Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}c^{13/4}} + \frac{b^{5/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{13/4}} - \frac{b^{5/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{13/4}} + \frac{2b\sqrt{x}(bB-Ac)}{c^3} - \frac{2x^{5/2}(bB-Ac)}{5c^2} + \frac{2Bx^{9/2}}{9c}$$

```
[In] Int[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*(b*B - A*c)*Sqrt[x])/c^3 - (2*(b*B - A*c)*x^(5/2))/(5*c^2) + (2*B*x^(9
/2))/(9*c) + (b^(5/4)*(b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(Sqrt[2]*c^(13/4)) - (b^(5/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)
*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(13/4)) + (b^(5/4)*(b*B - A*c)*Log[Sqrt[b] -
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(13/4)) - (b^(5
/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])
/(2*Sqrt[2]*c^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{x^{7/2}(A+Bx^2)}{b+cx^2}\,dx$$

$$= \frac{2Bx^{9/2}}{9c} - \frac{\left(2\left(\frac{9bB}{2}-\frac{9Ac}{2}\right)\right)\int \frac{x^{7/2}}{b+cx^2}\,dx}{9c}$$

$$= -\frac{2(bB-Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} + \frac{(b(bB-Ac))\int \frac{x^{3/2}}{b+cx^2}\,dx}{c^2}$$

$$= \frac{2b(bB-Ac)\sqrt{x}}{c^3} - \frac{2(bB-Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} - \frac{(b^2(bB-Ac))\int \frac{1}{\sqrt{x}(b+cx^2)}\,dx}{c^3}$$

$$= \frac{2b(bB-Ac)\sqrt{x}}{c^3} - \frac{2(bB-Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} - \frac{(2b^2(bB-Ac))\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{c^3}$$

$$= \frac{2b(bB-Ac)\sqrt{x}}{c^3} - \frac{2(bB-Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} - \frac{\left(b^{3/2}(bB-Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{c^3} - \frac{\left(b^{3/2}(bB-Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{c^3}$$

$$= \frac{2b(bB-Ac)\sqrt{x}}{c^3} - \frac{2(bB-Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} - \frac{\left(b^{3/2}(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{2c^{7/2}} - \frac{\left(b^{3/2}(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{2c^{7/2}} + \frac{\left(b^{5/4}(bB-Ac)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} - x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}c^{13/4}} + \frac{\left(b^{5/4}(bB-Ac)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}} - 2x}{-\frac{\sqrt{b}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} - x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}c^{13/4}}$$

$$= \frac{2b(bB-Ac)\sqrt{x}}{c^3} - \frac{2(bB-Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} + \frac{b^{5/4}(bB-Ac)\log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{2\sqrt{2}c^{13/4}} - \frac{b^{5/4}(bB-Ac)\log\left(\sqrt{b} + \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{2\sqrt{2}c^{13/4}} - \frac{\left(b^{5/4}(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{13/4}} + \frac{\left(b^{5/4}(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{13/4}}$$

$$
\begin{aligned}
&= \frac{2b(bB-Ac)\sqrt{x}}{c^3} - \frac{2(bB-Ac)x^{5/2}}{5c^2} + \frac{2Bx^{9/2}}{9c} \\
&+ \frac{b^{5/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{13/4}} - \frac{b^{5/4}(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{13/4}} \\
&+ \frac{b^{5/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{13/4}} \\
&- \frac{b^{5/4}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{13/4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.63

$$
\begin{aligned}
\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx &= \frac{2\sqrt{x}(45b^2B-45Abc-9bBcx^2+9Ac^2x^2+5Bc^2x^4)}{45c^3} \\
&+ \frac{b^{5/4}(bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}c^{13/4}} - \frac{b^{5/4}(bB-Ac)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}c^{13/4}}
\end{aligned}
$$

```
[In] Integrate[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[x]*(45*b^2*B - 45*A*b*c - 9*b*B*c*x^2 + 9*A*c^2*x^2 + 5*B*c^2*x^4))
/(45*c^3) + (b^(5/4)*(b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*c^(13/4)) - (b^(5/4)*(b*B - A*c)*ArcTanh[(S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*c^(13/4))
```

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.59

method	result
risch	$-\frac{2\left(-5B\,c^2x^4-9A\,c^2x^2+9Bbc\,x^2+45Abc-45B\,b^2\right)\sqrt{x}}{45c^3}+\frac{b(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\cdots\right.\right.}{4c^3}$
derivativedivides	$-\frac{2\left(-\frac{B\,x^{\frac{9}{2}}c^2}{9}-\frac{A\,c^2x^{\frac{5}{2}}}{5}+\frac{Bbc\,x^{\frac{5}{2}}}{5}+Abc\sqrt{x}-B\,b^2\sqrt{x}\right)}{c^3}+\frac{b(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\cdots\right.\right.}{4c^3}$
default	$-\frac{2\left(-\frac{B\,x^{\frac{9}{2}}c^2}{9}-\frac{A\,c^2x^{\frac{5}{2}}}{5}+\frac{Bbc\,x^{\frac{5}{2}}}{5}+Abc\sqrt{x}-B\,b^2\sqrt{x}\right)}{c^3}+\frac{b(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\cdots\right.\right.}{4c^3}$

```
[In] int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(-5*B*c^2*x^4-9*A*c^2*x^2+9*B*b*c*x^2+45*A*b*c-45*B*b^2)*x^(1/2)/c^3+
1/4*b*(A*c-B*b)/c^3*(1/c*b)^(1/4)*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1
/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arcta
n(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1
))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 647, normalized size of antiderivative = 2.34

$$\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{45\,c^3\left(-\frac{B^4b^9-4\,AB^3b^8c+6\,A^2B^2b^7c^2-4\,A^3Bb^6c^3+A^4b^5c^4}{c^{13}}\right)^{\frac{1}{4}}\log\left(c^3\left(-\frac{B^4b^9-4\,AB^3b^8c+6\,A^2B^2b^7c^2-}{c^{13}}\right.\right.}{}$$

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/90*(45*c^3*(-(B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c
^3 + A^4*b^5*c^4)/c^13)^(1/4)*log(c^3*(-(B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*B^
2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^13)^(1/4) - (B*b^2 - A*b*c)*sq
rt(x)) + 45*I*c^3*(-(B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*b^7*c^2 - 4*A^3*B*
b^6*c^3 + A^4*b^5*c^4)/c^13)^(1/4)*log(I*c^3*(-(B^4*b^9 - 4*A*B^3*b^8*c + 6
*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^13)^(1/4) - (B*b^2 - A*
b*c)*sqrt(x)) - 45*I*c^3*(-(B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*b^7*c^2 - 4
*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^13)^(1/4)*log(-I*c^3*(-(B^4*b^9 - 4*A*B^3*b
^8*c + 6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^13)^(1/4) - (B*
b^2 - A*b*c)*sqrt(x)) - 45*c^3*(-(B^4*b^9 - 4*A*B^3*b^8*c + 6*A^2*B^2*b^7*c
^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^13)^(1/4)*log(-c^3*(-(B^4*b^9 - 4*A*B
^3*b^8*c + 6*A^2*B^2*b^7*c^2 - 4*A^3*B*b^6*c^3 + A^4*b^5*c^4)/c^13)^(1/4) -
 (B*b^2 - A*b*c)*sqrt(x)) + 4*(5*B*c^2*x^4 + 45*B*b^2 - 45*A*b*c - 9*(B*b*c
 - A*c^2)*x^2)*sqrt(x))/c^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.94

$$\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{\left(\frac{2\sqrt{2}(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(Bb-Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}(Bb-Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right.}{4c^3} + \frac{2\left(5Bc^2x^{\frac{9}{2}}-9\,(Bbc-Ac^2)x^{\frac{5}{2}}+45\,(Bb^2-Abc)\sqrt{x}\right)}{45\,c^3}$$

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*(B*b - A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2
*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) +
2*sqrt(2)*(B*b - A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt
(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(
2)*(B*b - A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(
b^(3/4)*c^(1/4)) - sqrt(2)*(B*b - A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x)
 + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))*b^2/c^3 + 2/45*(5*B*c^2*x^(9/2)
- 9*(B*b*c - A*c^2)*x^(5/2) + 45*(B*b^2 - A*b*c)*sqrt(x))/c^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.08

$$\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx =$$

$$-\frac{\sqrt{2}\Big((bc^3)^{\frac{1}{4}}\,Bb^2-(bc^3)^{\frac{1}{4}}\,Abc\Big)\arctan\left(\frac{\sqrt{2}\Big(\sqrt{2}\big(\frac{b}{c}\big)^{\frac{1}{4}}+2\sqrt{x}\Big)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^4}$$
$$-\frac{\sqrt{2}\Big((bc^3)^{\frac{1}{4}}\,Bb^2-(bc^3)^{\frac{1}{4}}\,Abc\Big)\arctan\left(-\frac{\sqrt{2}\Big(\sqrt{2}\big(\frac{b}{c}\big)^{\frac{1}{4}}-2\sqrt{x}\Big)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^4}$$
$$-\frac{\sqrt{2}\Big((bc^3)^{\frac{1}{4}}\,Bb^2-(bc^3)^{\frac{1}{4}}\,Abc\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^4}$$
$$+\frac{\sqrt{2}\Big((bc^3)^{\frac{1}{4}}\,Bb^2-(bc^3)^{\frac{1}{4}}\,Abc\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^4}$$
$$+\frac{2\left(5\,Bc^8x^{\frac{9}{2}}-9\,Bbc^7x^{\frac{5}{2}}+9\,Ac^8x^{\frac{5}{2}}+45\,Bb^2c^6\sqrt{x}-45\,Abc^7\sqrt{x}\right)}{45\,c^9}$$

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((b*c^3)^(1/4)*B*b^2 - (b*c^3)^(1/4)*A*b*c)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^4 - 1/2*sqrt(2)*((b*c^3)^
(1/4)*B*b^2 - (b*c^3)^(1/4)*A*b*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
 - 2*sqrt(x))/(b/c)^(1/4))/c^4 - 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b^2 - (b*c^3)
^(1/4)*A*b*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 + 1/4*sq
rt(2)*((b*c^3)^(1/4)*B*b^2 - (b*c^3)^(1/4)*A*b*c)*log(-sqrt(2)*sqrt(x)*(b/c
)^(1/4) + x + sqrt(b/c))/c^4 + 2/45*(5*B*c^8*x^(9/2) - 9*B*b*c^7*x^(5/2) +
9*A*c^8*x^(5/2) + 45*B*b^2*c^6*sqrt(x) - 45*A*b*c^7*sqrt(x))/c^9
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 788, normalized size of antiderivative = 2.86

$$\int \frac{x^{11/2}(A+Bx^2)}{bx^2+cx^4}\,dx = x^{5/2}\left(\frac{2\,A}{5\,c}-\frac{2\,B\,b}{5\,c^2}\right)+\frac{2\,B\,x^{9/2}}{9\,c}$$

$$-\frac{(-b)^{5/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{5/4}\left(\frac{16\sqrt{x}\left(A^2\,b^4\,c^2-2\,A\,B\,b^5\,c+B^2\,b^6\right)}{c^3}-\frac{(-b)^{5/4}\,(A\,c-B\,b)\left(32\,B\,b^4-32\,A\,b^3\,c\right)1\mathrm{i}}{2\,c^{13/4}}\right)(A\,c-B\,b)}{2\,c^{13/4}}+\frac{(-b)^{5/4}\left(\frac{16\sqrt{x}\left(A^2\,b^4\,c^2-2\,A\,B\right.}{c^3}\right.}{}}{\frac{(-b)^{5/4}\left(\frac{16\sqrt{x}\left(A^2\,b^4\,c^2-2\,A\,B\,b^5\,c+B^2\,b^6\right)}{c^3}-\frac{(-b)^{5/4}\,(A\,c-B\,b)\left(32\,B\,b^4-32\,A\,b^3\,c\right)1\mathrm{i}}{2\,c^{13/4}}\right)(A\,c-B\,b)\,1\mathrm{i}}{2\,c^{13/4}}-\frac{(-b)^{5/4}\left(\frac{16\sqrt{x}\left(A^2\,b^4\,c^2-2\,A\,B\right.}{c^3}\right.}{}}\right)}{c^{13/4}}$$

$$-\frac{b\sqrt{x}\left(\frac{2\,A}{c}-\frac{2\,B\,b}{c^2}\right)}{c}$$

$$-\frac{(-b)^{5/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{5/4}\left(\frac{16\sqrt{x}\left(A^2\,b^4\,c^2-2\,A\,B\,b^5\,c+B^2\,b^6\right)}{c^3}-\frac{(-b)^{5/4}\,(A\,c-B\,b)\left(32\,B\,b^4-32\,A\,b^3\,c\right)}{2\,c^{13/4}}\right)(A\,c-B\,b)\,1\mathrm{i}}{2\,c^{13/4}}+\frac{(-b)^{5/4}\left(\frac{16\sqrt{x}\left(A^2\,b^4\,c^2-2\,A\,B\,b\right.}{c^3}\right.}{}}{\frac{(-b)^{5/4}\left(\frac{16\sqrt{x}\left(A^2\,b^4\,c^2-2\,A\,B\,b^5\,c+B^2\,b^6\right)}{c^3}-\frac{(-b)^{5/4}\,(A\,c-B\,b)\left(32\,B\,b^4-32\,A\,b^3\,c\right)}{2\,c^{13/4}}\right)(A\,c-B\,b)}{2\,c^{13/4}}-\frac{(-b)^{5/4}\left(\frac{16\sqrt{x}\left(A^2\,b^4\,c^2-2\,A\,B\,b\right.}{c^3}\right.}{}}\right)}{c^{13/4}}$$

```
[In] int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^(5/2)*((2*A)/(5*c) - (2*B*b)/(5*c^2)) + (2*B*x^(9/2))/(9*c) - ((-b)^(5/4)
*atan((((-b)^(5/4)*((16*x^(1/2)*(B^2*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3
- ((-b)^(5/4)*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c))/(2*c^(13/4)))*(A*c - B*b
)*1i)/(2*c^(13/4)) + ((-b)^(5/4)*((16*x^(1/2)*(B^2*b^6 + A^2*b^4*c^2 - 2*A*
B*b^5*c))/c^3 + ((-b)^(5/4)*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c))/(2*c^(13/4
)))*(A*c - B*b)*1i)/(2*c^(13/4)))/(((-b)^(5/4)*((16*x^(1/2)*(B^2*b^6 + A^2*
b^4*c^2 - 2*A*B*b^5*c))/c^3 - ((-b)^(5/4)*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*
c))/(2*c^(13/4)))*(A*c - B*b))/(2*c^(13/4)) - ((-b)^(5/4)*((16*x^(1/2)*(B^2
*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 + ((-b)^(5/4)*(A*c - B*b)*(32*B*b^4
- 32*A*b^3*c))/(2*c^(13/4)))*(A*c - B*b))/(2*c^(13/4))))*(A*c - B*b)*1i)/c^
(13/4) - ((-b)^(5/4)*atan((((-b)^(5/4)*((16*x^(1/2)*(B^2*b^6 + A^2*b^4*c^2
- 2*A*B*b^5*c))/c^3 - ((-b)^(5/4)*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c)*1i)/(
2*c^(13/4)))*(A*c - B*b))/(2*c^(13/4)) + ((-b)^(5/4)*((16*x^(1/2)*(B^2*b^6
+ A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 + ((-b)^(5/4)*(A*c - B*b)*(32*B*b^4 - 32*
A*b^3*c)*1i)/(2*c^(13/4)))*(A*c - B*b))/(2*c^(13/4)))/(((-b)^(5/4)*((16*x^(
1/2)*(B^2*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 - ((-b)^(5/4)*(A*c - B*b)*(
32*B*b^4 - 32*A*b^3*c)*1i)/(2*c^(13/4)))*(A*c - B*b)*1i)/(2*c^(13/4)) - ((-
b)^(5/4)*((16*x^(1/2)*(B^2*b^6 + A^2*b^4*c^2 - 2*A*B*b^5*c))/c^3 + ((-b)^(5
/4)*(A*c - B*b)*(32*B*b^4 - 32*A*b^3*c)*1i)/(2*c^(13/4)))*(A*c - B*b)*1i)/(
2*c^(13/4))))*(A*c - B*b))/c^(13/4) - (b*x^(1/2)*((2*A)/c - (2*B*b)/c^2))/c
```

3.185 $\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 257

$$\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{2(bB-Ac)x^{3/2}}{3c^2}+\frac{2Bx^{7/2}}{7c} - \frac{b^{3/4}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{11/4}}+\frac{b^{3/4}(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{11/4}} + \frac{b^{3/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}} - \frac{b^{3/4}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}}$$

```
[Out] -2/3*(-A*c+B*b)*x^(3/2)/c^2+2/7*B*x^(7/2)/c-1/2*b^(3/4)*(-A*c+B*b)*arctan(1
-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(11/4)*2^(1/2)+1/2*b^(3/4)*(-A*c+B*b)*a
rctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(11/4)*2^(1/2)+1/4*b^(3/4)*(-A*c
+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)*2^(1/2
)-1/4*b^(3/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/
2))/c^(11/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1598, 470, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{b^{3/4}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{11/4}} + \frac{b^{3/4}(bB-Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}c^{11/4}} + \frac{b^{3/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}} - \frac{b^{3/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}} - \frac{2x^{3/2}(bB-Ac)}{3c^2} + \frac{2Bx^{7/2}}{7c}$$

```
[In] Int[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*(b*B - A*c)*x^(3/2))/(3*c^2) + (2*B*x^(7/2))/(7*c) - (b^(3/4)*(b*B - A*
c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(11/4)) + (b^(
3/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^
(11/4)) + (b^(3/4)*(b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x])/(2*Sqrt[2]*c^(11/4)) - (b^(3/4)*(b*B - A*c)*Log[Sqrt[b] + S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(11/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{x^{5/2}(A+Bx^2)}{b+cx^2}\,dx$$

$$= \frac{2Bx^{7/2}}{7c} - \frac{\left(2\left(\frac{7bB}{2}-\frac{7Ac}{2}\right)\right)\int \frac{x^{5/2}}{b+cx^2}\,dx}{7c}$$

$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} + \frac{(b(bB-Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{c^2}$$

$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} + \frac{(2b(bB-Ac))\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2}$$

$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} - \frac{(b(bB-Ac))\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{5/2}} + \frac{(b(bB-Ac))\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^{5/2}}$$

$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} + \frac{(b(bB-Ac))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^3} + \frac{(b(bB-Ac))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^3} + \frac{\left(b^{3/4}(bB-Ac)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}c^{11/4}} + \frac{\left(b^{3/4}(bB-Ac)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}c^{11/4}}$$

$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} + \frac{b^{3/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}} - \frac{b^{3/4}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}} + \frac{\left(b^{3/4}(bB-Ac)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{11/4}} - \frac{\left(b^{3/4}(bB-Ac)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{11/4}}$$

$$= -\frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{7/2}}{7c} - \frac{b^{3/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{11/4}} + \frac{b^{3/4}(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{11/4}} + \frac{b^{3/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}} - \frac{b^{3/4}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.59

$$\int\frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}dx = \frac{2x^{3/2}(-7bB+7Ac+3Bcx^2)}{21c^2} - \frac{b^{3/4}(bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}c^{11/4}} - \frac{b^{3/4}(bB-Ac)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}c^{11/4}}$$

```
[In] Integrate[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(3/2)*(-7*b*B + 7*A*c + 3*B*c*x^2))/(21*c^2) - (b^(3/4)*(b*B - A*c)*Ar
cTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*c^(
11/4)) - (b^(3/4)*(b*B - A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sq
rt[b] + Sqrt[c]*x)])/(Sqrt[2]*c^(11/4))
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.54

method	result
risch	$\frac{2x^{\frac{3}{2}}\left(3Bc\,x^{2}+7Ac-7Bb\right)}{21c^{2}}-\frac{b(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
derivativedivides	$\frac{\frac{2Bc\,x^{\frac{7}{2}}}{7}+\frac{2(Ac-Bb)x^{\frac{3}{2}}}{3}}{c^{2}}-\frac{b(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$\frac{\frac{2Bc\,x^{\frac{7}{2}}}{7}+\frac{2(Ac-Bb)x^{\frac{3}{2}}}{3}}{c^{2}}-\frac{b(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^{3}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

```
[In] int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*x^(3/2)*(3*B*c*x^2+7*A*c-7*B*b)/c^2-1/4*b*(A*c-B*b)/c^3/(1/c*b)^(1/4)*
2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4
)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)
+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 749, normalized size of antiderivative = 2.91

$$\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx =$$

$$-\frac{21\,c^{2}\left(-\frac{B^{4}b^{7}-4\,AB^{3}b^{6}c+6\,A^{2}B^{2}b^{5}c^{2}-4\,A^{3}Bb^{4}c^{3}+A^{4}b^{3}c^{4}}{c^{11}}\right)^{\frac{1}{4}}\log\left(c^{8}\left(-\frac{B^{4}b^{7}-4\,AB^{3}b^{6}c+6\,A^{2}B^{2}b^{5}c^{2}-4\,A^{3}Bb^{4}c^{3}+A^{4}b^{3}c^{4}}{c^{11}}\right)^{\frac{3}{4}}-(B^{3}\right.}{}$$

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/42*(21*c^2*(-(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*
c^3 + A^4*b^3*c^4)/c^11)^(1/4)*log(c^8*(-(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B
^2*b^5*c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11)^(3/4) - (B^3*b^5 - 3*A*B^
2*b^4*c + 3*A^2*B*b^3*c^2 - A^3*b^2*c^3)*sqrt(x)) - 21*I*c^2*(-(B^4*b^7 - 4
*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11)^(1/
4)*log(I*c^8*(-(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c
^3 + A^4*b^3*c^4)/c^11)^(3/4) - (B^3*b^5 - 3*A*B^2*b^4*c + 3*A^2*B*b^3*c^2
- A^3*b^2*c^3)*sqrt(x)) + 21*I*c^2*(-(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b
```

```
^5*c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11)^(1/4)*log(-I*c^8*(-(B^4*b^7 -
 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11)^(
3/4) - (B^3*b^5 - 3*A*B^2*b^4*c + 3*A^2*B*b^3*c^2 - A^3*b^2*c^3)*sqrt(x)) -
 21*c^2*(-(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^5*c^2 - 4*A^3*B*b^4*c^3 +
A^4*b^3*c^4)/c^11)^(1/4)*log(-c^8*(-(B^4*b^7 - 4*A*B^3*b^6*c + 6*A^2*B^2*b^
5*c^2 - 4*A^3*B*b^4*c^3 + A^4*b^3*c^4)/c^11)^(3/4) - (B^3*b^5 - 3*A*B^2*b^4
*c + 3*A^2*B*b^3*c^2 - A^3*b^2*c^3)*sqrt(x)) - 4*(3*B*c*x^3 - 7*(B*b - A*c)
*x)*sqrt(x))/c^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.83

$$\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{(Bb^2-Abc)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log \cdots}{\cdots}\right.}{4c^2} + \frac{2\left(3Bcx^{\frac{7}{2}}-7(Bb-Ac)x^{\frac{3}{2}}\right)}{21c^2}$$

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*(B*b^2 - A*b*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4)
+ 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c))
 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(
x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sq
rt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sq
rt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*
c^(3/4)))/c^2 + 2/21*(3*B*c*x^(7/2) - 7*(B*b - A*c)*x^(3/2))/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.03

$$\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^5} + \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^5} - \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^5} + \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^5} + \frac{2\left(3Bc^6x^{\frac{7}{2}}-7Bbc^5x^{\frac{3}{2}}+7Ac^6x^{\frac{3}{2}}\right)}{21c^7}$$

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^5 + 1/2*sqrt(2)*((b*c^3)^(3/4)
*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt
(x))/(b/c)^(1/4))/c^5 - 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)
*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 + 1/4*sqrt(2)*((b*c^3
)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqr
t(b/c))/c^5 + 2/21*(3*B*c^6*x^(7/2) - 7*B*b*c^5*x^(3/2) + 7*A*c^6*x^(3/2))/
c^7
```

Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.36

$$\int \frac{x^{9/2}(A+Bx^2)}{bx^2+cx^4}\,dx = x^{3/2}\left(\frac{2A}{3c}-\frac{2Bb}{3c^2}\right)+\frac{2Bx^{7/2}}{7c} + \frac{(-b)^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)(Ac-Bb)}{c^{11/4}} + \frac{(-b)^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\sqrt{x}\,1\mathrm{i}}{(-b)^{1/4}}\right)(Ac-Bb)\,1\mathrm{i}}{c^{11/4}}$$

```
[In] int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^(3/2)*((2*A)/(3*c) - (2*B*b)/(3*c^2)) + (2*B*x^(7/2))/(7*c) + ((-b)^(3/4)
*atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/c^(11/4) + ((-b)^(3/4)*ata
n((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*(A*c - B*b)*1i)/c^(11/4)
```

3.186 $\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\begin{aligned}\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx =& -\frac{2(bB-Ac)\sqrt{x}}{c^2}+\frac{2Bx^{5/2}}{5c} \\ &-\frac{\sqrt[4]{b}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{9/4}}+\frac{\sqrt[4]{b}(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{9/4}} \\ &-\frac{\sqrt[4]{b}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}} \\ &+\frac{\sqrt[4]{b}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}}\end{aligned}$$

```
[Out] 2/5*B*x^(5/2)/c-1/2*b^(1/4)*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(
1/4))/c^(9/4)*2^(1/2)+1/2*b^(1/4)*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/
2)/b^(1/4))/c^(9/4)*2^(1/2)-1/4*b^(1/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(
1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)*2^(1/2)+1/4*b^(1/4)*(-A*c+B*b)*ln(b^(
1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)*2^(1/2)-2*(-A*c+B*b
)*x^(1/2)/c^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1598, 470, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{\sqrt[4]{b}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{9/4}} + \frac{\sqrt[4]{b}(bB-Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}c^{9/4}} - \frac{\sqrt[4]{b}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}} + \frac{\sqrt[4]{b}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}} - \frac{2\sqrt{x}(bB-Ac)}{c^2} + \frac{2Bx^{5/2}}{5c}$$

```
[In] Int[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*(b*B - A*c)*Sqrt[x])/c^2 + (2*B*x^(5/2))/(5*c) - (b^(1/4)*(b*B - A*c)*A
rcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)) + (b^(1/4)*
(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)
) - (b^(1/4)*(b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sq
rt[c]*x])/(2*Sqrt[2]*c^(9/4)) + (b^(1/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*
b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{x^{3/2}(A+Bx^2)}{b+cx^2}\,dx$$

$$= \frac{2Bx^{5/2}}{5c} - \frac{\left(2\left(\frac{5bB}{2}-\frac{5Ac}{2}\right)\right)\int \frac{x^{3/2}}{b+cx^2}\,dx}{5c}$$

$$= -\frac{2(bB-Ac)\sqrt{x}}{c^2} + \frac{2Bx^{5/2}}{5c} + \frac{(b(bB-Ac))\int \frac{1}{\sqrt{x}(b+cx^2)}\,dx}{c^2}$$

$$= -\frac{2(bB-Ac)\sqrt{x}}{c^2} + \frac{2Bx^{5/2}}{5c} + \frac{(2b(bB-Ac))\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2}$$

$$= -\frac{2(bB-Ac)\sqrt{x}}{c^2} + \frac{2Bx^{5/2}}{5c} + \frac{\left(\sqrt{b}(bB-Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2} + \frac{\left(\sqrt{b}(bB-Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c^2}$$

$$= -\frac{2(bB-Ac)\sqrt{x}}{c^2} + \frac{2Bx^{5/2}}{5c} + \frac{\left(\sqrt{b}(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^{5/2}} + \frac{\left(\sqrt{b}(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^{5/2}} - \frac{\left(\sqrt[4]{b}(bB-Ac)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}c^{9/4}} - \frac{\left(\sqrt[4]{b}(bB-Ac)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}c^{9/4}}$$

$$= -\frac{2(bB-Ac)\sqrt{x}}{c^2} + \frac{2Bx^{5/2}}{5c} - \frac{\sqrt[4]{b}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}} + \frac{\sqrt[4]{b}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}} + \frac{\left(\sqrt[4]{b}(bB-Ac)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{9/4}} - \frac{\left(\sqrt[4]{b}(bB-Ac)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{9/4}}$$

$$= -\frac{2(bB-Ac)\sqrt{x}}{c^2} + \frac{2Bx^{5/2}}{5c} - \frac{\sqrt[4]{b}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{9/4}} + \frac{\sqrt[4]{b}(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{9/4}} - \frac{\sqrt[4]{b}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}} + \frac{\sqrt[4]{b}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.59

$$\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{4\sqrt[4]{c}\sqrt{x}(-5bB+5Ac+Bcx^2) - 5\sqrt{2}\sqrt[4]{b}(bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) + 5\sqrt{2}\sqrt[4]{b}(}{10c^{9/4}}$$

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (4*c^(1/4)*Sqrt[x]*(-5*b*B + 5*A*c + B*c*x^2) - 5*Sqrt[2]*b^(1/4)*(b*B - A*
c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 5*Sqrt
[2]*b^(1/4)*(b*B - A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b]
+ Sqrt[c]*x)])/(10*c^(9/4))
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.54

method	result
risch	$\frac{2(Bc\,x^2+5Ac-5Bb)\sqrt{x}}{5c^2}-\frac{(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{4c^2}$
derivativedivides	$\frac{\frac{2Bc\,x^{\frac{5}{2}}}{5}+2Ac\sqrt{x}-2bB\sqrt{x}}{c^2}-\frac{(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{4c^2}$
default	$\frac{\frac{2Bc\,x^{\frac{5}{2}}}{5}+2Ac\sqrt{x}-2bB\sqrt{x}}{c^2}-\frac{(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{4c^2}$

```
[In] int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(B*c*x^2+5*A*c-5*B*b)*x^(1/2)/c^2-1/4*(A*c-B*b)/c^2*(1/c*b)^(1/4)*2^(1/
2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(
1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*ar
ctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 597, normalized size of antiderivative = 2.34

$$\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx =$$

$$-\frac{5\,c^2\left(-\frac{B^4b^5-4\,AB^3b^4c+6\,A^2B^2b^3c^2-4\,A^3Bb^2c^3+A^4bc^4}{c^9}\right)^{\frac{1}{4}}\log\left(c^2\left(-\frac{B^4b^5-4\,AB^3b^4c+6\,A^2B^2b^3c^2-4\,A^3Bb^2c^3+A^4bc^4}{c^9}\right)^{\frac{1}{4}}-(Bb\right.}{}$$

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/10*(5*c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c
^3 + A^4*b*c^4)/c^9)^(1/4)*log(c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b
^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^(1/4) - (B*b - A*c)*sqrt(x)) + 5
*I*c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A
^4*b*c^4)/c^9)^(1/4)*log(I*c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c
^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^(1/4) - (B*b - A*c)*sqrt(x)) - 5*I*c
^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b
*c^4)/c^9)^(1/4)*log(-I*c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2
- 4*A^3*B*b^2*c^3 + A^4*b*c^4)/c^9)^(1/4) - (B*b - A*c)*sqrt(x)) - 5*c^2*(-
(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3*B*b^2*c^3 + A^4*b*c^4)
```

```
/c^9)^(1/4)*log(-c^2*(-(B^4*b^5 - 4*A*B^3*b^4*c + 6*A^2*B^2*b^3*c^2 - 4*A^3
*B*b^2*c^3 + A^4*b*c^4)/c^9)^(1/4) - (B*b - A*c)*sqrt(x)) - 4*(B*c*x^2 - 5*
B*b + 5*A*c)*sqrt(x))/c^2
```

Sympy [A] (verification not implemented)

Time = 98.31 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.08

$$\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \begin{cases} \tilde{\infty}\left(2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}\right) \\ \frac{\frac{2Ax^{\frac{5}{2}}}{5}+\frac{2Bx^{\frac{9}{2}}}{9}}{b} \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}}{c} \\ \frac{2A\sqrt{x}}{c}+\frac{A\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2c}-\frac{A\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2c}-\frac{A\sqrt[4]{-\frac{b}{c}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{c}-\frac{2}{} \end{cases}$$

```
[In] integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*(2*A*sqrt(x) + 2*B*x**(5/2)/5), Eq(b, 0) & Eq(c, 0)), ((2*A*
x**(5/2)/5 + 2*B*x**(9/2)/9)/b, Eq(c, 0)), ((2*A*sqrt(x) + 2*B*x**(5/2)/5)/
c, Eq(b, 0)), (2*A*sqrt(x)/c + A*(-b/c)**(1/4)*log(sqrt(x) - (-b/c)**(1/4))
/(2*c) - A*(-b/c)**(1/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*c) - A*(-b/c)**(1/
4)*atan(sqrt(x)/(-b/c)**(1/4))/c - 2*B*b*sqrt(x)/c**2 - B*b*(-b/c)**(1/4)*l
og(sqrt(x) - (-b/c)**(1/4))/(2*c**2) + B*b*(-b/c)**(1/4)*log(sqrt(x) + (-b/
c)**(1/4))/(2*c**2) + B*b*(-b/c)**(1/4)*atan(sqrt(x)/(-b/c)**(1/4))/c**2 +
2*B*x**(5/2)/(5*c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.92

$$\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{\left(\frac{2\sqrt{2}(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(Bb-Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}(Bb}{}\right.}{4c^2} + \frac{2\left(Bcx^{\frac{5}{2}}-5(Bb-Ac)\sqrt{x}\right)}{5c^2}$$

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*(B*b - A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*
sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2
*sqrt(2)*(B*b - A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(
c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2
)*(B*b - A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b
^(3/4)*c^(1/4)) - sqrt(2)*(B*b - A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x)
+ sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))*b/c^2 + 2/5*(B*c*x^(5/2) - 5*(B*b
 - A*c)*sqrt(x))/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.03

$$\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^3} + \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^3} + \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^3} - \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^3} + \frac{2\left(Bc^4x^{\frac{5}{2}}-5\,Bbc^3\sqrt{x}+5\,Ac^4\sqrt{x}\right)}{5\,c^5}$$

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^3 + 1/2*sqrt(2)*((b*c^3)^(1/4)
*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt
(x))/(b/c)^(1/4))/c^3 + 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)
*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 - 1/4*sqrt(2)*((b*c^3
)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqr
t(b/c))/c^3 + 2/5*(B*c^4*x^(5/2) - 5*B*b*c^3*sqrt(x) + 5*A*c^4*sqrt(x))/c^5
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 789, normalized size of antiderivative = 3.09

$$\int \frac{x^{7/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \sqrt{x}\left(\frac{2\,A}{c}-\frac{2\,B\,b}{c^2}\right)+\frac{2\,B\,x^{5/2}}{5\,c}$$

$$-\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{1/4}\,(A\,c-B\,b)\,\left(\frac{16\,\sqrt{x}\,\left(A^2\,b^2\,c^2-2\,A\,B\,b^3\,c+B^2\,b^4\right)}{c}-\frac{(-b)^{1/4}\,\left(32\,A\,b^2\,c^2-32\,B\,b^3\,c\right)\,(A\,c-B\,b)}{2\,c^{9/4}}\right)\,1\mathrm{i}}{2\,c^{9/4}}+\frac{(-b)^{1/4}\,(A\,c-B\,b)\,\left(\frac{16\,\sqrt{x}\,\left(A^2\,b\right.}{}\right.}{}}{\frac{(-b)^{1/4}\,(A\,c-B\,b)\,\left(\frac{16\,\sqrt{x}\,\left(A^2\,b^2\,c^2-2\,A\,B\,b^3\,c+B^2\,b^4\right)}{c}-\frac{(-b)^{1/4}\,\left(32\,A\,b^2\,c^2-32\,B\,b^3\,c\right)\,(A\,c-B\,b)}{2\,c^{9/4}}\right)}{2\,c^{9/4}}-\frac{(-b)^{1/4}\,(A\,c-B\,b)\,\left(\frac{16\,\sqrt{x}\,\left(A^2\,b\right.}{}\right.}{}}\right.}{c^{9/4}}$$

$$-\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{1/4}\,(A\,c-B\,b)\,\left(\frac{16\,\sqrt{x}\,\left(A^2\,b^2\,c^2-2\,A\,B\,b^3\,c+B^2\,b^4\right)}{c}-\frac{(-b)^{1/4}\,\left(32\,A\,b^2\,c^2-32\,B\,b^3\,c\right)\,(A\,c-B\,b)\,1\mathrm{i}}{2\,c^{9/4}}\right)}{2\,c^{9/4}}+\frac{(-b)^{1/4}\,(A\,c-B\,b)\,\left(\frac{16\,\sqrt{x}\,\left(A\right.}{}\right.}{}}{\frac{(-b)^{1/4}\,(A\,c-B\,b)\,\left(\frac{16\,\sqrt{x}\,\left(A^2\,b^2\,c^2-2\,A\,B\,b^3\,c+B^2\,b^4\right)}{c}-\frac{(-b)^{1/4}\,\left(32\,A\,b^2\,c^2-32\,B\,b^3\,c\right)\,(A\,c-B\,b)\,1\mathrm{i}}{2\,c^{9/4}}\right)\,1\mathrm{i}}{2\,c^{9/4}}-\frac{(-b)^{1/4}\,(A\,c-B\,b)\,\left(\frac{16\,\sqrt{x}\,\left(A\right.}{}\right.}{}}\right.}{c^{9/4}}$$

```
[In] int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] x^(1/2)*((2*A)/c - (2*B*b)/c^2) + (2*B*x^(5/2))/(5*c) - ((-b)^(1/4)*atan(((
(-b)^(1/4)*(A*c - B*b)*((16*x^(1/2)*(B^2*b^4 + A^2*b^2*c^2 - 2*A*B*b^3*c))/
c - ((-b)^(1/4)*(32*A*b^2*c^2 - 32*B*b^3*c)*(A*c - B*b))/(2*c^(9/4)))*1i)/(
2*c^(9/4)) + ((-b)^(1/4)*(A*c - B*b)*((16*x^(1/2)*(B^2*b^4 + A^2*b^2*c^2 -
2*A*B*b^3*c))/c + ((-b)^(1/4)*(32*A*b^2*c^2 - 32*B*b^3*c)*(A*c - B*b))/(2*c
^(9/4)))*1i)/(2*c^(9/4)))/(((-b)^(1/4)*(A*c - B*b)*((16*x^(1/2)*(B^2*b^4 +
A^2*b^2*c^2 - 2*A*B*b^3*c))/c - ((-b)^(1/4)*(32*A*b^2*c^2 - 32*B*b^3*c)*(A*
c - B*b))/(2*c^(9/4))))/(2*c^(9/4)) - ((-b)^(1/4)*(A*c - B*b)*((16*x^(1/2)*
(B^2*b^4 + A^2*b^2*c^2 - 2*A*B*b^3*c))/c + ((-b)^(1/4)*(32*A*b^2*c^2 - 32*B
*b^3*c)*(A*c - B*b))/(2*c^(9/4))))/(2*c^(9/4))))*(A*c - B*b)*1i)/c^(9/4) -
((-b)^(1/4)*atan((((-b)^(1/4)*(A*c - B*b)*((16*x^(1/2)*(B^2*b^4 + A^2*b^2*c
^2 - 2*A*B*b^3*c))/c - ((-b)^(1/4)*(32*A*b^2*c^2 - 32*B*b^3*c)*(A*c - B*b)*
1i)/(2*c^(9/4))))/(2*c^(9/4)) + ((-b)^(1/4)*(A*c - B*b)*((16*x^(1/2)*(B^2*b
^4 + A^2*b^2*c^2 - 2*A*B*b^3*c))/c + ((-b)^(1/4)*(32*A*b^2*c^2 - 32*B*b^3*c
)*(A*c - B*b)*1i)/(2*c^(9/4))))/(2*c^(9/4)))/(((-b)^(1/4)*(A*c - B*b)*((16*
x^(1/2)*(B^2*b^4 + A^2*b^2*c^2 - 2*A*B*b^3*c))/c - ((-b)^(1/4)*(32*A*b^2*c^
2 - 32*B*b^3*c)*(A*c - B*b)*1i)/(2*c^(9/4)))*1i)/(2*c^(9/4)) - ((-b)^(1/4)*
(A*c - B*b)*((16*x^(1/2)*(B^2*b^4 + A^2*b^2*c^2 - 2*A*B*b^3*c))/c + ((-b)^(
1/4)*(32*A*b^2*c^2 - 32*B*b^3*c)*(A*c - B*b)*1i)/(2*c^(9/4)))*1i)/(2*c^(9/4
))))*(A*c - B*b))/c^(9/4)
```

3.187 $\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 237

$$\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2Bx^{3/2}}{3c}+\frac{(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{7/4}} - \frac{(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{7/4}} - \frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}} + \frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}}$$

```
[Out] 2/3*B*x^(3/2)/c+1/2*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^
(1/4)/c^(7/4)*2^(1/2)-1/2*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/
4))/b^(1/4)/c^(7/4)*2^(1/2)-1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(
1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(7/4)*2^(1/2)+1/4*(-A*c+B*b)*ln(b^(1/2)+x*c
^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(7/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used

$= \{1598, 470, 335, 303, 1176, 631, 210, 1179, 642\}$

$$\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{7/4}} - \frac{(bB-Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}c^{7/4}} - \frac{(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}} + \frac{(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}} + \frac{2Bx^{3/2}}{3c}$$

```
[In] Int[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*B*x^(3/2))/(3*c) + ((b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(
1/4)])/(Sqrt[2]*b^(1/4)*c^(7/4)) - ((b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)
*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(1/4)*c^(7/4)) - ((b*B - A*c)*Log[Sqrt[b] -
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(1/4)*c^(7/4)) +
 ((b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(
2*Sqrt[2]*b^(1/4)*c^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
```

```
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{x}(A+Bx^2)}{b+cx^2}\,dx$$

$$= \frac{2Bx^{3/2}}{3c} - \frac{\left(2\left(\frac{3bB}{2}-\frac{3Ac}{2}\right)\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{3c}$$

$$= \frac{2Bx^{3/2}}{3c} - \frac{\left(4\left(\frac{3bB}{2}-\frac{3Ac}{2}\right)\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{3c}$$

$$=\frac{2Bx^{3/2}}{3c}+\frac{(bB-Ac)\text{Subst}\left(\int\frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{c^{3/2}}-\frac{(bB-Ac)\text{Subst}\left(\int\frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{c^{3/2}}$$

$$=\frac{2Bx^{3/2}}{3c}-\frac{(bB-Ac)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{2c^2}$$
$$-\frac{(bB-Ac)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{2c^2}$$
$$-\frac{(bB-Ac)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}}$$
$$-\frac{(bB-Ac)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}}$$

$$=\frac{2Bx^{3/2}}{3c}-\frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}}$$
$$+\frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}}$$
$$-\frac{(bB-Ac)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{7/4}}$$
$$+\frac{(bB-Ac)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{7/4}}$$

$$=\frac{2Bx^{3/2}}{3c}+\frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{7/4}}-\frac{(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{7/4}}$$
$$-\frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}}$$
$$+\frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{7/4}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.57

$$\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2Bx^{3/2}}{3c} + \frac{(bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}\sqrt[4]{b}c^{7/4}} + \frac{(bB-Ac)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}\sqrt[4]{b}c^{7/4}}$$

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*B*x^(3/2))/(3*c) + ((b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^
(1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*b^(1/4)*c^(7/4)) + ((b*B - A*c)*ArcTanh[(
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(1/4)*c
^(7/4))
```

Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.52

method	result	size
derivativedivides	$\frac{2B\,x^{\frac{3}{2}}}{3c}+\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	124
default	$\frac{2B\,x^{\frac{3}{2}}}{3c}+\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	124
risch	$\frac{2B\,x^{\frac{3}{2}}}{3c}+\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	124

```
[In] int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*B*x^(3/2)/c+1/4*(A*c-B*b)/c^2/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4
)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(
1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(
1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 691, normalized size of antiderivative = 2.92

$$\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{4\,Bx^{\frac{3}{2}} + 3\,c\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{bc^7}\right)^{\frac{1}{4}}\log\left(bc^5\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b}{bc^7}\right.\right.}{}$$

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*B*x^(3/2) + 3*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A
^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(1/4)*log(b*c^5*(-(B^4*b^4 - 4*A*B^3*b^3*c +
 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(3/4) - (B^3*b^3 - 3
*A*B^2*b^2*c + 3*A^2*B*b*c^2 - A^3*c^3)*sqrt(x)) - 3*I*c*(-(B^4*b^4 - 4*A*B
^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(1/4)*log(
I*b*c^5*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^
4*c^4)/(b*c^7))^(3/4) - (B^3*b^3 - 3*A*B^2*b^2*c + 3*A^2*B*b*c^2 - A^3*c^3)
*sqrt(x)) + 3*I*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*
b*c^3 + A^4*c^4)/(b*c^7))^(1/4)*log(-I*b*c^5*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6
*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(3/4) - (B^3*b^3 - 3*A
*B^2*b^2*c + 3*A^2*B*b*c^2 - A^3*c^3)*sqrt(x)) - 3*c*(-(B^4*b^4 - 4*A*B^3*b
^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b*c^7))^(1/4)*log(-b*c
^5*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4
)/(b*c^7))^(3/4) - (B^3*b^3 - 3*A*B^2*b^2*c + 3*A^2*B*b*c^2 - A^3*c^3)*sqrt
(x)))/c
```

Sympy [A] (verification not implemented)

Time = 37.34 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.28

$$\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{3}{2}}}{3}\right) \\ \frac{-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{3}{2}}}{3}}{c} \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{7}{2}}}{7}}{b} \\ \frac{2A\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{c\sqrt[4]{-\frac{b}{c}}} - \frac{A\left(-\frac{b}{c}\right)^{\frac{3}{4}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b} + \frac{A\left(-\frac{b}{c}\right)^{\frac{3}{4}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b} + \frac{A\left(-\frac{b}{c}\right)^{\frac{3}{4}}\mathrm{atan}\left(\right.}{b} \end{cases}$$

```
[In] integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*(-2*A/sqrt(x) + 2*B*x**(3/2)/3), Eq(b, 0) & Eq(c, 0)), ((-2*
A/sqrt(x) + 2*B*x**(3/2)/3)/c, Eq(b, 0)), ((2*A*x**(3/2)/3 + 2*B*x**(7/2)/7
)/b, Eq(c, 0)), (2*A*atan(sqrt(x)/(-b/c)**(1/4))/(c*(-b/c)**(1/4)) - A*(-b/
c)**(3/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*b) + A*(-b/c)**(3/4)*log(sqrt(x)
+ (-b/c)**(1/4))/(2*b) + A*(-b/c)**(3/4)*atan(sqrt(x)/(-b/c)**(1/4))/b - 2*
B*b*atan(sqrt(x)/(-b/c)**(1/4))/(c**2*(-b/c)**(1/4)) + 2*B*x**(3/2)/(3*c) +
 B*(-b/c)**(3/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*c) - B*(-b/c)**(3/4)*log(s
qrt(x) + (-b/c)**(1/4))/(2*c) - B*(-b/c)**(3/4)*atan(sqrt(x)/(-b/c)**(1/4))
/c, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.82

$$\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2\,Bx^{\frac{3}{2}}}{3\,c} - \frac{(Bb-Ac)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \cdots\right)}{4\,c}$$

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2/3*B*x^(3/2)/c - 1/4*(B*b - A*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^
(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqr
t(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2
*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) -
sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)
*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt
(b))/(b^(1/4)*c^(3/4)))/c
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.06

$$\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2\,Bx^{\frac{3}{2}}}{3\,c} - \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}\,Bb-(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^4} - \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}\,Bb-(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^4} + \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}\,Bb-(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc^4} - \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}\,Bb-(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc^4}$$

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/c - 1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arcta
n(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) - 1/2*
sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) + 1/4*sqrt(2)*((b*c^3)^(3/4
)*B*b - (b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))
/(b*c^4) - 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-sqrt(2)
*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.30

$$\int \frac{x^{5/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2\,B\,x^{3/2}}{3\,c} + \frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(A\,c-B\,b)}{(-b)^{1/4}\,c^{7/4}} - \frac{\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(A\,c-B\,b)}{(-b)^{1/4}\,c^{7/4}}$$

```
[In] int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] (2*B*x^(3/2))/(3*c) + (atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/((-b
)^(1/4)*c^(7/4)) - (atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/((-b)^
(1/4)*c^(7/4))
```

3.188 $\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 235

$$\begin{aligned}\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4}\,dx &= \frac{2B\sqrt{x}}{c}+\frac{(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{3/4}c^{5/4}} \\ &\quad-\frac{(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{3/4}c^{5/4}}+\frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}c^{5/4}} \\ &\quad-\frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}c^{5/4}}\end{aligned}$$

```
[Out] 1/2*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^(5/4)*2^
(1/2)-1/2*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^(5
/4)*2^(1/2)+1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(
1/2))/b^(3/4)/c^(5/4)*2^(1/2)-1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c
^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(5/4)*2^(1/2)+2*B*x^(1/2)/c
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used

$= \{1598, 470, 335, 217, 1179, 642, 1176, 631, 210\}$

$$\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{3/4}c^{5/4}} - \frac{(bB-Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}b^{3/4}c^{5/4}} + \frac{(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}c^{5/4}} - \frac{(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}c^{5/4}} + \frac{2B\sqrt{x}}{c}$$

```
[In] Int[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*B*Sqrt[x])/c + ((b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)
])/(Sqrt[2]*b^(3/4)*c^(5/4)) - ((b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqr
t[x])/b^(1/4)])/(Sqrt[2]*b^(3/4)*c^(5/4)) + ((b*B - A*c)*Log[Sqrt[b] - Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(3/4)*c^(5/4)) - ((b
*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sq
rt[2]*b^(3/4)*c^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
```

```
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{\sqrt{x}\,(b+cx^2)}\,dx \\
&= \frac{2B\sqrt{x}}{c} - \frac{\left(2\left(\frac{bB}{2}-\frac{Ac}{2}\right)\right)\int \frac{1}{\sqrt{x}(b+cx^2)}\,dx}{c} \\
&= \frac{2B\sqrt{x}}{c} - \frac{\left(4\left(\frac{bB}{2}-\frac{Ac}{2}\right)\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{c}
\end{aligned}$$

$$=\frac{2B\sqrt{x}}{c}-\frac{(bB-Ac)\text{Subst}\left(\int\frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{\sqrt{b}c}-\frac{(bB-Ac)\text{Subst}\left(\int\frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{\sqrt{b}c}$$

$$=\frac{2B\sqrt{x}}{c}-\frac{(bB-Ac)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{2\sqrt{b}c^{3/2}}-\frac{(bB-Ac)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{2\sqrt{b}c^{3/2}}+\frac{(bB-Ac)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{2\sqrt{2}b^{3/4}c^{5/4}}+\frac{(bB-Ac)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{2\sqrt{2}b^{3/4}c^{5/4}}$$

$$=\frac{2B\sqrt{x}}{c}+\frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}c^{5/4}}-\frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}c^{5/4}}-\frac{(bB-Ac)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{3/4}c^{5/4}}+\frac{(bB-Ac)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{3/4}c^{5/4}}$$

$$=\frac{2B\sqrt{x}}{c}+\frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{3/4}c^{5/4}}-\frac{(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{3/4}c^{5/4}}+\frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}c^{5/4}}-\frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}c^{5/4}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.57

$$\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4}\,dx=\frac{2B\sqrt{x}}{c}+\frac{(bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}b^{3/4}c^{5/4}}-\frac{(bB-Ac)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}b^{3/4}c^{5/4}}$$

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (2*B*Sqrt[x])/c + ((b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4
)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*b^(3/4)*c^(5/4)) - ((b*B - A*c)*ArcTanh[(Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(3/4)*c^(5/
4))
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.54

method	result	size
derivativedivides	$\frac{2B\sqrt{x}}{c}+\frac{(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4cb}$	127
default	$\frac{2B\sqrt{x}}{c}+\frac{(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4cb}$	127
risch	$\frac{2B\sqrt{x}}{c}+\frac{(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4cb}$	127

```
[In] int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*B*x^(1/2)/c+1/4*(A*c-B*b)/c*(1/c*b)^(1/4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)*
x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/
2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/
4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.43

$$\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{c\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^3c^5}\right)^{\frac{1}{4}}\log\left(bc\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+}{b^3c^5}\right.\right.}{}$$

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4
*c^4)/(b^3*c^5))^(1/4)*log(b*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c
^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^5))^(1/4) - (B*b - A*c)*sqrt(x)) + I*c
*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/
(b^3*c^5))^(1/4)*log(I*b*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 -
 4*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^5))^(1/4) - (B*b - A*c)*sqrt(x)) - I*c*(-(
B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^3
*c^5))^(1/4)*log(-I*b*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*
A^3*B*b*c^3 + A^4*c^4)/(b^3*c^5))^(1/4) - (B*b - A*c)*sqrt(x)) - c*(-(B^4*b
^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^5)
)^(1/4)*log(-b*c*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b
*c^3 + A^4*c^4)/(b^3*c^5))^(1/4) - (B*b - A*c)*sqrt(x)) + 4*B*sqrt(x))/c
```

Sympy [A] (verification not implemented)

Time = 15.75 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.01

$$\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}\right) \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}}{c} \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}}{b} \\ -\frac{A\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b}+\frac{A\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b}+\frac{A\sqrt[4]{-\frac{b}{c}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b}+\frac{2B\sqrt{x}}{c}+ \end{cases}$$

```
[In] integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(3*x**(3/2)) + 2*B*sqrt(x)), Eq(b, 0) & Eq(c, 0)), ((-
2*A/(3*x**(3/2)) + 2*B*sqrt(x))/c, Eq(b, 0)), ((2*A*sqrt(x) + 2*B*x**(5/2)/
5)/b, Eq(c, 0)), (-A*(-b/c)**(1/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*b) + A*(
-b/c)**(1/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*b) + A*(-b/c)**(1/4)*atan(sqrt
```

```
(x)/(-b/c)**(1/4))/b + 2*B*sqrt(x)/c + B*(-b/c)**(1/4)*log(sqrt(x) - (-b/c)
**(1/4))/(2*c) - B*(-b/c)**(1/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*c) - B*(-b
/c)**(1/4)*atan(sqrt(x)/(-b/c)**(1/4))/c, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.93

$$\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2\,B\sqrt{x}}{c}$$
$$-\frac{\frac{2\,\sqrt{2}(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\,\sqrt{2}(Bb-Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}(Bb-Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{4\,c}$$

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 2*B*sqrt(x)/c - 1/4*(2*sqrt(2)*(B*b - A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1
/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(
b)*sqrt(c))) + 2*sqrt(2)*(B*b - A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c
^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sq
rt(c))) + sqrt(2)*(B*b - A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)
*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(B*b - A*c)*log(-sqrt(2)*b^(1/4)*
c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/c
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.07

$$\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2\,B\sqrt{x}}{c} - \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bb-\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^2} - \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bb-\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^2} - \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bb-\left(bc^3\right)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc^2} + \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bb-\left(bc^3\right)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc^2}$$

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/c - 1/2*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(
1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^2) - 1/2*sq
rt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*
(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^2) - 1/4*sqrt(2)*((b*c^3)^(1/4)*
B*b - (b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(
b*c^2) + 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(-sqrt(2)*s
qrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^2)
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 739, normalized size of antiderivative = 3.14

$$\int \frac{x^{3/2}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{2B\sqrt{x}}{c} - \frac{\operatorname{atan}\left(\frac{\frac{(Ac-Bb)\left(\sqrt{x}\left(16A^2c^3-32ABbc^2+16B^2b^2c\right)-\frac{\left(32Bb^2c^2-32Abc^3\right)(Ac-Bb)}{2(-b)^{3/4}c^{5/4}}\right)1i}{2(-b)^{3/4}c^{5/4}}+\frac{(Ac-Bb)\left(\sqrt{x}\left(16A^2c^3-32ABbc^2+16B^2b^2c\right)+\cdots\right.}{2(-b)^{3/4}c^{5/4}}}{\frac{(Ac-Bb)\left(\sqrt{x}\left(16A^2c^3-32ABbc^2+16B^2b^2c\right)-\frac{\left(32Bb^2c^2-32Abc^3\right)(Ac-Bb)}{2(-b)^{3/4}c^{5/4}}\right)}{2(-b)^{3/4}c^{5/4}}-\frac{(Ac-Bb)\left(\sqrt{x}\left(16A^2c^3-32ABbc^2+16B^2b^2c\right)+\cdots\right.}{2(-b)^{3/4}c^{5/4}}}\cdots\right)}{(-b)^{3/4}c^{5/4}} - \frac{\operatorname{atan}\left(\frac{\frac{(Ac-Bb)\left(\sqrt{x}\left(16A^2c^3-32ABbc^2+16B^2b^2c\right)-\frac{\left(32Bb^2c^2-32Abc^3\right)(Ac-Bb)1i}{2(-b)^{3/4}c^{5/4}}\right)}{2(-b)^{3/4}c^{5/4}}+\frac{(Ac-Bb)\left(\sqrt{x}\left(16A^2c^3-32ABbc^2+16B^2b^2c\right)+\cdots\right.}{2(-b)^{3/4}c^{5/4}}}{\frac{(Ac-Bb)\left(\sqrt{x}\left(16A^2c^3-32ABbc^2+16B^2b^2c\right)-\frac{\left(32Bb^2c^2-32Abc^3\right)(Ac-Bb)1i}{2(-b)^{3/4}c^{5/4}}\right)1i}{2(-b)^{3/4}c^{5/4}}-\frac{(Ac-Bb)\left(\sqrt{x}\left(16A^2c^3-32ABbc^2+16B^2b^2c\right)+\cdots\right.}{2(-b)^{3/4}c^{5/4}}}\cdots\right)}{(-b)^{3/4}c^{5/4}}$$

```
[In] int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] (2*B*x^(1/2))/c - (atan((((A*c - B*b)*(x^(1/2)*(16*A^2*c^3 + 16*B^2*b^2*c -
 32*A*B*b*c^2) - ((32*B*b^2*c^2 - 32*A*b*c^3)*(A*c - B*b))/(2*(-b)^(3/4)*c^
(5/4)))*1i)/(2*(-b)^(3/4)*c^(5/4)) + ((A*c - B*b)*(x^(1/2)*(16*A^2*c^3 + 16
*B^2*b^2*c - 32*A*B*b*c^2) + ((32*B*b^2*c^2 - 32*A*b*c^3)*(A*c - B*b))/(2*(
-b)^(3/4)*c^(5/4)))*1i)/(2*(-b)^(3/4)*c^(5/4)))/(((A*c - B*b)*(x^(1/2)*(16*
A^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) - ((32*B*b^2*c^2 - 32*A*b*c^3)*(A*c
- B*b))/(2*(-b)^(3/4)*c^(5/4))))/(2*(-b)^(3/4)*c^(5/4)) - ((A*c - B*b)*(x^(
1/2)*(16*A^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) + ((32*B*b^2*c^2 - 32*A*b*c
^3)*(A*c - B*b))/(2*(-b)^(3/4)*c^(5/4))))/(2*(-b)^(3/4)*c^(5/4))))*(A*c - B
*b)*1i)/((-b)^(3/4)*c^(5/4)) - (atan((((A*c - B*b)*(x^(1/2)*(16*A^2*c^3 + 1
6*B^2*b^2*c - 32*A*B*b*c^2) - ((32*B*b^2*c^2 - 32*A*b*c^3)*(A*c - B*b)*1i)/
(2*(-b)^(3/4)*c^(5/4))))/(2*(-b)^(3/4)*c^(5/4)) + ((A*c - B*b)*(x^(1/2)*(16
*A^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) + ((32*B*b^2*c^2 - 32*A*b*c^3)*(A*c
 - B*b)*1i)/(2*(-b)^(3/4)*c^(5/4))))/(2*(-b)^(3/4)*c^(5/4)))/(((A*c - B*b)*
(x^(1/2)*(16*A^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) - ((32*B*b^2*c^2 - 32*A
*b*c^3)*(A*c - B*b)*1i)/(2*(-b)^(3/4)*c^(5/4)))*1i)/(2*(-b)^(3/4)*c^(5/4))
- ((A*c - B*b)*(x^(1/2)*(16*A^2*c^3 + 16*B^2*b^2*c - 32*A*B*b*c^2) + ((32*B
*b^2*c^2 - 32*A*b*c^3)*(A*c - B*b)*1i)/(2*(-b)^(3/4)*c^(5/4)))*1i)/(2*(-b)^
(3/4)*c^(5/4))))*(A*c - B*b))/((-b)^(3/4)*c^(5/4))
```

3.189 $\int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 235

$$\int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{2A}{b\sqrt{x}} - \frac{(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{5/4}c^{3/4}} + \frac{(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{5/4}c^{3/4}} + \frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}} - \frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}}$$

```
[Out] -1/2*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^(3/4)*2
^(1/2)+1/2*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^(
3/4)*2^(1/2)+1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^
(1/2))/b^(5/4)/c^(3/4)*2^(1/2)-1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*
c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)/c^(3/4)*2^(1/2)-2*A/b/x^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1598, 464, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{5/4}c^{3/4}} + \frac{(bB-Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}b^{5/4}c^{3/4}} + \frac{(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}} - \frac{(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}} - \frac{2A}{b\sqrt{x}}$$

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*A)/(b*Sqrt[x]) - ((b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(Sqrt[2]*b^(5/4)*c^(3/4)) + ((b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*
Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(5/4)*c^(3/4)) + ((b*B - A*c)*Log[Sqrt[b] - S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(5/4)*c^(3/4)) -
((b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2
*Sqrt[2]*b^(5/4)*c^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{x^{3/2}\left(b+cx^2\right)}\,dx \\
&= -\frac{2A}{b\sqrt{x}} - \frac{\left(2\left(-\frac{bB}{2}+\frac{Ac}{2}\right)\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b} \\
&= -\frac{2A}{b\sqrt{x}} - \frac{\left(4\left(-\frac{bB}{2}+\frac{Ac}{2}\right)\right)\operatorname{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b} \\
&= -\frac{2A}{b\sqrt{x}} - \frac{(bB-Ac)\operatorname{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b\sqrt{c}} + \frac{(bB-Ac)\operatorname{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b\sqrt{c}} \\
&= -\frac{2A}{b\sqrt{x}} + \frac{(bB-Ac)\operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2bc} \\
&\quad + \frac{(bB-Ac)\operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2bc} \\
&\quad + \frac{(bB-Ac)\operatorname{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}b^{5/4}c^{3/4}} \\
&\quad + \frac{(bB-Ac)\operatorname{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}b^{5/4}c^{3/4}} \\
&= -\frac{2A}{b\sqrt{x}} + \frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}} \\
&\quad - \frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}} \\
&\quad + \frac{(bB-Ac)\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{5/4}c^{3/4}} \\
&\quad - \frac{(bB-Ac)\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{5/4}c^{3/4}}
\end{aligned}
$$

$$= -\frac{2A}{b\sqrt{x}} - \frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{5/4}c^{3/4}} + \frac{(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{5/4}c^{3/4}} + \frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}} - \frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}c^{3/4}}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{2A}{b\sqrt{x}} - \frac{(bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}b^{5/4}c^{3/4}} - \frac{(bB-Ac)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}b^{5/4}c^{3/4}}$$

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4),x]
[Out] (-2*A)/(b*Sqrt[x]) - ((b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*b^(5/4)*c^(3/4)) - ((b*B - A*c)*ArcTanh[(S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(5/4)*c^
(3/4))
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.54

method	result	size
derivativedivides	$-\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4bc\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{2A}{b\sqrt{x}}$	127
default	$-\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4bc\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{2A}{b\sqrt{x}}$	127
risch	$-\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4bc\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\frac{2A}{b\sqrt{x}}$	127

```
[In] int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(A*c-B*b)/b/c/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/
2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan
(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)
)-2*A/b/x^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 705, normalized size of antiderivative = 3.00

$$\int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{bx\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^5c^3}\right)^{\frac{1}{4}}\log\left(b^4c^2\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^5c^3}\right)^{\frac{3}{4}}-(B^3\right.}{}$$

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 +
A^4*c^4)/(b^5*c^3))^(1/4)*log(b^4*c^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^
2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3))^(3/4) - (B^3*b^3 - 3*A*B^2*
b^2*c + 3*A^2*B*b*c^2 - A^3*c^3)*sqrt(x)) - I*b*x*(-(B^4*b^4 - 4*A*B^3*b^3*
c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3))^(1/4)*log(I*b^4
*c^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c
^4)/(b^5*c^3))^(3/4) - (B^3*b^3 - 3*A*B^2*b^2*c + 3*A^2*B*b*c^2 - A^3*c^3)*
sqrt(x)) + I*b*x*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b
*c^3 + A^4*c^4)/(b^5*c^3))^(1/4)*log(-I*b^4*c^2*(-(B^4*b^4 - 4*A*B^3*b^3*c
+ 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3))^(3/4) - (B^3*b^3
- 3*A*B^2*b^2*c + 3*A^2*B*b*c^2 - A^3*c^3)*sqrt(x)) - b*x*(-(B^4*b^4 - 4*A*
B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^3))^(1/4)*l
og(-b^4*c^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3
+ A^4*c^4)/(b^5*c^3))^(3/4) - (B^3*b^3 - 3*A*B^2*b^2*c + 3*A^2*B*b*c^2 - A^
3*c^3)*sqrt(x)) + 4*A*sqrt(x))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 8.34 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{5x^{\frac{5}{2}}}-\frac{2B}{\sqrt{x}}\right) \\ \frac{-\frac{2A}{5x^{\frac{5}{2}}}-\frac{2B}{\sqrt{x}}}{c} \\ \frac{-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{3}{2}}}{3}}{b} \\ -\frac{2A\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b\sqrt[4]{-\frac{b}{c}}}-\frac{2A}{b\sqrt{x}}+\frac{Ac\left(-\frac{b}{c}\right)^{\frac{3}{4}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b^2}-\frac{Ac\left(-\frac{b}{c}\right)^{\frac{3}{4}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b^2}-\frac{Ac\left(-\frac{b}{c}\right)^{\frac{3}{4}}\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b^2}+ \end{cases}$$

```
[In] integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(5*x**(5/2)) - 2*B/sqrt(x)), Eq(b, 0) & Eq(c, 0)), ((-
2*A/(5*x**(5/2)) - 2*B/sqrt(x))/c, Eq(b, 0)), ((-2*A/sqrt(x) + 2*B*x**(3/2)
/3)/b, Eq(c, 0)), (-2*A*atan(sqrt(x)/(-b/c)**(1/4))/(b*(-b/c)**(1/4)) - 2*A
/(b*sqrt(x)) + A*c*(-b/c)**(3/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*b**2) - A*
c*(-b/c)**(3/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*b**2) - A*c*(-b/c)**(3/4)*a
tan(sqrt(x)/(-b/c)**(1/4))/b**2 + 2*B*atan(sqrt(x)/(-b/c)**(1/4))/(c*(-b/c)
**(1/4)) - B*(-b/c)**(3/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*b) + B*(-b/c)**(
3/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*b) + B*(-b/c)**(3/4)*atan(sqrt(x)/(-b/
c)**(1/4))/b, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{(Bb-Ac)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}}{}\right)}{4b} - \frac{2A}{b\sqrt{x}}$$

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*(B*b - A*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*
sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2
*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/
sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2
)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2
)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3
/4)))/b - 2*A/(b*sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{x}(A+Bx^2)}{bx^2+cx^4}\,dx = -\frac{2A}{b\sqrt{x}} + \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^2c^3} + \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^2c^3} - \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4b^2c^3} + \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4b^2c^3}$$

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -2*A/(b*sqrt(x)) + 1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arct
an(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) + 1
/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) - 1/4*sqrt(2)*((b*c^3)
^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(
b/c))/(b^2*c^3) + 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-
sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{x}(A + Bx^2)}{bx^2 + cx^4}\,dx = \frac{\mathrm{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)\,(A\,c - B\,b)}{(-b)^{5/4}\,c^{3/4}} - \frac{2\,A}{b\,\sqrt{x}} - \frac{\mathrm{atanh}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)\,(A\,c - B\,b)}{(-b)^{5/4}\,c^{3/4}}$$

```
[In] int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] (atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/((-b)^(5/4)*c^(3/4)) - (2*
A)/(b*x^(1/2)) - (atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c - B*b))/((-b)^(5
/4)*c^(3/4))
```

3.190 $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 237

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)}\,dx = -\frac{2A}{3bx^{3/2}} - \frac{(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{7/4}\sqrt[4]{c}} + \frac{(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{7/4}\sqrt[4]{c}} - \frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}\sqrt[4]{c}} + \frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}\sqrt[4]{c}}$$

```
[Out] -2/3*A/b/x^(3/2)-1/2*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b
^(7/4)/c^(1/4)*2^(1/2)+1/2*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1
/4))/b^(7/4)/c^(1/4)*2^(1/2)-1/4*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^
(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(1/4)*2^(1/2)+1/4*(-A*c+B*b)*ln(b^(1/2)+x*
c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(1/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1598, 464, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)}\,dx = -\frac{(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{7/4}\sqrt[4]{c}} + \frac{(bB-Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}b^{7/4}\sqrt[4]{c}} - \frac{(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}\sqrt[4]{c}} + \frac{(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}\sqrt[4]{c}} - \frac{2A}{3bx^{3/2}}$$

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*A)/(3*b*x^(3/2)) - ((b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^
(1/4)])/(Sqrt[2]*b^(7/4)*c^(1/4)) + ((b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(7/4)*c^(1/4)) - ((b*B - A*c)*Log[Sqrt[b] -
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(7/4)*c^(1/4))
+ ((b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/
(2*Sqrt[2]*b^(7/4)*c^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{A+Bx^2}{x^{5/2}\,(b+cx^2)}\,dx$$

$$= -\frac{2A}{3bx^{3/2}} - \frac{\left(2\left(-\frac{3bB}{2}+\frac{3Ac}{2}\right)\right)\int \frac{1}{\sqrt{x}(b+cx^2)}\,dx}{3b}$$

$$= -\frac{2A}{3bx^{3/2}} - \frac{\left(4\left(-\frac{3bB}{2}+\frac{3Ac}{2}\right)\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{3b}$$

$$= -\frac{2A}{3bx^{3/2}} + \frac{(bB-Ac)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}} + \frac{(bB-Ac)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}}$$

$$= -\frac{2A}{3bx^{3/2}} + \frac{(bB-Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{3/2}\sqrt{c}}$$
$$+ \frac{(bB-Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{3/2}\sqrt{c}}$$
$$- \frac{(bB-Ac)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}b^{7/4}\sqrt[4]{c}}$$
$$- \frac{(bB-Ac)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}b^{7/4}\sqrt[4]{c}}$$

$$= -\frac{2A}{3bx^{3/2}} - \frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}\sqrt[4]{c}}$$
$$+ \frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}\sqrt[4]{c}}$$
$$+ \frac{(bB-Ac)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{7/4}\sqrt[4]{c}}$$
$$- \frac{(bB-Ac)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{7/4}\sqrt[4]{c}}$$

$$= -\frac{2A}{3bx^{3/2}} - \frac{(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{7/4}\sqrt[4]{c}} + \frac{(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{7/4}\sqrt[4]{c}}$$
$$- \frac{(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}\sqrt[4]{c}}$$
$$+ \frac{(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}\sqrt[4]{c}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.57

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)}\,dx = -\frac{2A}{3bx^{3/2}} - \frac{(bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}b^{7/4}\sqrt[4]{c}} + \frac{(bB-Ac)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}b^{7/4}\sqrt[4]{c}}$$

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)),x]
[Out] (-2*A)/(3*b*x^(3/2)) - ((b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*b^(7/4)*c^(1/4)) + ((b*B - A*c)*ArcTanh[
(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(7/4)*
c^(1/4))
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.52

method	result	size
derivativedivides	$\frac{(-Ac+Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^2}-\frac{2A}{3b\,x^{\frac{3}{2}}}$	124
default	$\frac{(-Ac+Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^2}-\frac{2A}{3b\,x^{\frac{3}{2}}}$	124
risch	$-\frac{2A}{3b\,x^{\frac{3}{2}}}-\frac{(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^2}$	124

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-A*c+B*b)/b^2*(1/c*b)^(1/4)*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/
2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan
(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)
)-2/3*A/b/x^(3/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 590, normalized size of antiderivative = 2.49

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)}\,dx = -\frac{3\,bx^2\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^7c}\right)^{\frac{1}{4}}\log\left(b^2\left(-\frac{B^4b^4-4\,AB^3b^3c+6\,A^2B^2b^2c^2-4\,A^3Bbc^3+A^4c^4}{b^7c}\right)^{\frac{1}{4}}-(Bb\,\cdots\right.}{\cdots}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*x^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^
3 + A^4*c^4)/(b^7*c))^(1/4)*log(b^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*
b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(1/4) - (B*b - A*c)*sqrt(x)) +
3*I*b*x^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 +
A^4*c^4)/(b^7*c))^(1/4)*log(I*b^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^
2*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(1/4) - (B*b - A*c)*sqrt(x)) - 3*
I*b*x^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^
4*c^4)/(b^7*c))^(1/4)*log(-I*b^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2
*c^2 - 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(1/4) - (B*b - A*c)*sqrt(x)) - 3*b
*x^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2 - 4*A^3*B*b*c^3 + A^4*c
^4)/(b^7*c))^(1/4)*log(-b^2*(-(B^4*b^4 - 4*A*B^3*b^3*c + 6*A^2*B^2*b^2*c^2
- 4*A^3*B*b*c^3 + A^4*c^4)/(b^7*c))^(1/4) - (B*b - A*c)*sqrt(x)) + 4*A*sqrt
(x))/(b*x^2)
```

Sympy [A] (verification not implemented)

Time = 13.82 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.08

$$\int \frac{A + Bx^2}{\sqrt{x}\,(bx^2 + cx^4)}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(-\frac{2A}{7x^{\frac{7}{2}}} - \frac{2B}{3x^{\frac{3}{2}}}\right) \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}} - \frac{2B}{3x^{\frac{3}{2}}}}{c} \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}} + 2B\sqrt{x}}{b} \\ -\frac{2A}{3bx^{\frac{3}{2}}} + \frac{Ac\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{b}{c}}\right)}{2b^2} - \frac{Ac\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{b}{c}}\right)}{2b^2} - \frac{Ac\sqrt[4]{-\frac{b}{c}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b^2} - \frac{B\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x} - \sqrt[4]{\ldots}\right.}{2b} \end{cases}$$

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2))), Eq(b, 0) & Eq(c, 0))
, ((-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2)))/c, Eq(b, 0)), ((-2*A/(3*x**(3/2))
 + 2*B*sqrt(x))/b, Eq(c, 0)), (-2*A/(3*b*x**(3/2)) + A*c*(-b/c)**(1/4)*log(
sqrt(x) - (-b/c)**(1/4))/(2*b**2) - A*c*(-b/c)**(1/4)*log(sqrt(x) + (-b/c)*
*(1/4))/(2*b**2) - A*c*(-b/c)**(1/4)*atan(sqrt(x)/(-b/c)**(1/4))/b**2 - B*(
-b/c)**(1/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*b) + B*(-b/c)**(1/4)*log(sqrt(
x) + (-b/c)**(1/4))/(2*b) + B*(-b/c)**(1/4)*atan(sqrt(x)/(-b/c)**(1/4))/b,
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.92

$$\int \frac{A + Bx^2}{\sqrt{x}\,(bx^2 + cx^4)}\,dx$$
$$= \frac{\frac{2\sqrt{2}(Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}(Bb-Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}(Bb-Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\ldots\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{4b}$$
$$- \frac{2A}{3bx^{\frac{3}{2}}}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*(B*b - A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*
sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2
*sqrt(2)*(B*b - A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(
c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2
)*(B*b - A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b
^(3/4)*c^(1/4)) - sqrt(2)*(B*b - A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x)
+ sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b - 2/3*A/(b*x^(3/2))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)}\,dx = \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^2c} + \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^2c} + \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^2c} - \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^2c} - \frac{2\,A}{3\,bx^{\frac{3}{2}}}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c) + 1/2*sqrt(2)*((b*c^3)^(
1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*
sqrt(x))/(b/c)^(1/4))/(b^2*c) + 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1
/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c) - 1/4*sqr
t(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/
4) + x + sqrt(b/c))/(b^2*c) - 2/3*A/(b*x^(3/2))
```

Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 811, normalized size of antiderivative = 3.42

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)}\,dx = -\frac{2\,A}{3\,b\,x^{3/2}}$$

$$-\frac{\operatorname{atan}\left(\frac{\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,b^3\,c^5-32\,A\,B\,b^4\,c^4+16\,B^2\,b^5\,c^3\right)-\frac{(A\,c-B\,b)\left(32\,A\,b^5\,c^4-32\,B\,b^6\,c^3\right)}{2\,(-b)^{7/4}\,c^{1/4}}\right)1\mathrm{i}}{2\,(-b)^{7/4}\,c^{1/4}}+\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,b^3\,c^5-32\,A\,B\,b^4\,c^4+16\,\cdots\right.\right.}{2\,(-b)\cdots}}{\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,b^3\,c^5-32\,A\,B\,b^4\,c^4+16\,B^2\,b^5\,c^3\right)-\frac{(A\,c-B\,b)\left(32\,A\,b^5\,c^4-32\,B\,b^6\,c^3\right)}{2\,(-b)^{7/4}\,c^{1/4}}\right)}{2\,(-b)^{7/4}\,c^{1/4}}-\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,b^3\,c^5-32\,A\,B\,b^4\,c^4+16\,\cdots\right.\right.}{2\,(-b)^{7}\cdots}}\right.\cdots}{(-b)^{7/4}\,c^{1/4}}$$

$$-\frac{\operatorname{atan}\left(\frac{\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,b^3\,c^5-32\,A\,B\,b^4\,c^4+16\,B^2\,b^5\,c^3\right)-\frac{(A\,c-B\,b)\left(32\,A\,b^5\,c^4-32\,B\,b^6\,c^3\right)1\mathrm{i}}{2\,(-b)^{7/4}\,c^{1/4}}\right)}{2\,(-b)^{7/4}\,c^{1/4}}+\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,b^3\,c^5-32\,A\,B\,b^4\,c^4+1\cdots\right.\right.}{2\,(-\cdots}}{\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,b^3\,c^5-32\,A\,B\,b^4\,c^4+16\,B^2\,b^5\,c^3\right)-\frac{(A\,c-B\,b)\left(32\,A\,b^5\,c^4-32\,B\,b^6\,c^3\right)1\mathrm{i}}{2\,(-b)^{7/4}\,c^{1/4}}\right)1\mathrm{i}}{2\,(-b)^{7/4}\,c^{1/4}}-\frac{(A\,c-B\,b)\left(\sqrt{x}\left(16\,A^2\,b^3\,c^5-32\,A\,B\,b^4\,c^4+1\cdots\right.\right.}{2\,(\cdots}}\right.\cdots}{(-b)^{7/4}\,c^{1/4}}$$

```
[In] int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] - (2*A)/(3*b*x^(3/2)) - (atan((((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B
^2*b^5*c^3 - 32*A*B*b^4*c^4) - ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3))/
(2*(-b)^(7/4)*c^(1/4)))*1i)/(2*(-b)^(7/4)*c^(1/4)) + ((A*c - B*b)*(x^(1/2)*
(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 - 32*A*B*b^4*c^4) + ((A*c - B*b)*(32*A*b^5
*c^4 - 32*B*b^6*c^3))/(2*(-b)^(7/4)*c^(1/4)))*1i)/(2*(-b)^(7/4)*c^(1/4)))/(
((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 - 32*A*B*b^4*c^4) -
((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3))/(2*(-b)^(7/4)*c^(1/4))))/(2*(-b
)^(7/4)*c^(1/4)) - ((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 -
 32*A*B*b^4*c^4) + ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3))/(2*(-b)^(7/4
)*c^(1/4))))/(2*(-b)^(7/4)*c^(1/4))))*(A*c - B*b)*1i)/((-b)^(7/4)*c^(1/4))
- (atan((((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 - 32*A*B*b^
4*c^4) - ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3)*1i)/(2*(-b)^(7/4)*c^(1/
4))))/(2*(-b)^(7/4)*c^(1/4)) + ((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B
^2*b^5*c^3 - 32*A*B*b^4*c^4) + ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3)*1
i)/(2*(-b)^(7/4)*c^(1/4))))/(2*(-b)^(7/4)*c^(1/4)))/(((A*c - B*b)*(x^(1/2)*
(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 - 32*A*B*b^4*c^4) - ((A*c - B*b)*(32*A*b^5
*c^4 - 32*B*b^6*c^3)*1i)/(2*(-b)^(7/4)*c^(1/4)))*1i)/(2*(-b)^(7/4)*c^(1/4))
 - ((A*c - B*b)*(x^(1/2)*(16*A^2*b^3*c^5 + 16*B^2*b^5*c^3 - 32*A*B*b^4*c^4)
 + ((A*c - B*b)*(32*A*b^5*c^4 - 32*B*b^6*c^3)*1i)/(2*(-b)^(7/4)*c^(1/4)))*1
i)/(2*(-b)^(7/4)*c^(1/4))))*(A*c - B*b))/((-b)^(7/4)*c^(1/4))
```

3.191 $\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 255

$$\begin{aligned}\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)}\,dx &= -\frac{2A}{5bx^{5/2}}-\frac{2(bB-Ac)}{b^2\sqrt{x}}\\ &+\frac{\sqrt[4]{c}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{9/4}}-\frac{\sqrt[4]{c}(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{9/4}}\\ &-\frac{\sqrt[4]{c}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}}\\ &+\frac{\sqrt[4]{c}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}}\end{aligned}$$

```
[Out] -2/5*A/b/x^(5/2)+1/2*c^(1/4)*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^
(1/4))/b^(9/4)*2^(1/2)-1/2*c^(1/4)*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1
/2)/b^(1/4))/b^(9/4)*2^(1/2)-1/4*c^(1/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^
(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)*2^(1/2)+1/4*c^(1/4)*(-A*c+B*b)*ln(b^
(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)*2^(1/2)-2*(-A*c+B*
b)/b^2/x^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1598, 464, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)}\,dx = {} & \frac{\sqrt[4]{c}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{9/4}} \\ & -\frac{\sqrt[4]{c}(bB-Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}b^{9/4}} \\ & -\frac{\sqrt[4]{c}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}} \\ & +\frac{\sqrt[4]{c}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}}-\frac{2(bB-Ac)}{b^2\sqrt{x}}-\frac{2A}{5bx^{5/2}}\end{aligned}$$

```
[In] Int[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*A)/(5*b*x^(5/2)) - (2*(b*B - A*c))/(b^2*Sqrt[x]) + (c^(1/4)*(b*B - A*c)
*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/4)) - (c^(1/4
)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/
4)) - (c^(1/4)*(b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
Sqrt[c]*x])/(2*Sqrt[2]*b^(9/4)) + (c^(1/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{A+Bx^2}{x^{7/2}(b+cx^2)}\,dx$$

$$= -\frac{2A}{5bx^{5/2}} - \frac{\left(2\left(-\frac{5bB}{2}+\frac{5Ac}{2}\right)\right)\int \frac{1}{x^{3/2}(b+cx^2)}\,dx}{5b}$$

$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} - \frac{(c(bB-Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b^2}$$

$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} - \frac{(2c(bB-Ac))\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2}$$

$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} + \frac{\left(\sqrt{c}(bB-Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2} - \frac{\left(\sqrt{c}(bB-Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2}$$

$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} - \frac{(bB-Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^2} - \frac{(bB-Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^2} - \frac{\left(\sqrt[4]{c}(bB-Ac)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}b^{9/4}} - \frac{\left(\sqrt[4]{c}(bB-Ac)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}b^{9/4}}$$

$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} - \frac{\sqrt[4]{c}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}} + \frac{\sqrt[4]{c}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}} - \frac{\left(\sqrt[4]{c}(bB-Ac)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{9/4}} + \frac{\left(\sqrt[4]{c}(bB-Ac)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{9/4}}$$

$$= -\frac{2A}{5bx^{5/2}} - \frac{2(bB-Ac)}{b^2\sqrt{x}} + \frac{\sqrt[4]{c}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{9/4}} - \frac{\sqrt[4]{c}(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{9/4}} - \frac{\sqrt[4]{c}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}} + \frac{\sqrt[4]{c}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.60

$$\int\frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)}\,dx = -\frac{2(Ab+5bBx^2-5Acx^2)}{5b^2x^{5/2}} + \frac{\sqrt[4]{c}(bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}b^{9/4}} + \frac{\sqrt[4]{c}(bB-Ac)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}b^{9/4}}$$

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*(A*b + 5*b*B*x^2 - 5*A*c*x^2))/(5*b^2*x^(5/2)) + (c^(1/4)*(b*B - A*c)*A
rcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*b^
(9/4)) + (c^(1/4)*(b*B - A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sq
rt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(9/4))
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.55

method	result
derivativedivides	$-\frac{2A}{5b\,x^{\frac{5}{2}}}-\frac{2(-Ac+Bb)}{b^{2}\sqrt{x}}+\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$-\frac{2A}{5b\,x^{\frac{5}{2}}}-\frac{2(-Ac+Bb)}{b^{2}\sqrt{x}}+\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
risch	$-\frac{2(-5Ac\,x^{2}+5bB\,x^{2}+Ab)}{5b^{2}x^{\frac{5}{2}}}+\frac{(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

```
[In] int((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*A/b/x^(5/2)-2*(-A*c+B*b)/b^2/x^(1/2)+1/4*(A*c-B*b)/b^2/(1/c*b)^(1/4)*2
^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)
*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+
2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 738, normalized size of antiderivative = 2.89

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)}\,dx=\frac{5\,b^2x^3\left(-\frac{B^4b^4c-4\,AB^3b^3c^2+6\,A^2B^2b^2c^3-4\,A^3Bbc^4+A^4c^5}{b^9}\right)^{\frac{1}{4}}\log\left(b^7\left(-\frac{B^4b^4c-4\,AB^3b^3c^2+6\,A^2B^2b^2c}{b^9}\right.\right.}{}$$

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/10*(5*b^2*x^3*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*
B*b*c^4 + A^4*c^5)/b^9)^(1/4)*log(b^7*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^
2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(3/4) - (B^3*b^3*c - 3*A*B^2*
b^2*c^2 + 3*A^2*B*b*c^3 - A^3*c^4)*sqrt(x)) - 5*I*b^2*x^3*(-(B^4*b^4*c - 4*
A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(1/4)*log
(I*b^7*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 +
 A^4*c^5)/b^9)^(3/4) - (B^3*b^3*c - 3*A*B^2*b^2*c^2 + 3*A^2*B*b*c^3 - A^3*c
^4)*sqrt(x)) + 5*I*b^2*x^3*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c
^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(1/4)*log(-I*b^7*(-(B^4*b^4*c - 4*A*B^3*
b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^9)^(3/4) - (B^3*b^
```

```
3*c - 3*A*B^2*b^2*c^2 + 3*A^2*B*b*c^3 - A^3*c^4)*sqrt(x)) - 5*b^2*x^3*(-(B^
4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^3*B*b*c^4 + A^4*c^5)/b^
9)^(1/4)*log(-b^7*(-(B^4*b^4*c - 4*A*B^3*b^3*c^2 + 6*A^2*B^2*b^2*c^3 - 4*A^
3*B*b*c^4 + A^4*c^5)/b^9)^(3/4) - (B^3*b^3*c - 3*A*B^2*b^2*c^2 + 3*A^2*B*b*
c^3 - A^3*c^4)*sqrt(x)) - 4*(5*(B*b - A*c)*x^2 + A*b)*sqrt(x))/(b^2*x^3)
```

Sympy [A] (verification not implemented)

Time = 73.16 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)}\,dx = A\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for } b = \\ -\frac{2}{9cx^{\frac{9}{2}}} & \text{for } b = \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } c = \\ -\frac{2}{5bx^{\frac{5}{2}}} + \frac{c\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b^2\sqrt[4]{-\frac{b}{c}}} - \frac{c\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b^2\sqrt[4]{-\frac{b}{c}}} + \frac{c\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b^2\sqrt[4]{-\frac{b}{c}}} + \frac{2c}{b^2\sqrt{x}} & \text{otherw} \end{cases}\right.$$

$$+ B\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } b = 0 \wedge c = 0 \\ -\frac{2}{5cx^{\frac{5}{2}}} & \text{for } b = 0 \\ -\frac{2}{b\sqrt{x}} & \text{for } c = 0 \\ -\frac{\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b\sqrt[4]{-\frac{b}{c}}} + \frac{\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b\sqrt[4]{-\frac{b}{c}}} - \frac{\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{b}{c}}}\right)}{b\sqrt[4]{-\frac{b}{c}}} - \frac{2}{b\sqrt{x}} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((B*x**2+A)/x**(3/2)/(c*x**4+b*x**2),x)
```

```
[Out] A*Piecewise((zoo/x**(9/2), Eq(b, 0) & Eq(c, 0)), (-2/(9*c*x**(9/2)), Eq(b,
0)), (-2/(5*b*x**(5/2)), Eq(c, 0)), (-2/(5*b*x**(5/2)) + c*log(sqrt(x) - (-
b/c)**(1/4))/(2*b**2*(-b/c)**(1/4)) - c*log(sqrt(x) + (-b/c)**(1/4))/(2*b**
2*(-b/c)**(1/4)) + c*atan(sqrt(x)/(-b/c)**(1/4))/(b**2*(-b/c)**(1/4)) + 2*c
/(b**2*sqrt(x)), True)) + B*Piecewise((zoo/x**(5/2), Eq(b, 0) & Eq(c, 0)),
(-2/(5*c*x**(5/2)), Eq(b, 0)), (-2/(b*sqrt(x)), Eq(c, 0)), (-log(sqrt(x) -
(-b/c)**(1/4))/(2*b*(-b/c)**(1/4)) + log(sqrt(x) + (-b/c)**(1/4))/(2*b*(-b/
c)**(1/4)) - atan(sqrt(x)/(-b/c)**(1/4))/(b*(-b/c)**(1/4)) - 2/(b*sqrt(x)),
 True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)}\,dx = -\frac{(Bbc-Ac^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\right.}{4\,b^2} - \frac{2\left(5\left(Bb-Ac\right)x^2+Ab\right)}{5\,b^2x^{\frac{5}{2}}}$$

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*(B*b*c - A*c^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4)
 + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt
(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(s
qrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + s
qrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)
*c^(3/4)))/b^2 - 2/5*(5*(B*b - A*c)*x^2 + A*b)/(b^2*x^(5/2))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)}\,dx = -\frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^3c^2} - \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^3c^2} + \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^3c^2} - \frac{\sqrt{2}\left((bc^3)^{\frac{3}{4}}Bb-(bc^3)^{\frac{3}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^3c^2} - \frac{2\left(5\,Bbx^2-5\,Acx^2+Ab\right)}{5\,b^2x^{\frac{5}{2}}}$$

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) - 1/2*sqrt(2)*((b*c^3
)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) -
 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) + 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^
3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2) -
1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b
/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2) - 2/5*(5*B*b*x^2 - 5*A*c*x^2 + A*b)/(b
^2*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.35

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)}\,dx = \frac{(-c)^{1/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)\,(A\,c-B\,b)}{b^{9/4}} - \frac{\frac{2\,A}{5\,b}-\frac{2\,x^2\,(A\,c-B\,b)}{b^2}}{x^{5/2}} - \frac{(-c)^{1/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)\,(A\,c-B\,b)}{b^{9/4}}$$

```
[In] int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] ((-c)^(1/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4))*(A*c - B*b))/b^(9/4) - ((2*A
)/(5*b) - (2*x^2*(A*c - B*b))/b^2)/x^(5/2) - ((-c)^(1/4)*atanh(((-c)^(1/4)*
x^(1/2))/b^(1/4))*(A*c - B*b))/b^(9/4)
```

3.192 $\int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)} dx$

Optimal result

Integrand size = 26, antiderivative size = 257

$$\begin{aligned}\int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)} dx =& -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} \\ &+ \frac{c^{3/4}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{11/4}} - \frac{c^{3/4}(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{11/4}} \\ &+ \frac{c^{3/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}} \\ &- \frac{c^{3/4}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}}\end{aligned}$$

```
[Out] -2/7*A/b/x^(7/2)-2/3*(-A*c+B*b)/b^2/x^(3/2)+1/2*c^(3/4)*(-A*c+B*b)*arctan(1
-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/4)*2^(1/2)-1/2*c^(3/4)*(-A*c+B*b)*a
rctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/4)*2^(1/2)+1/4*c^(3/4)*(-A*c
+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(11/4)*2^(1/2
)-1/4*c^(3/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/
2))/b^(11/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1598, 464, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)}\,dx = \frac{c^{3/4}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{11/4}} - \frac{c^{3/4}(bB-Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}b^{11/4}} + \frac{c^{3/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}} - \frac{c^{3/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} - \frac{2A}{7bx^{7/2}}$$

```
[In] Int[(A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*A)/(7*b*x^(7/2)) - (2*(b*B - A*c))/(3*b^2*x^(3/2)) + (c^(3/4)*(b*B - A*
c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(11/4)) - (c^(
3/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^
(11/4)) + (c^(3/4)*(b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x])/(2*Sqrt[2]*b^(11/4)) - (c^(3/4)*(b*B - A*c)*Log[Sqrt[b] + S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(11/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{A+Bx^2}{x^{9/2}\left(b+cx^2\right)}\,dx$$

$$= -\frac{2A}{7bx^{7/2}} - \frac{\left(2\left(-\frac{7bB}{2}+\frac{7Ac}{2}\right)\right)\int \frac{1}{x^{5/2}(b+cx^2)}\,dx}{7b}$$

$$= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} - \frac{(c(bB-Ac))\int \frac{1}{\sqrt{x}(b+cx^2)}\,dx}{b^2}$$

$$= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} - \frac{(2c(bB-Ac))\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^2}$$

$$= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} - \frac{(c(bB-Ac))\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^{5/2}} - \frac{(c(bB-Ac))\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^{5/2}}$$

$$= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} - \frac{\left(\sqrt{c}(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{5/2}} - \frac{\left(\sqrt{c}(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{5/2}} + \frac{\left(c^{3/4}(bB-Ac)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}b^{11/4}} + \frac{\left(c^{3/4}(bB-Ac)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}b^{11/4}}$$

$$= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} + \frac{c^{3/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}}$$
$$- \frac{c^{3/4}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}}$$
$$- \frac{\left(c^{3/4}(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{11/4}}$$
$$+ \frac{\left(c^{3/4}(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{11/4}}$$

$$= -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{3b^2x^{3/2}} + \frac{c^{3/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{11/4}}$$
$$- \frac{c^{3/4}(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{11/4}}$$
$$+ \frac{c^{3/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}}$$
$$- \frac{c^{3/4}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)}\,dx = -\frac{2(3Ab+7bBx^2-7Acx^2)}{21b^2x^{7/2}}$$
$$+ \frac{c^{3/4}(bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}b^{11/4}} - \frac{c^{3/4}(bB-Ac)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}b^{11/4}}$$

```
[In] Integrate[(A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*(3*A*b + 7*b*B*x^2 - 7*A*c*x^2))/(21*b^2*x^(7/2)) + (c^(3/4)*(b*B - A*c
)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]
*b^(11/4)) - (c^(3/4)*(b*B - A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])
/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*b^(11/4))
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.55

method	result
derivativedivides	$-\frac{2A}{7b\,x^{\frac{7}{2}}}-\frac{2(-Ac+Bb)}{3b^{2}x^{\frac{3}{2}}}+\frac{c(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{4b^{3}}$
default	$-\frac{2A}{7b\,x^{\frac{7}{2}}}-\frac{2(-Ac+Bb)}{3b^{2}x^{\frac{3}{2}}}+\frac{c(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{4b^{3}}$
risch	$-\frac{2(-7Ac\,x^{2}+7bB\,x^{2}+3Ab)}{21b^{2}x^{\frac{7}{2}}}+\frac{c(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{b}{c}\right.}\right.\right.}{4b^{3}}$

```
[In] int((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7*A/b/x^(7/2)-2/3*(-A*c+B*b)/b^2/x^(3/2)+1/4*c*(A*c-B*b)/b^3*(1/c*b)^(1/
4)*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(
1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)
+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 643, normalized size of antiderivative = 2.50

$$\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)}\,dx=\frac{21\,b^2x^4\left(-\frac{B^4b^4c^3-4\,AB^3b^3c^4+6\,A^2B^2b^2c^5-4\,A^3Bbc^6+A^4c^7}{b^{11}}\right)^{\frac{1}{4}}\log\left(b^3\left(-\frac{B^4b^4c^3-4\,AB^3b^3c^4+6\,A^2B^2b}{b^{11}}\right.\right.}{}$$

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/42*(21*b^2*x^4*(-(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A
^3*B*b*c^6 + A^4*c^7)/b^11)^(1/4)*log(b^3*(-(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4
+ 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)/b^11)^(1/4) - (B*b*c - A*c^2
)*sqrt(x)) + 21*I*b^2*x^4*(-(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*A^2*B^2*b^2*
c^5 - 4*A^3*B*b*c^6 + A^4*c^7)/b^11)^(1/4)*log(I*b^3*(-(B^4*b^4*c^3 - 4*A*B
^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)/b^11)^(1/4) - (B*
b*c - A*c^2)*sqrt(x)) - 21*I*b^2*x^4*(-(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*A
^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)/b^11)^(1/4)*log(-I*b^3*(-(B^4*b^4
*c^3 - 4*A*B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)/b^11)
^(1/4) - (B*b*c - A*c^2)*sqrt(x)) - 21*b^2*x^4*(-(B^4*b^4*c^3 - 4*A*B^3*b^3
*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^7)/b^11)^(1/4)*log(-b^3*(-
```

```
(B^4*b^4*c^3 - 4*A*B^3*b^3*c^4 + 6*A^2*B^2*b^2*c^5 - 4*A^3*B*b*c^6 + A^4*c^
7)/b^11)^(1/4) - (B*b*c - A*c^2)*sqrt(x)) - 4*(7*(B*b - A*c)*x^2 + 3*A*b)*s
qrt(x))/(b^2*x^4)
```

Sympy [A] (verification not implemented)

Time = 67.26 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{11x^{\frac{11}{2}}}-\frac{2B}{7x^{\frac{7}{2}}}\right) \\ \frac{-\frac{2A}{11x^{\frac{11}{2}}}-\frac{2B}{7x^{\frac{7}{2}}}}{c} \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{b} \\ -\frac{2A}{7bx^{\frac{7}{2}}}+\frac{2Ac}{3b^2x^{\frac{3}{2}}}-\frac{Ac^2\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{b}{c}}\right)}{2b^3}+\frac{Ac^2\sqrt[4]{-\frac{b}{c}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{b}{c}}\right)}{2b^3}+\frac{Ac^2\sqrt[4]{-\frac{b}{c}}\,a}{} \end{cases}$$

```
[In] integrate((B*x**2+A)/x**(5/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(11*x**(11/2)) - 2*B/(7*x**(7/2))), Eq(b, 0) & Eq(c, 0
)), ((-2*A/(11*x**(11/2)) - 2*B/(7*x**(7/2)))/c, Eq(b, 0)), ((-2*A/(7*x**(7
/2)) - 2*B/(3*x**(3/2)))/b, Eq(c, 0)), (-2*A/(7*b*x**(7/2)) + 2*A*c/(3*b**2
*x**(3/2)) - A*c**2*(-b/c)**(1/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*b**3) + A
*c**2*(-b/c)**(1/4)*log(sqrt(x) + (-b/c)**(1/4))/(2*b**3) + A*c**2*(-b/c)**
(1/4)*atan(sqrt(x)/(-b/c)**(1/4))/b**3 - 2*B/(3*b*x**(3/2)) + B*c*(-b/c)**(
1/4)*log(sqrt(x) - (-b/c)**(1/4))/(2*b**2) - B*c*(-b/c)**(1/4)*log(sqrt(x)
+ (-b/c)**(1/4))/(2*b**2) - B*c*(-b/c)**(1/4)*atan(sqrt(x)/(-b/c)**(1/4))/b
**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)}\,dx =$$

$$-\frac{\frac{2\sqrt{2}(Bbc-Ac^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(Bbc-Ac^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}(Bbc-Ac^2)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}-\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{4b^2}$$

$$-\frac{2\left(7\left(Bb-Ac\right)x^2+3\,Ab\right)}{21\,b^2x^{\frac{7}{2}}}$$

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*(B*b*c - A*c^2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4)
 + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))
) + 2*sqrt(2)*(B*b*c - A*c^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4)
- 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c)))
 + sqrt(2)*(B*b*c - A*c^2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x
+ sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(B*b*c - A*c^2)*log(-sqrt(2)*b^(1/4)
*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^2 - 2/21*(7*(B
*b - A*c)*x^2 + 3*A*b)/(b^2*x^(7/2))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)}\,dx = -\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bb-\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^3} - \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bb-\left(bc^3\right)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^3} - \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bb-\left(bc^3\right)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4b^3} + \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{1}{4}}Bb-\left(bc^3\right)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4b^3} - \frac{2\left(7Bbx^2-7Acx^2+3Ab\right)}{21b^2x^{\frac{7}{2}}}$$

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^3 - 1/2*sqrt(2)*((b*c^3)^(1/4
)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqr
t(x))/(b/c)^(1/4))/b^3 - 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c
)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 + 1/4*sqrt(2)*((b*c^
3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sq
rt(b/c))/b^3 - 2/21*(7*B*b*x^2 - 7*A*c*x^2 + 3*A*b)/(b^2*x^(7/2))
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 555, normalized size of antiderivative = 2.16

$$\int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)}\,dx = -\frac{\frac{2A}{7b}-\frac{2x^2(Ac-Bb)}{3b^2}}{x^{7/2}} + \frac{(-c)^{3/4}\operatorname{atan}\left(\frac{\frac{(-c)^{3/4}(Ac-Bb)\left(\sqrt{x}\left(16A^2b^6c^7-32ABb^7c^6+16B^2b^8c^5\right)-\frac{(-c)^{3/4}(Ac-Bb)\left(32Ab^9c^5-32Bb^{10}c^4\right)1\mathrm{i}}{2b^{11/4}}\right)}{2b^{11/4}}+\frac{(-c)^{3/4}(Ac-Bb)\cdots}{\cdots}}{\frac{(-c)^{3/4}(Ac-Bb)\left(\sqrt{x}\left(16A^2b^6c^7-32ABb^7c^6+16B^2b^8c^5\right)-\frac{(-c)^{3/4}(Ac-Bb)\left(32Ab^9c^5-32Bb^{10}c^4\right)1\mathrm{i}}{2b^{11/4}}\right)1\mathrm{i}}{2b^{11/4}}-\frac{(-c)^{3/4}(Ac-Bb)\cdots}{\cdots}}\right)\cdots}{b^{11/4}}$$
$$-\frac{(-c)^{3/4}\operatorname{atan}\left(\frac{A^3c^8\sqrt{x}\,1\mathrm{i}-B^3b^3c^5\sqrt{x}\,1\mathrm{i}-A^2Bbc^7\sqrt{x}\,3\mathrm{i}+AB^2b^2c^6\sqrt{x}\,3\mathrm{i}}{b^{1/4}(-c)^{19/4}\left(c\left(c\left(A^3c-3A^2Bb\right)+3AB^2b^2\right)-B^3b^3\right)}\right)(Ac-Bb)\,1\mathrm{i}}{b^{11/4}}$$

```
[In] int((A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] ((-c)^(3/4)*atan((((-c)^(3/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*b^6*c^7 + 16*B^2
*b^8*c^5 - 32*A*B*b^7*c^6) - ((-c)^(3/4)*(A*c - B*b)*(32*A*b^9*c^5 - 32*B*b
^10*c^4)*1i)/(2*b^(11/4))))/(2*b^(11/4)) + ((-c)^(3/4)*(A*c - B*b)*(x^(1/2)
*(16*A^2*b^6*c^7 + 16*B^2*b^8*c^5 - 32*A*B*b^7*c^6) + ((-c)^(3/4)*(A*c - B*
b)*(32*A*b^9*c^5 - 32*B*b^10*c^4)*1i)/(2*b^(11/4))))/(2*b^(11/4)))/(((-c)^(
3/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*b^6*c^7 + 16*B^2*b^8*c^5 - 32*A*B*b^7*c^6
) - ((-c)^(3/4)*(A*c - B*b)*(32*A*b^9*c^5 - 32*B*b^10*c^4)*1i)/(2*b^(11/4))
)*1i)/(2*b^(11/4)) - ((-c)^(3/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*b^6*c^7 + 16*
B^2*b^8*c^5 - 32*A*B*b^7*c^6) + ((-c)^(3/4)*(A*c - B*b)*(32*A*b^9*c^5 - 32*
B*b^10*c^4)*1i)/(2*b^(11/4)))*1i)/(2*b^(11/4))))*(A*c - B*b))/b^(11/4) - ((
2*A)/(7*b) - (2*x^2*(A*c - B*b))/(3*b^2))/x^(7/2) - ((-c)^(3/4)*atan((A^3*c
^8*x^(1/2)*1i - B^3*b^3*c^5*x^(1/2)*1i - A^2*B*b*c^7*x^(1/2)*3i + A*B^2*b^2
*c^6*x^(1/2)*3i)/(b^(1/4)*(-c)^(19/4)*(c*(c*(A^3*c - 3*A^2*B*b) + 3*A*B^2*b
^2) - B^3*b^3)))*(A*c - B*b)*1i)/b^(11/4)
```

3.193 $\int \frac{A+Bx^2}{x^{7/2}(bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 276

$$\begin{aligned}
\int \frac{A+Bx^2}{x^{7/2}\left(bx^2+cx^4\right)}\,dx &= -\frac{2A}{9bx^{9/2}}-\frac{2(bB-Ac)}{5b^2x^{5/2}}+\frac{2c(bB-Ac)}{b^3\sqrt{x}} \\
&- \frac{c^{5/4}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{13/4}}+\frac{c^{5/4}(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{13/4}} \\
&+ \frac{c^{5/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{13/4}} \\
&- \frac{c^{5/4}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{13/4}}
\end{aligned}$$

```
[Out] -2/9*A/b/x^(9/2)-2/5*(-A*c+B*b)/b^2/x^(5/2)-1/2*c^(5/4)*(-A*c+B*b)*arctan(1
-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(13/4)*2^(1/2)+1/2*c^(5/4)*(-A*c+B*b)*a
rctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(13/4)*2^(1/2)+1/4*c^(5/4)*(-A*c
+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^(1/2
)-1/4*c^(5/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/
2))/b^(13/4)*2^(1/2)+2*c*(-A*c+B*b)/b^3/x^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1598, 464, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{A+Bx^2}{x^{7/2}\left(bx^2+cx^4\right)}\,dx = -\frac{c^{5/4}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{13/4}} + \frac{c^{5/4}(bB-Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}b^{13/4}} + \frac{c^{5/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{13/4}} - \frac{c^{5/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{13/4}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} - \frac{2A}{9bx^{9/2}}$$

```
[In] Int[(A + B*x^2)/(x^(7/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*A)/(9*b*x^(9/2)) - (2*(b*B - A*c))/(5*b^2*x^(5/2)) + (2*c*(b*B - A*c))/
(b^3*Sqrt[x]) - (c^(5/4)*(b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b
^(1/4)])/(Sqrt[2]*b^(13/4)) + (c^(5/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(13/4)) + (c^(5/4)*(b*B - A*c)*Log[Sqrt[b
] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(13/4)) - (c
^(5/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*
x])/(2*Sqrt[2]*b^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{x^{11/2}(b+cx^2)}\,dx \\
&= -\frac{2A}{9bx^{9/2}} - \frac{\left(2\left(-\frac{9bB}{2}+\frac{9Ac}{2}\right)\right)\int \frac{1}{x^{7/2}(b+cx^2)}\,dx}{9b} \\
&= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} - \frac{(c(bB-Ac))\int \frac{1}{x^{3/2}(b+cx^2)}\,dx}{b^2} \\
&= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} + \frac{(c^2(bB-Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{b^3} \\
&= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} + \frac{(2c^2(bB-Ac))\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^3} \\
&= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} \\
&\quad - \frac{(c^{3/2}(bB-Ac))\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^3} \\
&\quad + \frac{(c^{3/2}(bB-Ac))\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{b^3}
\end{aligned}
$$

$$= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} + \frac{(c(bB-Ac))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{2b^3} + \frac{(c(bB-Ac))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{2b^3} + \frac{\left(c^{5/4}(bB-Ac)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} - x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}b^{13/4}} + \frac{\left(c^{5/4}(bB-Ac)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}} - 2x}{-\frac{\sqrt{b}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} - x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}b^{13/4}}$$

$$= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} + \frac{c^{5/4}(bB-Ac)\log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{2\sqrt{2}b^{13/4}} - \frac{c^{5/4}(bB-Ac)\log\left(\sqrt{b} + \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{2\sqrt{2}b^{13/4}} + \frac{\left(c^{5/4}(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{13/4}} - \frac{\left(c^{5/4}(bB-Ac)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{13/4}}$$

$$= -\frac{2A}{9bx^{9/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{b^3\sqrt{x}} - \frac{c^{5/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{13/4}}$$
$$+ \frac{c^{5/4}(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{13/4}}$$
$$+ \frac{c^{5/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{13/4}}$$
$$- \frac{c^{5/4}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{13/4}}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx^2}{x^{7/2}(bx^2+cx^4)}\,dx = \frac{-\frac{4\sqrt[4]{b}(9bBx^2(b-5cx^2)+A(5b^2-9bcx^2+45c^2x^4))}{x^{9/2}} + 45\sqrt{2}c^{5/4}(-bB+Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right.}{90b^{13/4}}$$

```
[In] Integrate[(A + B*x^2)/(x^(7/2)*(b*x^2 + c*x^4)),x]
[Out] ((-4*b^(1/4)*(9*b*B*x^2*(b - 5*c*x^2) + A*(5*b^2 - 9*b*c*x^2 + 45*c^2*x^4))
)/x^(9/2) + 45*Sqrt[2]*c^(5/4)*(-(b*B) + A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/
(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 45*Sqrt[2]*c^(5/4)*(-(b*B) + A*c)*ArcT
anh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(90*b^(13/4))
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.57

method	result
derivativedivides	$-\frac{2A}{9b\,x^{\frac{9}{2}}}-\frac{2(-Ac+Bb)}{5b^2x^{\frac{5}{2}}}-\frac{2c(Ac-Bb)}{b^3\sqrt{x}}-\frac{c(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{arct}\right.}{4b^3\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$-\frac{2A}{9b\,x^{\frac{9}{2}}}-\frac{2(-Ac+Bb)}{5b^2x^{\frac{5}{2}}}-\frac{2c(Ac-Bb)}{b^3\sqrt{x}}-\frac{c(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{arct}\right.}{4b^3\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
risch	$-\frac{2(45A\,c^2x^4-45x^4Bbc-9Abc\,x^2+9b^2B\,x^2+5b^2A)}{45b^3x^{\frac{9}{2}}}-\frac{c(Ac-Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+\right.\right.}{4b^3\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

```
[In] int((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*A/b/x^(9/2)-2/5*(-A*c+B*b)/b^2/x^(5/2)-2*c*(A*c-B*b)/b^3/x^(1/2)-1/4*c
*(A*c-B*b)/b^3/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(
1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(
1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 784, normalized size of antiderivative = 2.84

$$\int \frac{A+Bx^2}{x^{7/2}\left(bx^2+cx^4\right)}\,dx =$$

$$-\frac{45\,b^3x^5\left(-\frac{B^4b^4c^5-4\,AB^3b^3c^6+6\,A^2B^2b^2c^7-4\,A^3Bbc^8+A^4c^9}{b^{13}}\right)^{\frac{1}{4}}\log\left(b^{10}\left(-\frac{B^4b^4c^5-4\,AB^3b^3c^6+6\,A^2B^2b^2c^7-4\,A^3Bbc^8+A^4c^9}{b^{13}}\right)^{\frac{3}{4}}-\right.}{}$$

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/90*(45*b^3*x^5*(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*
A^3*B*b*c^8 + A^4*c^9)/b^13)^(1/4)*log(b^10*(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^
6 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13)^(3/4) - (B^3*b^3*c^4
 - 3*A*B^2*b^2*c^5 + 3*A^2*B*b*c^6 - A^3*c^7)*sqrt(x)) - 45*I*b^3*x^5*(-(B^
4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/
b^13)^(1/4)*log(I*b^10*(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7
 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13)^(3/4) - (B^3*b^3*c^4 - 3*A*B^2*b^2*c^5 +
3*A^2*B*b*c^6 - A^3*c^7)*sqrt(x)) + 45*I*b^3*x^5*(-(B^4*b^4*c^5 - 4*A*B^3*b
^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13)^(1/4)*log(-I*b^
10*(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A
^4*c^9)/b^13)^(3/4) - (B^3*b^3*c^4 - 3*A*B^2*b^2*c^5 + 3*A^2*B*b*c^6 - A^3*
c^7)*sqrt(x)) - 45*b^3*x^5*(-(B^4*b^4*c^5 - 4*A*B^3*b^3*c^6 + 6*A^2*B^2*b^2
*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13)^(1/4)*log(-b^10*(-(B^4*b^4*c^5 - 4*A*
B^3*b^3*c^6 + 6*A^2*B^2*b^2*c^7 - 4*A^3*B*b*c^8 + A^4*c^9)/b^13)^(3/4) - (B
^3*b^3*c^4 - 3*A*B^2*b^2*c^5 + 3*A^2*B*b*c^6 - A^3*c^7)*sqrt(x)) - 4*(45*(B
*b*c - A*c^2)*x^4 - 5*A*b^2 - 9*(B*b^2 - A*b*c)*x^2)*sqrt(x))/(b^3*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^2}{x^{7/2} (bx^2 + cx^4)} \, dx = \text{Timed out}$$

```
[In] integrate((B*x**2+A)/x**(7/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.86

$$\int \frac{A + Bx^2}{x^{7/2} (bx^2 + cx^4)} \, dx = \frac{(Bbc^2 - Ac^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}} + 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}} - 2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\cdots}{\cdots}\right)}{4b^3} + \frac{2\left(45\left(Bbc - Ac^2\right)x^4 - 5Ab^2 - 9\left(Bb^2 - Abc\right)x^2\right)}{45b^3x^{\frac{9}{2}}}$$

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*(B*b*c^2 - A*c^3)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4
) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c
)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqr
t(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(
sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) +
sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4
)*c^(3/4)))/b^3 + 2/45*(45*(B*b*c - A*c^2)*x^4 - 5*A*b^2 - 9*(B*b^2 - A*b*c
)*x^2)/(b^3*x^(9/2))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2}{x^{7/2}\left(bx^2+cx^4\right)}\,dx=\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb-\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^4c}+\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb-\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^4c}-\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb-\left(bc^3\right)^{\frac{3}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^4c}+\frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb-\left(bc^3\right)^{\frac{3}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^4c}+\frac{2\left(45\,Bbcx^4-45\,Ac^2x^4-9\,Bb^2x^2+9\,Abcx^2-5\,Ab^2\right)}{45\,b^3x^{\frac{9}{2}}}$$

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c) + 1/2*sqrt(2)*((b*c^3)^(
3/4)*B*b - (b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*
sqrt(x))/(b/c)^(1/4))/(b^4*c) - 1/4*sqrt(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3
/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c) + 1/4*sqr
t(2)*((b*c^3)^(3/4)*B*b - (b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/
4) + x + sqrt(b/c))/(b^4*c) + 2/45*(45*B*b*c*x^4 - 45*A*c^2*x^4 - 9*B*b^2*x
^2 + 9*A*b*c*x^2 - 5*A*b^2)/(b^3*x^(9/2))
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.39

$$\int \frac{A+Bx^2}{x^{7/2}\left(bx^2+cx^4\right)}\,dx=\frac{(-c)^{5/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)\,\left(A\,c-B\,b\right)}{b^{13/4}}-\frac{\frac{2\,A}{9\,b}-\frac{2\,x^2\,\left(A\,c-B\,b\right)}{5\,b^2}+\frac{2\,c\,x^4\,\left(A\,c-B\,b\right)}{b^3}}{x^{9/2}}-\frac{(-c)^{5/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\sqrt{x}}{b^{1/4}}\right)\,\left(A\,c-B\,b\right)}{b^{13/4}}$$

```
[In] int((A + B*x^2)/(x^(7/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] ((-c)^(5/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4))*(A*c - B*b))/b^(13/4) - ((2*
A)/(9*b) - (2*x^2*(A*c - B*b))/(5*b^2) + (2*c*x^4*(A*c - B*b))/b^3)/x^(9/2)
 - ((-c)^(5/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4))*(A*c - B*b))/b^(13/4)
```

3.194 $\int \frac{A+Bx^2}{x^{9/2}(bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 278

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^{9/2}\left(bx^2+cx^4\right)}\,dx = &-\frac{2A}{11bx^{11/2}}-\frac{2(bB-Ac)}{7b^2x^{7/2}}+\frac{2c(bB-Ac)}{3b^3x^{3/2}} \\
&-\frac{c^{7/4}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{15/4}}+\frac{c^{7/4}(bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{15/4}} \\
&-\frac{c^{7/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{15/4}} \\
&+\frac{c^{7/4}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{15/4}}
\end{aligned}
$$

```
[Out] -2/11*A/b/x^(11/2)-2/7*(-A*c+B*b)/b^2/x^(7/2)+2/3*c*(-A*c+B*b)/b^3/x^(3/2)-
1/2*c^(7/4)*(-A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)*2
^(1/2)+1/2*c^(7/4)*(-A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(
15/4)*2^(1/2)-1/4*c^(7/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2
^(1/2)*x^(1/2))/b^(15/4)*2^(1/2)+1/4*c^(7/4)*(-A*c+B*b)*ln(b^(1/2)+x*c^(1/2
)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(15/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1598, 464, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{A+Bx^2}{x^{9/2}\left(bx^2+cx^4\right)}\,dx = -\frac{c^{7/4}(bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{15/4}} + \frac{c^{7/4}(bB-Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}b^{15/4}} - \frac{c^{7/4}(bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{15/4}} + \frac{c^{7/4}(bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{15/4}} + \frac{2c(bB-Ac)}{3b^3x^{3/2}} - \frac{2(bB-Ac)}{7b^2x^{7/2}} - \frac{2A}{11bx^{11/2}}$$

```
[In] Int[(A + B*x^2)/(x^(9/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*A)/(11*b*x^(11/2)) - (2*(b*B - A*c))/(7*b^2*x^(7/2)) + (2*c*(b*B - A*c)
)/(3*b^3*x^(3/2)) - (c^(7/4)*(b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x
])/b^(1/4)])/(Sqrt[2]*b^(15/4)) + (c^(7/4)*(b*B - A*c)*ArcTan[1 + (Sqrt[2]*
c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(15/4)) - (c^(7/4)*(b*B - A*c)*Log[Sq
rt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(15/4))
+ (c^(7/4)*(b*B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt
[c]*x])/(2*Sqrt[2]*b^(15/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{x^{13/2}\,(b+cx^2)}\,dx \\
&= -\frac{2A}{11bx^{11/2}}-\frac{\left(2\left(-\frac{11bB}{2}+\frac{11Ac}{2}\right)\right)\int \frac{1}{x^{9/2}(b+cx^2)}\,dx}{11b} \\
&= -\frac{2A}{11bx^{11/2}}-\frac{2(bB-Ac)}{7b^2x^{7/2}}-\frac{(c(bB-Ac))\int \frac{1}{x^{5/2}(b+cx^2)}\,dx}{b^2} \\
&= -\frac{2A}{11bx^{11/2}}-\frac{2(bB-Ac)}{7b^2x^{7/2}}+\frac{2c(bB-Ac)}{3b^3x^{3/2}}+\frac{(c^2(bB-Ac))\int \frac{1}{\sqrt{x}(b+cx^2)}\,dx}{b^3} \\
&= -\frac{2A}{11bx^{11/2}}-\frac{2(bB-Ac)}{7b^2x^{7/2}}+\frac{2c(bB-Ac)}{3b^3x^{3/2}}+\frac{(2c^2(bB-Ac))\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{b^3} \\
&= -\frac{2A}{11bx^{11/2}}-\frac{2(bB-Ac)}{7b^2x^{7/2}}+\frac{2c(bB-Ac)}{3b^3x^{3/2}} \\
&\quad+\frac{(c^2(bB-Ac))\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{b^{7/2}} \\
&\quad+\frac{(c^2(bB-Ac))\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{b^{7/2}}
\end{aligned}$$

$$= -\frac{2A}{11bx^{11/2}} - \frac{2(bB-Ac)}{7b^2x^{7/2}} + \frac{2c(bB-Ac)}{3b^3x^{3/2}} + \frac{\left(c^{3/2}(bB-Ac)\right)\operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{2b^{7/2}} + \frac{\left(c^{3/2}(bB-Ac)\right)\operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{2b^{7/2}} - \frac{\left(c^{7/4}(bB-Ac)\right)\operatorname{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} - x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}b^{15/4}} - \frac{\left(c^{7/4}(bB-Ac)\right)\operatorname{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}} - 2x}{-\frac{\sqrt{b}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} - x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{2}b^{15/4}}$$

$$= -\frac{2A}{11bx^{11/2}} - \frac{2(bB-Ac)}{7b^2x^{7/2}} + \frac{2c(bB-Ac)}{3b^3x^{3/2}} - \frac{c^{7/4}(bB-Ac)\log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{2\sqrt{2}b^{15/4}} + \frac{c^{7/4}(bB-Ac)\log\left(\sqrt{b} + \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{2\sqrt{2}b^{15/4}} + \frac{\left(c^{7/4}(bB-Ac)\right)\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{15/4}} - \frac{\left(c^{7/4}(bB-Ac)\right)\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{15/4}}$$

$$= -\frac{2A}{11bx^{11/2}} - \frac{2(bB-Ac)}{7b^2x^{7/2}} + \frac{2c(bB-Ac)}{3b^3x^{3/2}} - \frac{c^{7/4}(bB-Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{15/4}} + \frac{c^{7/4}(bB-Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{15/4}} - \frac{c^{7/4}(bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{15/4}} + \frac{c^{7/4}(bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{2\sqrt{2}b^{15/4}}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx^2}{x^{9/2}(bx^2+cx^4)}\,dx = -\frac{2(21Ab^2+33b^2Bx^2-33Abcx^2-77bBcx^4+77Ac^2x^4)}{231b^3x^{11/2}} - \frac{c^{7/4}(bB-Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt{2}b^{15/4}} + \frac{c^{7/4}(bB-Ac)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b}+\sqrt{c}x}\right)}{\sqrt{2}b^{15/4}}$$

```
[In] Integrate[(A + B*x^2)/(x^(9/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-2*(21*A*b^2 + 33*b^2*B*x^2 - 33*A*b*c*x^2 - 77*b*B*c*x^4 + 77*A*c^2*x^4))
/(231*b^3*x^(11/2)) - (c^(7/4)*(b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/(Sqrt[2]*b^(15/4)) + (c^(7/4)*(b*B - A*c)*
ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(Sqrt[2]*
b^(15/4))
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.58

method	result
derivativedivides	$-\frac{2A}{11b\,x^{\frac{11}{2}}}-\frac{2(-Ac+Bb)}{7b^{2}x^{\frac{7}{2}}}-\frac{2c(Ac-Bb)}{3b^{3}x^{\frac{3}{2}}}-\frac{c^{2}(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right.\right.}{4b^{4}}$
default	$-\frac{2A}{11b\,x^{\frac{11}{2}}}-\frac{2(-Ac+Bb)}{7b^{2}x^{\frac{7}{2}}}-\frac{2c(Ac-Bb)}{3b^{3}x^{\frac{3}{2}}}-\frac{c^{2}(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right.\right.}{4b^{4}}$
risch	$-\frac{2\left(77A\,c^{2}x^{4}-77x^{4}Bbc-33Abc\,x^{2}+33b^{2}B\,x^{2}+21b^{2}A\right)}{231b^{3}x^{\frac{11}{2}}}-\frac{c^{2}(Ac-Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\right.}{4b^{4}}$

```
[In] int((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/11*A/b/x^(11/2)-2/7*(-A*c+B*b)/b^2/x^(7/2)-2/3*c*(A*c-B*b)/b^3/x^(3/2)-1
/4*c^2*(A*c-B*b)/b^4*(1/c*b)^(1/4)*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(
1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arct
an(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-
1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.42

$$\int \frac{A+Bx^2}{x^{9/2}\left(bx^2+cx^4\right)}\,dx =$$

$$-\frac{231\,b^{3}x^{6}\left(-\frac{B^{4}b^{4}c^{7}-4\,AB^{3}b^{3}c^{8}+6\,A^{2}B^{2}b^{2}c^{9}-4\,A^{3}Bbc^{10}+A^{4}c^{11}}{b^{15}}\right)^{\frac{1}{4}}\log\left(b^{4}\left(-\frac{B^{4}b^{4}c^{7}-4\,AB^{3}b^{3}c^{8}+6\,A^{2}B^{2}b^{2}c^{9}-4\,A^{3}Bbc^{10}+A^{4}c^{11}}{b^{15}}\right)\right.}{}$$

```
[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/462*(231*b^3*x^6*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 -
4*A^3*B*b*c^10 + A^4*c^11)/b^15)^(1/4)*log(b^4*(-(B^4*b^4*c^7 - 4*A*B^3*b^3
*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^15)^(1/4) - (B*b*c^
2 - A*c^3)*sqrt(x)) + 231*I*b^3*x^6*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^
2*B^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^15)^(1/4)*log(I*b^4*(-(B^4*b^4
*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^1
5)^(1/4) - (B*b*c^2 - A*c^3)*sqrt(x)) - 231*I*b^3*x^6*(-(B^4*b^4*c^7 - 4*A*
B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^15)^(1/4)*lo
```

```
g(-I*b^4*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c
^10 + A^4*c^11)/b^15)^(1/4) - (B*b*c^2 - A*c^3)*sqrt(x)) - 231*b^3*x^6*(-(B
^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^9 - 4*A^3*B*b*c^10 + A^4*c^1
1)/b^15)^(1/4)*log(-b^4*(-(B^4*b^4*c^7 - 4*A*B^3*b^3*c^8 + 6*A^2*B^2*b^2*c^
9 - 4*A^3*B*b*c^10 + A^4*c^11)/b^15)^(1/4) - (B*b*c^2 - A*c^3)*sqrt(x)) - 4
*(77*(B*b*c - A*c^2)*x^4 - 21*A*b^2 - 33*(B*b^2 - A*b*c)*x^2)*sqrt(x))/(b^3
*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^{9/2}\left(bx^2+cx^4\right)}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**2+A)/x**(9/2)/(c*x**4+b*x**2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^2}{x^{9/2}\left(bx^2+cx^4\right)}\,dx = \frac{\frac{2\sqrt{2}(Bbc^2-Ac^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(Bbc^2-Ac^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+}{4b^3} + \frac{2\left(77\left(Bbc-Ac^2\right)x^4-21\,Ab^2-33\left(Bb^2-Abc\right)x^2\right)}{231\,b^3x^{\frac{11}{2}}}$$

```
[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 1/4*(2*sqrt(2)*(B*b*c^2 - A*c^3)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4
) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c)
)) + 2*sqrt(2)*(B*b*c^2 - A*c^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/
4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c
))) + sqrt(2)*(B*b*c^2 - A*c^3)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(
c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(B*b*c^2 - A*c^3)*log(-sqrt(2)*
b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^3 + 2/2
31*(77*(B*b*c - A*c^2)*x^4 - 21*A*b^2 - 33*(B*b^2 - A*b*c)*x^2)/(b^3*x^(11/
2))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2}{x^{9/2}\,(bx^2+cx^4)}\,dx = \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}\,Bbc-(bc^3)^{\frac{1}{4}}\,Ac^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^4} + \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}\,Bbc-(bc^3)^{\frac{1}{4}}\,Ac^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^4} + \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}\,Bbc-(bc^3)^{\frac{1}{4}}\,Ac^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^4} - \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}\,Bbc-(bc^3)^{\frac{1}{4}}\,Ac^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^4} + \frac{2\,(77\,Bbcx^4-77\,Ac^2x^4-33\,Bb^2x^2+33\,Abcx^2-21\,Ab^2)}{231\,b^3x^{\frac{11}{2}}}$$

```
[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((b*c^3)^(1/4)*B*b*c - (b*c^3)^(1/4)*A*c^2)*arctan(1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^4 + 1/2*sqrt(2)*((b*c^3)^(
1/4)*B*b*c - (b*c^3)^(1/4)*A*c^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
- 2*sqrt(x))/(b/c)^(1/4))/b^4 + 1/4*sqrt(2)*((b*c^3)^(1/4)*B*b*c - (b*c^3)^
(1/4)*A*c^2)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 1/4*sqr
t(2)*((b*c^3)^(1/4)*B*b*c - (b*c^3)^(1/4)*A*c^2)*log(-sqrt(2)*sqrt(x)*(b/c)
^(1/4) + x + sqrt(b/c))/b^4 + 2/231*(77*B*b*c*x^4 - 77*A*c^2*x^4 - 33*B*b^2
*x^2 + 33*A*b*c*x^2 - 21*A*b^2)/(b^3*x^(11/2))
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 563, normalized size of antiderivative = 2.03

$$\int \frac{A+Bx^2}{x^{9/2}\,(bx^2+cx^4)}\,dx = \frac{(-c)^{7/4}\,\mathrm{atan}\left(\frac{A^3\,c^{10}\,\sqrt{x}-B^3\,b^3\,c^7\,\sqrt{x}-3\,A^2\,B\,b\,c^9\,\sqrt{x}+3\,A\,B^2\,b^2\,c^8\,\sqrt{x}}{b^{1/4}\,(-c)^{27/4}\,(c\,(c\,(A^3\,c-3\,A^2\,B\,b)+3\,A\,B^2\,b^2)-B^3\,b^3)}\right)\,(A\,c-B\,b)}{b^{15/4}}$$
$$-\frac{\frac{2\,A}{11\,b}-\frac{2\,x^2\,(A\,c-B\,b)}{7\,b^2}+\frac{2\,c\,x^4\,(A\,c-B\,b)}{3\,b^3}}{x^{11/2}}$$
$$-\frac{(-c)^{7/4}\,\mathrm{atan}\left(\frac{\frac{(-c)^{7/4}\,(A\,c-B\,b)\,\left(\sqrt{x}\,\left(16\,A^2\,b^9\,c^9-32\,A\,B\,b^{10}\,c^8+16\,B^2\,b^{11}\,c^7\right)-\frac{(-c)^{7/4}\,(A\,c-B\,b)\,\left(32\,A\,b^{13}\,c^6-32\,B\,b^{14}\,c^5\right)}{2\,b^{15/4}}\right)\,1i}{2\,b^{15/4}}+\frac{(-c)^{7/4}\,(A\,c-B\,b)}{}}{\frac{(-c)^{7/4}\,(A\,c-B\,b)\,\left(\sqrt{x}\,\left(16\,A^2\,b^9\,c^9-32\,A\,B\,b^{10}\,c^8+16\,B^2\,b^{11}\,c^7\right)-\frac{(-c)^{7/4}\,(A\,c-B\,b)\,\left(32\,A\,b^{13}\,c^6-32\,B\,b^{14}\,c^5\right)}{2\,b^{15/4}}\right)}{2\,b^{15/4}}-\frac{(-c)^{7/4}\,(A\,c-B\,b)}{}}\right.}{b^{15/4}}$$

```
[In] int((A + B*x^2)/(x^(9/2)*(b*x^2 + c*x^4)),x)
```

```
[Out] ((-c)^(7/4)*atan((A^3*c^10*x^(1/2) - B^3*b^3*c^7*x^(1/2) - 3*A^2*B*b*c^9*x^
(1/2) + 3*A*B^2*b^2*c^8*x^(1/2))/(b^(1/4)*(-c)^(27/4)*(c*(c*(A^3*c - 3*A^2*
B*b) + 3*A*B^2*b^2) - B^3*b^3)))*(A*c - B*b))/b^(15/4) - ((-c)^(7/4)*atan((
((-c)^(7/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*b^9*c^9 + 16*B^2*b^11*c^7 - 32*A*B
*b^10*c^8) - ((-c)^(7/4)*(A*c - B*b)*(32*A*b^13*c^6 - 32*B*b^14*c^5))/(2*b^
(15/4)))*1i)/(2*b^(15/4)) + ((-c)^(7/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*b^9*c^
9 + 16*B^2*b^11*c^7 - 32*A*B*b^10*c^8) + ((-c)^(7/4)*(A*c - B*b)*(32*A*b^13
*c^6 - 32*B*b^14*c^5))/(2*b^(15/4)))*1i)/(2*b^(15/4)))/(((-c)^(7/4)*(A*c -
B*b)*(x^(1/2)*(16*A^2*b^9*c^9 + 16*B^2*b^11*c^7 - 32*A*B*b^10*c^8) - ((-c)^
(7/4)*(A*c - B*b)*(32*A*b^13*c^6 - 32*B*b^14*c^5))/(2*b^(15/4))))/(2*b^(15/
4)) - ((-c)^(7/4)*(A*c - B*b)*(x^(1/2)*(16*A^2*b^9*c^9 + 16*B^2*b^11*c^7 -
32*A*B*b^10*c^8) + ((-c)^(7/4)*(A*c - B*b)*(32*A*b^13*c^6 - 32*B*b^14*c^5))
/(2*b^(15/4))))/(2*b^(15/4))))*(A*c - B*b)*1i)/b^(15/4) - ((2*A)/(11*b) - (
2*x^2*(A*c - B*b))/(7*b^2) + (2*c*x^4*(A*c - B*b))/(3*b^3))/x^(11/2)
```

3.195 $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 332

$$\begin{aligned}\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx &= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4}-\frac{(13bB-9Ac)x^{5/2}}{10c^3}\\
&+\frac{(13bB-9Ac)x^{9/2}}{18bc^2}-\frac{(bB-Ac)x^{13/2}}{2bc\,(b+cx^2)}+\frac{b^{5/4}(13bB-9Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{17/4}}\\
&-\frac{b^{5/4}(13bB-9Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{17/4}}\\
&+\frac{b^{5/4}(13bB-9Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{17/4}}\\
&-\frac{b^{5/4}(13bB-9Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{17/4}}\end{aligned}$$

```
[Out] -1/10*(-9*A*c+13*B*b)*x^(5/2)/c^3+1/18*(-9*A*c+13*B*b)*x^(9/2)/b/c^2-1/2*(-
A*c+B*b)*x^(13/2)/b/c/(c*x^2+b)+1/8*b^(5/4)*(-9*A*c+13*B*b)*arctan(1-c^(1/4
)*2^(1/2)*x^(1/2)/b^(1/4))/c^(17/4)*2^(1/2)-1/8*b^(5/4)*(-9*A*c+13*B*b)*arc
tan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(17/4)*2^(1/2)+1/16*b^(5/4)*(-9*A*
c+13*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(17/4)*2^
(1/2)-1/16*b^(5/4)*(-9*A*c+13*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(
1/2)*x^(1/2))/c^(17/4)*2^(1/2)+1/2*b*(-9*A*c+13*B*b)*x^(1/2)/c^4
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1598, 468, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{b^{5/4}(13bB-9Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{17/4}} - \frac{b^{5/4}(13bB-9Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}c^{17/4}} + \frac{b^{5/4}(13bB-9Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{17/4}} - \frac{b^{5/4}(13bB-9Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{17/4}} + \frac{b\sqrt{x}(13bB-9Ac)}{2c^4} - \frac{x^{5/2}(13bB-9Ac)}{10c^3} + \frac{x^{9/2}(13bB-9Ac)}{18bc^2} - \frac{x^{13/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

```
[In] Int[(x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*(13*b*B - 9*A*c)*Sqrt[x])/(2*c^4) - ((13*b*B - 9*A*c)*x^(5/2))/(10*c^3)
+ ((13*b*B - 9*A*c)*x^(9/2))/(18*b*c^2) - ((b*B - A*c)*x^(13/2))/(2*b*c*(b
+ c*x^2)) + (b^(5/4)*(13*b*B - 9*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/
b^(1/4)])/(4*Sqrt[2]*c^(17/4)) - (b^(5/4)*(13*b*B - 9*A*c)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(17/4)) + (b^(5/4)*(13*b*B - 9*
A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]
*c^(17/4)) - (b^(5/4)*(13*b*B - 9*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(17/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{x^{11/2}(A+Bx^2)}{(b+cx^2)^2}\,dx$$

$$= -\frac{(bB-Ac)x^{13/2}}{2bc(b+cx^2)} + \frac{\left(\frac{13bB}{2}-\frac{9Ac}{2}\right)\int \frac{x^{11/2}}{b+cx^2}\,dx}{2bc}$$

$$= \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc(b+cx^2)} - \frac{(13bB-9Ac)\int \frac{x^{7/2}}{b+cx^2}\,dx}{4c^2}$$

$$= -\frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc(b+cx^2)} + \frac{(b(13bB-9Ac))\int \frac{x^{3/2}}{b+cx^2}\,dx}{4c^3}$$

$$= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4} - \frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc(b+cx^2)} - \frac{(b^2(13bB-9Ac))\int \frac{1}{\sqrt{x}(b+cx^2)}\,dx}{4c^4}$$

$$= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4} - \frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc(b+cx^2)} - \frac{(b^2(13bB-9Ac))\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c^4}$$

$$= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4} - \frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} - \frac{(bB-Ac)x^{13/2}}{2bc(b+cx^2)} - \frac{\left(b^{3/2}(13bB-9Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^4} - \frac{\left(b^{3/2}(13bB-9Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^4}$$

$$=\frac{b(13bB-9Ac)\sqrt{x}}{2c^4}-\frac{(13bB-9Ac)x^{5/2}}{10c^3}+\frac{(13bB-9Ac)x^{9/2}}{18bc^2}-\frac{(bB-Ac)x^{13/2}}{2bc\left(b+cx^2\right)}-\frac{\left(b^{3/2}(13bB-9Ac)\right)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8c^{9/2}}-\frac{\left(b^{3/2}(13bB-9Ac)\right)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8c^{9/2}}+\frac{\left(b^{5/4}(13bB-9Ac)\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{2}c^{17/4}}+\frac{\left(b^{5/4}(13bB-9Ac)\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{2}c^{17/4}}$$

$$=\frac{b(13bB-9Ac)\sqrt{x}}{2c^4}-\frac{(13bB-9Ac)x^{5/2}}{10c^3}+\frac{(13bB-9Ac)x^{9/2}}{18bc^2}-\frac{(bB-Ac)x^{13/2}}{2bc\left(b+cx^2\right)}+\frac{b^{5/4}(13bB-9Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{17/4}}-\frac{b^{5/4}(13bB-9Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{17/4}}-\frac{\left(b^{5/4}(13bB-9Ac)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{17/4}}+\frac{\left(b^{5/4}(13bB-9Ac)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{17/4}}$$

$$
\begin{aligned}
&= \frac{b(13bB-9Ac)\sqrt{x}}{2c^4} - \frac{(13bB-9Ac)x^{5/2}}{10c^3} + \frac{(13bB-9Ac)x^{9/2}}{18bc^2} \\
&- \frac{(bB-Ac)x^{13/2}}{2bc\,(b+cx^2)} + \frac{b^{5/4}(13bB-9Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{17/4}} \\
&- \frac{b^{5/4}(13bB-9Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{17/4}} \\
&+ \frac{b^{5/4}(13bB-9Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{17/4}} \\
&- \frac{b^{5/4}(13bB-9Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{17/4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.62

$$
\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\frac{4\sqrt[4]{c}\sqrt{x}(585b^3B-9b^2c(45A-52Bx^2)+4c^3x^4(9A+5Bx^2)-4bc^2x^2(81A+13Bx^2))}{b+cx^2} + 45\sqrt{2}b^{5/4}(13bB-9A
}{360c^{17/4}}
$$

```
[In] Integrate[(x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*c^(1/4)*Sqrt[x]*(585*b^3*B - 9*b^2*c*(45*A - 52*B*x^2) + 4*c^3*x^4*(9*A
 + 5*B*x^2) - 4*b*c^2*x^2*(81*A + 13*B*x^2)))/(b + c*x^2) + 45*Sqrt[2]*b^(5
/4)*(13*b*B - 9*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*
Sqrt[x])] - 45*Sqrt[2]*b^(5/4)*(13*b*B - 9*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(360*c^(17/4))
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.58

method	result
risch	$-\frac{2(-5Bc^2x^4-9Ac^2x^2+18Bbcx^2+90Abc-135Bb^2)\sqrt{x}}{45c^4}+\frac{b^2\left(\frac{2\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)\sqrt{x}}{cx^2+b}+\frac{(9Ac-13Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{}\right.}{}$
derivativedivides	$-\frac{2\left(-\frac{Bx^{\frac{9}{2}}c^2}{9}-\frac{Ac^2x^{\frac{5}{2}}}{5}+\frac{2Bbcx^{\frac{5}{2}}}{5}+2Abc\sqrt{x}-3Bb^2\sqrt{x}\right)}{c^4}+\frac{2b^2\left(\frac{\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)\sqrt{x}}{cx^2+b}+\frac{(9Ac-13Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{}\right.}{}$
default	$-\frac{2\left(-\frac{Bx^{\frac{9}{2}}c^2}{9}-\frac{Ac^2x^{\frac{5}{2}}}{5}+\frac{2Bbcx^{\frac{5}{2}}}{5}+2Abc\sqrt{x}-3Bb^2\sqrt{x}\right)}{c^4}+\frac{2b^2\left(\frac{\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)\sqrt{x}}{cx^2+b}+\frac{(9Ac-13Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{}\right.}{}$

```
[In] int(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(-5*B*c^2*x^4-9*A*c^2*x^2+18*B*b*c*x^2+90*A*b*c-135*B*b^2)*x^(1/2)/c^
4+b^2/c^4*(2*(-1/4*A*c+1/4*B*b)*x^(1/2)/(c*x^2+b)+1/16*(9*A*c-13*B*b)*(1/c*
b)^(1/4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(
1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)
*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 748, normalized size of antiderivative = 2.25

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{45\,(c^5x^2+bc^4)\left(-\frac{28561\,B^4b^9-79092\,AB^3b^8c+82134\,A^2B^2b^7c^2-37908\,A^3Bb^6c^3+6561\,A^4b^5c^4}{c^{17}}\right)^{\frac{1}{4}}\log\left(\right.}{}$$

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/360*(45*(c^5*x^2 + b*c^4)*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^
2*B^2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4)*log(c^4
*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^2*B^2*b^7*c^2 - 37908*A^3*B
*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4) - (13*B*b^2 - 9*A*b*c)*sqrt(x)) -
45*(-I*c^5*x^2 - I*b*c^4)*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^2*
```

```
B^2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4)*log(I*c^4
*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^2*B^2*b^7*c^2 - 37908*A^3*B
*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4) - (13*B*b^2 - 9*A*b*c)*sqrt(x)) -
45*(I*c^5*x^2 + I*b*c^4)*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^2*B
^2*b^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4)*log(-I*c^4
*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^2*B^2*b^7*c^2 - 37908*A^3*B
*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4) - (13*B*b^2 - 9*A*b*c)*sqrt(x)) -
45*(c^5*x^2 + b*c^4)*(-(28561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^2*B^2*b
^7*c^2 - 37908*A^3*B*b^6*c^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4)*log(-c^4*(-(28
561*B^4*b^9 - 79092*A*B^3*b^8*c + 82134*A^2*B^2*b^7*c^2 - 37908*A^3*B*b^6*c
^3 + 6561*A^4*b^5*c^4)/c^17)^(1/4) - (13*B*b^2 - 9*A*b*c)*sqrt(x)) + 4*(20*
B*c^3*x^6 - 4*(13*B*b*c^2 - 9*A*c^3)*x^4 + 585*B*b^3 - 405*A*b^2*c + 36*(13
*B*b^2*c - 9*A*b*c^2)*x^2)*sqrt(x))/(c^5*x^2 + b*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**(19/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.90

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(Bb^3-Ab^2c)\sqrt{x}}{2\,(c^5x^2+bc^4)} - \frac{\left(\frac{2\sqrt{2}(13\,Bb-9\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}(13\,Bb-9\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}(13\,Bb-9\,Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c\cdots\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\cdots\right)}{16\,c^4} + \frac{2\left(5\,Bc^2x^{\frac{9}{2}}-9\,(2\,Bbc-Ac^2)x^{\frac{5}{2}}+45\,(3\,Bb^2-2\,Abc)\sqrt{x}\right)}{45\,c^4}$$

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
[Out] 1/2*(B*b^3 - A*b^2*c)*sqrt(x)/(c^5*x^2 + b*c^4) - 1/16*(2*sqrt(2)*(13*B*b -
 9*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sq
```

```
rt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(13*B*b -
9*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sq
rt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(13*B*b - 9*
A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^
(1/4)) - sqrt(2)*(13*B*b - 9*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sq
rt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))*b^2/c^4 + 2/45*(5*B*c^2*x^(9/2) - 9*(
2*B*b*c - A*c^2)*x^(5/2) + 45*(3*B*b^2 - 2*A*b*c)*sqrt(x))/c^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.01

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{\sqrt{2}\Big(13\,(bc^3)^{\frac{1}{4}}\,Bb^2-9\,(bc^3)^{\frac{1}{4}}\,Abc\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^5} - \frac{\sqrt{2}\Big(13\,(bc^3)^{\frac{1}{4}}\,Bb^2-9\,(bc^3)^{\frac{1}{4}}\,Abc\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^5} - \frac{\sqrt{2}\Big(13\,(bc^3)^{\frac{1}{4}}\,Bb^2-9\,(bc^3)^{\frac{1}{4}}\,Abc\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^5} + \frac{\sqrt{2}\Big(13\,(bc^3)^{\frac{1}{4}}\,Bb^2-9\,(bc^3)^{\frac{1}{4}}\,Abc\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^5} + \frac{Bb^3\sqrt{x}-Ab^2c\sqrt{x}}{2\,(cx^2+b)c^4} + \frac{2\left(5\,Bc^{16}x^{\frac{9}{2}}-18\,Bbc^{15}x^{\frac{5}{2}}+9\,Ac^{16}x^{\frac{5}{2}}+135\,Bb^2c^{14}\sqrt{x}-90\,Abc^{15}\sqrt{x}\right)}{45\,c^{18}}$$

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(13*(b*c^3)^(1/4)*B*b^2 - 9*(b*c^3)^(1/4)*A*b*c)*arctan(1/2*sq
rt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^5 - 1/8*sqrt(2)*(13*
(b*c^3)^(1/4)*B*b^2 - 9*(b*c^3)^(1/4)*A*b*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^5 - 1/16*sqrt(2)*(13*(b*c^3)^(1/4)*B
*b^2 - 9*(b*c^3)^(1/4)*A*b*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/
c))/c^5 + 1/16*sqrt(2)*(13*(b*c^3)^(1/4)*B*b^2 - 9*(b*c^3)^(1/4)*A*b*c)*log
(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 + 1/2*(B*b^3*sqrt(x) - A
```

*b^2*c*sqrt(x))/((c*x^2 + b)*c^4) + 2/45*(5*B*c^16*x^(9/2) - 18*B*b*c^15*x^(5/2) + 9*A*c^16*x^(5/2) + 135*B*b^2*c^14*sqrt(x) - 90*A*b*c^15*sqrt(x))/c^18

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 857, normalized size of antiderivative = 2.58

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = x^{5/2}\left(\frac{2\,A}{5\,c^2}-\frac{4\,B\,b}{5\,c^3}\right) - \sqrt{x}\left(\frac{2\,b\,\left(\frac{2\,A}{c^2}-\frac{4\,B\,b}{c^3}\right)}{c}+\frac{2\,B\,b^2}{c^4}\right)+\frac{2\,B\,x^{9/2}}{9\,c^2}+\frac{\sqrt{x}\,\left(\frac{B\,b^3}{2}-\frac{A\,b^2\,c}{2}\right)}{c^5\,x^2+b\,c^4}$$
$$+\frac{(-b)^{5/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81\,A^2\,b^4\,c^2-234\,A\,B\,b^5\,c+169\,B^2\,b^6\right)}{c^5}+\frac{(-b)^{5/4}\,(9\,A\,c-13\,B\,b)\left(13\,B\,b^4-9\,A\,b^3\,c\right)}{c^{21/4}}\right)(9\,A\,c-13\,B\,b)\,1\mathrm{i}}{8\,c^{17/4}}+\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81\,A^2\,b\cdots\right.}{\cdots}\right.}{\cdots}}{\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81\,A^2\,b^4\,c^2-234\,A\,B\,b^5\,c+169\,B^2\,b^6\right)}{c^5}+\frac{(-b)^{5/4}\,(9\,A\,c-13\,B\,b)\left(13\,B\,b^4-9\,A\,b^3\,c\right)}{c^{21/4}}\right)(9\,A\,c-13\,B\,b)}{8\,c^{17/4}}-\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81\,A^2\,b\cdots\right.}{\cdots}\right.}{\cdots}}\right.\cdots}{4\,c^{17/4}}$$
$$-\frac{(-b)^{5/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81\,A^2\,b^4\,c^2-234\,A\,B\,b^5\,c+169\,B^2\,b^6\right)}{c^5}-\frac{(-b)^{5/4}\,(9\,A\,c-13\,B\,b)\left(13\,B\,b^4-9\,A\,b^3\,c\right)\,1\mathrm{i}}{c^{21/4}}\right)(9\,A\,c-13\,B\,b)}{8\,c^{17/4}}+\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81\,A\cdots\right.}{\cdots}\right.}{\cdots}}{\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81\,A^2\,b^4\,c^2-234\,A\,B\,b^5\,c+169\,B^2\,b^6\right)}{c^5}-\frac{(-b)^{5/4}\,(9\,A\,c-13\,B\,b)\left(13\,B\,b^4-9\,A\,b^3\,c\right)\,1\mathrm{i}}{c^{21/4}}\right)(9\,A\,c-13\,B\,b)\,1\mathrm{i}}{8\,c^{17/4}}-\frac{(-b)^{5/4}\left(\frac{\sqrt{x}\left(81\,A\cdots\right.}{\cdots}\right.}{\cdots}}\right.\cdots}{4\,c^{17/4}}$$

[In] int((x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)

[Out] x^(5/2)*((2*A)/(5*c^2) - (4*B*b)/(5*c^3)) - x^(1/2)*((2*b*((2*A)/c^2 - (4*B*b)/c^3))/c + (2*B*b^2)/c^4) + (2*B*x^(9/2))/(9*c^2) + (x^(1/2)*((B*b^3)/2 - (A*b^2*c)/2))/(b*c^4 + c^5*x^2) + ((-b)^(5/4)*atan((((-b)^(5/4)*((x^(1/2)*(169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5 + ((-b)^(5/4)*(9*A*c - 13*B*b)*(13*B*b^4 - 9*A*b^3*c))/c^(21/4))*(9*A*c - 13*B*b)*1i)/(8*c^(17/4)) + ((-b)^(5/4)*((x^(1/2)*(169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5 - ((-b)^(5/4)*(9*A*c - 13*B*b)*(13*B*b^4 - 9*A*b^3*c))/c^(21/4))*(9*A*c - 13*B*b)*1i)/(8*c^(17/4)))/(((-b)^(5/4)*((x^(1/2)*(169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5 + ((-b)^(5/4)*(9*A*c - 13*B*b)*(13*B*b^4 - 9*A*b^3*c))/c^(21/4))*(9*A*c - 13*B*b))/(8*c^(17/4)) - ((-b)^(5/4)*((x^(1/2)*(169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5 - ((-b)^(5/4)*(9*A*c - 13*B*b)*(13*B*b^4 - 9*A*b^3*c))/c^(21/4))*(9*A*c - 13*B*b))/(8*c^(17/4))))*(9*A*c - 13*B*b)*1i)/(4*c^(17/4)) - ((-b)^(5/4)*atan((((-b)^(5/4)*((x^(1/2)*(169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5 - ((-b)^(5/4)*(9*A*c - 13*B*b)*(13*B*b^4 - 9*A*b^3*c)*1i)/c^(21/4))*(9*A*c - 13*B*b))/(8*c^(17/4)) + ((-b)^(5/4)*((x^(1/2)*(169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5 + ((-b)^(5/4)*(9*A*c - 13*B*b)*(13*B*b^4 - 9*A*b^3*c)*1i)/c^(21/4))*(9*A*c - 13*B*b))/(8*c^(17/4)))/(((-b)^(5/4)*((x^(1/2)*(169*B^2*b^6 + 81*A^2*b^4*

```
c^2 - 234*A*B*b^5*c))/c^5 - ((-b)^(5/4)*(9*A*c - 13*B*b)*(13*B*b^4 - 9*A*b^
3*c)*1i)/c^(21/4))*(9*A*c - 13*B*b)*1i)/(8*c^(17/4)) - ((-b)^(5/4)*((x^(1/2
)*(169*B^2*b^6 + 81*A^2*b^4*c^2 - 234*A*B*b^5*c))/c^5 + ((-b)^(5/4)*(9*A*c
- 13*B*b)*(13*B*b^4 - 9*A*b^3*c)*1i)/c^(21/4))*(9*A*c - 13*B*b)*1i)/(8*c^(1
7/4))))*(9*A*c - 13*B*b))/(4*c^(17/4))
```

3.196 $\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 310

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(11bB-7Ac)x^{3/2}}{6c^3}+\frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\,(b+cx^2)} - \frac{b^{3/4}(11bB-7Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{15/4}} + \frac{b^{3/4}(11bB-7Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{15/4}} + \frac{b^{3/4}(11bB-7Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{15/4}} - \frac{b^{3/4}(11bB-7Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{15/4}}$$

```
[Out] -1/6*(-7*A*c+11*B*b)*x^(3/2)/c^3+1/14*(-7*A*c+11*B*b)*x^(7/2)/b/c^2-1/2*(-A
*c+B*b)*x^(11/2)/b/c/(c*x^2+b)-1/8*b^(3/4)*(-7*A*c+11*B*b)*arctan(1-c^(1/4)
*2^(1/2)*x^(1/2)/b^(1/4))/c^(15/4)*2^(1/2)+1/8*b^(3/4)*(-7*A*c+11*B*b)*arct
an(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(15/4)*2^(1/2)+1/16*b^(3/4)*(-7*A*c
+11*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(15/4)*2^(
1/2)-1/16*b^(3/4)*(-7*A*c+11*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1
/2)*x^(1/2))/c^(15/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 468, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{b^{3/4}(11bB-7Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{15/4}} + \frac{b^{3/4}(11bB-7Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}c^{15/4}} + \frac{b^{3/4}(11bB-7Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{15/4}} - \frac{b^{3/4}(11bB-7Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{15/4}} - \frac{x^{3/2}(11bB-7Ac)}{6c^3} + \frac{x^{7/2}(11bB-7Ac)}{14bc^2} - \frac{x^{11/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

```
[In] Int[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/6*((11*b*B - 7*A*c)*x^(3/2))/c^3 + ((11*b*B - 7*A*c)*x^(7/2))/(14*b*c^2)
 - ((b*B - A*c)*x^(11/2))/(2*b*c*(b + c*x^2)) - (b^(3/4)*(11*b*B - 7*A*c)*A
rcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(15/4)) + (b^(3/
4)*(11*b*B - 7*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[
2]*c^(15/4)) + (b^(3/4)*(11*b*B - 7*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1
/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(15/4)) - (b^(3/4)*(11*b*B - 7*A*c)*
Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(1
5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{x^{9/2}(A+Bx^2)}{(b+cx^2)^2}\,dx$$

$$= -\frac{(bB-Ac)x^{11/2}}{2bc\,(b+cx^2)} + \frac{\left(\frac{11bB}{2}-\frac{7Ac}{2}\right)\int \frac{x^{9/2}}{b+cx^2}\,dx}{2bc}$$

$$= \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\,(b+cx^2)} - \frac{(11bB-7Ac)\int \frac{x^{5/2}}{b+cx^2}\,dx}{4c^2}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\,(b+cx^2)} + \frac{(b(11bB-7Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4c^3}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\,(b+cx^2)} + \frac{(b(11bB-7Ac))\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c^3}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\,(b+cx^2)} - \frac{(b(11bB-7Ac))\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^{7/2}} + \frac{(b(11bB-7Ac))\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^{7/2}}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\,(b+cx^2)} + \frac{(b(11bB-7Ac))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8c^4} + \frac{(b(11bB-7Ac))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8c^4} + \frac{\left(b^{3/4}(11bB-7Ac)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{2}c^{15/4}} + \frac{\left(b^{3/4}(11bB-7Ac)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{2}c^{15/4}}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc\,(b+cx^2)} + \frac{b^{3/4}(11bB-7Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{15/4}} - \frac{b^{3/4}(11bB-7Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{15/4}} + \frac{\left(b^{3/4}(11bB-7Ac)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{15/4}} - \frac{\left(b^{3/4}(11bB-7Ac)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{15/4}}$$

$$= -\frac{(11bB-7Ac)x^{3/2}}{6c^3} + \frac{(11bB-7Ac)x^{7/2}}{14bc^2} - \frac{(bB-Ac)x^{11/2}}{2bc(b+cx^2)} - \frac{b^{3/4}(11bB-7Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{15/4}} + \frac{b^{3/4}(11bB-7Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{15/4}} + \frac{b^{3/4}(11bB-7Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{15/4}} - \frac{b^{3/4}(11bB-7Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{15/4}}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.59

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\frac{4c^{3/4}x^{3/2}(-77b^2B+bc(49A-44Bx^2)+4c^2x^2(7A+3Bx^2))}{b+cx^2} - 21\sqrt{2}b^{3/4}(11bB-7Ac)\arctan\left(\frac{\sqrt{}}{\sqrt{2}\sqrt[4]{}}\right.}{168c^{15/4}}$$

```
[In] Integrate[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*c^(3/4)*x^(3/2)*(-77*b^2*B + b*c*(49*A - 44*B*x^2) + 4*c^2*x^2*(7*A + 3
*B*x^2)))/(b + c*x^2) - 21*Sqrt[2]*b^(3/4)*(11*b*B - 7*A*c)*ArcTan[(Sqrt[b]
 - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - 21*Sqrt[2]*b^(3/4)*(11*b
*B - 7*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)
])/(168*c^(15/4))
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.55

method	result
risch	$\frac{2x^{\frac{3}{2}}\left(3Bc\,x^2+7Ac-14Bb\right)}{21c^3}-\frac{b\left(\frac{2\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^2+b}+\frac{\left(\frac{7Ac}{4}-\frac{11Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{a}\cdots\right.}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\cdots\right)}{c^3}$
derivativedivides	$\frac{\frac{2Bc\,x^{\frac{7}{2}}}{7}+\frac{2(Ac-2Bb)x^{\frac{3}{2}}}{3}}{c^3}-\frac{2b\left(\frac{\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^2+b}+\frac{\left(\frac{7Ac}{4}-\frac{11Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{arcta}\cdots\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\cdots\right)}{c^3}$
default	$\frac{\frac{2Bc\,x^{\frac{7}{2}}}{7}+\frac{2(Ac-2Bb)x^{\frac{3}{2}}}{3}}{c^3}-\frac{2b\left(\frac{\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^2+b}+\frac{\left(\frac{7Ac}{4}-\frac{11Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{arcta}\cdots\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\cdots\right)}{c^3}$

```
[In] int(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*x^(3/2)*(3*B*c*x^2+7*A*c-14*B*b)/c^3-b/c^3*(2*(-1/4*A*c+1/4*B*b)*x^(3/
2)/(c*x^2+b)+1/4*(7/4*A*c-11/4*B*b)/c/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^
(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*
b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*
b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 848, normalized size of antiderivative = 2.74

$$\int \frac{x^{17/2}(A+Bx^2)}{\left(bx^2+cx^4\right)^2}\,dx =$$

$$-\frac{21\left(c^4x^2+bc^3\right)\left(-\frac{14641\,B^4b^7-37268\,AB^3b^6c+35574\,A^2B^2b^5c^2-15092\,A^3Bb^4c^3+2401\,A^4b^3c^4}{c^{15}}\right)^{\frac{1}{4}}\log\left(c^{11}\left(-\frac{14641\,B^4b^7-37268\,AB^3\cdots}{\cdots}\right.\right.}{\cdots}$$

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/168*(21*(c^4*x^2 + b*c^3)*(-(14641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A
^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15)^(1/4)*log(c^
11*(-(14641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*A^3
```

```
*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15)^(3/4) - (1331*B^3*b^5 - 2541*A*B^2*b^4
*c + 1617*A^2*B*b^3*c^2 - 343*A^3*b^2*c^3)*sqrt(x)) + 21*(-I*c^4*x^2 - I*b*
c^3)*(-(14641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*A
^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15)^(1/4)*log(I*c^11*(-(14641*B^4*b^7 -
37268*A*B^3*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4*
b^3*c^4)/c^15)^(3/4) - (1331*B^3*b^5 - 2541*A*B^2*b^4*c + 1617*A^2*B*b^3*c^
2 - 343*A^3*b^2*c^3)*sqrt(x)) + 21*(I*c^4*x^2 + I*b*c^3)*(-(14641*B^4*b^7 -
 37268*A*B^3*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4
*b^3*c^4)/c^15)^(1/4)*log(-I*c^11*(-(14641*B^4*b^7 - 37268*A*B^3*b^6*c + 35
574*A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15)^(3/4) -
 (1331*B^3*b^5 - 2541*A*B^2*b^4*c + 1617*A^2*B*b^3*c^2 - 343*A^3*b^2*c^3)*s
qrt(x)) - 21*(c^4*x^2 + b*c^3)*(-(14641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574
*A^2*B^2*b^5*c^2 - 15092*A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15)^(1/4)*log(
-c^11*(-(14641*B^4*b^7 - 37268*A*B^3*b^6*c + 35574*A^2*B^2*b^5*c^2 - 15092*
A^3*B*b^4*c^3 + 2401*A^4*b^3*c^4)/c^15)^(3/4) - (1331*B^3*b^5 - 2541*A*B^2*
b^4*c + 1617*A^2*B*b^3*c^2 - 343*A^3*b^2*c^3)*sqrt(x)) - 4*(12*B*c^2*x^5 -
4*(11*B*b*c - 7*A*c^2)*x^3 - 7*(11*B*b^2 - 7*A*b*c)*x)*sqrt(x))/(c^4*x^2 +
b*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**(17/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.80

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(Bb^2-Abc)x^{\frac{3}{2}}}{2\,(c^4x^2+bc^3)} + \frac{(11\,Bb^2-7\,Abc)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right.}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right.}{16\,c^3} + \frac{2\left(3\,Bcx^{\frac{7}{2}}-7\,(2\,Bb-Ac)x^{\frac{3}{2}}\right)}{21\,c^3}$$

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b^2 - A*b*c)*x^(3/2)/(c^4*x^2 + b*c^3) + 1/16*(11*B*b^2 - 7*A*b*c)*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))
/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqr
t(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4
)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b
^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/c^3 + 2/21
*(3*B*c*x^(7/2) - 7*(2*B*b - A*c)*x^(3/2))/c^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.96

$$
\begin{aligned}
\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = &-\frac{Bb^2x^{\frac{3}{2}}-Abcx^{\frac{3}{2}}}{2\,(cx^2+b)c^3}\\
&+\frac{\sqrt{2}\Big(11\,(bc^3)^{\frac{3}{4}}\,Bb-7\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^6}\\
&+\frac{\sqrt{2}\Big(11\,(bc^3)^{\frac{3}{4}}\,Bb-7\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^6}\\
&-\frac{\sqrt{2}\Big(11\,(bc^3)^{\frac{3}{4}}\,Bb-7\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^6}\\
&+\frac{\sqrt{2}\Big(11\,(bc^3)^{\frac{3}{4}}\,Bb-7\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^6}\\
&+\frac{2\left(3\,Bc^{12}x^{\frac{7}{2}}-14\,Bbc^{11}x^{\frac{3}{2}}+7\,Ac^{12}x^{\frac{3}{2}}\right)}{21\,c^{14}}
\end{aligned}
$$

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*b^2*x^(3/2) - A*b*c*x^(3/2))/((c*x^2 + b)*c^3) + 1/8*sqrt(2)*(11*(b
*c^3)^(3/4)*B*b - 7*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1
/4) + 2*sqrt(x))/(b/c)^(1/4))/c^6 + 1/8*sqrt(2)*(11*(b*c^3)^(3/4)*B*b - 7*(
b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/
c)^(1/4))/c^6 - 1/16*sqrt(2)*(11*(b*c^3)^(3/4)*B*b - 7*(b*c^3)^(3/4)*A*c)*l
og(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^6 + 1/16*sqrt(2)*(11*(b*c
^3)^(3/4)*B*b - 7*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
```

```
 sqrt(b/c))/c^6 + 2/21*(3*B*c^12*x^(7/2) - 14*B*b*c^11*x^(3/2) + 7*A*c^12*x
^(3/2))/c^14
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.41

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = x^{3/2}\left(\frac{2\,A}{3\,c^2}-\frac{4\,B\,b}{3\,c^3}\right)+\frac{2\,B\,x^{7/2}}{7\,c^2}-\frac{x^{3/2}\,\left(\frac{B\,b^2}{2}-\frac{A\,b\,c}{2}\right)}{c^4\,x^2+b\,c^3} + \frac{(-b)^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(7\,A\,c-11\,B\,b)}{4\,c^{15/4}}+\frac{(-b)^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1\mathrm{i}}{(-b)^{1/4}}\right)\,(7\,A\,c-11\,B\,b)\,1\mathrm{i}}{4\,c^{15/4}}$$

```
[In] int((x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(3/2)*((2*A)/(3*c^2) - (4*B*b)/(3*c^3)) + (2*B*x^(7/2))/(7*c^2) - (x^(3/2
)*((B*b^2)/2 - (A*b*c)/2))/(b*c^3 + c^4*x^2) + ((-b)^(3/4)*atan((c^(1/4)*x^
(1/2))/(-b)^(1/4))*(7*A*c - 11*B*b))/(4*c^(15/4)) + ((-b)^(3/4)*atan((c^(1/
4)*x^(1/2)*1i)/(-b)^(1/4))*(7*A*c - 11*B*b)*1i)/(4*c^(15/4))
```

3.197 $\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 310

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(9bB-5Ac)\sqrt{x}}{2c^3}+\frac{(9bB-5Ac)x^{5/2}}{10bc^2}-\frac{(bB-Ac)x^{9/2}}{2bc\,(b+cx^2)} -\frac{\sqrt[4]{b}(9bB-5Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{13/4}}+\frac{\sqrt[4]{b}(9bB-5Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{13/4}} -\frac{\sqrt[4]{b}(9bB-5Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}} +\frac{\sqrt[4]{b}(9bB-5Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}$$

```
[Out] 1/10*(-5*A*c+9*B*b)*x^(5/2)/b/c^2-1/2*(-A*c+B*b)*x^(9/2)/b/c/(c*x^2+b)-1/8*
b^(1/4)*(-5*A*c+9*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(13/4)*2
^(1/2)+1/8*b^(1/4)*(-5*A*c+9*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))
/c^(13/4)*2^(1/2)-1/16*b^(1/4)*(-5*A*c+9*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*
c^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)*2^(1/2)+1/16*b^(1/4)*(-5*A*c+9*B*b)*ln(b^
(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)*2^(1/2)-1/2*(-5*A
*c+9*B*b)*x^(1/2)/c^3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1598, 468, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{\sqrt[4]{b}(9bB-5Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{13/4}} + \frac{\sqrt[4]{b}(9bB-5Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}c^{13/4}} - \frac{\sqrt[4]{b}(9bB-5Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}} + \frac{\sqrt[4]{b}(9bB-5Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}} - \frac{\sqrt{x}(9bB-5Ac)}{2c^3} + \frac{x^{5/2}(9bB-5Ac)}{10bc^2} - \frac{x^{9/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

```
[In] Int[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*((9*b*B - 5*A*c)*Sqrt[x])/c^3 + ((9*b*B - 5*A*c)*x^(5/2))/(10*b*c^2) -
 ((b*B - A*c)*x^(9/2))/(2*b*c*(b + c*x^2)) - (b^(1/4)*(9*b*B - 5*A*c)*ArcTa
n[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(13/4)) + (b^(1/4)*(
9*b*B - 5*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^
(13/4)) - (b^(1/4)*(9*b*B - 5*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sq
rt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(13/4)) + (b^(1/4)*(9*b*B - 5*A*c)*Log[Sqr
t[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^{7/2}(A+Bx^2)}{(b+cx^2)^2}\,dx \\
&= -\frac{(bB-Ac)x^{9/2}}{2bc\,(b+cx^2)} + \frac{\left(\frac{9bB}{2}-\frac{5Ac}{2}\right)\int \frac{x^{7/2}}{b+cx^2}\,dx}{2bc} \\
&= \frac{(9bB-5Ac)x^{5/2}}{10bc^2} - \frac{(bB-Ac)x^{9/2}}{2bc\,(b+cx^2)} - \frac{(9bB-5Ac)\int \frac{x^{3/2}}{b+cx^2}\,dx}{4c^2} \\
&= -\frac{(9bB-5Ac)\sqrt{x}}{2c^3} + \frac{(9bB-5Ac)x^{5/2}}{10bc^2} - \frac{(bB-Ac)x^{9/2}}{2bc\,(b+cx^2)} + \frac{(b(9bB-5Ac))\int \frac{1}{\sqrt{x}(b+cx^2)}\,dx}{4c^3} \\
&= -\frac{(9bB-5Ac)\sqrt{x}}{2c^3} + \frac{(9bB-5Ac)x^{5/2}}{10bc^2} - \frac{(bB-Ac)x^{9/2}}{2bc\,(b+cx^2)} \\
&\quad + \frac{(b(9bB-5Ac))\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2c^3} \\
&= -\frac{(9bB-5Ac)\sqrt{x}}{2c^3} + \frac{(9bB-5Ac)x^{5/2}}{10bc^2} - \frac{(bB-Ac)x^{9/2}}{2bc\,(b+cx^2)} \\
&\quad + \frac{\left(\sqrt{b}(9bB-5Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^3} \\
&\quad + \frac{\left(\sqrt{b}(9bB-5Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4c^3}
\end{aligned}$$

$$= -\frac{(9bB-5Ac)\sqrt{x}}{2c^3}+\frac{(9bB-5Ac)x^{5/2}}{10bc^2}-\frac{(bB-Ac)x^{9/2}}{2bc\,(b+cx^2)}$$
$$+\frac{\left(\sqrt{b}(9bB-5Ac)\right)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8c^{7/2}}$$
$$+\frac{\left(\sqrt{b}(9bB-5Ac)\right)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8c^{7/2}}$$
$$-\frac{\left(\sqrt[4]{b}(9bB-5Ac)\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{2}c^{13/4}}$$
$$-\frac{\left(\sqrt[4]{b}(9bB-5Ac)\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{2}c^{13/4}}$$

$$= -\frac{(9bB-5Ac)\sqrt{x}}{2c^3}+\frac{(9bB-5Ac)x^{5/2}}{10bc^2}-\frac{(bB-Ac)x^{9/2}}{2bc\,(b+cx^2)}$$
$$-\frac{\sqrt[4]{b}(9bB-5Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}$$
$$+\frac{\sqrt[4]{b}(9bB-5Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}$$
$$+\frac{\left(\sqrt[4]{b}(9bB-5Ac)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{13/4}}$$
$$-\frac{\left(\sqrt[4]{b}(9bB-5Ac)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{13/4}}$$

$$
\begin{aligned}
&= -\frac{(9bB-5Ac)\sqrt{x}}{2c^3}+\frac{(9bB-5Ac)x^{5/2}}{10bc^2}-\frac{(bB-Ac)x^{9/2}}{2bc\,(b+cx^2)} \\
&\quad -\frac{\sqrt[4]{b}(9bB-5Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{13/4}}+\frac{\sqrt[4]{b}(9bB-5Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{13/4}} \\
&\quad -\frac{\sqrt[4]{b}(9bB-5Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}} \\
&\quad +\frac{\sqrt[4]{b}(9bB-5Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.59

$$
\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx=\frac{\frac{4\sqrt[4]{c}\sqrt{x}(-45b^2B+bc(25A-36Bx^2)+4c^2x^2(5A+Bx^2))}{b+cx^2}-5\sqrt{2}\sqrt[4]{b}(9bB-5Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\cdots}\right.\cdots}{40c^{13/4}}
$$

```
[In] Integrate[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*c^(1/4)*Sqrt[x]*(-45*b^2*B + b*c*(25*A - 36*B*x^2) + 4*c^2*x^2*(5*A + B
*x^2)))/(b + c*x^2) - 5*Sqrt[2]*b^(1/4)*(9*b*B - 5*A*c)*ArcTan[(Sqrt[b] - S
qrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 5*Sqrt[2]*b^(1/4)*(9*b*B - 5
*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(40
*c^(13/4))
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.55

method	result
risch	$\frac{2(Bc\,x^2+5Ac-10Bb)\sqrt{x}}{5c^3}-\frac{b\left(\frac{2\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(5Ac-9Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2}{16b}\right.}{c^3}$
derivativedivides	$\frac{\frac{2Bc\,x^{\frac{5}{2}}}{5}+2Ac\sqrt{x}-4bB\sqrt{x}}{c^3}-\frac{2b\left(\frac{\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(5Ac-9Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2}{32b}\right.}{c^3}$
default	$\frac{\frac{2Bc\,x^{\frac{5}{2}}}{5}+2Ac\sqrt{x}-4bB\sqrt{x}}{c^3}-\frac{2b\left(\frac{\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(5Ac-9Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2}{32b}\right.}{c^3}$

```
[In] int(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(B*c*x^2+5*A*c-10*B*b)*x^(1/2)/c^3-b/c^3*(2*(-1/4*A*c+1/4*B*b)*x^(1/2)/
(c*x^2+b)+1/16*(5*A*c-9*B*b)*(1/c*b)^(1/4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x
^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2
)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4
)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 696, normalized size of antiderivative = 2.25

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx =$$

$$-\frac{5\left(c^4x^2+bc^3\right)\left(-\frac{6561\,B^4b^5-14580\,AB^3b^4c+12150\,A^2B^2b^3c^2-4500\,A^3Bb^2c^3+625\,A^4bc^4}{c^{13}}\right)^{\frac{1}{4}}\log\left(c^3\left(-\frac{6561\,B^4b^5-14580\,AB^3b^4c+121}{}\right.\right.}{}$$

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(5*(c^4*x^2 + b*c^3)*(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2*
B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13)^(1/4)*log(c^3*(-(65
61*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3
```

```
 + 625*A^4*b*c^4)/c^13)^(1/4) - (9*B*b - 5*A*c)*sqrt(x)) + 5*(I*c^4*x^2 + I
*b*c^3)*(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4500*
A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13)^(1/4)*log(I*c^3*(-(6561*B^4*b^5 - 1458
0*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)
/c^13)^(1/4) - (9*B*b - 5*A*c)*sqrt(x)) + 5*(-I*c^4*x^2 - I*b*c^3)*(-(6561*
B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 +
625*A^4*b*c^4)/c^13)^(1/4)*log(-I*c^3*(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c +
 12150*A^2*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13)^(1/4) -
(9*B*b - 5*A*c)*sqrt(x)) - 5*(c^4*x^2 + b*c^3)*(-(6561*B^4*b^5 - 14580*A*B^
3*b^4*c + 12150*A^2*B^2*b^3*c^2 - 4500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13)
^(1/4)*log(-c^3*(-(6561*B^4*b^5 - 14580*A*B^3*b^4*c + 12150*A^2*B^2*b^3*c^2
 - 4500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^13)^(1/4) - (9*B*b - 5*A*c)*sqrt(x
)) - 4*(4*B*c^2*x^4 - 45*B*b^2 + 25*A*b*c - 4*(9*B*b*c - 5*A*c^2)*x^2)*sqrt
(x))/(c^4*x^2 + b*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**(15/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.87

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(Bb^2-Abc)\sqrt{x}}{2\,(c^4x^2+bc^3)} + \frac{\left(\frac{2\sqrt{2}(9\,Bb-5\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}(9\,Bb-5\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}(9\,Bb-5\,Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right.}{16\,c^3} + \frac{2\left(Bcx^{\frac{5}{2}}-5\,(2\,Bb-Ac)\sqrt{x}\right)}{5\,c^3}$$

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b^2 - A*b*c)*sqrt(x)/(c^4*x^2 + b*c^3) + 1/16*(2*sqrt(2)*(9*B*b - 5
*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt
(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(9*B*b - 5*A
*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(
sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(9*B*b - 5*A*c)
*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4
)) - sqrt(2)*(9*B*b - 5*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)
*x + sqrt(b))/(b^(3/4)*c^(1/4)))*b/c^3 + 2/5*(B*c*x^(5/2) - 5*(2*B*b - A*c)
*sqrt(x))/c^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.96

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\sqrt{2}\Big(9\,(bc^3)^{\frac{1}{4}}\,Bb-5\,(bc^3)^{\frac{1}{4}}\,Ac\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^4} + \frac{\sqrt{2}\Big(9\,(bc^3)^{\frac{1}{4}}\,Bb-5\,(bc^3)^{\frac{1}{4}}\,Ac\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^4} + \frac{\sqrt{2}\Big(9\,(bc^3)^{\frac{1}{4}}\,Bb-5\,(bc^3)^{\frac{1}{4}}\,Ac\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^4} - \frac{\sqrt{2}\Big(9\,(bc^3)^{\frac{1}{4}}\,Bb-5\,(bc^3)^{\frac{1}{4}}\,Ac\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^4} - \frac{Bb^2\sqrt{x}-Abc\sqrt{x}}{2\,(cx^2+b)c^3} + \frac{2\left(Bc^8x^{\frac{5}{2}}-10\,Bbc^7\sqrt{x}+5\,Ac^8\sqrt{x}\right)}{5\,c^{10}}$$

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^4 + 1/8*sqrt(2)*(9*(b*c^3)
^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
- 2*sqrt(x))/(b/c)^(1/4))/c^4 + 1/16*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - 5*(b*c^
3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 - 1/16*s
qrt(2)*(9*(b*c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/
c)^(1/4) + x + sqrt(b/c))/c^4 - 1/2*(B*b^2*sqrt(x) - A*b*c*sqrt(x))/((c*x^2
 + b)*c^3) + 2/5*(B*c^8*x^(5/2) - 10*B*b*c^7*sqrt(x) + 5*A*c^8*sqrt(x))/c^1
0
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 823, normalized size of antiderivative = 2.65

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \sqrt{x}\left(\frac{2\,A}{c^2}-\frac{4\,B\,b}{c^3}\right)+\frac{2\,B\,x^{5/2}}{5\,c^2}-\frac{\sqrt{x}\left(\frac{B\,b^2}{2}-\frac{A\,b\,c}{2}\right)}{c^4\,x^2+b\,c^3}$$

$$+\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25\,A^2\,b^2\,c^2-90\,A\,B\,b^3\,c+81\,B^2\,b^4\right)}{c^3}-\frac{(-b)^{1/4}\,(5\,A\,c-9\,B\,b)\left(72\,B\,b^3-40\,A\,b^2\,c\right)}{8\,c^{13/4}}\right)(5\,A\,c-9\,B\,b)\,1\mathrm{i}}{8\,c^{13/4}}+\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25\,A^2\,b^2\right.}{}\right.}{}}{\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25\,A^2\,b^2\,c^2-90\,A\,B\,b^3\,c+81\,B^2\,b^4\right)}{c^3}-\frac{(-b)^{1/4}\,(5\,A\,c-9\,B\,b)\left(72\,B\,b^3-40\,A\,b^2\,c\right)}{8\,c^{13/4}}\right)(5\,A\,c-9\,B\,b)}{8\,c^{13/4}}-\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25\,A^2\,b^2\right.}{}\right.}{}}\right.}{4\,c^{13/4}}$$

$$+\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25\,A^2\,b^2\,c^2-90\,A\,B\,b^3\,c+81\,B^2\,b^4\right)}{c^3}-\frac{(-b)^{1/4}\,(5\,A\,c-9\,B\,b)\left(72\,B\,b^3-40\,A\,b^2\,c\right)1\mathrm{i}}{8\,c^{13/4}}\right)(5\,A\,c-9\,B\,b)}{8\,c^{13/4}}+\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25\,A^2\right.}{}\right.}{}}{\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25\,A^2\,b^2\,c^2-90\,A\,B\,b^3\,c+81\,B^2\,b^4\right)}{c^3}-\frac{(-b)^{1/4}\,(5\,A\,c-9\,B\,b)\left(72\,B\,b^3-40\,A\,b^2\,c\right)1\mathrm{i}}{8\,c^{13/4}}\right)(5\,A\,c-9\,B\,b)\,1\mathrm{i}}{8\,c^{13/4}}-\frac{(-b)^{1/4}\left(\frac{\sqrt{x}\left(25\,A^2\right.}{}\right.}{}}\right.}{4\,c^{13/4}}$$

```
[In] int((x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] x^(1/2)*((2*A)/c^2 - (4*B*b)/c^3) + (2*B*x^(5/2))/(5*c^2) - (x^(1/2)*((B*b^
2)/2 - (A*b*c)/2))/(b*c^3 + c^4*x^2) + ((-b)^(1/4)*atan((((-b)^(1/4)*((x^(1
/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c))/c^3 - ((-b)^(1/4)*(5*A*c
- 9*B*b)*(72*B*b^3 - 40*A*b^2*c))/(8*c^(13/4)))*(5*A*c - 9*B*b)*1i)/(8*c^(1
3/4)) + ((-b)^(1/4)*((x^(1/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c))
/c^3 + ((-b)^(1/4)*(5*A*c - 9*B*b)*(72*B*b^3 - 40*A*b^2*c))/(8*c^(13/4)))*(
5*A*c - 9*B*b)*1i)/(8*c^(13/4)))/(((-b)^(1/4)*((x^(1/2)*(81*B^2*b^4 + 25*A^
2*b^2*c^2 - 90*A*B*b^3*c))/c^3 - ((-b)^(1/4)*(5*A*c - 9*B*b)*(72*B*b^3 - 40
*A*b^2*c))/(8*c^(13/4)))*(5*A*c - 9*B*b))/(8*c^(13/4)) - ((-b)^(1/4)*((x^(1
/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c))/c^3 + ((-b)^(1/4)*(5*A*c
- 9*B*b)*(72*B*b^3 - 40*A*b^2*c))/(8*c^(13/4)))*(5*A*c - 9*B*b))/(8*c^(13/4
))))*(5*A*c - 9*B*b)*1i)/(4*c^(13/4)) + ((-b)^(1/4)*atan((((-b)^(1/4)*((x^(
1/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c))/c^3 - ((-b)^(1/4)*(5*A*c
 - 9*B*b)*(72*B*b^3 - 40*A*b^2*c)*1i)/(8*c^(13/4)))*(5*A*c - 9*B*b))/(8*c^(
13/4)) + ((-b)^(1/4)*((x^(1/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c)
)/c^3 + ((-b)^(1/4)*(5*A*c - 9*B*b)*(72*B*b^3 - 40*A*b^2*c)*1i)/(8*c^(13/4)
))*(5*A*c - 9*B*b))/(8*c^(13/4)))/(((-b)^(1/4)*((x^(1/2)*(81*B^2*b^4 + 25*A
^2*b^2*c^2 - 90*A*B*b^3*c))/c^3 - ((-b)^(1/4)*(5*A*c - 9*B*b)*(72*B*b^3 - 4
0*A*b^2*c)*1i)/(8*c^(13/4)))*(5*A*c - 9*B*b)*1i)/(8*c^(13/4)) - ((-b)^(1/4)
*((x^(1/2)*(81*B^2*b^4 + 25*A^2*b^2*c^2 - 90*A*B*b^3*c))/c^3 + ((-b)^(1/4)*
(5*A*c - 9*B*b)*(72*B*b^3 - 40*A*b^2*c)*1i)/(8*c^(13/4)))*(5*A*c - 9*B*b)*1
i)/(8*c^(13/4))))*(5*A*c - 9*B*b))/(4*c^(13/4))
```

3.198 $\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 289

$$\begin{aligned}\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx &= \frac{(7bB-3Ac)x^{3/2}}{6bc^2}-\frac{(bB-Ac)x^{7/2}}{2bc\,(b+cx^2)}\\
&+\frac{(7bB-3Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}c^{11/4}}-\frac{(7bB-3Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}c^{11/4}}\\
&-\frac{(7bB-3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}}\\
&+\frac{(7bB-3Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}}\end{aligned}$$

```
[Out] 1/6*(-3*A*c+7*B*b)*x^(3/2)/b/c^2-1/2*(-A*c+B*b)*x^(7/2)/b/c/(c*x^2+b)+1/8*(
-3*A*c+7*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/c^(11/4)*2^
(1/2)-1/8*(-3*A*c+7*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/
c^(11/4)*2^(1/2)-1/16*(-3*A*c+7*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2
^(1/2)*x^(1/2))/b^(1/4)/c^(11/4)*2^(1/2)+1/16*(-3*A*c+7*B*b)*ln(b^(1/2)+x*c
^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(11/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 468, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(7bB-3Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}c^{11/4}} - \frac{(7bB-3Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\sqrt[4]{b}c^{11/4}} - \frac{(7bB-3Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}} + \frac{(7bB-3Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}} + \frac{x^{3/2}(7bB-3Ac)}{6bc^2} - \frac{x^{7/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

```
[In] Int[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((7*b*B - 3*A*c)*x^(3/2))/(6*b*c^2) - ((b*B - A*c)*x^(7/2))/(2*b*c*(b + c*x
^2)) + ((7*b*B - 3*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*S
qrt[2]*b^(1/4)*c^(11/4)) - ((7*b*B - 3*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqr
t[x])/b^(1/4)])/(4*Sqrt[2]*b^(1/4)*c^(11/4)) - ((7*b*B - 3*A*c)*Log[Sqrt[b]
 - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(1/4)*c^(11/4
)) + ((7*b*B - 3*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[
c]*x])/(8*Sqrt[2]*b^(1/4)*c^(11/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{x^{5/2}(A+Bx^2)}{(b+cx^2)^2}\,dx$$

$$= -\frac{(bB-Ac)x^{7/2}}{2bc\,(b+cx^2)} + \frac{\left(\frac{7bB}{2}-\frac{3Ac}{2}\right)\int\frac{x^{5/2}}{b+cx^2}\,dx}{2bc}$$

$$= \frac{(7bB-3Ac)x^{3/2}}{6bc^2} - \frac{(bB-Ac)x^{7/2}}{2bc\,(b+cx^2)} - \frac{(7bB-3Ac)\int\frac{\sqrt{x}}{b+cx^2}\,dx}{4c^2}$$

$$= \frac{(7bB-3Ac)x^{3/2}}{6bc^2} - \frac{(bB-Ac)x^{7/2}}{2bc\,(b+cx^2)} - \frac{(7bB-3Ac)\text{Subst}\left(\int\frac{x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{2c^2}$$

$$= \frac{(7bB-3Ac)x^{3/2}}{6bc^2} - \frac{(bB-Ac)x^{7/2}}{2bc\,(b+cx^2)} + \frac{(7bB-3Ac)\text{Subst}\left(\int\frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4c^{5/2}}$$
$$- \frac{(7bB-3Ac)\text{Subst}\left(\int\frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4c^{5/2}}$$

$$= \frac{(7bB-3Ac)x^{3/2}}{6bc^2} - \frac{(bB-Ac)x^{7/2}}{2bc\left(b+cx^2\right)}$$
$$- \frac{(7bB-3Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8c^3}$$
$$- \frac{(7bB-3Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8c^3}$$
$$- \frac{(7bB-3Ac)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}}$$
$$- \frac{(7bB-3Ac)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}}$$

$$= \frac{(7bB-3Ac)x^{3/2}}{6bc^2} - \frac{(bB-Ac)x^{7/2}}{2bc\left(b+cx^2\right)} - \frac{(7bB-3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}}$$
$$+ \frac{(7bB-3Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}}$$
$$- \frac{(7bB-3Ac)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}c^{11/4}}$$
$$+ \frac{(7bB-3Ac)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}c^{11/4}}$$

$$= \frac{(7bB-3Ac)x^{3/2}}{6bc^2} - \frac{(bB-Ac)x^{7/2}}{2bc\,(b+cx^2)} + \frac{(7bB-3Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}c^{11/4}}$$
$$- \frac{(7bB-3Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}c^{11/4}}$$
$$- \frac{(7bB-3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}}$$
$$+ \frac{(7bB-3Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{11/4}}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.56

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\frac{4c^{3/4}x^{3/2}(7bB-3Ac+4Bcx^2)}{b+cx^2} + \frac{3\sqrt{2}(7bB-3Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt[4]{b}} + \frac{3\sqrt{2}(7bB-3Ac)\text{arctanh}\left(\frac{\sqrt{2}\cdots}{\cdots}\right)}{\sqrt[4]{b}}}{24c^{11/4}}$$

```
[In] Integrate[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*c^(3/4)*x^(3/2)*(7*b*B - 3*A*c + 4*B*c*x^2))/(b + c*x^2) + (3*Sqrt[2]*(
7*b*B - 3*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
]])]/b^(1/4) + (3*Sqrt[2]*(7*b*B - 3*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*
Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)]])/b^(1/4))/(24*c^(11/4))
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.53

method	result
derivativedivides	$\frac{2Bx^{\frac{3}{2}}}{3c^{2}}+\frac{\frac{2\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)x^{\frac{3}{2}}}{cx^{2}+b}+\frac{\left(-\frac{7Bb}{4}+\frac{3Ac}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{c^{2}}$
default	$\frac{2Bx^{\frac{3}{2}}}{3c^{2}}+\frac{\frac{2\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)x^{\frac{3}{2}}}{cx^{2}+b}+\frac{\left(-\frac{7Bb}{4}+\frac{3Ac}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{c^{2}}$
risch	$\frac{2Bx^{\frac{3}{2}}}{3c^{2}}+\frac{\frac{2\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)x^{\frac{3}{2}}}{cx^{2}+b}+\frac{\left(-\frac{7Bb}{4}+\frac{3Ac}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{c^{2}}$

```
[In] int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*B/c^2*x^(3/2)+2/c^2*((-1/4*A*c+1/4*B*b)*x^(3/2)/(c*x^2+b)+1/8*(-7/4*B*b
+3/4*A*c)/c/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c
*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2
)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 793, normalized size of antiderivative = 2.74

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{3\left(c^3x^2+bc^2\right)\left(-\frac{2401\,B^4b^4-4116\,AB^3b^3c+2646\,A^2B^2b^2c^2-756\,A^3Bbc^3+81\,A^4c^4}{bc^{11}}\right)^{\frac{1}{4}}\log\left(bc^8\left(-\frac{2401\,\ldots}{}\right.\right.}{}$$

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(c^3*x^2 + b*c^2)*(-(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2
*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(1/4)*log(b*c^8*(-(2401*
B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^
4*c^4)/(b*c^11))^(3/4) - (343*B^3*b^3 - 441*A*B^2*b^2*c + 189*A^2*B*b*c^2 -
 27*A^3*c^3)*sqrt(x)) - 3*(I*c^3*x^2 + I*b*c^2)*(-(2401*B^4*b^4 - 4116*A*B^
3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(1
/4)*log(I*b*c^8*(-(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 -
 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(3/4) - (343*B^3*b^3 - 441*A*B^2*b
^2*c + 189*A^2*B*b*c^2 - 27*A^3*c^3)*sqrt(x)) - 3*(-I*c^3*x^2 - I*b*c^2)*(-
```

```
(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*B*b*c^3 +
 81*A^4*c^4)/(b*c^11))^(1/4)*log(-I*b*c^8*(-(2401*B^4*b^4 - 4116*A*B^3*b^3*
c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(3/4) -
(343*B^3*b^3 - 441*A*B^2*b^2*c + 189*A^2*B*b*c^2 - 27*A^3*c^3)*sqrt(x)) - 3
*(c^3*x^2 + b*c^2)*(-(2401*B^4*b^4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^
2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^11))^(1/4)*log(-b*c^8*(-(2401*B^4*b^
4 - 4116*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 756*A^3*B*b*c^3 + 81*A^4*c^4)
/(b*c^11))^(3/4) - (343*B^3*b^3 - 441*A*B^2*b^2*c + 189*A^2*B*b*c^2 - 27*A^
3*c^3)*sqrt(x)) + 4*(4*B*c*x^3 + (7*B*b - 3*A*c)*x)*sqrt(x))/(c^3*x^2 + b*c
^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.77

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(Bb-Ac)x^{\frac{3}{2}}}{2\,(c^3x^2+bc^2)} + \frac{2\,Bx^{\frac{3}{2}}}{3\,c^2} - \frac{(7\,Bb-3\,Ac)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} \cdots\right)}{16\,c^2}$$

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
[Out] 1/2*(B*b - A*c)*x^(3/2)/(c^3*x^2 + b*c^2) + 2/3*B*x^(3/2)/c^2 - 1/16*(7*B*b
 - 3*A*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c
)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(s
qrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1
/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(
-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/
c^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.98

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{2\,Bx^{\frac{3}{2}}}{3\,c^2} + \frac{Bbx^{\frac{3}{2}} - Acx^{\frac{3}{2}}}{2\,(cx^2+b)c^2} - \frac{\sqrt{2}\Big(7\,(bc^3)^{\frac{3}{4}}\,Bb - 3\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^5} - \frac{\sqrt{2}\Big(7\,(bc^3)^{\frac{3}{4}}\,Bb - 3\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^5} + \frac{\sqrt{2}\Big(7\,(bc^3)^{\frac{3}{4}}\,Bb - 3\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{16\,bc^5} - \frac{\sqrt{2}\Big(7\,(bc^3)^{\frac{3}{4}}\,Bb - 3\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{16\,bc^5}$$

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/c^2 + 1/2*(B*b*x^(3/2) - A*c*x^(3/2))/((c*x^2 + b)*c^2) - 1/8
*sqrt(2)*(7*(b*c^3)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^5) - 1/8*sqrt(2)*(7*(b*c^3
)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
 - 2*sqrt(x))/(b/c)^(1/4))/(b*c^5) + 1/16*sqrt(2)*(7*(b*c^3)^(3/4)*B*b - 3*
(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^5)
 - 1/16*sqrt(2)*(7*(b*c^3)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sq
rt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^5)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.37

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{2\,B\,x^{3/2}}{3\,c^2} - \frac{x^{3/2}\left(\frac{A\,c}{2}-\frac{B\,b}{2}\right)}{c^3\,x^2+b\,c^2} + \frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,\left(3\,A\,c-7\,B\,b\right)}{4\,{(-b)}^{1/4}\,c^{11/4}} + \frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{(-b)^{1/4}}\right)\,\left(3\,A\,c-7\,B\,b\right)\,1{}\mathrm{i}}{4\,{(-b)}^{1/4}\,c^{11/4}}$$

```
[In] int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (2*B*x^(3/2))/(3*c^2) - (x^(3/2)*((A*c)/2 - (B*b)/2))/(b*c^2 + c^3*x^2) + (
atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(3*A*c - 7*B*b))/(4*(-b)^(1/4)*c^(11/4))
 + (atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*(3*A*c - 7*B*b)*1i)/(4*(-b)^(1/4)
*c^(11/4))
```

3.199 $\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 289

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(5bB-Ac)\sqrt{x}}{2bc^2} - \frac{(bB-Ac)x^{5/2}}{2bc\,(b+cx^2)} + \frac{(5bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{3/4}c^{9/4}} - \frac{(5bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{3/4}c^{9/4}} + \frac{(5bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{9/4}} - \frac{(5bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{9/4}}$$

```
[Out] -1/2*(-A*c+B*b)*x^(5/2)/b/c/(c*x^2+b)+1/8*(-A*c+5*B*b)*arctan(1-c^(1/4)*2^(
1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^(9/4)*2^(1/2)-1/8*(-A*c+5*B*b)*arctan(1+c^(
1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/c^(9/4)*2^(1/2)+1/16*(-A*c+5*B*b)*ln(
b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(9/4)*2^(1/2)-
1/16*(-A*c+5*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(
3/4)/c^(9/4)*2^(1/2)+1/2*(-A*c+5*B*b)*x^(1/2)/b/c^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1598, 468, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(5bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{3/4}c^{9/4}} - \frac{(5bB-Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{3/4}c^{9/4}} + \frac{(5bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{9/4}} - \frac{(5bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{9/4}} + \frac{\sqrt{x}(5bB-Ac)}{2bc^2} - \frac{x^{5/2}(bB-Ac)}{2bc(b+cx^2)}$$

```
[In] Int[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((5*b*B - A*c)*Sqrt[x])/(2*b*c^2) - ((b*B - A*c)*x^(5/2))/(2*b*c*(b + c*x^2
)) + ((5*b*B - A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[
2]*b^(3/4)*c^(9/4)) - ((5*b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b
^(1/4)])/(4*Sqrt[2]*b^(3/4)*c^(9/4)) + ((5*b*B - A*c)*Log[Sqrt[b] - Sqrt[2]
*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(3/4)*c^(9/4)) - ((5*b*
B - A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqr
t[2]*b^(3/4)*c^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{x^{3/2}(A+Bx^2)}{(b+cx^2)^2}\,dx$$

$$= -\frac{(bB-Ac)x^{5/2}}{2bc\,(b+cx^2)} + \frac{\left(\frac{5bB}{2}-\frac{Ac}{2}\right)\int \frac{x^{3/2}}{b+cx^2}\,dx}{2bc}$$

$$= \frac{(5bB-Ac)\sqrt{x}}{2bc^2} - \frac{(bB-Ac)x^{5/2}}{2bc\,(b+cx^2)} - \frac{(5bB-Ac)\int \frac{1}{\sqrt{x}(b+cx^2)}\,dx}{4c^2}$$

$$= \frac{(5bB-Ac)\sqrt{x}}{2bc^2} - \frac{(bB-Ac)x^{5/2}}{2bc\,(b+cx^2)} - \frac{(5bB-Ac)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{2c^2}$$

$$= \frac{(5bB-Ac)\sqrt{x}}{2bc^2} - \frac{(bB-Ac)x^{5/2}}{2bc\,(b+cx^2)} - \frac{(5bB-Ac)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4\sqrt{b}c^2} - \frac{(5bB-Ac)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4\sqrt{b}c^2}$$

$$= \frac{(5bB-Ac)\sqrt{x}}{2bc^2} - \frac{(bB-Ac)x^{5/2}}{2bc\,(b+cx^2)} - \frac{(5bB-Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{b}c^{5/2}} - \frac{(5bB-Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{b}c^{5/2}} + \frac{(5bB-Ac)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{2}b^{3/4}c^{9/4}} + \frac{(5bB-Ac)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{2}b^{3/4}c^{9/4}}$$

$$= \frac{(5bB - Ac)\sqrt{x}}{2bc^2} - \frac{(bB - Ac)x^{5/2}}{2bc\,(b + cx^2)} + \frac{(5bB - Ac)\log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{9/4}} - \frac{(5bB - Ac)\log\left(\sqrt{b} + \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{9/4}} - \frac{(5bB - Ac)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{3/4}c^{9/4}} + \frac{(5bB - Ac)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{3/4}c^{9/4}}$$

$$= \frac{(5bB - Ac)\sqrt{x}}{2bc^2} - \frac{(bB - Ac)x^{5/2}}{2bc\,(b + cx^2)} + \frac{(5bB - Ac)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{3/4}c^{9/4}} - \frac{(5bB - Ac)\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{3/4}c^{9/4}} + \frac{(5bB - Ac)\log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{9/4}} - \frac{(5bB - Ac)\log\left(\sqrt{b} + \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{9/4}}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.56

$$\int \frac{x^{11/2}(A + Bx^2)}{(bx^2 + cx^4)^2}\,dx = \frac{\frac{4\sqrt[4]{c}\sqrt{x}(5bB - Ac + 4Bcx^2)}{b + cx^2} + \frac{\sqrt{2}(5bB - Ac)\arctan\left(\frac{\sqrt{b} - \sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{b^{3/4}} - \frac{\sqrt{2}(5bB - Ac)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}{\sqrt{b} + \sqrt{c}x}\right)}{b^{3/4}}}{8c^{9/4}}$$

```
[In] Integrate[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*c^(1/4)*Sqrt[x]*(5*b*B - A*c + 4*B*c*x^2))/(b + c*x^2) + (Sqrt[2]*(5*b*
B - A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/b
^(3/4) - (Sqrt[2]*(5*b*B - A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(
Sqrt[b] + Sqrt[c]*x)])/b^(3/4))/(8*c^(9/4))
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.53

method	result
derivativedivides	$\frac{2B\sqrt{x}}{c^2}+\frac{\frac{2\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(Ac-5Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16b}}{c^2}$
default	$\frac{2B\sqrt{x}}{c^2}+\frac{\frac{2\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(Ac-5Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16b}}{c^2}$
risch	$\frac{2B\sqrt{x}}{c^2}+\frac{\frac{2\left(-\frac{Ac}{4}+\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(Ac-5Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16b}}{c^2}$

```
[In] int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*B/c^2*x^(1/2)+2/c^2*((-1/4*A*c+1/4*B*b)*x^(1/2)/(c*x^2+b)+1/32*(A*c-5*B*b
)*(1/c*b)^(1/4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2
))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b
)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 669, normalized size of antiderivative = 2.31

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(c^3x^2+bc^2)\left(-\frac{625\,B^4b^4-500\,AB^3b^3c+150\,A^2B^2b^2c^2-20\,A^3Bbc^3+A^4c^4}{b^3c^9}\right)^{\frac{1}{4}}\log\left(bc^2\left(-\frac{625\,B^4b^4-50}{}\right.\right.}{}$$

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((c^3*x^2 + b*c^2)*(-(625*B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c
^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^9))^(1/4)*log(b*c^2*(-(625*B^4*b^4 -
500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^9)
)^(1/4) - (5*B*b - A*c)*sqrt(x)) - (-I*c^3*x^2 - I*b*c^2)*(-(625*B^4*b^4 -
500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^9)
)^(1/4)*log(I*b*c^2*(-(625*B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2
- 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^9))^(1/4) - (5*B*b - A*c)*sqrt(x)) - (I*
c^3*x^2 + I*b*c^2)*(-(625*B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 -
 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^9))^(1/4)*log(-I*b*c^2*(-(625*B^4*b^4 - 5
00*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^9))
```

```
^(1/4) - (5*B*b - A*c)*sqrt(x)) - (c^3*x^2 + b*c^2)*(-(625*B^4*b^4 - 500*A*
B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^9))^(1/4
)*log(-b*c^2*(-(625*B^4*b^4 - 500*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 20*A^
3*B*b*c^3 + A^4*c^4)/(b^3*c^9))^(1/4) - (5*B*b - A*c)*sqrt(x)) + 4*(4*B*c*x
^2 + 5*B*b - A*c)*sqrt(x))/(c^3*x^2 + b*c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.87

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(Bb-Ac)\sqrt{x}}{2(c^3x^2+bc^2)} + \frac{2B\sqrt{x}}{c^2} - \frac{\frac{2\sqrt{2}(5Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}(5Bb-Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}(5Bb-Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{16c^2}$$

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
[Out] 1/2*(B*b - A*c)*sqrt(x)/(c^3*x^2 + b*c^2) + 2*B*sqrt(x)/c^2 - 1/16*(2*sqrt(
2)*(5*B*b - A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sq
rt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(
5*B*b - A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(
x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(5*B*b
 - A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)
*c^(1/4)) - sqrt(2)*(5*B*b - A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sq
rt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.98

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{2\,B\sqrt{x}}{c^2} - \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^3} - \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^3} - \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,bc^3} + \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,bc^3} + \frac{Bb\sqrt{x}-Ac\sqrt{x}}{2\,(cx^2+b)c^2}$$

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/c^2 - 1/8*sqrt(2)*(5*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arc
tan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^3) - 1/
8*sqrt(2)*(5*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^3) - 1/16*sqrt(2)*(5*(b*c^
3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqr
t(b/c))/(b*c^3) + 1/16*sqrt(2)*(5*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*lo
g(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^3) + 1/2*(B*b*sqrt(x)
- A*c*sqrt(x))/((c*x^2 + b)*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 744, normalized size of antiderivative = 2.57

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{2\,B\,\sqrt{x}}{c^2} - \frac{\sqrt{x}\,\left(\frac{A\,c}{2}-\frac{B\,b}{2}\right)}{c^3\,x^2+b\,c^2} + \frac{\mathrm{atan}\left(\frac{\frac{(A\,c-5\,B\,b)\,\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}-\frac{(A\,c-5\,B\,b)\,\left(8\,A\,b\,c^2-40\,B\,b^2\,c\right)}{8\,(-b)^{3/4}\,c^{9/4}}\right)\,1\mathrm{i}}{8\,(-b)^{3/4}\,c^{9/4}}+\frac{(A\,c-5\,B\,b)\,\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}+\frac{(A\,c-5\,B\,b)\,(8\cdots}{8\,(-b)\cdots}\right.}{8\,(-b)^{3/4}\,c^{9/4}}}{\frac{(A\,c-5\,B\,b)\,\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}-\frac{(A\,c-5\,B\,b)\,\left(8\,A\,b\,c^2-40\,B\,b^2\,c\right)}{8\,(-b)^{3/4}\,c^{9/4}}\right)}{8\,(-b)^{3/4}\,c^{9/4}}-\frac{(A\,c-5\,B\,b)\,\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}+\frac{(A\,c-5\,B\,b)\,(8\cdots}{8\,(-b)\cdots}\right.}{8\,(-b)^{3/4}\,c^{9/4}}}\right.\cdots}{4\,(-b)^{3/4}\,c^{9/4}} + \frac{\mathrm{atan}\left(\frac{\frac{(A\,c-5\,B\,b)\,\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}-\frac{(A\,c-5\,B\,b)\,\left(8\,A\,b\,c^2-40\,B\,b^2\,c\right)\,1\mathrm{i}}{8\,(-b)^{3/4}\,c^{9/4}}\right)}{8\,(-b)^{3/4}\,c^{9/4}}+\frac{(A\,c-5\,B\,b)\,\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}+\frac{(A\,c-5\,B\,b)\,(\cdots}{8\,(\cdots}\right.}{8\,(-b)^{3/4}\,c^{9/4}}}{\frac{(A\,c-5\,B\,b)\,\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}-\frac{(A\,c-5\,B\,b)\,\left(8\,A\,b\,c^2-40\,B\,b^2\,c\right)\,1\mathrm{i}}{8\,(-b)^{3/4}\,c^{9/4}}\right)\,1\mathrm{i}}{8\,(-b)^{3/4}\,c^{9/4}}-\frac{(A\,c-5\,B\,b)\,\left(\frac{\sqrt{x}\,\left(A^2\,c^2-10\,A\,B\,b\,c+25\,B^2\,b^2\right)}{c}+\frac{(A\,c-5\,B\,b)\,(\cdots}{8\,(\cdots}\right.}{8\,(-b)^{3/4}\,c^{9/4}}}\right.\cdots}{4\,(-b)^{3/4}\,c^{9/4}}$$

```
[In] int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (2*B*x^(1/2))/c^2 - (x^(1/2)*((A*c)/2 - (B*b)/2))/(b*c^2 + c^3*x^2) + (atan
((((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b^2 - 10*A*B*b*c))/c - ((A*c -
 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c))/(8*(-b)^(3/4)*c^(9/4)))*1i)/(8*(-b)^(3/4)
*c^(9/4)) + ((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b^2 - 10*A*B*b*c))/c
 + ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c))/(8*(-b)^(3/4)*c^(9/4)))*1i)/(8*
(-b)^(3/4)*c^(9/4)))/(((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b^2 - 10*A
*B*b*c))/c - ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c))/(8*(-b)^(3/4)*c^(9/4)
)))/(8*(-b)^(3/4)*c^(9/4)) - ((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b^2
 - 10*A*B*b*c))/c + ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c))/(8*(-b)^(3/4)*
c^(9/4))))/(8*(-b)^(3/4)*c^(9/4))))*(A*c - 5*B*b)*1i)/(4*(-b)^(3/4)*c^(9/4)
) + (atan((((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b^2 - 10*A*B*b*c))/c
- ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c)*1i)/(8*(-b)^(3/4)*c^(9/4))))/(8*(
-b)^(3/4)*c^(9/4)) + ((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b^2 - 10*A*
B*b*c))/c + ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c)*1i)/(8*(-b)^(3/4)*c^(9/
4))))/(8*(-b)^(3/4)*c^(9/4)))/(((A*c - 5*B*b)*((x^(1/2)*(A^2*c^2 + 25*B^2*b
^2 - 10*A*B*b*c))/c - ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c)*1i)/(8*(-b)^(
3/4)*c^(9/4)))*1i)/(8*(-b)^(3/4)*c^(9/4)) - ((A*c - 5*B*b)*((x^(1/2)*(A^2*c
^2 + 25*B^2*b^2 - 10*A*B*b*c))/c + ((A*c - 5*B*b)*(8*A*b*c^2 - 40*B*b^2*c)*
1i)/(8*(-b)^(3/4)*c^(9/4)))*1i)/(8*(-b)^(3/4)*c^(9/4))))*(A*c - 5*B*b))/(4*
(-b)^(3/4)*c^(9/4))
```

3.200 $\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 261

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(bB-Ac)x^{3/2}}{2bc\,(b+cx^2)} - \frac{(3bB+Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{5/4}c^{7/4}} + \frac{(3bB+Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{5/4}c^{7/4}} + \frac{(3bB+Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{7/4}} - \frac{(3bB+Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{7/4}}$$

```
[Out] -1/2*(-A*c+B*b)*x^(3/2)/b/c/(c*x^2+b)-1/8*(A*c+3*B*b)*arctan(1-c^(1/4)*2^(1
/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^(7/4)*2^(1/2)+1/8*(A*c+3*B*b)*arctan(1+c^(1/
4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^(7/4)*2^(1/2)+1/16*(A*c+3*B*b)*ln(b^(
1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)/c^(7/4)*2^(1/2)-1/1
6*(A*c+3*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)
/c^(7/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1598, 468, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(Ac+3bB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{5/4}c^{7/4}} + \frac{(Ac+3bB)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{5/4}c^{7/4}} + \frac{(Ac+3bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{7/4}} - \frac{(Ac+3bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{7/4}} - \frac{x^{3/2}(bB-Ac)}{2bc\,(b+cx^2)}$$

```
[In] Int[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*((b*B - A*c)*x^(3/2))/(b*c*(b + c*x^2)) - ((3*b*B + A*c)*ArcTan[1 - (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(5/4)*c^(7/4)) + ((3*b*B + A
*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(5/4)*c^(7/
4)) + ((3*b*B + A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x])/(8*Sqrt[2]*b^(5/4)*c^(7/4)) - ((3*b*B + A*c)*Log[Sqrt[b] + Sqrt[2]*b^
(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(5/4)*c^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{x}(A+Bx^2)}{(b+cx^2)^2}\,dx$$

$$= -\frac{(bB-Ac)x^{3/2}}{2bc\,(b+cx^2)} + \frac{\left(\frac{3bB}{2}+\frac{Ac}{2}\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{2bc}$$

$$= -\frac{(bB-Ac)x^{3/2}}{2bc\,(b+cx^2)} + \frac{\left(\frac{3bB}{2}+\frac{Ac}{2}\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{bc}$$

$$= -\frac{(bB-Ac)x^{3/2}}{2bc\,(b+cx^2)} - \frac{(3bB+Ac)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4bc^{3/2}} + \frac{(3bB+Ac)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4bc^{3/2}}$$

$$= -\frac{(bB-Ac)x^{3/2}}{2bc\,(b+cx^2)} + \frac{(3bB+Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8bc^2} + \frac{(3bB+Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8bc^2} + \frac{(3bB+Ac)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{2}b^{5/4}c^{7/4}} + \frac{(3bB+Ac)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{2}b^{5/4}c^{7/4}}$$

$$= -\frac{(bB-Ac)x^{3/2}}{2bc\,(b+cx^2)} + \frac{(3bB+Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{7/4}} - \frac{(3bB+Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{7/4}} + \frac{(3bB+Ac)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{5/4}c^{7/4}} - \frac{(3bB+Ac)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{5/4}c^{7/4}}$$

$$= -\frac{(bB-Ac)x^{3/2}}{2bc\,(b+cx^2)} - \frac{(3bB+Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{5/4}c^{7/4}} + \frac{(3bB+Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{5/4}c^{7/4}} + \frac{(3bB+Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{7/4}} - \frac{(3bB+Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{7/4}}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.58

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\frac{4\sqrt[4]{b}c^{3/4}(-bB+Ac)x^{3/2}}{b+cx^2} - \sqrt{2}(3bB+Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) - \sqrt{2}(3bB+Ac)\text{arctan}}{8b^{5/4}c^{7/4}}$$

```
[In] Integrate[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*b^(1/4)*c^(3/4)*(-(b*B) + A*c)*x^(3/2))/(b + c*x^2) - Sqrt[2]*(3*b*B +
A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - Sqrt
[2]*(3*b*B + A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt
[c]*x)])/(8*b^(5/4)*c^(7/4))
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.56

method	result	size
derivativedivides	$\frac{(Ac-Bb)x^{\frac{3}{2}}}{2bc(c\,x^2+b)} + \frac{(Ac+3Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16b\,c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	146
default	$\frac{(Ac-Bb)x^{\frac{3}{2}}}{2bc(c\,x^2+b)} + \frac{(Ac+3Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16b\,c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}$	146

```
[In] int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(A*c-B*b)/b/c*x^(3/2)/(c*x^2+b)+1/16*(A*c+3*B*b)/b/c^2/(1/c*b)^(1/4)*2^
(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*
x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2
*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 776, normalized size of antiderivative = 2.97

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx =$$

$$-\frac{4\,(Bb-Ac)x^{\frac{3}{2}}-(bc^2x^2+b^2c)\left(-\frac{81\,B^4b^4+108\,AB^3b^3c+54\,A^2B^2b^2c^2+12\,A^3Bbc^3+A^4c^4}{b^5c^7}\right)^{\frac{1}{4}}\log\left(b^4c^5\left(-\frac{81\,B^4b^4+108\,AB^3b^3c}{}\right.\right.}{}$$

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(B*b - A*c)*x^(3/2) - (b*c^2*x^2 + b^2*c)*(-(81*B^4*b^4 + 108*A*B^3
*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^(1/4)*lo
g(b^4*c^5*(-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b
*c^3 + A^4*c^4)/(b^5*c^7))^(3/4) + (27*B^3*b^3 + 27*A*B^2*b^2*c + 9*A^2*B*b
*c^2 + A^3*c^3)*sqrt(x)) + (I*b*c^2*x^2 + I*b^2*c)*(-(81*B^4*b^4 + 108*A*B^
3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^(1/4)*l
og(I*b^4*c^5*(-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*
B*b*c^3 + A^4*c^4)/(b^5*c^7))^(3/4) + (27*B^3*b^3 + 27*A*B^2*b^2*c + 9*A^2*
B*b*c^2 + A^3*c^3)*sqrt(x)) + (-I*b*c^2*x^2 - I*b^2*c)*(-(81*B^4*b^4 + 108*
A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^(1/
4)*log(-I*b^4*c^5*(-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12
*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^(3/4) + (27*B^3*b^3 + 27*A*B^2*b^2*c + 9
*A^2*B*b*c^2 + A^3*c^3)*sqrt(x)) + (b*c^2*x^2 + b^2*c)*(-(81*B^4*b^4 + 108*
A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^(1/
4)*log(-b^4*c^5*(-(81*B^4*b^4 + 108*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 12*A
^3*B*b*c^3 + A^4*c^4)/(b^5*c^7))^(3/4) + (27*B^3*b^3 + 27*A*B^2*b^2*c + 9*A
^2*B*b*c^2 + A^3*c^3)*sqrt(x)))/(b*c^2*x^2 + b^2*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.83

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(Bb-Ac)x^{\frac{3}{2}}}{2\,(bc^2x^2+b^2c)} + \frac{(3\,Bb+Ac)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \cdots\right)}{16\,bc}$$

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b - A*c)*x^(3/2)/(b*c^2*x^2 + b^2*c) + 1/16*(3*B*b + A*c)*(2*sqrt(2
)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqr
t(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt
(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(s
qrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x)
 + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^
(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.05

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{Bbx^{\frac{3}{2}}-Acx^{\frac{3}{2}}}{2\,(cx^2+b)bc} + \frac{\sqrt{2}\left(3\,(bc^3)^{\frac{3}{4}}\,Bb+(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^4} + \frac{\sqrt{2}\left(3\,(bc^3)^{\frac{3}{4}}\,Bb+(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^4} - \frac{\sqrt{2}\left(3\,(bc^3)^{\frac{3}{4}}\,Bb+(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^2c^4} + \frac{\sqrt{2}\left(3\,(bc^3)^{\frac{3}{4}}\,Bb+(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^2c^4}$$

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*b*x^(3/2) - A*c*x^(3/2))/((c*x^2 + b)*b*c) + 1/8*sqrt(2)*(3*(b*c^3)
^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2
*sqrt(x))/(b/c)^(1/4))/(b^2*c^4) + 1/8*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + (b*c^
3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(
1/4))/(b^2*c^4) - 1/16*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*lo
g(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^4) + 1/16*sqrt(2)*(3*
(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x
 + sqrt(b/c))/(b^2*c^4)
```

Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.35

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\operatorname{atanh}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)\,(A\,c+3\,B\,b)}{4\,(-b)^{5/4}\,c^{7/4}} - \frac{\operatorname{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)\,(A\,c+3\,B\,b)}{4\,(-b)^{5/4}\,c^{7/4}} + \frac{x^{3/2}\,(A\,c-B\,b)}{2\,b\,c\,(c\,x^2+b)}$$

```
[In] int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c + 3*B*b))/(4*(-b)^(5/4)*c^(7/4))
- (atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c + 3*B*b))/(4*(-b)^(5/4)*c^(7/4))
 + (x^(3/2)*(A*c - B*b))/(2*b*c*(b + c*x^2))
```

3.201 $\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 26, antiderivative size = 261

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^2} dx = -\frac{(bB-Ac)\sqrt{x}}{2bc(b+cx^2)} - \frac{(bB+3Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}c^{5/4}} + \frac{(bB+3Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}c^{5/4}} - \frac{(bB+3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}c^{5/4}} + \frac{(bB+3Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}c^{5/4}}$$

```
[Out] -1/8*(3*A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)/c^(5/4)*
2^(1/2)+1/8*(3*A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)/c
^(5/4)*2^(1/2)-1/16*(3*A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2
)*x^(1/2))/b^(7/4)/c^(5/4)*2^(1/2)+1/16*(3*A*c+B*b)*ln(b^(1/2)+x*c^(1/2)+b^
(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(5/4)*2^(1/2)-1/2*(-A*c+B*b)*x^(1/
2)/b/c/(c*x^2+b)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1598, 468, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(3Ac+bB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}c^{5/4}} + \frac{(3Ac+bB)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{7/4}c^{5/4}} - \frac{(3Ac+bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}c^{5/4}} + \frac{(3Ac+bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}c^{5/4}} - \frac{\sqrt{x}(bB-Ac)}{2bc\,(b+cx^2)}$$

```
[In] Int[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*((b*B - A*c)*Sqrt[x])/(b*c*(b + c*x^2)) - ((b*B + 3*A*c)*ArcTan[1 - (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(7/4)*c^(5/4)) + ((b*B + 3*A
*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(7/4)*c^(5/
4)) - ((b*B + 3*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x])/(8*Sqrt[2]*b^(7/4)*c^(5/4)) + ((b*B + 3*A*c)*Log[Sqrt[b] + Sqrt[2]*b^
(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(7/4)*c^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{A + Bx^2}{\sqrt{x}\,(b + cx^2)^2}\,dx$$

$$= -\frac{(bB-Ac)\sqrt{x}}{2bc\,(b+cx^2)} + \frac{\left(\frac{bB}{2}+\frac{3Ac}{2}\right)\int \frac{1}{\sqrt{x}(b+cx^2)}\,dx}{2bc}$$

$$= -\frac{(bB-Ac)\sqrt{x}}{2bc\,(b+cx^2)} + \frac{\left(\frac{bB}{2}+\frac{3Ac}{2}\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{bc}$$

$$= -\frac{(bB-Ac)\sqrt{x}}{2bc\,(b+cx^2)} + \frac{(bB+3Ac)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^{3/2}c} + \frac{(bB+3Ac)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^{3/2}c}$$

$$= -\frac{(bB-Ac)\sqrt{x}}{2bc\,(b+cx^2)} + \frac{(bB+3Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^{3/2}c^{3/2}} + \frac{(bB+3Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^{3/2}c^{3/2}} - \frac{(bB+3Ac)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{2}b^{7/4}c^{5/4}} - \frac{(bB+3Ac)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{2}b^{7/4}c^{5/4}}$$

$$= -\frac{(bB-Ac)\sqrt{x}}{2bc\,(b+cx^2)} - \frac{(bB+3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}c^{5/4}} + \frac{(bB+3Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}c^{5/4}} + \frac{(bB+3Ac)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}c^{5/4}} - \frac{(bB+3Ac)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}c^{5/4}}$$

$$= -\frac{(bB-Ac)\sqrt{x}}{2bc\,(b+cx^2)} - \frac{(bB+3Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}c^{5/4}} + \frac{(bB+3Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}c^{5/4}} - \frac{(bB+3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}c^{5/4}} + \frac{(bB+3Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}c^{5/4}}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.58

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\frac{4b^{3/4}\sqrt[4]{c}(-bB+Ac)\sqrt{x}}{b+cx^2} - \sqrt{2}(bB+3Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) + \sqrt{2}(bB+3Ac)\text{arctanh}}{8b^{7/4}c^{5/4}}$$

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*b^(3/4)*c^(1/4)*(-(b*B) + A*c)*Sqrt[x])/(b + c*x^2) - Sqrt[2]*(b*B + 3*
A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + Sqrt
[2]*(b*B + 3*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt
[c]*x)])/(8*b^(7/4)*c^(5/4))
```

Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.56

method	result
derivativedivides	$\frac{(Ac-Bb)\sqrt{x}}{2bc(c\,x^2+b)} + \frac{(3Ac+Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16b^2c}$
default	$\frac{(Ac-Bb)\sqrt{x}}{2bc(c\,x^2+b)} + \frac{(3Ac+Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16b^2c}$

```
[In] int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(A*c-B*b)/b/c*x^(1/2)/(c*x^2+b)+1/16*(3*A*c+B*b)/b^2/c*(1/c*b)^(1/4)*2^
(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*
```

```
x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2
*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 658, normalized size of antiderivative = 2.52

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(bc^2x^2+b^2c)\left(-\frac{B^4b^4+12\,AB^3b^3c+54\,A^2B^2b^2c^2+108\,A^3Bbc^3+81\,A^4c^4}{b^7c^5}\right)^{\frac{1}{4}}\log\left(b^2c\left(-\frac{B^4b^4+12\,AB^3b^3c}{}\right.\right.}{}$$

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((b*c^2*x^2 + b^2*c)*(-(B^4*b^4 + 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 +
 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(1/4)*log(b^2*c*(-(B^4*b^4 + 12*A
*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^
(1/4) + (B*b + 3*A*c)*sqrt(x)) - (-I*b*c^2*x^2 - I*b^2*c)*(-(B^4*b^4 + 12*A
*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^
(1/4)*log(I*b^2*c*(-(B^4*b^4 + 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^
3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(1/4) + (B*b + 3*A*c)*sqrt(x)) - (I*b*c^
2*x^2 + I*b^2*c)*(-(B^4*b^4 + 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3
*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(1/4)*log(-I*b^2*c*(-(B^4*b^4 + 12*A*B^3*
b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(1/4)
 + (B*b + 3*A*c)*sqrt(x)) - (b*c^2*x^2 + b^2*c)*(-(B^4*b^4 + 12*A*B^3*b^3*c
 + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^5))^(1/4)*log(
-b^2*c*(-(B^4*b^4 + 12*A*B^3*b^3*c + 54*A^2*B^2*b^2*c^2 + 108*A^3*B*b*c^3 +
 81*A^4*c^4)/(b^7*c^5))^(1/4) + (B*b + 3*A*c)*sqrt(x)) - 4*(B*b - A*c)*sqrt
(x))/(b*c^2*x^2 + b^2*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.92

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(Bb-Ac)\sqrt{x}}{2\,(bc^2x^2+b^2c)} + \frac{\frac{2\sqrt{2}(Bb+3\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}(Bb+3\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}(Bb+3\,Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{16\,bc}$$

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*b - A*c)*sqrt(x)/(b*c^2*x^2 + b^2*c) + 1/16*(2*sqrt(2)*(B*b + 3*A*c
)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqr
t(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(B*b + 3*A*c)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b
)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(B*b + 3*A*c)*log(sqr
t(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqr
t(2)*(B*b + 3*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(
b))/(b^(3/4)*c^(1/4)))/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.05

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+3\,(bc^3)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^2} + \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+3\,(bc^3)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^2} + \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+3\,(bc^3)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^2c^2} - \frac{\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+3\,(bc^3)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^2c^2} - \frac{Bb\sqrt{x}-Ac\sqrt{x}}{2\,(cx^2+b)bc}$$

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*((b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(s
qrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) + 1/8*sqrt(2)*((b*c^
3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4
) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) + 1/16*sqrt(2)*((b*c^3)^(1/4)*B*b + 3
*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c
^2) - 1/16*sqrt(2)*((b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*log(-sqrt(2)*s
qrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^2) - 1/2*(B*b*sqrt(x) - A*c*sqrt
(x))/((c*x^2 + b)*b*c)
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 750, normalized size of antiderivative = 2.87

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\operatorname{atan}\left(\frac{\frac{(3Ac+Bb)\left(\frac{\sqrt{x}\left(9A^2c^3+6ABbc^2+B^2b^2c\right)}{b^2}-\frac{(3Ac+Bb)\left(24Ac^3+8Bbc^2\right)1i}{8(-b)^{7/4}c^{5/4}}\right)}{8(-b)^{7/4}c^{5/4}}+\frac{(3Ac+Bb)\left(\frac{\sqrt{x}\left(9A^2c^3+6\cdots\right.}{\cdots}\right.}{\cdots}}{\frac{(3Ac+Bb)\left(\frac{\sqrt{x}\left(9A^2c^3+6ABbc^2+B^2b^2c\right)}{b^2}-\frac{(3Ac+Bb)\left(24Ac^3+8Bbc^2\right)1i}{8(-b)^{7/4}c^{5/4}}\right)1i}{8(-b)^{7/4}c^{5/4}}-\frac{(3Ac+Bb)\left(\frac{\sqrt{x}\left(9A^2c^3+6\cdots\right.}{\cdots}\right.}{\cdots}}\right)\cdots}{4(-b)^{7/4}c^{5/4}} + \frac{\sqrt{x}\,(Ac-Bb)}{2bc\,(cx^2+b)} + \frac{\operatorname{atan}\left(\frac{\frac{(3Ac+Bb)\left(\frac{\sqrt{x}\left(9A^2c^3+6ABbc^2+B^2b^2c\right)}{b^2}-\frac{(3Ac+Bb)\left(24Ac^3+8Bbc^2\right)}{8(-b)^{7/4}c^{5/4}}\right)1i}{8(-b)^{7/4}c^{5/4}}+\frac{(3Ac+Bb)\left(\frac{\sqrt{x}\left(9A^2c^3+6ABbc^2+B^2b^2c\right)}{b^2}+\frac{(3Ac+Bb)\left(24\cdots\right.}{8(-b)^{7/4}\cdots}\right.}{8(-b)^{7/4}c^{5/4}}}{\frac{(3Ac+Bb)\left(\frac{\sqrt{x}\left(9A^2c^3+6ABbc^2+B^2b^2c\right)}{b^2}-\frac{(3Ac+Bb)\left(24Ac^3+8Bbc^2\right)}{8(-b)^{7/4}c^{5/4}}\right)}{8(-b)^{7/4}c^{5/4}}-\frac{(3Ac+Bb)\left(\frac{\sqrt{x}\left(9A^2c^3+6ABbc^2+B^2b^2c\right)}{b^2}+\frac{(3Ac+Bb)\left(24\cdots\right.}{8(-b)^{7/4}\cdots}\right.}{8(-b)^{7/4}c^{5/4}}}\right)\cdots}{4(-b)^{7/4}c^{5/4}}$$

```
[In] int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (atan((((3*A*c + B*b)*((x^(1/2)*(9*A^2*c^3 + B^2*b^2*c + 6*A*B*b*c^2))/b^2
- ((3*A*c + B*b)*(24*A*c^3 + 8*B*b*c^2))/(8*(-b)^(7/4)*c^(5/4)))*1i)/(8*(-b
)^(7/4)*c^(5/4)) + ((3*A*c + B*b)*((x^(1/2)*(9*A^2*c^3 + B^2*b^2*c + 6*A*B*
b*c^2))/b^2 + ((3*A*c + B*b)*(24*A*c^3 + 8*B*b*c^2))/(8*(-b)^(7/4)*c^(5/4))
)*1i)/(8*(-b)^(7/4)*c^(5/4)))/(((3*A*c + B*b)*((x^(1/2)*(9*A^2*c^3 + B^2*b^
2*c + 6*A*B*b*c^2))/b^2 - ((3*A*c + B*b)*(24*A*c^3 + 8*B*b*c^2))/(8*(-b)^(7
/4)*c^(5/4))))/(8*(-b)^(7/4)*c^(5/4)) - ((3*A*c + B*b)*((x^(1/2)*(9*A^2*c^3
 + B^2*b^2*c + 6*A*B*b*c^2))/b^2 + ((3*A*c + B*b)*(24*A*c^3 + 8*B*b*c^2))/(
8*(-b)^(7/4)*c^(5/4))))/(8*(-b)^(7/4)*c^(5/4))))*(3*A*c + B*b)*1i)/(4*(-b)^
(7/4)*c^(5/4)) + (atan((((3*A*c + B*b)*((x^(1/2)*(9*A^2*c^3 + B^2*b^2*c + 6
*A*B*b*c^2))/b^2 - ((3*A*c + B*b)*(24*A*c^3 + 8*B*b*c^2)*1i)/(8*(-b)^(7/4)*
c^(5/4))))/(8*(-b)^(7/4)*c^(5/4)) + ((3*A*c + B*b)*((x^(1/2)*(9*A^2*c^3 + B
^2*b^2*c + 6*A*B*b*c^2))/b^2 + ((3*A*c + B*b)*(24*A*c^3 + 8*B*b*c^2)*1i)/(8
*(-b)^(7/4)*c^(5/4))))/(8*(-b)^(7/4)*c^(5/4)))/(((3*A*c + B*b)*((x^(1/2)*(9
```

```
*A^2*c^3 + B^2*b^2*c + 6*A*B*b*c^2))/b^2 - ((3*A*c + B*b)*(24*A*c^3 + 8*B*b
*c^2)*1i)/(8*(-b)^(7/4)*c^(5/4)))*1i)/(8*(-b)^(7/4)*c^(5/4)) - ((3*A*c + B*
b)*((x^(1/2)*(9*A^2*c^3 + B^2*b^2*c + 6*A*B*b*c^2))/b^2 + ((3*A*c + B*b)*(2
4*A*c^3 + 8*B*b*c^2)*1i)/(8*(-b)^(7/4)*c^(5/4)))*1i)/(8*(-b)^(7/4)*c^(5/4))
))*(3*A*c + B*b))/(4*(-b)^(7/4)*c^(5/4)) + (x^(1/2)*(A*c - B*b))/(2*b*c*(b
+ c*x^2))
```

3.202 $\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 284

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{bB-5Ac}{2b^2c\sqrt{x}} - \frac{bB-Ac}{2bc\sqrt{x}\,(b+cx^2)} - \frac{(bB-5Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{9/4}c^{3/4}} + \frac{(bB-5Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{9/4}c^{3/4}} + \frac{(bB-5Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}c^{3/4}} - \frac{(bB-5Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}c^{3/4}}$$

```
[Out] -1/8*(-5*A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(9/4)/c^(3/4)
*2^(1/2)+1/8*(-5*A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(9/4)
/c^(3/4)*2^(1/2)+1/16*(-5*A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(
1/2)*x^(1/2))/b^(9/4)/c^(3/4)*2^(1/2)-1/16*(-5*A*c+B*b)*ln(b^(1/2)+x*c^(1/2
)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)/c^(3/4)*2^(1/2)+1/2*(-5*A*c+B*b)
/b^2/c/x^(1/2)+1/2*(A*c-B*b)/b/c/(c*x^2+b)/x^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1598, 468, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(bB-5Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{9/4}c^{3/4}} + \frac{(bB-5Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{9/4}c^{3/4}} + \frac{(bB-5Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}c^{3/4}} - \frac{(bB-5Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}c^{3/4}} + \frac{bB-5Ac}{2b^2c\sqrt{x}} - \frac{bB-Ac}{2bc\sqrt{x}\,(b+cx^2)}$$

```
[In] Int[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b*B - 5*A*c)/(2*b^2*c*Sqrt[x]) - (b*B - A*c)/(2*b*c*Sqrt[x]*(b + c*x^2)) -
 ((b*B - 5*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b
^(9/4)*c^(3/4)) + ((b*B - 5*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/
4)])/(4*Sqrt[2]*b^(9/4)*c^(3/4)) + ((b*B - 5*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(9/4)*c^(3/4)) - ((b*B - 5*
A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]
*b^(9/4)*c^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{A+Bx^2}{x^{3/2}(b+cx^2)^2}\,dx$$

$$= -\frac{bB-Ac}{2bc\sqrt{x}(b+cx^2)} + \frac{\left(-\frac{bB}{2}+\frac{5Ac}{2}\right)\int \frac{1}{x^{3/2}(b+cx^2)}\,dx}{2bc}$$

$$= \frac{bB-5Ac}{2b^2c\sqrt{x}} - \frac{bB-Ac}{2bc\sqrt{x}(b+cx^2)} + \frac{(bB-5Ac)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b^2}$$

$$= \frac{bB-5Ac}{2b^2c\sqrt{x}} - \frac{bB-Ac}{2bc\sqrt{x}(b+cx^2)} + \frac{(bB-5Ac)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^2}$$

$$= \frac{bB-5Ac}{2b^2c\sqrt{x}} - \frac{bB-Ac}{2bc\sqrt{x}(b+cx^2)} - \frac{(bB-5Ac)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^2\sqrt{c}} + \frac{(bB-5Ac)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^2\sqrt{c}}$$

$$= \frac{bB-5Ac}{2b^2c\sqrt{x}} - \frac{bB-Ac}{2bc\sqrt{x}(b+cx^2)} + \frac{(bB-5Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^2c} + \frac{(bB-5Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^2c} + \frac{(bB-5Ac)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{2}b^{9/4}c^{3/4}} + \frac{(bB-5Ac)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{2}b^{9/4}c^{3/4}}$$

$$= \frac{bB-5Ac}{2b^2c\sqrt{x}} - \frac{bB-Ac}{2bc\sqrt{x}\,(b+cx^2)} + \frac{(bB-5Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}c^{3/4}} - \frac{(bB-5Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}c^{3/4}} + \frac{(bB-5Ac)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{9/4}c^{3/4}} - \frac{(bB-5Ac)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{9/4}c^{3/4}}$$

$$= \frac{bB-5Ac}{2b^2c\sqrt{x}} - \frac{bB-Ac}{2bc\sqrt{x}\,(b+cx^2)} - \frac{(bB-5Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{9/4}c^{3/4}} + \frac{(bB-5Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{9/4}c^{3/4}} + \frac{(bB-5Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}c^{3/4}} - \frac{(bB-5Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}c^{3/4}}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.57

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\frac{4\sqrt[4]{b}(-4Ab+bBx^2-5Acx^2)}{\sqrt{x}(b+cx^2)} + \frac{\sqrt{2}(-bB+5Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{c^{3/4}} + \frac{\sqrt{2}(-bB+5Ac)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt{b}+}\right.}{c^{3/4}}}{8b^{9/4}}$$

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*b^(1/4)*(-4*A*b + b*B*x^2 - 5*A*c*x^2))/(Sqrt[x]*(b + c*x^2)) + (Sqrt[2
]*(-(b*B) + 5*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sq
rt[x])])/c^(3/4) + (Sqrt[2]*(-(b*B) + 5*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/
4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/c^(3/4))/(8*b^(9/4))
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.54

method	result
derivativedivides	$-\frac{2A}{b^{2}\sqrt{x}}-\frac{2\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^{2}+b}+\frac{\left(\frac{5Ac}{4}-\frac{Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^{2}}$
default	$-\frac{2A}{b^{2}\sqrt{x}}-\frac{2\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^{2}+b}+\frac{\left(\frac{5Ac}{4}-\frac{Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^{2}}$
risch	$-\frac{2A}{b^{2}\sqrt{x}}-\frac{\frac{2\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^{2}+b}+\frac{\left(\frac{5Ac}{4}-\frac{Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{b^{2}}$

```
[In] int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*A/b^2/x^(1/2)-2/b^2*((1/4*A*c-1/4*B*b)*x^(3/2)/(c*x^2+b)+1/8*(5/4*A*c-1/
4*B*b)/c/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)
^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(
1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 789, normalized size of antiderivative = 2.78

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx =$$

$$-\frac{(b^2cx^3+b^3x)\left(-\frac{B^4b^4-20\,AB^3b^3c+150\,A^2B^2b^2c^2-500\,A^3Bbc^3+625\,A^4c^4}{b^9c^3}\right)^{\frac{1}{4}}\log\left(b^7c^2\left(-\frac{B^4b^4-20\,AB^3b^3c+150\,A^2B^2b^2c^2-500\,A}{b^9c^3}\right.\right.}{}$$

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((b^2*c*x^3 + b^3*x)*(-(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2
 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(1/4)*log(b^7*c^2*(-(B^4*b^4 -
 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9
*c^3))^(3/4) - (B^3*b^3 - 15*A*B^2*b^2*c + 75*A^2*B*b*c^2 - 125*A^3*c^3)*sq
```

```
rt(x)) + (-I*b^2*c*x^3 - I*b^3*x)*(-(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2
*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(1/4)*log(I*b^7*c^2*(-
(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4
*c^4)/(b^9*c^3))^(3/4) - (B^3*b^3 - 15*A*B^2*b^2*c + 75*A^2*B*b*c^2 - 125*A
^3*c^3)*sqrt(x)) + (I*b^2*c*x^3 + I*b^3*x)*(-(B^4*b^4 - 20*A*B^3*b^3*c + 15
0*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(1/4)*log(-I*
b^7*c^2*(-(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3
 + 625*A^4*c^4)/(b^9*c^3))^(3/4) - (B^3*b^3 - 15*A*B^2*b^2*c + 75*A^2*B*b*c
^2 - 125*A^3*c^3)*sqrt(x)) - (b^2*c*x^3 + b^3*x)*(-(B^4*b^4 - 20*A*B^3*b^3*
c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^3))^(1/4)*l
og(-b^7*c^2*(-(B^4*b^4 - 20*A*B^3*b^3*c + 150*A^2*B^2*b^2*c^2 - 500*A^3*B*b
*c^3 + 625*A^4*c^4)/(b^9*c^3))^(3/4) - (B^3*b^3 - 15*A*B^2*b^2*c + 75*A^2*B
*b*c^2 - 125*A^3*c^3)*sqrt(x)) - 4*((B*b - 5*A*c)*x^2 - 4*A*b)*sqrt(x))/(b^
2*c*x^3 + b^3*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.78

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(Bb-5\,Ac)x^2-4\,Ab}{2\left(b^2cx^{\frac{5}{2}}+b^3\sqrt{x}\right)} + \frac{(Bb-5\,Ac)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \cdots\right)}{16\,b^2}$$

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*b - 5*A*c)*x^2 - 4*A*b)/(b^2*c*x^(5/2) + b^3*sqrt(x)) + 1/16*(B*b -
 5*A*c)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*
```

```
sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqr
t(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4
)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-s
qrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^
2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.98

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{Bbx^2-5\,Acx^2-4\,Ab}{2\left(cx^{\frac{5}{2}}+b\sqrt{x}\right)b^2} + \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb-5\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3c^3} + \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb-5\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3c^3} - \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb-5\left(bc^3\right)^{\frac{3}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^3c^3} + \frac{\sqrt{2}\left(\left(bc^3\right)^{\frac{3}{4}}Bb-5\left(bc^3\right)^{\frac{3}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^3c^3}$$

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*b*x^2 - 5*A*c*x^2 - 4*A*b)/((c*x^(5/2) + b*sqrt(x))*b^2) + 1/8*sqrt(
2)*((b*c^3)^(3/4)*B*b - 5*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b
/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^3) + 1/8*sqrt(2)*((b*c^3)^(3/4)*
B*b - 5*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqr
t(x))/(b/c)^(1/4))/(b^3*c^3) - 1/16*sqrt(2)*((b*c^3)^(3/4)*B*b - 5*(b*c^3)^
(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^3) + 1/1
6*sqrt(2)*((b*c^3)^(3/4)*B*b - 5*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b
/c)^(1/4) + x + sqrt(b/c))/(b^3*c^3)
```

Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.37

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\operatorname{atanh}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)\,(5\,A\,c-B\,b)}{4\,(-b)^{9/4}\,c^{3/4}} - \frac{\operatorname{atan}\left(\frac{c^{1/4}\sqrt{x}}{(-b)^{1/4}}\right)\,(5\,A\,c-B\,b)}{4\,(-b)^{9/4}\,c^{3/4}} - \frac{\frac{2\,A}{b}+\frac{x^2\,(5\,A\,c-B\,b)}{2\,b^2}}{b\,\sqrt{x}+c\,x^{5/2}}$$

```
[In] int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] (atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(5*A*c - B*b))/(4*(-b)^(9/4)*c^(3/4))
- (atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(5*A*c - B*b))/(4*(-b)^(9/4)*c^(3/4))
 - ((2*A)/b + (x^2*(5*A*c - B*b))/(2*b^2))/(b*x^(1/2) + c*x^(5/2))
```

3.203 $\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 289

$$\begin{aligned}\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx &= \frac{3bB-7Ac}{6b^2cx^{3/2}}-\frac{bB-Ac}{2bcx^{3/2}(b+cx^2)}\\ &-\frac{(3bB-7Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{11/4}\sqrt[4]{c}}+\frac{(3bB-7Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{11/4}\sqrt[4]{c}}\\ &-\frac{(3bB-7Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}\sqrt[4]{c}}\\ &+\frac{(3bB-7Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}\sqrt[4]{c}}\end{aligned}$$

```
[Out] 1/6*(-7*A*c+3*B*b)/b^2/c/x^(3/2)+1/2*(A*c-B*b)/b/c/x^(3/2)/(c*x^2+b)-1/8*(-
7*A*c+3*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/4)/c^(1/4)*2^(
1/2)+1/8*(-7*A*c+3*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/4)/
c^(1/4)*2^(1/2)-1/16*(-7*A*c+3*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^
(1/2)*x^(1/2))/b^(11/4)/c^(1/4)*2^(1/2)+1/16*(-7*A*c+3*B*b)*ln(b^(1/2)+x*c^
(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(11/4)/c^(1/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1598, 468, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(3bB-7Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{11/4}\sqrt[4]{c}} + \frac{(3bB-7Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{11/4}\sqrt[4]{c}} - \frac{(3bB-7Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}\sqrt[4]{c}} + \frac{(3bB-7Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}\sqrt[4]{c}} + \frac{3bB-7Ac}{6b^2cx^{3/2}} - \frac{bB-Ac}{2bcx^{3/2}(b+cx^2)}$$

```
[In] Int[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (3*b*B - 7*A*c)/(6*b^2*c*x^(3/2)) - (b*B - A*c)/(2*b*c*x^(3/2)*(b + c*x^2))
 - ((3*b*B - 7*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[
2]*b^(11/4)*c^(1/4)) + ((3*b*B - 7*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/(4*Sqrt[2]*b^(11/4)*c^(1/4)) - ((3*b*B - 7*A*c)*Log[Sqrt[b] - S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(11/4)*c^(1/4)) +
 ((3*b*B - 7*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x
])/(8*Sqrt[2]*b^(11/4)*c^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{A+Bx^2}{x^{5/2}\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)} + \frac{\left(-\frac{3bB}{2}+\frac{7Ac}{2}\right)\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{2bc}$$

$$= \frac{3bB-7Ac}{6b^2cx^{3/2}} - \frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)} + \frac{(3bB-7Ac)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4b^2}$$

$$= \frac{3bB-7Ac}{6b^2cx^{3/2}} - \frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)} + \frac{(3bB-7Ac)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^2}$$

$$= \frac{3bB-7Ac}{6b^2cx^{3/2}} - \frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)} + \frac{(3bB-7Ac)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^{5/2}}$$
$$+ \frac{(3bB-7Ac)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^{5/2}}$$

$$=\frac{3bB-7Ac}{6b^2cx^{3/2}}-\frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)}+\frac{(3bB-7Ac)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8b^{5/2}\sqrt{c}}$$

$$+\frac{(3bB-7Ac)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8b^{5/2}\sqrt{c}}$$

$$-\frac{(3bB-7Ac)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{2}b^{11/4}\sqrt[4]{c}}$$

$$-\frac{(3bB-7Ac)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{2}b^{11/4}\sqrt[4]{c}}$$

$$=\frac{3bB-7Ac}{6b^2cx^{3/2}}-\frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)}-\frac{(3bB-7Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}\sqrt[4]{c}}$$

$$+\frac{(3bB-7Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}\sqrt[4]{c}}$$

$$+\frac{(3bB-7Ac)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{11/4}\sqrt[4]{c}}$$

$$-\frac{(3bB-7Ac)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{11/4}\sqrt[4]{c}}$$

$$=\frac{3bB-7Ac}{6b^2cx^{3/2}}-\frac{bB-Ac}{2bcx^{3/2}\left(b+cx^2\right)}-\frac{(3bB-7Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{11/4}\sqrt[4]{c}}$$

$$+\frac{(3bB-7Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{11/4}\sqrt[4]{c}}$$

$$-\frac{(3bB-7Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}\sqrt[4]{c}}$$

$$+\frac{(3bB-7Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}\sqrt[4]{c}}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.57

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\frac{4b^{3/4}(-4Ab+3bBx^2-7Acx^2)}{x^{3/2}(b+cx^2)}+\frac{3\sqrt{2}(-3bB+7Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt[4]{c}}+\frac{3\sqrt{2}(3bB-7Ac)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}\right)}{\sqrt[4]{c}}}{24b^{11/4}}$$

[In] Integrate[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]

[Out] ((4*b^(3/4)*(-4*A*b + 3*b*B*x^2 - 7*A*c*x^2))/(x^(3/2)*(b + c*x^2)) + (3*Sqrt[2]*(-3*b*B + 7*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/c^(1/4) + (3*Sqrt[2]*(3*b*B - 7*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/c^(1/4))/(24*b^(11/4))

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.53

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(7Ac-3Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32b}\right)}{b^2}-\frac{2\cdots}{3b^{2}\cdots}$
default	$-\frac{2\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(7Ac-3Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32b}\right)}{b^2}-\frac{2\cdots}{3b^{2}\cdots}$
risch	$-\frac{2A}{3b^2x^{\frac{3}{2}}}-\frac{\frac{2\left(\frac{Ac}{4}-\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(7Ac-3Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{16b}}{b^2}$

[In] int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)

[Out] -2/b^2*((1/4*A*c-1/4*B*b)*x^(1/2)/(c*x^2+b)+1/32*(7*A*c-3*B*b)*(1/c*b)^(1/4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))-2/3*A/b^2/x^(3/2)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 691, normalized size of antiderivative = 2.39

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx =$$

$$-\frac{3\,(b^2cx^4+b^3x^2)\left(-\frac{81\,B^4b^4-756\,AB^3b^3c+2646\,A^2B^2b^2c^2-4116\,A^3Bbc^3+2401\,A^4c^4}{b^{11}c}\right)^{\frac{1}{4}}\log\left(b^3\left(-\frac{81\,B^4b^4-756\,AB^3b^3c+2646\,A^2B}{b^{11}}\right.\right.}{}$$

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(3*(b^2*c*x^4 + b^3*x^2)*(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B
^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(1/4)*log(b^3*(-(81
*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401
*A^4*c^4)/(b^11*c))^(1/4) - (3*B*b - 7*A*c)*sqrt(x)) + 3*(I*b^2*c*x^4 + I*b
^3*x^2)*(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B
*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(1/4)*log(I*b^3*(-(81*B^4*b^4 - 756*A*B^3*
b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(
1/4) - (3*B*b - 7*A*c)*sqrt(x)) + 3*(-I*b^2*c*x^4 - I*b^3*x^2)*(-(81*B^4*b^
4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^
4)/(b^11*c))^(1/4)*log(-I*b^3*(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^
2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(1/4) - (3*B*b - 7*A
*c)*sqrt(x)) - 3*(b^2*c*x^4 + b^3*x^2)*(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 26
46*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c))^(1/4)*log(-
b^3*(-(81*B^4*b^4 - 756*A*B^3*b^3*c + 2646*A^2*B^2*b^2*c^2 - 4116*A^3*B*b*c
^3 + 2401*A^4*c^4)/(b^11*c))^(1/4) - (3*B*b - 7*A*c)*sqrt(x)) - 4*((3*B*b -
 7*A*c)*x^2 - 4*A*b)*sqrt(x))/(b^2*c*x^4 + b^3*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.87

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{(3\,Bb-7\,Ac)x^2-4\,Ab}{6\left(b^2cx^{\frac{7}{2}}+b^3x^{\frac{3}{2}}\right)} + \frac{\frac{2\sqrt{2}(3\,Bb-7\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}(3\,Bb-7\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}(3\,Bb-7\,Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{16\,b^2}$$

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/6*((3*B*b - 7*A*c)*x^2 - 4*A*b)/(b^2*c*x^(7/2) + b^3*x^(3/2)) + 1/16*(2*s
qrt(2)*(3*B*b - 7*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt
(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqr
t(2)*(3*B*b - 7*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(
c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2
)*(3*B*b - 7*A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b)
)/(b^(3/4)*c^(1/4)) - sqrt(2)*(3*B*b - 7*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*
sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx &= \frac{\sqrt{2}\left(3\,(bc^3)^{\frac{1}{4}}\,Bb-7\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3c} \\ &+ \frac{\sqrt{2}\left(3\,(bc^3)^{\frac{1}{4}}\,Bb-7\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3c} \\ &+ \frac{\sqrt{2}\left(3\,(bc^3)^{\frac{1}{4}}\,Bb-7\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^3c} \\ &- \frac{\sqrt{2}\left(3\,(bc^3)^{\frac{1}{4}}\,Bb-7\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^3c} \\ &+ \frac{Bb\sqrt{x}-Ac\sqrt{x}}{2\,(cx^2+b)b^2} - \frac{2\,A}{3\,b^2x^{\frac{3}{2}}}\end{aligned}$$

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(3*(b*c^3)^(1/4)*B*b - 7*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*c) + 1/8*sqrt(2)*(3*(b*
c^3)^(1/4)*B*b - 7*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1
/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c) + 1/16*sqrt(2)*(3*(b*c^3)^(1/4)*B*b -
 7*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3
*c) - 1/16*sqrt(2)*(3*(b*c^3)^(1/4)*B*b - 7*(b*c^3)^(1/4)*A*c)*log(-sqrt(2)
*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c) + 1/2*(B*b*sqrt(x) - A*c*sqrt
(x))/((c*x^2 + b)*b^2) - 2/3*A/(b^2*x^(3/2))
```

Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 859, normalized size of antiderivative = 2.97

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{\frac{2A}{3b}+\frac{x^2\,(7\,A\,c-3\,B\,b)}{6\,b^2}}{b\,x^{3/2}+c\,x^{7/2}}$$
$$-\frac{\operatorname{atan}\left(\frac{\frac{(7\,A\,c-3\,B\,b)\left(\sqrt{x}\left(1568\,A^2\,b^6\,c^5-1344\,A\,B\,b^7\,c^4+288\,B^2\,b^8\,c^3\right)-\frac{(7\,A\,c-3\,B\,b)\left(1792\,A\,b^9\,c^4-768\,B\,b^{10}\,c^3\right)}{8\,(-b)^{11/4}\,c^{1/4}}\right)1\mathrm{i}}{8\,(-b)^{11/4}\,c^{1/4}}+\frac{(7\,A\,c-3\,B\,b)\left(\sqrt{x}\left(1568\,A^2\,b^6\,c^5-\cdots\right.\right.}{\cdots}}{\frac{(7\,A\,c-3\,B\,b)\left(\sqrt{x}\left(1568\,A^2\,b^6\,c^5-1344\,A\,B\,b^7\,c^4+288\,B^2\,b^8\,c^3\right)-\frac{(7\,A\,c-3\,B\,b)\left(1792\,A\,b^9\,c^4-768\,B\,b^{10}\,c^3\right)}{8\,(-b)^{11/4}\,c^{1/4}}\right)}{8\,(-b)^{11/4}\,c^{1/4}}-\frac{(7\,A\,c-3\,B\,b)\left(\sqrt{x}\left(1568\,A^2\,b^6\,c^5-\cdots\right.\right.}{\cdots}}\right)\cdots}{4\,(-b)^{11/4}\,c^{1/4}}$$
$$-\frac{\operatorname{atan}\left(\frac{\frac{(7\,A\,c-3\,B\,b)\left(\sqrt{x}\left(1568\,A^2\,b^6\,c^5-1344\,A\,B\,b^7\,c^4+288\,B^2\,b^8\,c^3\right)-\frac{(7\,A\,c-3\,B\,b)\left(1792\,A\,b^9\,c^4-768\,B\,b^{10}\,c^3\right)1\mathrm{i}}{8\,(-b)^{11/4}\,c^{1/4}}\right)}{8\,(-b)^{11/4}\,c^{1/4}}+\frac{(7\,A\,c-3\,B\,b)\left(\sqrt{x}\left(1568\,A^2\,b^6\,c^5\cdots\right.\right.}{\cdots}}{\frac{(7\,A\,c-3\,B\,b)\left(\sqrt{x}\left(1568\,A^2\,b^6\,c^5-1344\,A\,B\,b^7\,c^4+288\,B^2\,b^8\,c^3\right)-\frac{(7\,A\,c-3\,B\,b)\left(1792\,A\,b^9\,c^4-768\,B\,b^{10}\,c^3\right)1\mathrm{i}}{8\,(-b)^{11/4}\,c^{1/4}}\right)1\mathrm{i}}{8\,(-b)^{11/4}\,c^{1/4}}-\frac{(7\,A\,c-3\,B\,b)\left(\sqrt{x}\left(1568\,A^2\,b^6\,c^5\cdots\right.\right.}{\cdots}}\right)\cdots}{4\,(-b)^{11/4}\,c^{1/4}}$$

```
[In] int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((2*A)/(3*b) + (x^2*(7*A*c - 3*B*b))/(6*b^2))/(b*x^(3/2) + c*x^(7/2)) - (
atan((((7*A*c - 3*B*b)*(x^(1/2)*(1568*A^2*b^6*c^5 + 288*B^2*b^8*c^3 - 1344*
A*B*b^7*c^4) - ((7*A*c - 3*B*b)*(1792*A*b^9*c^4 - 768*B*b^10*c^3))/(8*(-b)^
(11/4)*c^(1/4)))*1i)/(8*(-b)^(11/4)*c^(1/4)) + ((7*A*c - 3*B*b)*(x^(1/2)*(1
568*A^2*b^6*c^5 + 288*B^2*b^8*c^3 - 1344*A*B*b^7*c^4) + ((7*A*c - 3*B*b)*(1
792*A*b^9*c^4 - 768*B*b^10*c^3))/(8*(-b)^(11/4)*c^(1/4)))*1i)/(8*(-b)^(11/4
)*c^(1/4)))/(((7*A*c - 3*B*b)*(x^(1/2)*(1568*A^2*b^6*c^5 + 288*B^2*b^8*c^3
- 1344*A*B*b^7*c^4) - ((7*A*c - 3*B*b)*(1792*A*b^9*c^4 - 768*B*b^10*c^3))/(
8*(-b)^(11/4)*c^(1/4))))/(8*(-b)^(11/4)*c^(1/4)) - ((7*A*c - 3*B*b)*(x^(1/2
)*(1568*A^2*b^6*c^5 + 288*B^2*b^8*c^3 - 1344*A*B*b^7*c^4) + ((7*A*c - 3*B*b
)*(1792*A*b^9*c^4 - 768*B*b^10*c^3))/(8*(-b)^(11/4)*c^(1/4))))/(8*(-b)^(11/
4)*c^(1/4))))*(7*A*c - 3*B*b)*1i)/(4*(-b)^(11/4)*c^(1/4)) - (atan((((7*A*c
- 3*B*b)*(x^(1/2)*(1568*A^2*b^6*c^5 + 288*B^2*b^8*c^3 - 1344*A*B*b^7*c^4) -
```

```
((7*A*c - 3*B*b)*(1792*A*b^9*c^4 - 768*B*b^10*c^3)*1i)/(8*(-b)^(11/4)*c^(1
/4))))/(8*(-b)^(11/4)*c^(1/4)) + ((7*A*c - 3*B*b)*(x^(1/2)*(1568*A^2*b^6*c^
5 + 288*B^2*b^8*c^3 - 1344*A*B*b^7*c^4) + ((7*A*c - 3*B*b)*(1792*A*b^9*c^4
- 768*B*b^10*c^3)*1i)/(8*(-b)^(11/4)*c^(1/4))))/(8*(-b)^(11/4)*c^(1/4)))/((
(7*A*c - 3*B*b)*(x^(1/2)*(1568*A^2*b^6*c^5 + 288*B^2*b^8*c^3 - 1344*A*B*b^7
*c^4) - ((7*A*c - 3*B*b)*(1792*A*b^9*c^4 - 768*B*b^10*c^3)*1i)/(8*(-b)^(11/
4)*c^(1/4)))*1i)/(8*(-b)^(11/4)*c^(1/4)) - ((7*A*c - 3*B*b)*(x^(1/2)*(1568*
A^2*b^6*c^5 + 288*B^2*b^8*c^3 - 1344*A*B*b^7*c^4) + ((7*A*c - 3*B*b)*(1792*
A*b^9*c^4 - 768*B*b^10*c^3)*1i)/(8*(-b)^(11/4)*c^(1/4)))*1i)/(8*(-b)^(11/4)
*c^(1/4))))*(7*A*c - 3*B*b))/(4*(-b)^(11/4)*c^(1/4))
```

3.204 $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 310

$$\begin{aligned}\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx &= \frac{5bB-9Ac}{10b^2cx^{5/2}}-\frac{5bB-9Ac}{2b^3\sqrt{x}}-\frac{bB-Ac}{2bcx^{5/2}(b+cx^2)}\\
&+\frac{\sqrt[4]{c}(5bB-9Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{13/4}}\\
&-\frac{\sqrt[4]{c}(5bB-9Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{13/4}}\\
&-\frac{\sqrt[4]{c}(5bB-9Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}}\\
&+\frac{\sqrt[4]{c}(5bB-9Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}}\end{aligned}$$

```
[Out] 1/10*(-9*A*c+5*B*b)/b^2/c/x^(5/2)+1/2*(A*c-B*b)/b/c/x^(5/2)/(c*x^2+b)+1/8*c
^(1/4)*(-9*A*c+5*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(13/4)*2^
(1/2)-1/8*c^(1/4)*(-9*A*c+5*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/
b^(13/4)*2^(1/2)-1/16*c^(1/4)*(-9*A*c+5*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c
^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^(1/2)+1/16*c^(1/4)*(-9*A*c+5*B*b)*ln(b^(
1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^(1/2)+1/2*(9*A*c
-5*B*b)/b^3/x^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1598, 468, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\sqrt[4]{c}(5bB-9Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{13/4}} - \frac{\sqrt[4]{c}(5bB-9Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{13/4}} - \frac{\sqrt[4]{c}(5bB-9Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}} + \frac{\sqrt[4]{c}(5bB-9Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} + \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{bB-Ac}{2bcx^{5/2}(b+cx^2)}$$

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (5*b*B - 9*A*c)/(10*b^2*c*x^(5/2)) - (5*b*B - 9*A*c)/(2*b^3*Sqrt[x]) - (b*B
 - A*c)/(2*b*c*x^(5/2)*(b + c*x^2)) + (c^(1/4)*(5*b*B - 9*A*c)*ArcTan[1 - (
Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(13/4)) - (c^(1/4)*(5*b*B -
 9*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(13/4))
 - (c^(1/4)*(5*b*B - 9*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
 Sqrt[c]*x])/(8*Sqrt[2]*b^(13/4)) + (c^(1/4)*(5*b*B - 9*A*c)*Log[Sqrt[b] +
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{x^{7/2}\left(b+cx^2\right)^2}\,dx \\
&= -\frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)}+\frac{\left(-\frac{5bB}{2}+\frac{9Ac}{2}\right)\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{2bc} \\
&= \frac{5bB-9Ac}{10b^2cx^{5/2}}-\frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)}+\frac{(5bB-9Ac)\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{4b^2} \\
&= \frac{5bB-9Ac}{10b^2cx^{5/2}}-\frac{5bB-9Ac}{2b^3\sqrt{x}}-\frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)}-\frac{(c(5bB-9Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b^3} \\
&= \frac{5bB-9Ac}{10b^2cx^{5/2}}-\frac{5bB-9Ac}{2b^3\sqrt{x}}-\frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)}-\frac{(c(5bB-9Ac))\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{2b^3} \\
&= \frac{5bB-9Ac}{10b^2cx^{5/2}}-\frac{5bB-9Ac}{2b^3\sqrt{x}}-\frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} \\
&\quad+\frac{\left(\sqrt{c}(5bB-9Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4b^3} \\
&\quad-\frac{\left(\sqrt{c}(5bB-9Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{4b^3}
\end{aligned}$$

$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} - \frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} - \frac{(5bB-9Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8b^3} - \frac{(5bB-9Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{8b^3} - \frac{\left(\sqrt[4]{c}(5bB-9Ac)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{2}b^{13/4}} - \frac{\left(\sqrt[4]{c}(5bB-9Ac)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{8\sqrt{2}b^{13/4}}$$

$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} - \frac{bB-Ac}{2bcx^{5/2}\left(b+cx^2\right)} - \frac{\sqrt[4]{c}(5bB-9Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}} + \frac{\sqrt[4]{c}(5bB-9Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}} - \frac{\left(\sqrt[4]{c}(5bB-9Ac)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{13/4}} + \frac{\left(\sqrt[4]{c}(5bB-9Ac)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{13/4}}$$

$$= \frac{5bB-9Ac}{10b^2cx^{5/2}} - \frac{5bB-9Ac}{2b^3\sqrt{x}} - \frac{bB-Ac}{2bcx^{5/2}(b+cx^2)} + \frac{\sqrt[4]{c}(5bB-9Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{13/4}} - \frac{\sqrt[4]{c}(5bB-9Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{13/4}} - \frac{\sqrt[4]{c}(5bB-9Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}} + \frac{\sqrt[4]{c}(5bB-9Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{-\frac{4\sqrt[4]{b}(5bBx^2(4b+5cx^2)+A(4b^2-36bcx^2-45c^2x^4))}{x^{5/2}(b+cx^2)} + 5\sqrt{2}\sqrt[4]{c}(5bB-9Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) + 5\sqrt{2}\sqrt[4]{c}(5bB-9A}{40b^{13/4}}$$

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-4*b^(1/4)*(5*b*B*x^2*(4*b + 5*c*x^2) + A*(4*b^2 - 36*b*c*x^2 - 45*c^2*x^
4)))/(x^(5/2)*(b + c*x^2)) + 5*Sqrt[2]*c^(1/4)*(5*b*B - 9*A*c)*ArcTan[(Sqrt
[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 5*Sqrt[2]*c^(1/4)*(5*
b*B - 9*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x
)])/(40*b^(13/4))
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.55

method	result
derivativedivides	$-\frac{2A}{5b^2x^{\frac{5}{2}}}-\frac{2(-2Ac+Bb)}{b^3\sqrt{x}}+\frac{2c\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^2+b}+\frac{\left(\frac{9Ac}{4}-\frac{5Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\cdots\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{b^3}$
default	$-\frac{2A}{5b^2x^{\frac{5}{2}}}-\frac{2(-2Ac+Bb)}{b^3\sqrt{x}}+\frac{2c\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^2+b}+\frac{\left(\frac{9Ac}{4}-\frac{5Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\cdots\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{b^3}$
risch	$-\frac{2(-10Ac\,x^2+5bB\,x^2+Ab)}{5b^3x^{\frac{5}{2}}}+\frac{c\left(\frac{2\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^2+b}+\frac{\left(\frac{9Ac}{4}-\frac{5Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+\cdots\right.}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{b^3}$

```
[In] int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*A/b^2/x^(5/2)-2*(-2*A*c+B*b)/b^3/x^(1/2)+2/b^3*c*((1/4*A*c-1/4*B*b)*x^
(3/2)/(c*x^2+b)+1/8*(9/4*A*c-5/4*B*b)/c/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b
)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/
c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/
c*b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 838, normalized size of antiderivative = 2.70

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$$
$$= 5\left(b^3cx^5+b^4x^3\right)\left(-\frac{625\,B^4b^4c-4500\,AB^3b^3c^2+12150\,A^2B^2b^2c^3-14580\,A^3Bbc^4+6561\,A^4c^5}{b^{13}}\right)^{\frac{1}{4}}\log\left(b^{10}\left(-\frac{625\,B^4b^4c-4500\,AB^3b^3c\cdots}{}\right.\right.$$

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/40*(5*(b^3*c*x^5 + b^4*x^3)*(-(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150
*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^13)^(1/4)*log(b^10*(
```

```
-(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*
b*c^4 + 6561*A^4*c^5)/b^13)^(3/4) - (125*B^3*b^3*c - 675*A*B^2*b^2*c^2 + 12
15*A^2*B*b*c^3 - 729*A^3*c^4)*sqrt(x)) - 5*(I*b^3*c*x^5 + I*b^4*x^3)*(-(625
*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4
 + 6561*A^4*c^5)/b^13)^(1/4)*log(I*b^10*(-(625*B^4*b^4*c - 4500*A*B^3*b^3*c
^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^13)^(3/4)
- (125*B^3*b^3*c - 675*A*B^2*b^2*c^2 + 1215*A^2*B*b*c^3 - 729*A^3*c^4)*sqrt
(x)) - 5*(-I*b^3*c*x^5 - I*b^4*x^3)*(-(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 +
 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^13)^(1/4)*log(
-I*b^10*(-(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 145
80*A^3*B*b*c^4 + 6561*A^4*c^5)/b^13)^(3/4) - (125*B^3*b^3*c - 675*A*B^2*b^2
*c^2 + 1215*A^2*B*b*c^3 - 729*A^3*c^4)*sqrt(x)) - 5*(b^3*c*x^5 + b^4*x^3)*(
-(625*B^4*b^4*c - 4500*A*B^3*b^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*
b*c^4 + 6561*A^4*c^5)/b^13)^(1/4)*log(-b^10*(-(625*B^4*b^4*c - 4500*A*B^3*b
^3*c^2 + 12150*A^2*B^2*b^2*c^3 - 14580*A^3*B*b*c^4 + 6561*A^4*c^5)/b^13)^(3
/4) - (125*B^3*b^3*c - 675*A*B^2*b^2*c^2 + 1215*A^2*B*b*c^3 - 729*A^3*c^4)*
sqrt(x)) - 4*(5*(5*B*b*c - 9*A*c^2)*x^4 + 4*A*b^2 + 4*(5*B*b^2 - 9*A*b*c)*x
^2)*sqrt(x))/(b^3*c*x^5 + b^4*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx^2)}{\left(bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{x}(A+Bx^2)}{\left(bx^2+cx^4\right)^2}\,dx = -\frac{5\,(5\,Bbc-9\,Ac^2)x^4+4\,Ab^2+4\,(5\,Bb^2-9\,Abc)x^2}{10\left(b^3cx^{\frac{9}{2}}+b^4x^{\frac{5}{2}}\right)} - \frac{(5\,Bbc-9\,Ac^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{\right.}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right.}{16\,b^3}$$

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/10*(5*(5*B*b*c - 9*A*c^2)*x^4 + 4*A*b^2 + 4*(5*B*b^2 - 9*A*b*c)*x^2)/(b^
3*c*x^(9/2) + b^4*x^(5/2)) - 1/16*(5*B*b*c - 9*A*c^2)*(2*sqrt(2)*arctan(1/2
*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)
))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)
*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*
sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x
 + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x
) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{Bbcx^{\frac{3}{2}}-Ac^2x^{\frac{3}{2}}}{2\,(cx^2+b)b^3} - \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{3}{4}}\,Bb-9\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4c^2} - \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{3}{4}}\,Bb-9\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4c^2} + \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{3}{4}}\,Bb-9\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4c^2} - \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{3}{4}}\,Bb-9\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4c^2} - \frac{2\,(5\,Bbx^2-10\,Acx^2+Ab)}{5\,b^3x^{\frac{5}{2}}}$$

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*b*c*x^(3/2) - A*c^2*x^(3/2))/((c*x^2 + b)*b^3) - 1/8*sqrt(2)*(5*(b*
c^3)^(3/4)*B*b - 9*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/
4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c^2) - 1/8*sqrt(2)*(5*(b*c^3)^(3/4)*B*b -
 9*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))
/(b/c)^(1/4))/(b^4*c^2) + 1/16*sqrt(2)*(5*(b*c^3)^(3/4)*B*b - 9*(b*c^3)^(3/
4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c^2) - 1/16*s
qrt(2)*(5*(b*c^3)^(3/4)*B*b - 9*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/
c)^(1/4) + x + sqrt(b/c))/(b^4*c^2) - 2/5*(5*B*b*x^2 - 10*A*c*x^2 + A*b)/(b
^3*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{\frac{2\,x^2\,(9\,A\,c-5\,B\,b)}{5\,b^2}-\frac{2\,A}{5\,b}+\frac{c\,x^4\,(9\,A\,c-5\,B\,b)}{2\,b^3}}{b\,x^{5/2}+c\,x^{9/2}} + \frac{(-c)^{1/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(9\,A\,c-5\,B\,b)}{4\,b^{13/4}} - \frac{(-c)^{1/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(9\,A\,c-5\,B\,b)}{4\,b^{13/4}}$$

```
[In] int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
[Out] ((2*x^2*(9*A*c - 5*B*b))/(5*b^2) - (2*A)/(5*b) + (c*x^4*(9*A*c - 5*B*b))/(2
*b^3))/(b*x^(5/2) + c*x^(9/2)) + ((-c)^(1/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1
/4))*(9*A*c - 5*B*b))/(4*b^(13/4)) - ((-c)^(1/4)*atanh(((-c)^(1/4)*x^(1/2))
/b^(1/4))*(9*A*c - 5*B*b))/(4*b^(13/4))
```

3.205 $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 310

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^2}\,dx = \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{7bB-11Ac}{6b^3x^{3/2}} - \frac{bB-Ac}{2bcx^{7/2}\,(b+cx^2)} + \frac{c^{3/4}(7bB-11Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{15/4}} - \frac{c^{3/4}(7bB-11Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{15/4}} + \frac{c^{3/4}(7bB-11Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}} - \frac{c^{3/4}(7bB-11Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}}$$

```
[Out] 1/14*(-11*A*c+7*B*b)/b^2/c/x^(7/2)+1/6*(11*A*c-7*B*b)/b^3/x^(3/2)+1/2*(A*c-
B*b)/b/c/x^(7/2)/(c*x^2+b)+1/8*c^(3/4)*(-11*A*c+7*B*b)*arctan(1-c^(1/4)*2^(
1/2)*x^(1/2)/b^(1/4))/b^(15/4)*2^(1/2)-1/8*c^(3/4)*(-11*A*c+7*B*b)*arctan(1
+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)*2^(1/2)+1/16*c^(3/4)*(-11*A*c+7*
B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(15/4)*2^(1/2)
-1/16*c^(3/4)*(-11*A*c+7*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*
x^(1/2))/b^(15/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1598, 468, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^2}\,dx = \frac{c^{3/4}(7bB-11Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{15/4}} - \frac{c^{3/4}(7bB-11Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{15/4}} + \frac{c^{3/4}(7bB-11Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}} - \frac{c^{3/4}(7bB-11Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}} - \frac{7bB-11Ac}{6b^3x^{3/2}} + \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{bB-Ac}{2bcx^{7/2}\,(b+cx^2)}$$

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^2),x]
```

```
[Out] (7*b*B - 11*A*c)/(14*b^2*c*x^(7/2)) - (7*b*B - 11*A*c)/(6*b^3*x^(3/2)) - (b
*B - A*c)/(2*b*c*x^(7/2)*(b + c*x^2)) + (c^(3/4)*(7*b*B - 11*A*c)*ArcTan[1
- (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(15/4)) - (c^(3/4)*(7*b*
B - 11*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(15
/4)) + (c^(3/4)*(7*b*B - 11*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt
[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(15/4)) - (c^(3/4)*(7*b*B - 11*A*c)*Log[Sqrt
[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(15/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{A + Bx^2}{x^{9/2}\left(b + cx^2\right)^2}\,dx$$

$$= -\frac{bB - Ac}{2bcx^{7/2}\left(b + cx^2\right)} + \frac{\left(-\frac{7bB}{2} + \frac{11Ac}{2}\right)\int \frac{1}{x^{9/2}\left(b+cx^2\right)}\,dx}{2bc}$$

$$= \frac{7bB - 11Ac}{14b^2cx^{7/2}} - \frac{bB - Ac}{2bcx^{7/2}\left(b + cx^2\right)} + \frac{(7bB - 11Ac)\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{4b^2}$$

$$= \frac{7bB - 11Ac}{14b^2cx^{7/2}} - \frac{7bB - 11Ac}{6b^3x^{3/2}} - \frac{bB - Ac}{2bcx^{7/2}\left(b + cx^2\right)} - \frac{(c(7bB - 11Ac))\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{4b^3}$$

$$= \frac{7bB - 11Ac}{14b^2cx^{7/2}} - \frac{7bB - 11Ac}{6b^3x^{3/2}} - \frac{bB - Ac}{2bcx^{7/2}\left(b + cx^2\right)} - \frac{(c(7bB - 11Ac))\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^3}$$

$$= \frac{7bB - 11Ac}{14b^2cx^{7/2}} - \frac{7bB - 11Ac}{6b^3x^{3/2}} - \frac{bB - Ac}{2bcx^{7/2}\left(b + cx^2\right)} - \frac{(c(7bB - 11Ac))\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^{7/2}} - \frac{(c(7bB - 11Ac))\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^{7/2}}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{7bB-11Ac}{6b^3x^{3/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} - \frac{\left(\sqrt{c}(7bB-11Ac)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^{7/2}} - \frac{\left(\sqrt{c}(7bB-11Ac)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^{7/2}} + \frac{\left(c^{3/4}(7bB-11Ac)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{2}b^{15/4}} + \frac{\left(c^{3/4}(7bB-11Ac)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{2}b^{15/4}}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{7bB-11Ac}{6b^3x^{3/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)} + \frac{c^{3/4}(7bB-11Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}} - \frac{c^{3/4}(7bB-11Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}} - \frac{\left(c^{3/4}(7bB-11Ac)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{15/4}} + \frac{\left(c^{3/4}(7bB-11Ac)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{15/4}}$$

$$= \frac{7bB-11Ac}{14b^2cx^{7/2}} - \frac{7bB-11Ac}{6b^3x^{3/2}} - \frac{bB-Ac}{2bcx^{7/2}\left(b+cx^2\right)}$$
$$+ \frac{c^{3/4}(7bB-11Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{15/4}}$$
$$- \frac{c^{3/4}(7bB-11Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{15/4}}$$
$$+ \frac{c^{3/4}(7bB-11Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}}$$
$$- \frac{c^{3/4}(7bB-11Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}}$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^2}\,dx$$
$$= \frac{-\frac{4b^{3/4}\left(7bBx^2\left(4b+7cx^2\right)+A\left(12b^2-44bcx^2-77c^2x^4\right)\right)}{x^{7/2}\left(b+cx^2\right)} + 21\sqrt{2}c^{3/4}(7bB-11Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) + 21\sqrt{2}c^{3/4}(-7b}{168b^{15/4}}$$

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*b^(3/4)*(7*b*B*x^2*(4*b + 7*c*x^2) + A*(12*b^2 - 44*b*c*x^2 - 77*c^2*x
^4)))/(x^(7/2)*(b + c*x^2)) + 21*Sqrt[2]*c^(3/4)*(7*b*B - 11*A*c)*ArcTan[(S
qrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 21*Sqrt[2]*c^(3/4)
*(-7*b*B + 11*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqr
t[c]*x)])/(168*b^(15/4))
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.55

method	result
derivativedivides	$\frac{2c\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(11Ac-7Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32b}\right)}{b^3}-\frac{}{7}$
default	$\frac{2c\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(11Ac-7Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32b}\right)}{b^3}-\frac{}{7}$
risch	$-\frac{2\left(-14Ac\,x^2+7bB\,x^2+3Ab\right)}{21b^3x^{\frac{7}{2}}}+\frac{c\left(\frac{2\left(\frac{Ac}{4}-\frac{Bb}{4}\right)\sqrt{x}}{c\,x^2+b}+\frac{(11Ac-7Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{16b}\right.}{b^3}$

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*c*((1/4*A*c-1/4*B*b)*x^(1/2)/(c*x^2+b)+1/32*(11*A*c-7*B*b)*(1/c*b)^(1
/4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b
)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1
/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))-2/7*A/b^2/x^(7/2)-2/3*(-
2*A*c+B*b)/b^3/x^(3/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.39

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^2}\,dx$$

$$=\frac{21\,(b^3cx^6+b^4x^4)\left(-\frac{2401\,B^4b^4c^3-15092\,AB^3b^3c^4+35574\,A^2B^2b^2c^5-37268\,A^3Bbc^6+14641\,A^4c^7}{b^{15}}\right)^{\frac{1}{4}}\log\left(b^4\left(-\frac{2401\,B^4b^4c^3-15092}{}\right.\right.}{}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/168*(21*(b^3*c*x^6 + b^4*x^4)*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 +
 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(1/4)*log
(b^4*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^5 - 37
268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(1/4) - (7*B*b*c - 11*A*c^2)*sqrt(x)
) - 21*(-I*b^3*c*x^6 - I*b^4*x^4)*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4
```

```
 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(1/4)*l
og(I*b^4*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^5
- 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(1/4) - (7*B*b*c - 11*A*c^2)*sqr
t(x)) - 21*(I*b^3*c*x^6 + I*b^4*x^4)*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*
c^4 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(1/4
)*log(-I*b^4*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*
c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(1/4) - (7*B*b*c - 11*A*c^2)
*sqrt(x)) - 21*(b^3*c*x^6 + b^4*x^4)*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*
c^4 + 35574*A^2*B^2*b^2*c^5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(1/4
)*log(-b^4*(-(2401*B^4*b^4*c^3 - 15092*A*B^3*b^3*c^4 + 35574*A^2*B^2*b^2*c^
5 - 37268*A^3*B*b*c^6 + 14641*A^4*c^7)/b^15)^(1/4) - (7*B*b*c - 11*A*c^2)*s
qrt(x)) - 4*(7*(7*B*b*c - 11*A*c^2)*x^4 + 12*A*b^2 + 4*(7*B*b^2 - 11*A*b*c)
*x^2)*sqrt(x))/(b^3*c*x^6 + b^4*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^2}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**2/x**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^2}\,dx = -\frac{7\,(7\,Bbc-11\,Ac^2)x^4+12\,Ab^2+4\,(7\,Bb^2-11\,Abc)x^2}{42\,\left(b^3cx^{\frac{11}{2}}+b^4x^{\frac{7}{2}}\right)} - \frac{\frac{2\sqrt{2}(7\,Bbc-11\,Ac^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}(7\,Bbc-11\,Ac^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}(7\,Bbc-11\,Ac^2)\log \cdots}{b \cdots}}{16\,b^3}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="maxima")
[Out] -1/42*(7*(7*B*b*c - 11*A*c^2)*x^4 + 12*A*b^2 + 4*(7*B*b^2 - 11*A*b*c)*x^2)/
(b^3*c*x^(11/2) + b^4*x^(7/2)) - 1/16*(2*sqrt(2)*(7*B*b*c - 11*A*c^2)*arcta
n(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sq
rt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(7*B*b*c - 11*A*c^2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b
```

```
)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(7*B*b*c - 11*A*c^2)*
log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)
) - sqrt(2)*(7*B*b*c - 11*A*c^2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqr
t(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^2}\,dx = -\frac{\sqrt{2}\Big(7\,(bc^3)^{\frac{1}{4}}\,Bb-11\,(bc^3)^{\frac{1}{4}}\,Ac\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4} - \frac{\sqrt{2}\Big(7\,(bc^3)^{\frac{1}{4}}\,Bb-11\,(bc^3)^{\frac{1}{4}}\,Ac\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4} - \frac{\sqrt{2}\Big(7\,(bc^3)^{\frac{1}{4}}\,Bb-11\,(bc^3)^{\frac{1}{4}}\,Ac\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4} + \frac{\sqrt{2}\Big(7\,(bc^3)^{\frac{1}{4}}\,Bb-11\,(bc^3)^{\frac{1}{4}}\,Ac\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4} - \frac{Bbc\sqrt{x}-Ac^2\sqrt{x}}{2\,(cx^2+b)b^3} - \frac{2\,(7\,Bbx^2-14\,Acx^2+3\,Ab)}{21\,b^3x^{\frac{7}{2}}}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^4 - 1/8*sqrt(2)*(7*(b*c^
3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/
4) - 2*sqrt(x))/(b/c)^(1/4))/b^4 - 1/16*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 11*(
b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 + 1/
16*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqrt(x
)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 1/2*(B*b*c*sqrt(x) - A*c^2*sqrt(x))/((
c*x^2 + b)*b^3) - 2/21*(7*B*b*x^2 - 14*A*c*x^2 + 3*A*b)/(b^3*x^(7/2))
```

Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.92

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^2}\,dx = \frac{\frac{2\,x^2\,(11\,A\,c-7\,B\,b)}{21\,b^2}-\frac{2\,A}{7\,b}+\frac{c\,x^4\,(11\,A\,c-7\,B\,b)}{6\,b^3}}{b\,x^{7/2}+c\,x^{11/2}}$$
$$+\frac{(-c)^{3/4}\,\mathrm{atan}\left(\frac{\frac{(-c)^{3/4}\,(11\,A\,c-7\,B\,b)\left(\sqrt{x}\left(3872\,A^2\,b^9\,c^7-4928\,A\,B\,b^{10}\,c^6+1568\,B^2\,b^{11}\,c^5\right)-\frac{(-c)^{3/4}\,(11\,A\,c-7\,B\,b)\left(2816\,A\,b^{13}\,c^5-1792\,B\,b^{14}\,c^4\right)\,1\mathrm{i}}{8\,b^{15/4}}\right)}{8\,b^{15/4}}}{\frac{(-c)^{3/4}\,(11\,A\,c-7\,B\,b)\left(\sqrt{x}\left(3872\,A^2\,b^9\,c^7-4928\,A\,B\,b^{10}\,c^6+1568\,B^2\,b^{11}\,c^5\right)-\frac{(-c)^{3/4}\,(11\,A\,c-7\,B\,b)\left(2816\,A\,b^{13}\,c^5-1792\,B\,b^{14}\,c^4\right)\,1\mathrm{i}}{8\,b^{15/4}}\right)}{8\,b^{15/4}}}\right)}{}$$
$$-\frac{(-c)^{3/4}\,\mathrm{atan}\left(\frac{A^3\,c^8\,\sqrt{x}\,1331\mathrm{i}-B^3\,b^3\,c^5\,\sqrt{x}\,343\mathrm{i}-A^2\,B\,b\,c^7\,\sqrt{x}\,2541\mathrm{i}+A\,B^2\,b^2\,c^6\,\sqrt{x}\,1617\mathrm{i}}{b^{1/4}\,(-c)^{19/4}\,\left(c\,\left(c\,\left(1331\,A^3\,c-2541\,A^2\,B\,b\right)+1617\,A\,B^2\,b^2\right)-343\,B^3\,b^3\right)}\right)\,(11\,A\,c-7\,B\,b)\,1\mathrm{i}}{4\,b^{15/4}}$$

```
[In] int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^2),x)
```

```
[Out] ((2*x^2*(11*A*c - 7*B*b))/(21*b^2) - (2*A)/(7*b) + (c*x^4*(11*A*c - 7*B*b))
/(6*b^3))/(b*x^(7/2) + c*x^(11/2)) + ((-c)^(3/4)*atan((((-c)^(3/4)*(11*A*c
- 7*B*b)*(x^(1/2)*(3872*A^2*b^9*c^7 + 1568*B^2*b^11*c^5 - 4928*A*B*b^10*c^6
) - ((-c)^(3/4)*(11*A*c - 7*B*b)*(2816*A*b^13*c^5 - 1792*B*b^14*c^4)*1i)/(8
*b^(15/4))))/(8*b^(15/4)) + ((-c)^(3/4)*(11*A*c - 7*B*b)*(x^(1/2)*(3872*A^2
*b^9*c^7 + 1568*B^2*b^11*c^5 - 4928*A*B*b^10*c^6) + ((-c)^(3/4)*(11*A*c - 7
*B*b)*(2816*A*b^13*c^5 - 1792*B*b^14*c^4)*1i)/(8*b^(15/4))))/(8*b^(15/4)))/
(((-c)^(3/4)*(11*A*c - 7*B*b)*(x^(1/2)*(3872*A^2*b^9*c^7 + 1568*B^2*b^11*c^
5 - 4928*A*B*b^10*c^6) - ((-c)^(3/4)*(11*A*c - 7*B*b)*(2816*A*b^13*c^5 - 17
92*B*b^14*c^4)*1i)/(8*b^(15/4)))*1i)/(8*b^(15/4)) - ((-c)^(3/4)*(11*A*c - 7
*B*b)*(x^(1/2)*(3872*A^2*b^9*c^7 + 1568*B^2*b^11*c^5 - 4928*A*B*b^10*c^6) +
 ((-c)^(3/4)*(11*A*c - 7*B*b)*(2816*A*b^13*c^5 - 1792*B*b^14*c^4)*1i)/(8*b^
(15/4)))*1i)/(8*b^(15/4))))*(11*A*c - 7*B*b))/(4*b^(15/4)) - ((-c)^(3/4)*at
an((A^3*c^8*x^(1/2)*1331i - B^3*b^3*c^5*x^(1/2)*343i - A^2*B*b*c^7*x^(1/2)*
2541i + A*B^2*b^2*c^6*x^(1/2)*1617i)/(b^(1/4)*(-c)^(19/4)*(c*(c*(1331*A^3*c
 - 2541*A^2*B*b) + 1617*A*B^2*b^2) - 343*B^3*b^3)))*(11*A*c - 7*B*b)*1i)/(4
*b^(15/4))
```

3.206 $\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 332

$$\begin{aligned}\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx &= \frac{9bB-13Ac}{18b^2cx^{9/2}}-\frac{9bB-13Ac}{10b^3x^{5/2}}+\frac{c(9bB-13Ac)}{2b^4\sqrt{x}}\\
&-\frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)}-\frac{c^{5/4}(9bB-13Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{17/4}}\\
&+\frac{c^{5/4}(9bB-13Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{17/4}}\\
&+\frac{c^{5/4}(9bB-13Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}}\\
&-\frac{c^{5/4}(9bB-13Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}}\end{aligned}$$

```
[Out] 1/18*(-13*A*c+9*B*b)/b^2/c/x^(9/2)+1/10*(13*A*c-9*B*b)/b^3/x^(5/2)+1/2*(A*c
-B*b)/b/c/x^(9/2)/(c*x^2+b)-1/8*c^(5/4)*(-13*A*c+9*B*b)*arctan(1-c^(1/4)*2^
(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)+1/8*c^(5/4)*(-13*A*c+9*B*b)*arctan(
1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)+1/16*c^(5/4)*(-13*A*c+9
*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(17/4)*2^(1/2
)-1/16*c^(5/4)*(-13*A*c+9*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)
*x^(1/2))/b^(17/4)*2^(1/2)+1/2*c*(-13*A*c+9*B*b)/b^4/x^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1598, 468, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx = -\frac{c^{5/4}(9bB-13Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{17/4}} + \frac{c^{5/4}(9bB-13Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{17/4}} + \frac{c^{5/4}(9bB-13Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}} - \frac{c^{5/4}(9bB-13Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)}$$

```
[In] Int[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^2),x]
```

```
[Out] (9*b*B - 13*A*c)/(18*b^2*c*x^(9/2)) - (9*b*B - 13*A*c)/(10*b^3*x^(5/2)) + (
c*(9*b*B - 13*A*c))/(2*b^4*Sqrt[x]) - (b*B - A*c)/(2*b*c*x^(9/2)*(b + c*x^2
)) - (c^(5/4)*(9*b*B - 13*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)
])/(4*Sqrt[2]*b^(17/4)) + (c^(5/4)*(9*b*B - 13*A*c)*ArcTan[1 + (Sqrt[2]*c^(
1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(17/4)) + (c^(5/4)*(9*b*B - 13*A*c)*Lo
g[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(17/
4)) - (c^(5/4)*(9*b*B - 13*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[
x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(17/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{A+Bx^2}{x^{11/2}\left(b+cx^2\right)^2}\,dx$$

$$= -\frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} + \frac{\left(-\frac{9bB}{2}+\frac{13Ac}{2}\right)\int \frac{1}{x^{11/2}(b+cx^2)}\,dx}{2bc}$$

$$= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} + \frac{(9bB-13Ac)\int \frac{1}{x^{7/2}(b+cx^2)}\,dx}{4b^2}$$

$$= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} - \frac{(c(9bB-13Ac))\int \frac{1}{x^{3/2}(b+cx^2)}\,dx}{4b^3}$$

$$= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} + \frac{\left(c^2(9bB-13Ac)\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{4b^4}$$

$$= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} + \frac{\left(c^2(9bB-13Ac)\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{2b^4}$$

$$= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}} - \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} - \frac{\left(c^{3/2}(9bB-13Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^4} + \frac{\left(c^{3/2}(9bB-13Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{4b^4}$$

$$=\frac{9bB-13Ac}{18b^2cx^{9/2}}-\frac{9bB-13Ac}{10b^3x^{5/2}}+\frac{c(9bB-13Ac)}{2b^4\sqrt{x}}-\frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)}$$
$$+\frac{(c(9bB-13Ac))\text{Subst}\left(\int\frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}dx,x,\sqrt{x}\right)}{8b^4}$$
$$+\frac{(c(9bB-13Ac))\text{Subst}\left(\int\frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}dx,x,\sqrt{x}\right)}{8b^4}$$
$$+\frac{\left(c^{5/4}(9bB-13Ac)\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}dx,x,\sqrt{x}\right)}{8\sqrt{2}b^{17/4}}$$
$$+\frac{\left(c^{5/4}(9bB-13Ac)\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}dx,x,\sqrt{x}\right)}{8\sqrt{2}b^{17/4}}$$

$$=\frac{9bB-13Ac}{18b^2cx^{9/2}}-\frac{9bB-13Ac}{10b^3x^{5/2}}+\frac{c(9bB-13Ac)}{2b^4\sqrt{x}}-\frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)}$$
$$+\frac{c^{5/4}(9bB-13Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}}$$
$$-\frac{c^{5/4}(9bB-13Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}}$$
$$+\frac{\left(c^{5/4}(9bB-13Ac)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{17/4}}$$
$$-\frac{\left(c^{5/4}(9bB-13Ac)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{17/4}}$$

$$= \frac{9bB-13Ac}{18b^2cx^{9/2}} - \frac{9bB-13Ac}{10b^3x^{5/2}} + \frac{c(9bB-13Ac)}{2b^4\sqrt{x}}$$
$$- \frac{bB-Ac}{2bcx^{9/2}\left(b+cx^2\right)} - \frac{c^{5/4}(9bB-13Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{17/4}}$$
$$+ \frac{c^{5/4}(9bB-13Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{17/4}}$$
$$+ \frac{c^{5/4}(9bB-13Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}}$$
$$- \frac{c^{5/4}(9bB-13Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}}$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx = \frac{-\frac{4\sqrt[4]{b}\left(9bBx^2\left(4b^2-36bcx^2-45c^2x^4\right)+A\left(20b^3-52b^2cx^2+468bc^2x^4+585c^3x^6\right)\right)}{x^{9/2}\left(b+cx^2\right)} + 45\sqrt{2}c^{5/4}(-9bB+13}{360b^{17}}$$

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*b^(1/4)*(9*b*B*x^2*(4*b^2 - 36*b*c*x^2 - 45*c^2*x^4) + A*(20*b^3 - 52*
b^2*c*x^2 + 468*b*c^2*x^4 + 585*c^3*x^6)))/(x^(9/2)*(b + c*x^2)) + 45*Sqrt[
2]*c^(5/4)*(-9*b*B + 13*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*
c^(1/4)*Sqrt[x])] + 45*Sqrt[2]*c^(5/4)*(-9*b*B + 13*A*c)*ArcTanh[(Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(360*b^(17/4))
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.57

method	result
derivativedivides	$-\frac{2A}{9b^2x^{\frac{9}{2}}}-\frac{2(-2Ac+Bb)}{5b^3x^{\frac{5}{2}}}-\frac{2c(3Ac-2Bb)}{b^4\sqrt{x}}-\frac{2c^2\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^2+b}+\frac{\left(\frac{13Ac}{4}-\frac{9Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\,\mathrm{a}\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{b^4}$
default	$-\frac{2A}{9b^2x^{\frac{9}{2}}}-\frac{2(-2Ac+Bb)}{5b^3x^{\frac{5}{2}}}-\frac{2c(3Ac-2Bb)}{b^4\sqrt{x}}-\frac{2c^2\left(\frac{\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^2+b}+\frac{\left(\frac{13Ac}{4}-\frac{9Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\,\mathrm{a}\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.}{b^4}$
risch	$-\frac{2\left(135A\,c^2x^4-90x^4Bbc-18Abc\,x^2+9b^2B\,x^2+5b^2A\right)}{45b^4x^{\frac{9}{2}}}-\frac{c^2\left(\frac{2\left(\frac{Ac}{4}-\frac{Bb}{4}\right)x^{\frac{3}{2}}}{c\,x^2+b}+\frac{\left(\frac{13Ac}{4}-\frac{9Bb}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{}\right.}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{}}\right.}{}\right.}{b^4}$

```
[In] int((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*A/b^2/x^(9/2)-2/5*(-2*A*c+B*b)/b^3/x^(5/2)-2*c*(3*A*c-2*B*b)/b^4/x^(1/
2)-2/b^4*c^2*((1/4*A*c-1/4*B*b)*x^(3/2)/(c*x^2+b)+1/8*(13/4*A*c-9/4*B*b)/c/
(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(
x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1
/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 886, normalized size of antiderivative = 2.67

$$\int\frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx=$$
$$-\frac{45\left(b^4cx^7+b^5x^5\right)\left(-\frac{6561\,B^4b^4c^5-37908\,AB^3b^3c^6+82134\,A^2B^2b^2c^7-79092\,A^3Bbc^8+28561\,A^4c^9}{b^{17}}\right)^{\frac{1}{4}}\log\left(b^{13}\left(-\frac{6561\,B^4b^4c^5-3790}{}\right.\right.}{}$$

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/360*(45*(b^4*c*x^7 + b^5*x^5)*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6
+ 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(1/4)*lo
```

```
g(b^13*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 -
79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(3/4) - (729*B^3*b^3*c^4 - 3159*A*
B^2*b^2*c^5 + 4563*A^2*B*b*c^6 - 2197*A^3*c^7)*sqrt(x)) + 45*(-I*b^4*c*x^7
- I*b^5*x^5)*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*
c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(1/4)*log(I*b^13*(-(6561*B^4
*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8
+ 28561*A^4*c^9)/b^17)^(3/4) - (729*B^3*b^3*c^4 - 3159*A*B^2*b^2*c^5 + 4563
*A^2*B*b*c^6 - 2197*A^3*c^7)*sqrt(x)) + 45*(I*b^4*c*x^7 + I*b^5*x^5)*(-(656
1*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b
*c^8 + 28561*A^4*c^9)/b^17)^(1/4)*log(-I*b^13*(-(6561*B^4*b^4*c^5 - 37908*A
*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b
^17)^(3/4) - (729*B^3*b^3*c^4 - 3159*A*B^2*b^2*c^5 + 4563*A^2*B*b*c^6 - 219
7*A^3*c^7)*sqrt(x)) - 45*(b^4*c*x^7 + b^5*x^5)*(-(6561*B^4*b^4*c^5 - 37908*
A*B^3*b^3*c^6 + 82134*A^2*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/
b^17)^(1/4)*log(-b^13*(-(6561*B^4*b^4*c^5 - 37908*A*B^3*b^3*c^6 + 82134*A^2
*B^2*b^2*c^7 - 79092*A^3*B*b*c^8 + 28561*A^4*c^9)/b^17)^(3/4) - (729*B^3*b^
3*c^4 - 3159*A*B^2*b^2*c^5 + 4563*A^2*B*b*c^6 - 2197*A^3*c^7)*sqrt(x)) - 4*
(45*(9*B*b*c^2 - 13*A*c^3)*x^6 + 36*(9*B*b^2*c - 13*A*b*c^2)*x^4 - 20*A*b^3
 - 4*(9*B*b^3 - 13*A*b^2*c)*x^2)*sqrt(x))/(b^4*c*x^7 + b^5*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**2+A)/x**(3/2)/(c*x**4+b*x**2)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx = \frac{45\,(9\,Bbc^2-13\,Ac^3)x^6+36\,(9\,Bb^2c-13\,Abc^2)x^4-20\,Ab^3-4\,(9\,Bb^3-13\,Ab^2c)x^2}{90\left(b^4cx^{\frac{13}{2}}+b^5x^{\frac{9}{2}}\right)} + \frac{(9\,Bbc^2-13\,Ac^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right.}{16\,b^4}$$

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/90*(45*(9*B*b*c^2 - 13*A*c^3)*x^6 + 36*(9*B*b^2*c - 13*A*b*c^2)*x^4 - 20*
A*b^3 - 4*(9*B*b^3 - 13*A*b^2*c)*x^2)/(b^4*c*x^(13/2) + b^5*x^(9/2)) + 1/16
*(9*B*b*c^2 - 13*A*c^3)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1
/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt
(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*s
qrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*lo
g(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4))
+ sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1
/4)*c^(3/4)))/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx = \frac{\sqrt{2}\left(9\left(bc^3\right)^{\frac{3}{4}}Bb-13\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^5c} + \frac{\sqrt{2}\left(9\left(bc^3\right)^{\frac{3}{4}}Bb-13\left(bc^3\right)^{\frac{3}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^5c} - \frac{\sqrt{2}\left(9\left(bc^3\right)^{\frac{3}{4}}Bb-13\left(bc^3\right)^{\frac{3}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^5c} + \frac{\sqrt{2}\left(9\left(bc^3\right)^{\frac{3}{4}}Bb-13\left(bc^3\right)^{\frac{3}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^5c} + \frac{Bbc^2x^{\frac{3}{2}}-Ac^3x^{\frac{3}{2}}}{2\left(cx^2+b\right)b^4} + \frac{2\left(90\,Bbcx^4-135\,Ac^2x^4-9\,Bb^2x^2+18\,Abcx^2-5\,Ab^2\right)}{45\,b^4x^{\frac{9}{2}}}$$

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(9*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^5*c) + 1/8*sqrt(2)*(9*(b
*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^
(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^5*c) - 1/16*sqrt(2)*(9*(b*c^3)^(3/4)*B*b
 - 13*(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(
b^5*c) + 1/16*sqrt(2)*(9*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*log(-sqr
t(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^5*c) + 1/2*(B*b*c^2*x^(3/2) -
A*c^3*x^(3/2))/((c*x^2 + b)*b^4) + 2/45*(90*B*b*c*x^4 - 135*A*c^2*x^4 - 9*B
*b^2*x^2 + 18*A*b*c*x^2 - 5*A*b^2)/(b^4*x^(9/2))
```

Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.43

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^2}\,dx = \frac{{(-c)}^{5/4}\,\mathrm{atan}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,\left(13\,A\,c-9\,B\,b\right)}{4\,b^{17/4}} - \frac{\frac{2\,A}{9\,b}-\frac{2\,x^2\,\left(13\,A\,c-9\,B\,b\right)}{45\,b^2}+\frac{c^2\,x^6\,\left(13\,A\,c-9\,B\,b\right)}{2\,b^4}+\frac{2\,c\,x^4\,\left(13\,A\,c-9\,B\,b\right)}{5\,b^3}}{b\,x^{9/2}+c\,x^{13/2}} - \frac{{(-c)}^{5/4}\,\mathrm{atanh}\left(\frac{{(-c)}^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,\left(13\,A\,c-9\,B\,b\right)}{4\,b^{17/4}}$$

```
[In] int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^2),x)
```

```
[Out] ((-c)^(5/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4))*(13*A*c - 9*B*b))/(4*b^(17/4
)) - ((2*A)/(9*b) - (2*x^2*(13*A*c - 9*B*b))/(45*b^2) + (c^2*x^6*(13*A*c -
9*B*b))/(2*b^4) + (2*c*x^4*(13*A*c - 9*B*b))/(5*b^3))/(b*x^(9/2) + c*x^(13/
2)) - ((-c)^(5/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4))*(13*A*c - 9*B*b))/(4*
b^(17/4))
```

3.207 $\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 343

$$\begin{aligned}\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx &= -\frac{9(13bB-5Ac)\sqrt{x}}{16c^4}+\frac{9(13bB-5Ac)x^{5/2}}{80bc^3}\\ &-\frac{(bB-Ac)x^{13/2}}{4bc\,(b+cx^2)^2}-\frac{(13bB-5Ac)x^{9/2}}{16bc^2\,(b+cx^2)}-\frac{9\sqrt[4]{b}(13bB-5Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}c^{17/4}}\\ &+\frac{9\sqrt[4]{b}(13bB-5Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}c^{17/4}}\\ &-\frac{9\sqrt[4]{b}(13bB-5Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}c^{17/4}}\\ &+\frac{9\sqrt[4]{b}(13bB-5Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}c^{17/4}}\end{aligned}$$

```
[Out] 9/80*(-5*A*c+13*B*b)*x^(5/2)/b/c^3-1/4*(-A*c+B*b)*x^(13/2)/b/c/(c*x^2+b)^2-
1/16*(-5*A*c+13*B*b)*x^(9/2)/b/c^2/(c*x^2+b)-9/64*b^(1/4)*(-5*A*c+13*B*b)*a
rctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(17/4)*2^(1/2)+9/64*b^(1/4)*(-5*
A*c+13*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/c^(17/4)*2^(1/2)-9/12
8*b^(1/4)*(-5*A*c+13*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1
/2))/c^(17/4)*2^(1/2)+9/128*b^(1/4)*(-5*A*c+13*B*b)*ln(b^(1/2)+x*c^(1/2)+b^
(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/c^(17/4)*2^(1/2)-9/16*(-5*A*c+13*B*b)*x^(1/2
)/c^4
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {1598, 468, 294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{9\sqrt[4]{b}(13bB-5Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}c^{17/4}} + \frac{9\sqrt[4]{b}(13bB-5Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}c^{17/4}} - \frac{9\sqrt[4]{b}(13bB-5Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{17/4}} + \frac{9\sqrt[4]{b}(13bB-5Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{17/4}} - \frac{9\sqrt{x}(13bB-5Ac)}{16c^4} + \frac{9x^{5/2}(13bB-5Ac)}{80bc^3} - \frac{x^{9/2}(13bB-5Ac)}{16bc^2(b+cx^2)} - \frac{x^{13/2}(bB-Ac)}{4bc(b+cx^2)^2}$$

```
[In] Int[(x^(23/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-9*(13*b*B - 5*A*c)*Sqrt[x])/(16*c^4) + (9*(13*b*B - 5*A*c)*x^(5/2))/(80*b
*c^3) - ((b*B - A*c)*x^(13/2))/(4*b*c*(b + c*x^2)^2) - ((13*b*B - 5*A*c)*x^
(9/2))/(16*b*c^2*(b + c*x^2)) - (9*b^(1/4)*(13*b*B - 5*A*c)*ArcTan[1 - (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*c^(17/4)) + (9*b^(1/4)*(13*b*B
- 5*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*c^(17/4
)) - (9*b^(1/4)*(13*b*B - 5*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt
[x] + Sqrt[c]*x])/(64*Sqrt[2]*c^(17/4)) + (9*b^(1/4)*(13*b*B - 5*A*c)*Log[S
qrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*c^(17/4)
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^{11/2}(A+Bx^2)}{(b+cx^2)^3}\,dx \\
&= -\frac{(bB-Ac)x^{13/2}}{4bc\,(b+cx^2)^2}+\frac{\left(\frac{13bB}{2}-\frac{5Ac}{2}\right)\int \frac{x^{11/2}}{(b+cx^2)^2}\,dx}{4bc} \\
&= -\frac{(bB-Ac)x^{13/2}}{4bc\,(b+cx^2)^2}-\frac{(13bB-5Ac)x^{9/2}}{16bc^2\,(b+cx^2)}+\frac{(9(13bB-5Ac))\int \frac{x^{7/2}}{b+cx^2}\,dx}{32bc^2} \\
&= \frac{9(13bB-5Ac)x^{5/2}}{80bc^3}-\frac{(bB-Ac)x^{13/2}}{4bc\,(b+cx^2)^2}-\frac{(13bB-5Ac)x^{9/2}}{16bc^2\,(b+cx^2)}-\frac{(9(13bB-5Ac))\int \frac{x^{3/2}}{b+cx^2}\,dx}{32c^3} \\
&= -\frac{9(13bB-5Ac)\sqrt{x}}{16c^4}+\frac{9(13bB-5Ac)x^{5/2}}{80bc^3}-\frac{(bB-Ac)x^{13/2}}{4bc\,(b+cx^2)^2} \\
&\quad -\frac{(13bB-5Ac)x^{9/2}}{16bc^2\,(b+cx^2)}+\frac{(9b(13bB-5Ac))\int \frac{1}{\sqrt{x}(b+cx^2)}\,dx}{32c^4} \\
&= -\frac{9(13bB-5Ac)\sqrt{x}}{16c^4}+\frac{9(13bB-5Ac)x^{5/2}}{80bc^3}-\frac{(bB-Ac)x^{13/2}}{4bc\,(b+cx^2)^2} \\
&\quad -\frac{(13bB-5Ac)x^{9/2}}{16bc^2\,(b+cx^2)}+\frac{(9b(13bB-5Ac))\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{16c^4}
\end{aligned}
$$

$$= -\frac{9(13bB-5Ac)\sqrt{x}}{16c^4}+\frac{9(13bB-5Ac)x^{5/2}}{80bc^3}-\frac{(bB-Ac)x^{13/2}}{4bc\left(b+cx^2\right)^2}-\frac{(13bB-5Ac)x^{9/2}}{16bc^2\left(b+cx^2\right)}+\frac{\left(9\sqrt{b}(13bB-5Ac)\right)\text{Subst}\left(\int\frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{32c^4}+\frac{\left(9\sqrt{b}(13bB-5Ac)\right)\text{Subst}\left(\int\frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{32c^4}$$

$$= -\frac{9(13bB-5Ac)\sqrt{x}}{16c^4}+\frac{9(13bB-5Ac)x^{5/2}}{80bc^3}-\frac{(bB-Ac)x^{13/2}}{4bc\left(b+cx^2\right)^2}-\frac{(13bB-5Ac)x^{9/2}}{16bc^2\left(b+cx^2\right)}+\frac{\left(9\sqrt{b}(13bB-5Ac)\right)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64c^{9/2}}+\frac{\left(9\sqrt{b}(13bB-5Ac)\right)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64c^{9/2}}-\frac{\left(9\sqrt[4]{b}(13bB-5Ac)\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{64\sqrt{2}c^{17/4}}-\frac{\left(9\sqrt[4]{b}(13bB-5Ac)\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{64\sqrt{2}c^{17/4}}$$

$$= -\frac{9(13bB-5Ac)\sqrt{x}}{16c^4}+\frac{9(13bB-5Ac)x^{5/2}}{80bc^3}-\frac{(bB-Ac)x^{13/2}}{4bc\left(b+cx^2\right)^2}-\frac{(13bB-5Ac)x^{9/2}}{16bc^2\left(b+cx^2\right)}-\frac{9\sqrt[4]{b}(13bB-5Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}c^{17/4}}+\frac{9\sqrt[4]{b}(13bB-5Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}c^{17/4}}+\frac{\left(9\sqrt[4]{b}(13bB-5Ac)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}c^{17/4}}-\frac{\left(9\sqrt[4]{b}(13bB-5Ac)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}c^{17/4}}$$

$$= -\frac{9(13bB-5Ac)\sqrt{x}}{16c^4} + \frac{9(13bB-5Ac)x^{5/2}}{80bc^3} - \frac{(bB-Ac)x^{13/2}}{4bc\,(b+cx^2)^2}$$
$$- \frac{(13bB-5Ac)x^{9/2}}{16bc^2\,(b+cx^2)} - \frac{9\sqrt[4]{b}(13bB-5Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}c^{17/4}}$$
$$+ \frac{9\sqrt[4]{b}(13bB-5Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}c^{17/4}}$$
$$- \frac{9\sqrt[4]{b}(13bB-5Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}c^{17/4}}$$
$$+ \frac{9\sqrt[4]{b}(13bB-5Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}c^{17/4}}$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.59

$$\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{4\sqrt[4]{c}\sqrt{x}(-585b^3B+bc^2x^2(405A-416Bx^2)+9b^2c(25A-117Bx^2)+32c^3x^4(5A+Bx^2))}{(b+cx^2)^2} - 45\sqrt{2}\sqrt[4]{b}(13bB-}{320c^{17/4}}$$

```
[In] Integrate[(x^(23/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*c^(1/4)*Sqrt[x]*(-585*b^3*B + b*c^2*x^2*(405*A - 416*B*x^2) + 9*b^2*c*(
25*A - 117*B*x^2) + 32*c^3*x^4*(5*A + B*x^2)))/(b + c*x^2)^2 - 45*Sqrt[2]*b
^(1/4)*(13*b*B - 5*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/
4)*Sqrt[x])] + 45*Sqrt[2]*b^(1/4)*(13*b*B - 5*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)
*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(320*c^(17/4))
```

Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.55

method	result
risch	$\frac{2(Bc\,x^2+5Ac-15Bb)\sqrt{x}}{5c^4}-\frac{b\left(\frac{2\left(-\frac{17}{32}A\,c^2+\frac{25}{32}Bbc\right)x^{\frac{5}{2}}-\frac{b(13Ac-21Bb)\sqrt{x}}{16}}{\left(c\,x^2+b\right)^2}+\frac{9(5Ac-13Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+}\right.\right.}{}\right.}{c^4}$
derivativedivides	$\frac{\frac{2Bc\,x^{\frac{5}{2}}}{5}+2Ac\sqrt{x}-6bB\sqrt{x}}{c^4}-\frac{2b\left(\frac{\left(-\frac{17}{32}A\,c^2+\frac{25}{32}Bbc\right)x^{\frac{5}{2}}-\frac{b(13Ac-21Bb)\sqrt{x}}{32}}{\left(c\,x^2+b\right)^2}+\frac{9(5Ac-13Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+}\right.\right.}{}\right.}{c^4}$
default	$\frac{\frac{2Bc\,x^{\frac{5}{2}}}{5}+2Ac\sqrt{x}-6bB\sqrt{x}}{c^4}-\frac{2b\left(\frac{\left(-\frac{17}{32}A\,c^2+\frac{25}{32}Bbc\right)x^{\frac{5}{2}}-\frac{b(13Ac-21Bb)\sqrt{x}}{32}}{\left(c\,x^2+b\right)^2}+\frac{9(5Ac-13Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+}\right.\right.}{}\right.}{c^4}$

```
[In] int(x^(23/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(B*c*x^2+5*A*c-15*B*b)*x^(1/2)/c^4-b/c^4*(2*((-17/32*A*c^2+25/32*B*b*c)
*x^(5/2)-1/32*b*(13*A*c-21*B*b)*x^(1/2))/(c*x^2+b)^2+9/128*(5*A*c-13*B*b)*(
1/c*b)^(1/4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/
(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(
1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 776, normalized size of antiderivative = 2.26

$$\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx =$$

$$-\frac{45\left(c^6x^4+2\,bc^5x^2+b^2c^4\right)\left(-\frac{28561\,B^4b^5-43940\,AB^3b^4c+25350\,A^2B^2b^3c^2-6500\,A^3Bb^2c^3+625\,A^4bc^4}{c^{17}}\right)^{\frac{1}{4}}\log\left(9\,c^4\left(-\frac{28561\,B^4b}{}\right.\right.}{}$$

```
[In] integrate(x^(23/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/320*(45*(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)*(-(28561*B^4*b^5 - 43940*A*B^3
*b^4*c + 25350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17)^
(1/4)*log(9*c^4*(-(28561*B^4*b^5 - 43940*A*B^3*b^4*c + 25350*A^2*B^2*b^3*c^
```

```
2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17)^(1/4) - 9*(13*B*b - 5*A*c)*sq
rt(x)) + 45*(I*c^6*x^4 + 2*I*b*c^5*x^2 + I*b^2*c^4)*(-(28561*B^4*b^5 - 4394
0*A*B^3*b^4*c + 25350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)
/c^17)^(1/4)*log(9*I*c^4*(-(28561*B^4*b^5 - 43940*A*B^3*b^4*c + 25350*A^2*B
^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17)^(1/4) - 9*(13*B*b -
5*A*c)*sqrt(x)) + 45*(-I*c^6*x^4 - 2*I*b*c^5*x^2 - I*b^2*c^4)*(-(28561*B^4*
b^5 - 43940*A*B^3*b^4*c + 25350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*
A^4*b*c^4)/c^17)^(1/4)*log(-9*I*c^4*(-(28561*B^4*b^5 - 43940*A*B^3*b^4*c +
25350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17)^(1/4) - 9
*(13*B*b - 5*A*c)*sqrt(x)) - 45*(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)*(-(28561*
B^4*b^5 - 43940*A*B^3*b^4*c + 25350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 +
625*A^4*b*c^4)/c^17)^(1/4)*log(-9*c^4*(-(28561*B^4*b^5 - 43940*A*B^3*b^4*c
+ 25350*A^2*B^2*b^3*c^2 - 6500*A^3*B*b^2*c^3 + 625*A^4*b*c^4)/c^17)^(1/4) -
 9*(13*B*b - 5*A*c)*sqrt(x)) - 4*(32*B*c^3*x^6 - 32*(13*B*b*c^2 - 5*A*c^3)*
x^4 - 585*B*b^3 + 225*A*b^2*c - 81*(13*B*b^2*c - 5*A*b*c^2)*x^2)*sqrt(x))/(
c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**(23/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.89

$$\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{(25\,Bb^2c-17\,Abc^2)x^{\frac{5}{2}}+(21\,Bb^3-13\,Ab^2c)\sqrt{x}}{16\,(c^6x^4+2\,bc^5x^2+b^2c^4)} + \frac{9\left(\frac{2\sqrt{2}(13\,Bb-5\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{2\sqrt{2}(13\,Bb-5\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}} + \frac{\sqrt{2}(13\,Bb-5\,Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right.}{128\,c^4} + \frac{2\left(Bcx^{\frac{5}{2}}-5\,(3\,Bb-Ac)\sqrt{x}\right)}{5\,c^4}$$

```
[In] integrate(x^(23/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((25*B*b^2*c - 17*A*b*c^2)*x^(5/2) + (21*B*b^3 - 13*A*b^2*c)*sqrt(x))
/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4) + 9/128*(2*sqrt(2)*(13*B*b - 5*A*c)*arct
an(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*s
qrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(13*B*b - 5*A*c)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*s
qrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(13*B*b - 5*A*c)*log(sqr
t(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqr
t(2)*(13*B*b - 5*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sq
rt(b))/(b^(3/4)*c^(1/4)))*b/c^4 + 2/5*(B*c*x^(5/2) - 5*(3*B*b - A*c)*sqrt(x
))/c^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.94

$$\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{9\sqrt{2}\Big(13\,(bc^3)^{\frac{1}{4}}\,Bb-5\,(bc^3)^{\frac{1}{4}}\,Ac\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,c^5} + \frac{9\sqrt{2}\Big(13\,(bc^3)^{\frac{1}{4}}\,Bb-5\,(bc^3)^{\frac{1}{4}}\,Ac\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,c^5} + \frac{9\sqrt{2}\Big(13\,(bc^3)^{\frac{1}{4}}\,Bb-5\,(bc^3)^{\frac{1}{4}}\,Ac\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,c^5} - \frac{9\sqrt{2}\Big(13\,(bc^3)^{\frac{1}{4}}\,Bb-5\,(bc^3)^{\frac{1}{4}}\,Ac\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,c^5} - \frac{25\,Bb^2cx^{\frac{5}{2}}-17\,Abc^2x^{\frac{5}{2}}+21\,Bb^3\sqrt{x}-13\,Ab^2c\sqrt{x}}{16\,(cx^2+b)^2c^4} + \frac{2\left(Bc^{12}x^{\frac{5}{2}}-15\,Bbc^{11}\sqrt{x}+5\,Ac^{12}\sqrt{x}\right)}{5\,c^{15}}$$

```
[In] integrate(x^(23/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 9/64*sqrt(2)*(13*(b*c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^5 + 9/64*sqrt(2)*(13*(b*
c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1
/4) - 2*sqrt(x))/(b/c)^(1/4))/c^5 + 9/128*sqrt(2)*(13*(b*c^3)^(1/4)*B*b - 5
*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 -
9/128*sqrt(2)*(13*(b*c^3)^(1/4)*B*b - 5*(b*c^3)^(1/4)*A*c)*log(-sqrt(2)*sqr
```

```
t(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 - 1/16*(25*B*b^2*c*x^(5/2) - 17*A*b*c
^2*x^(5/2) + 21*B*b^3*sqrt(x) - 13*A*b^2*c*sqrt(x))/((c*x^2 + b)^2*c^4) + 2
/5*(B*c^12*x^(5/2) - 15*B*b*c^11*sqrt(x) + 5*A*c^12*sqrt(x))/c^15
```

Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 865, normalized size of antiderivative = 2.52

$$\int \frac{x^{23/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] int((x^(23/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (x^(5/2)*((17*A*b*c^2)/16 - (25*B*b^2*c)/16) - x^(1/2)*((21*B*b^3)/16 - (13
*A*b^2*c)/16))/(b^2*c^4 + c^6*x^4 + 2*b*c^5*x^2) + x^(1/2)*((2*A)/c^3 - (6*
B*b)/c^4) + (2*B*x^(5/2))/(5*c^3) + ((-b)^(1/4)*atan((((-b)^(1/4)*((81*x^(1
/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) - (81*(-b)^(1/
4)*(5*A*c - 13*B*b)*(13*B*b^3 - 5*A*b^2*c))/(64*c^(21/4)))*(5*A*c - 13*B*b)
*9i)/(64*c^(17/4)) + ((-b)^(1/4)*((81*x^(1/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2
 - 130*A*B*b^3*c))/(64*c^5) + (81*(-b)^(1/4)*(5*A*c - 13*B*b)*(13*B*b^3 - 5
*A*b^2*c))/(64*c^(21/4)))*(5*A*c - 13*B*b)*9i)/(64*c^(17/4)))/((9*(-b)^(1/4
)*((81*x^(1/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) - (
81*(-b)^(1/4)*(5*A*c - 13*B*b)*(13*B*b^3 - 5*A*b^2*c))/(64*c^(21/4)))*(5*A*
c - 13*B*b))/(64*c^(17/4)) - (9*(-b)^(1/4)*((81*x^(1/2)*(169*B^2*b^4 + 25*A
^2*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) + (81*(-b)^(1/4)*(5*A*c - 13*B*b)*(13
*B*b^3 - 5*A*b^2*c))/(64*c^(21/4)))*(5*A*c - 13*B*b))/(64*c^(17/4))))*(5*A*
c - 13*B*b)*9i)/(32*c^(17/4)) + (9*(-b)^(1/4)*atan(((9*(-b)^(1/4)*((81*x^(1
/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) - ((-b)^(1/4)*
(5*A*c - 13*B*b)*(13*B*b^3 - 5*A*b^2*c)*81i)/(64*c^(21/4)))*(5*A*c - 13*B*b
))/(64*c^(17/4)) + (9*(-b)^(1/4)*((81*x^(1/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2
 - 130*A*B*b^3*c))/(64*c^5) + ((-b)^(1/4)*(5*A*c - 13*B*b)*(13*B*b^3 - 5*A*
b^2*c)*81i)/(64*c^(21/4)))*(5*A*c - 13*B*b))/(64*c^(17/4)))/(((-b)^(1/4)*((
81*x^(1/2)*(169*B^2*b^4 + 25*A^2*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) - ((-b)
^(1/4)*(5*A*c - 13*B*b)*(13*B*b^3 - 5*A*b^2*c)*81i)/(64*c^(21/4)))*(5*A*c -
 13*B*b)*9i)/(64*c^(17/4)) - ((-b)^(1/4)*((81*x^(1/2)*(169*B^2*b^4 + 25*A^2
*b^2*c^2 - 130*A*B*b^3*c))/(64*c^5) + ((-b)^(1/4)*(5*A*c - 13*B*b)*(13*B*b^
3 - 5*A*b^2*c)*81i)/(64*c^(21/4)))*(5*A*c - 13*B*b)*9i)/(64*c^(17/4))))*(5*
A*c - 13*B*b))/(32*c^(17/4))
```

3.208 $\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 322

$$\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{7(11bB-3Ac)x^{3/2}}{48bc^3} - \frac{(bB-Ac)x^{11/2}}{4bc\,(b+cx^2)^2} - \frac{(11bB-3Ac)x^{7/2}}{16bc^2\,(b+cx^2)}$$
$$+ \frac{7(11bB-3Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}c^{15/4}} - \frac{7(11bB-3Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}c^{15/4}}$$
$$- \frac{7(11bB-3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{15/4}}$$
$$+ \frac{7(11bB-3Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{15/4}}$$

```
[Out] 7/48*(-3*A*c+11*B*b)*x^(3/2)/b/c^3-1/4*(-A*c+B*b)*x^(11/2)/b/c/(c*x^2+b)^2-
1/16*(-3*A*c+11*B*b)*x^(7/2)/b/c^2/(c*x^2+b)+7/64*(-3*A*c+11*B*b)*arctan(1-
c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/c^(15/4)*2^(1/2)-7/64*(-3*A*c+11*B
*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1/4)/c^(15/4)*2^(1/2)-7/12
8*(-3*A*c+11*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(
1/4)/c^(15/4)*2^(1/2)+7/128*(-3*A*c+11*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^
(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/c^(15/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1598, 468, 294, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{7(11bB-3Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}c^{15/4}} - \frac{7(11bB-3Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\sqrt[4]{b}c^{15/4}} - \frac{7(11bB-3Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{15/4}} + \frac{7(11bB-3Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{15/4}} + \frac{7x^{3/2}(11bB-3Ac)}{48bc^3} - \frac{x^{7/2}(11bB-3Ac)}{16bc^2(b+cx^2)} - \frac{x^{11/2}(bB-Ac)}{4bc(b+cx^2)^2}$$

```
[In] Int[(x^(21/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (7*(11*b*B - 3*A*c)*x^(3/2))/(48*b*c^3) - ((b*B - A*c)*x^(11/2))/(4*b*c*(b
+ c*x^2)^2) - ((11*b*B - 3*A*c)*x^(7/2))/(16*b*c^2*(b + c*x^2)) + (7*(11*b*
B - 3*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(1/
4)*c^(15/4)) - (7*(11*b*B - 3*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(
1/4)])/(32*Sqrt[2]*b^(1/4)*c^(15/4)) - (7*(11*b*B - 3*A*c)*Log[Sqrt[b] - Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(1/4)*c^(15/4)) +
 (7*(11*b*B - 3*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x])/(64*Sqrt[2]*b^(1/4)*c^(15/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{x^{9/2}(A+Bx^2)}{(b+cx^2)^3}\,dx$$

$$= -\frac{(bB-Ac)x^{11/2}}{4bc(b+cx^2)^2} + \frac{\left(\frac{11bB}{2}-\frac{3Ac}{2}\right)\int \frac{x^{9/2}}{(b+cx^2)^2}\,dx}{4bc}$$

$$= -\frac{(bB-Ac)x^{11/2}}{4bc(b+cx^2)^2} - \frac{(11bB-3Ac)x^{7/2}}{16bc^2(b+cx^2)} + \frac{(7(11bB-3Ac))\int \frac{x^{5/2}}{b+cx^2}\,dx}{32bc^2}$$

$$= \frac{7(11bB-3Ac)x^{3/2}}{48bc^3} - \frac{(bB-Ac)x^{11/2}}{4bc(b+cx^2)^2} - \frac{(11bB-3Ac)x^{7/2}}{16bc^2(b+cx^2)} - \frac{(7(11bB-3Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32c^3}$$

$$= \frac{7(11bB-3Ac)x^{3/2}}{48bc^3} - \frac{(bB-Ac)x^{11/2}}{4bc(b+cx^2)^2} - \frac{(11bB-3Ac)x^{7/2}}{16bc^2(b+cx^2)} - \frac{(7(11bB-3Ac))\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16c^3}$$

$$= \frac{7(11bB-3Ac)x^{3/2}}{48bc^3} - \frac{(bB-Ac)x^{11/2}}{4bc(b+cx^2)^2} - \frac{(11bB-3Ac)x^{7/2}}{16bc^2(b+cx^2)} + \frac{(7(11bB-3Ac))\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32c^{7/2}} - \frac{(7(11bB-3Ac))\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32c^{7/2}}$$

$$= \frac{7(11bB-3Ac)x^{3/2}}{48bc^3} - \frac{(bB-Ac)x^{11/2}}{4bc\left(b+cx^2\right)^2} - \frac{(11bB-3Ac)x^{7/2}}{16bc^2\left(b+cx^2\right)} - \frac{(7(11bB-3Ac))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64c^4} - \frac{(7(11bB-3Ac))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64c^4} - \frac{(7(11bB-3Ac))\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{64\sqrt{2}\sqrt[4]{b}c^{15/4}} - \frac{(7(11bB-3Ac))\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{64\sqrt{2}\sqrt[4]{b}c^{15/4}}$$

$$= \frac{7(11bB-3Ac)x^{3/2}}{48bc^3} - \frac{(bB-Ac)x^{11/2}}{4bc\left(b+cx^2\right)^2} - \frac{(11bB-3Ac)x^{7/2}}{16bc^2\left(b+cx^2\right)} - \frac{7(11bB-3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{15/4}} + \frac{7(11bB-3Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{15/4}} - \frac{(7(11bB-3Ac))\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}c^{15/4}} + \frac{(7(11bB-3Ac))\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}c^{15/4}}$$

$$= \frac{7(11bB-3Ac)x^{3/2}}{48bc^3} - \frac{(bB-Ac)x^{11/2}}{4bc(b+cx^2)^2} - \frac{(11bB-3Ac)x^{7/2}}{16bc^2(b+cx^2)} + \frac{7(11bB-3Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}c^{15/4}} - \frac{7(11bB-3Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}c^{15/4}} - \frac{7(11bB-3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{15/4}} + \frac{7(11bB-3Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{15/4}}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.57

$$\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{4c^{3/4}x^{3/2}(77b^2B-21Abc+121bBcx^2-33Ac^2x^2+32Bc^2x^4)}{(b+cx^2)^2} + \frac{21\sqrt{2}(11bB-3Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt[4]{b}} + \frac{21\cdots}{\cdots}}{192c^{15/4}}$$

```
[In] Integrate[(x^(21/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*c^(3/4)*x^(3/2)*(77*b^2*B - 21*A*b*c + 121*b*B*c*x^2 - 33*A*c^2*x^2 + 3
2*B*c^2*x^4))/(b + c*x^2)^2 + (21*Sqrt[2]*(11*b*B - 3*A*c)*ArcTan[(Sqrt[b]
- Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/b^(1/4) + (21*Sqrt[2]*(11*
b*B - 3*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x
)])/b^(1/4))/(192*c^(15/4))
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.54

method	result
derivativedivides	$\frac{2B x^{\frac{3}{2}}}{3c^{3}}+\frac{\frac{2\left(-\frac{c(11Ac-19Bb)x^{\frac{7}{2}}}{32}+\left(-\frac{7}{32}Abc+\frac{15}{32}B\,b^{2}\right)x^{\frac{3}{2}}\right)}{\left(c\,x^{2}+b\right)^{2}}+\frac{\left(\frac{21Ac}{32}-\frac{77Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{}}{\left(\frac{b}{c}\right)^{}}\right.\right.}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{c^{3}}$
default	$\frac{2B x^{\frac{3}{2}}}{3c^{3}}+\frac{\frac{2\left(-\frac{c(11Ac-19Bb)x^{\frac{7}{2}}}{32}+\left(-\frac{7}{32}Abc+\frac{15}{32}B\,b^{2}\right)x^{\frac{3}{2}}\right)}{\left(c\,x^{2}+b\right)^{2}}+\frac{\left(\frac{21Ac}{32}-\frac{77Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{}}{\left(\frac{b}{c}\right)^{}}\right.\right.}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{c^{3}}$
risch	$\frac{2B x^{\frac{3}{2}}}{3c^{3}}+\frac{\frac{-\frac{c(11Ac-19Bb)x^{\frac{7}{2}}}{16}+2\left(-\frac{7}{32}Abc+\frac{15}{32}B\,b^{2}\right)x^{\frac{3}{2}}}{\left(c\,x^{2}+b\right)^{2}}+\frac{\left(\frac{21Ac}{32}-\frac{77Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+\right.\right.}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{c^{3}}$

```
[In] int(x^(21/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*B/c^3*x^(3/2)+2/c^3*((-1/32*c*(11*A*c-19*B*b)*x^(7/2)+(-7/32*A*b*c+15/3
2*B*b^2)*x^(3/2))/(c*x^2+b)^2+1/8*(21/32*A*c-77/32*B*b)/c/(1/c*b)^(1/4)*2^(
1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x
^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*
arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 872, normalized size of antiderivative = 2.71

$$\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx=\frac{21\left(c^5x^4+2\,bc^4x^2+b^2c^3\right)\left(-\frac{14641\,B^4b^4-15972\,AB^3b^3c+6534\,A^2B^2b^2c^2-1188\,A^3Bbc^3+81\,A^4c^4}{bc^{15}}\right)^{\frac{1}{4}}}{}$$

```
[In] integrate(x^(21/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/192*(21*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-(14641*B^4*b^4 - 15972*A*B^3*
b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(1/
4)*log(343*b*c^11*(-(14641*B^4*b^4 - 15972*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c
^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(3/4) - 343*(1331*B^3*b^3 - 1
089*A*B^2*b^2*c + 297*A^2*B*b*c^2 - 27*A^3*c^3)*sqrt(x)) - 21*(I*c^5*x^4 +
2*I*b*c^4*x^2 + I*b^2*c^3)*(-(14641*B^4*b^4 - 15972*A*B^3*b^3*c + 6534*A^2*
B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(1/4)*log(343*I*b*c^
11*(-(14641*B^4*b^4 - 15972*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B
```

```
*b*c^3 + 81*A^4*c^4)/(b*c^15))^(3/4) - 343*(1331*B^3*b^3 - 1089*A*B^2*b^2*c
 + 297*A^2*B*b*c^2 - 27*A^3*c^3)*sqrt(x)) - 21*(-I*c^5*x^4 - 2*I*b*c^4*x^2
- I*b^2*c^3)*(-(14641*B^4*b^4 - 15972*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 -
1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(1/4)*log(-343*I*b*c^11*(-(14641*B
^4*b^4 - 15972*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81*A
^4*c^4)/(b*c^15))^(3/4) - 343*(1331*B^3*b^3 - 1089*A*B^2*b^2*c + 297*A^2*B*
b*c^2 - 27*A^3*c^3)*sqrt(x)) - 21*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-(1464
1*B^4*b^4 - 15972*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 8
1*A^4*c^4)/(b*c^15))^(1/4)*log(-343*b*c^11*(-(14641*B^4*b^4 - 15972*A*B^3*b
^3*c + 6534*A^2*B^2*b^2*c^2 - 1188*A^3*B*b*c^3 + 81*A^4*c^4)/(b*c^15))^(3/4
) - 343*(1331*B^3*b^3 - 1089*A*B^2*b^2*c + 297*A^2*B*b*c^2 - 27*A^3*c^3)*sq
rt(x)) + 4*(32*B*c^2*x^5 + 11*(11*B*b*c - 3*A*c^2)*x^3 + 7*(11*B*b^2 - 3*A*
b*c)*x)*sqrt(x))/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**(21/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.80

$$\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(19\,Bbc-11\,Ac^2)x^{\frac{7}{2}}+(15\,Bb^2-7\,Abc)x^{\frac{3}{2}}}{16\,(c^5x^4+2\,bc^4x^2+b^2c^3)}+\frac{2\,Bx^{\frac{3}{2}}}{3\,c^3} - \frac{7\,(11\,Bb-3\,Ac)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right.}{128\,c^3}$$

```
[In] integrate(x^(21/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
[Out] 1/16*((19*B*b*c - 11*A*c^2)*x^(7/2) + (15*B*b^2 - 7*A*b*c)*x^(3/2))/(c^5*x^
4 + 2*b*c^4*x^2 + b^2*c^3) + 2/3*B*x^(3/2)/c^3 - 7/128*(11*B*b - 3*A*c)*(2*
sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sq
rt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/
```

```
2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c
)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*s
qrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1
/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/c^3
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.94

$$\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{2\,Bx^{\frac{3}{2}}}{3\,c^3} + \frac{19\,Bbcx^{\frac{7}{2}} - 11\,Ac^2x^{\frac{7}{2}} + 15\,Bb^2x^{\frac{3}{2}} - 7\,Abcx^{\frac{3}{2}}}{16\,(cx^2+b)^2c^3} - \frac{7\sqrt{2}\Big(11\,(bc^3)^{\frac{3}{4}}\,Bb - 3\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^6} - \frac{7\sqrt{2}\Big(11\,(bc^3)^{\frac{3}{4}}\,Bb - 3\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^6} + \frac{7\sqrt{2}\Big(11\,(bc^3)^{\frac{3}{4}}\,Bb - 3\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{128\,bc^6} - \frac{7\sqrt{2}\Big(11\,(bc^3)^{\frac{3}{4}}\,Bb - 3\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{128\,bc^6}$$

```
[In] integrate(x^(21/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/c^3 + 1/16*(19*B*b*c*x^(7/2) - 11*A*c^2*x^(7/2) + 15*B*b^2*x^
(3/2) - 7*A*b*c*x^(3/2))/((c*x^2 + b)^2*c^3) - 7/64*sqrt(2)*(11*(b*c^3)^(3/
4)*B*b - 3*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*s
qrt(x))/(b/c)^(1/4))/(b*c^6) - 7/64*sqrt(2)*(11*(b*c^3)^(3/4)*B*b - 3*(b*c^
3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(
1/4))/(b*c^6) + 7/128*sqrt(2)*(11*(b*c^3)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*
log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^6) - 7/128*sqrt(2)*(1
1*(b*c^3)^(3/4)*B*b - 3*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4)
 + x + sqrt(b/c))/(b*c^6)
```

Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.43

$$\int \frac{x^{21/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{x^{3/2}\left(\frac{15\,B\,b^2}{16}-\frac{7\,A\,b\,c}{16}\right)-x^{7/2}\left(\frac{11\,A\,c^2}{16}-\frac{19\,B\,b\,c}{16}\right)}{b^2\,c^3+2\,b\,c^4\,x^2+c^5\,x^4}+\frac{2\,B\,x^{3/2}}{3\,c^3} + \frac{7\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(3\,A\,c-11\,B\,b)}{32\,(-b)^{1/4}\,c^{15/4}}+\frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}\,1\mathrm{i}}{(-b)^{1/4}}\right)\,(3\,A\,c-11\,B\,b)\,7\mathrm{i}}{32\,(-b)^{1/4}\,c^{15/4}}$$

```
[In] int((x^(21/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (x^(3/2)*((15*B*b^2)/16 - (7*A*b*c)/16) - x^(7/2)*((11*A*c^2)/16 - (19*B*b*
c)/16))/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^2) + (2*B*x^(3/2))/(3*c^3) + (7*atan
((c^(1/4)*x^(1/2))/(-b)^(1/4))*(3*A*c - 11*B*b))/(32*(-b)^(1/4)*c^(15/4)) +
 (atan((c^(1/4)*x^(1/2)*1i)/(-b)^(1/4))*(3*A*c - 11*B*b)*7i)/(32*(-b)^(1/4)
*c^(15/4))
```

3.209 $\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$

Optimal result

Integrand size = 26, antiderivative size = 322

$$\begin{aligned}\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx &= \frac{5(9bB-Ac)\sqrt{x}}{16bc^3} - \frac{(bB-Ac)x^{9/2}}{4bc\,(b+cx^2)^2} - \frac{(9bB-Ac)x^{5/2}}{16bc^2\,(b+cx^2)} \\ &+ \frac{5(9bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{3/4}c^{13/4}} - \frac{5(9bB-Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{3/4}c^{13/4}} \\ &+ \frac{5(9bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{13/4}} \\ &- \frac{5(9bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{13/4}}\end{aligned}$$

```
[Out] -1/4*(-A*c+B*b)*x^(9/2)/b/c/(c*x^2+b)^2-1/16*(-A*c+9*B*b)*x^(5/2)/b/c^2/(c*
x^2+b)+5/64*(-A*c+9*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(3/4)/
c^(13/4)*2^(1/2)-5/64*(-A*c+9*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4)
)/b^(3/4)/c^(13/4)*2^(1/2)+5/128*(-A*c+9*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*
c^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(13/4)*2^(1/2)-5/128*(-A*c+9*B*b)*ln(b^(
1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/c^(13/4)*2^(1/2)+5/
16*(-A*c+9*B*b)*x^(1/2)/b/c^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1598, 468, 294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{5(9bB-Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{3/4}c^{13/4}} - \frac{5(9bB-Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{3/4}c^{13/4}} + \frac{5(9bB-Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{13/4}} - \frac{5(9bB-Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{13/4}} + \frac{5\sqrt{x}(9bB-Ac)}{16bc^3} - \frac{x^{5/2}(9bB-Ac)}{16bc^2(b+cx^2)} - \frac{x^{9/2}(bB-Ac)}{4bc(b+cx^2)^2}$$

```
[In] Int[(x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (5*(9*b*B - A*c)*Sqrt[x])/(16*b*c^3) - ((b*B - A*c)*x^(9/2))/(4*b*c*(b + c*
x^2)^2) - ((9*b*B - A*c)*x^(5/2))/(16*b*c^2*(b + c*x^2)) + (5*(9*b*B - A*c)
*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(3/4)*c^(13/4
)) - (5*(9*b*B - A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sq
rt[2]*b^(3/4)*c^(13/4)) + (5*(9*b*B - A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(3/4)*c^(13/4)) - (5*(9*b*B - A*c
)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b
^(3/4)*c^(13/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{x^{7/2}(A+Bx^2)}{(b+cx^2)^3}\,dx$$

$$= -\frac{(bB-Ac)x^{9/2}}{4bc\,(b+cx^2)^2} + \frac{\left(\frac{9bB}{2}-\frac{Ac}{2}\right)\int \frac{x^{7/2}}{(b+cx^2)^2}\,dx}{4bc}$$

$$= -\frac{(bB-Ac)x^{9/2}}{4bc\,(b+cx^2)^2} - \frac{(9bB-Ac)x^{5/2}}{16bc^2\,(b+cx^2)} + \frac{(5(9bB-Ac))\int \frac{x^{3/2}}{b+cx^2}\,dx}{32bc^2}$$

$$= \frac{5(9bB-Ac)\sqrt{x}}{16bc^3} - \frac{(bB-Ac)x^{9/2}}{4bc\,(b+cx^2)^2} - \frac{(9bB-Ac)x^{5/2}}{16bc^2\,(b+cx^2)} - \frac{(5(9bB-Ac))\int \frac{1}{\sqrt{x}(b+cx^2)}\,dx}{32c^3}$$

$$= \frac{5(9bB-Ac)\sqrt{x}}{16bc^3} - \frac{(bB-Ac)x^{9/2}}{4bc\,(b+cx^2)^2} - \frac{(9bB-Ac)x^{5/2}}{16bc^2\,(b+cx^2)} - \frac{(5(9bB-Ac))\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16c^3}$$

$$= \frac{5(9bB-Ac)\sqrt{x}}{16bc^3} - \frac{(bB-Ac)x^{9/2}}{4bc\,(b+cx^2)^2} - \frac{(9bB-Ac)x^{5/2}}{16bc^2\,(b+cx^2)} - \frac{(5(9bB-Ac))\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32\sqrt{b}c^3} - \frac{(5(9bB-Ac))\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32\sqrt{b}c^3}$$

$$
\begin{aligned}
&= \frac{5(9bB-Ac)\sqrt{x}}{16bc^3} - \frac{(bB-Ac)x^{9/2}}{4bc\left(b+cx^2\right)^2} - \frac{(9bB-Ac)x^{5/2}}{16bc^2\left(b+cx^2\right)} \\
&\quad - \frac{(5(9bB-Ac))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64\sqrt{b}c^{7/2}} \\
&\quad - \frac{(5(9bB-Ac))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64\sqrt{b}c^{7/2}} \\
&\quad + \frac{(5(9bB-Ac))\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{64\sqrt{2}b^{3/4}c^{13/4}} \\
&\quad + \frac{(5(9bB-Ac))\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{64\sqrt{2}b^{3/4}c^{13/4}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{5(9bB-Ac)\sqrt{x}}{16bc^3} - \frac{(bB-Ac)x^{9/2}}{4bc\left(b+cx^2\right)^2} - \frac{(9bB-Ac)x^{5/2}}{16bc^2\left(b+cx^2\right)} \\
&\quad + \frac{5(9bB-Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{13/4}} \\
&\quad - \frac{5(9bB-Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{13/4}} \\
&\quad - \frac{(5(9bB-Ac))\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{3/4}c^{13/4}} \\
&\quad + \frac{(5(9bB-Ac))\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{3/4}c^{13/4}}
\end{aligned}
$$

$$= \frac{5(9bB - Ac)\sqrt{x}}{16bc^3} - \frac{(bB - Ac)x^{9/2}}{4bc\,(b + cx^2)^2} - \frac{(9bB - Ac)x^{5/2}}{16bc^2\,(b + cx^2)} + \frac{5(9bB - Ac)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{3/4}c^{13/4}} - \frac{5(9bB - Ac)\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{3/4}c^{13/4}} + \frac{5(9bB - Ac)\log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{13/4}} - \frac{5(9bB - Ac)\log\left(\sqrt{b} + \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{13/4}}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.57

$$\int \frac{x^{19/2}(A + Bx^2)}{(bx^2 + cx^4)^3}\,dx = \frac{\frac{4\sqrt[4]{c}\sqrt{x}(45b^2B - 5Abc + 81bBcx^2 - 9Ac^2x^2 + 32Bc^2x^4)}{(b+cx^2)^2} + \frac{5\sqrt{2}(9bB - Ac)\arctan\left(\frac{\sqrt{b} - \sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{b^{3/4}} - \frac{5\sqrt{2}(9bB-}{}}{64c^{13/4}}$$

```
[In] Integrate[(x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*c^(1/4)*Sqrt[x]*(45*b^2*B - 5*A*b*c + 81*b*B*c*x^2 - 9*A*c^2*x^2 + 32*B
*c^2*x^4))/(b + c*x^2)^2 + (5*Sqrt[2]*(9*b*B - A*c)*ArcTan[(Sqrt[b] - Sqrt[
c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/b^(3/4) - (5*Sqrt[2]*(9*b*B - A*c
)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/b^(3/4)
)/(64*c^(13/4))
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.53

method	result
derivativedivides	$\frac{2B\sqrt{x}}{c^3}+\frac{\frac{2\left(\left(-\frac{9}{32}Ac^2+\frac{17}{32}Bbc\right)x^{\frac{5}{2}}-\frac{b(5Ac-13Bb)\sqrt{x}}{32}\right)}{(cx^2+b)^2}+\frac{5(Ac-9Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{128b}}{c^3}$
default	$\frac{2B\sqrt{x}}{c^3}+\frac{\frac{2\left(\left(-\frac{9}{32}Ac^2+\frac{17}{32}Bbc\right)x^{\frac{5}{2}}-\frac{b(5Ac-13Bb)\sqrt{x}}{32}\right)}{(cx^2+b)^2}+\frac{5(Ac-9Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{128b}}{c^3}$
risch	$\frac{2B\sqrt{x}}{c^3}+\frac{\frac{2\left(-\frac{9}{32}Ac^2+\frac{17}{32}Bbc\right)x^{\frac{5}{2}}-\frac{b(5Ac-13Bb)\sqrt{x}}{16}}{(cx^2+b)^2}+\frac{5(Ac-9Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right.\right.}{128b}}{c^3}$

```
[In] int(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*B/c^3*x^(1/2)+2/c^3*(((-9/32*A*c^2+17/32*B*b*c)*x^(5/2)-1/32*b*(5*A*c-13*
B*b)*x^(1/2))/(c*x^2+b)^2+5/256*(A*c-9*B*b)*(1/c*b)^(1/4)/b*2^(1/2)*(ln((x+
(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/
2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/
2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 748, normalized size of antiderivative = 2.32

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx=\frac{5\,(c^5x^4+2\,bc^4x^2+b^2c^3)\left(-\frac{6561\,B^4b^4-2916\,AB^3b^3c+486\,A^2B^2b^2c^2-36\,A^3Bbc^3+A^4c^4}{b^3c^{13}}\right)^{\frac{1}{4}}\log\left(5\,b\right.}{}$$

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(5*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-(6561*B^4*b^4 - 2916*A*B^3*b^3*
c + 486*A^2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(1/4)*log(5
*b*c^3*(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 36*A^3*B*
b*c^3 + A^4*c^4)/(b^3*c^13))^(1/4) - 5*(9*B*b - A*c)*sqrt(x)) - 5*(-I*c^5*x
^4 - 2*I*b*c^4*x^2 - I*b^2*c^3)*(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^
2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(1/4)*log(5*I*b*c^3*(
-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 +
A^4*c^4)/(b^3*c^13))^(1/4) - 5*(9*B*b - A*c)*sqrt(x)) - 5*(I*c^5*x^4 + 2*I*
```

```
b*c^4*x^2 + I*b^2*c^3)*(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2
*c^2 - 36*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(1/4)*log(-5*I*b*c^3*(-(6561*B
^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 + A^4*c^4)
/(b^3*c^13))^(1/4) - 5*(9*B*b - A*c)*sqrt(x)) - 5*(c^5*x^4 + 2*b*c^4*x^2 +
b^2*c^3)*(-(6561*B^4*b^4 - 2916*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 36*A^3*
B*b*c^3 + A^4*c^4)/(b^3*c^13))^(1/4)*log(-5*b*c^3*(-(6561*B^4*b^4 - 2916*A*
B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 36*A^3*B*b*c^3 + A^4*c^4)/(b^3*c^13))^(1/
4) - 5*(9*B*b - A*c)*sqrt(x)) + 4*(32*B*c^2*x^4 + 45*B*b^2 - 5*A*b*c + 9*(9
*B*b*c - A*c^2)*x^2)*sqrt(x))/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**(19/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.88

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(17\,Bbc-9\,Ac^2)x^{\frac{5}{2}}+(13\,Bb^2-5\,Abc)\sqrt{x}}{16\,(c^5x^4+2\,bc^4x^2+b^2c^3)}+\frac{2\,B\sqrt{x}}{c^3} - \frac{5\left(\frac{2\sqrt{2}(9\,Bb-Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(9\,Bb-Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}(9\,Bb-Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right.}{128\,c^3}$$

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
[Out] 1/16*((17*B*b*c - 9*A*c^2)*x^(5/2) + (13*B*b^2 - 5*A*b*c)*sqrt(x))/(c^5*x^4
 + 2*b*c^4*x^2 + b^2*c^3) + 2*B*sqrt(x)/c^3 - 5/128*(2*sqrt(2)*(9*B*b - A*c
)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqr
t(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(9*B*b - A*c)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b
)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(9*B*b - A*c)*log(sqr
t(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqr
t(2)*(9*B*b - A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(
b))/(b^(3/4)*c^(1/4)))/c^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.94

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{2\,B\sqrt{x}}{c^3} - \frac{5\,\sqrt{2}\left(9\,(bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^4} - \frac{5\,\sqrt{2}\left(9\,(bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^4} - \frac{5\,\sqrt{2}\left(9\,(bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,bc^4} + \frac{5\,\sqrt{2}\left(9\,(bc^3)^{\frac{1}{4}}\,Bb-(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,bc^4} + \frac{17\,Bbcx^{\frac{5}{2}}-9\,Ac^2x^{\frac{5}{2}}+13\,Bb^2\sqrt{x}-5\,Abc\sqrt{x}}{16\,(cx^2+b)^2c^3}$$

```
[In] integrate(x^(19/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/c^3 - 5/64*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) - 5
/64*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) - 5/128*sqrt(2)*(9*(b
*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
sqrt(b/c))/(b*c^4) + 5/128*sqrt(2)*(9*(b*c^3)^(1/4)*B*b - (b*c^3)^(1/4)*A*c
)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^4) + 1/16*(17*B*b*
c*x^(5/2) - 9*A*c^2*x^(5/2) + 13*B*b^2*sqrt(x) - 5*A*b*c*sqrt(x))/((c*x^2 +
 b)^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 760, normalized size of antiderivative = 2.36

$$\int \frac{x^{19/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\sqrt{x}\left(\frac{13\,B\,b^2}{16}-\frac{5\,A\,b\,c}{16}\right)-x^{5/2}\left(\frac{9\,A\,c^2}{16}-\frac{17\,B\,b\,c}{16}\right)}{b^2\,c^3+2\,b\,c^4\,x^2+c^5\,x^4}+\frac{2\,B\,\sqrt{x}}{c^3}$$

$$-\frac{\mathrm{atan}\left(\frac{\frac{(A\,c-9\,B\,b)\left(\frac{25\,\sqrt{x}\,\left(A^2\,c^2-18\,A\,B\,b\,c+81\,B^2\,b^2\right)}{64\,c^3}-\frac{5\,\left(45\,B\,b^2-5\,A\,b\,c\right)\,(A\,c-9\,B\,b)}{64\,(-b)^{3/4}\,c^{13/4}}\right)5\mathrm{i}}{64\,(-b)^{3/4}\,c^{13/4}}+\frac{(A\,c-9\,B\,b)\left(\frac{25\,\sqrt{x}\,\left(A^2\,c^2-18\,A\,B\,b\,c+81\,B^2\,b^2\right)}{64\,c^3}+\frac{5\,(45\,B\,b^2 \cdots}{64 \cdots}\right.}{64\,(-b)^{3/4}\,c^{13/4}}}{\frac{5\,(A\,c-9\,B\,b)\left(\frac{25\,\sqrt{x}\,\left(A^2\,c^2-18\,A\,B\,b\,c+81\,B^2\,b^2\right)}{64\,c^3}-\frac{5\,\left(45\,B\,b^2-5\,A\,b\,c\right)\,(A\,c-9\,B\,b)}{64\,(-b)^{3/4}\,c^{13/4}}\right)}{64\,(-b)^{3/4}\,c^{13/4}}-\frac{5\,(A\,c-9\,B\,b)\left(\frac{25\,\sqrt{x}\,\left(A^2\,c^2-18\,A\,B\,b\,c+81\,B^2\,b^2\right)}{64\,c^3}+\frac{5\,(45\,B\,b \cdots}{6 \cdots}\right.}{64\,(-b)^{3/4}\,c^{13/4}}}\right) \cdots}{32\,(-b)^{3/4}\,c^{13/4}}$$

$$-\frac{5\,\mathrm{atan}\left(\frac{\frac{5\,(A\,c-9\,B\,b)\left(\frac{25\,\sqrt{x}\,\left(A^2\,c^2-18\,A\,B\,b\,c+81\,B^2\,b^2\right)}{64\,c^3}-\frac{\left(45\,B\,b^2-5\,A\,b\,c\right)\,(A\,c-9\,B\,b)\,5\mathrm{i}}{64\,(-b)^{3/4}\,c^{13/4}}\right)}{64\,(-b)^{3/4}\,c^{13/4}}+\frac{5\,(A\,c-9\,B\,b)\left(\frac{25\,\sqrt{x}\,\left(A^2\,c^2-18\,A\,B\,b\,c+81\,B^2\,b^2\right)}{64\,c^3}+\frac{(45\,B \cdots}{\cdots}\right.}{64\,(-b)^{3/4}\,c^{13/4}}}{\frac{(A\,c-9\,B\,b)\left(\frac{25\,\sqrt{x}\,\left(A^2\,c^2-18\,A\,B\,b\,c+81\,B^2\,b^2\right)}{64\,c^3}-\frac{\left(45\,B\,b^2-5\,A\,b\,c\right)\,(A\,c-9\,B\,b)\,5\mathrm{i}}{64\,(-b)^{3/4}\,c^{13/4}}\right)5\mathrm{i}}{64\,(-b)^{3/4}\,c^{13/4}}-\frac{(A\,c-9\,B\,b)\left(\frac{25\,\sqrt{x}\,\left(A^2\,c^2-18\,A\,B\,b\,c+81\,B^2\,b^2\right)}{64\,c^3}+\frac{(45\,B\,b \cdots}{\cdots}\right.}{64\,(-b)^{3/4}\,c^{13/4}}}\right) \cdots}{32\,(-b)^{3/4}\,c^{13/4}}$$

```
[In] int((x^(19/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (x^(1/2)*((13*B*b^2)/16 - (5*A*b*c)/16) - x^(5/2)*((9*A*c^2)/16 - (17*B*b*c
)/16))/(b^2*c^3 + c^5*x^4 + 2*b*c^4*x^2) + (2*B*x^(1/2))/c^3 - (atan((((A*c
 - 9*B*b)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*b^2 - 18*A*B*b*c))/(64*c^3) - (5*(
45*B*b^2 - 5*A*b*c)*(A*c - 9*B*b))/(64*(-b)^(3/4)*c^(13/4)))*5i)/(64*(-b)^(
3/4)*c^(13/4)) + ((A*c - 9*B*b)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*b^2 - 18*A*B
*b*c))/(64*c^3) + (5*(45*B*b^2 - 5*A*b*c)*(A*c - 9*B*b))/(64*(-b)^(3/4)*c^(
13/4)))*5i)/(64*(-b)^(3/4)*c^(13/4)))/((5*(A*c - 9*B*b)*((25*x^(1/2)*(A^2*c
^2 + 81*B^2*b^2 - 18*A*B*b*c))/(64*c^3) - (5*(45*B*b^2 - 5*A*b*c)*(A*c - 9*
B*b))/(64*(-b)^(3/4)*c^(13/4))))/(64*(-b)^(3/4)*c^(13/4)) - (5*(A*c - 9*B*b
)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*b^2 - 18*A*B*b*c))/(64*c^3) + (5*(45*B*b^2
 - 5*A*b*c)*(A*c - 9*B*b))/(64*(-b)^(3/4)*c^(13/4))))/(64*(-b)^(3/4)*c^(13/
4))))*(A*c - 9*B*b)*5i)/(32*(-b)^(3/4)*c^(13/4)) - (5*atan(((5*(A*c - 9*B*b
)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*b^2 - 18*A*B*b*c))/(64*c^3) - ((45*B*b^2 -
 5*A*b*c)*(A*c - 9*B*b)*5i)/(64*(-b)^(3/4)*c^(13/4))))/(64*(-b)^(3/4)*c^(13
/4)) + (5*(A*c - 9*B*b)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*b^2 - 18*A*B*b*c))/(
64*c^3) + ((45*B*b^2 - 5*A*b*c)*(A*c - 9*B*b)*5i)/(64*(-b)^(3/4)*c^(13/4)))
)/(64*(-b)^(3/4)*c^(13/4)))/(((A*c - 9*B*b)*((25*x^(1/2)*(A^2*c^2 + 81*B^2*
b^2 - 18*A*B*b*c))/(64*c^3) - ((45*B*b^2 - 5*A*b*c)*(A*c - 9*B*b)*5i)/(64*(
-b)^(3/4)*c^(13/4)))*5i)/(64*(-b)^(3/4)*c^(13/4)) - ((A*c - 9*B*b)*((25*x^(
1/2)*(A^2*c^2 + 81*B^2*b^2 - 18*A*B*b*c))/(64*c^3) + ((45*B*b^2 - 5*A*b*c)*
(A*c - 9*B*b)*5i)/(64*(-b)^(3/4)*c^(13/4)))*5i)/(64*(-b)^(3/4)*c^(13/4))))*
(A*c - 9*B*b))/(32*(-b)^(3/4)*c^(13/4))
```

3.210 $\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 293

$$\begin{aligned}\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3} \, dx = & -\frac{(bB-Ac)x^{7/2}}{4bc\,(b+cx^2)^2}-\frac{(7bB+Ac)x^{3/2}}{16bc^2\,(b+cx^2)} \\ & -\frac{3(7bB+Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{5/4}c^{11/4}}+\frac{3(7bB+Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{5/4}c^{11/4}} \\ & +\frac{3(7bB+Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{11/4}} \\ & -\frac{3(7bB+Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{11/4}}\end{aligned}$$

```
[Out] -1/4*(-A*c+B*b)*x^(7/2)/b/c/(c*x^2+b)^2-1/16*(A*c+7*B*b)*x^(3/2)/b/c^2/(c*x
^2+b)-3/64*(A*c+7*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(5/4)/c^
(11/4)*2^(1/2)+3/64*(A*c+7*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b
^(5/4)/c^(11/4)*2^(1/2)+3/128*(A*c+7*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1
/4)*2^(1/2)*x^(1/2))/b^(5/4)/c^(11/4)*2^(1/2)-3/128*(A*c+7*B*b)*ln(b^(1/2)+
x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)/c^(11/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1598, 468, 294, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{3(Ac+7bB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{5/4}c^{11/4}} + \frac{3(Ac+7bB)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{5/4}c^{11/4}} + \frac{3(Ac+7bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{11/4}} - \frac{3(Ac+7bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{11/4}} - \frac{x^{3/2}(Ac+7bB)}{16bc^2(b+cx^2)} - \frac{x^{7/2}(bB-Ac)}{4bc(b+cx^2)^2}$$

```
[In] Int[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*((b*B - A*c)*x^(7/2))/(b*c*(b + c*x^2)^2) - ((7*b*B + A*c)*x^(3/2))/(1
6*b*c^2*(b + c*x^2)) - (3*(7*b*B + A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/(32*Sqrt[2]*b^(5/4)*c^(11/4)) + (3*(7*b*B + A*c)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(5/4)*c^(11/4)) + (3*(7*b*B
+ A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt
[2]*b^(5/4)*c^(11/4)) - (3*(7*b*B + A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1
/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(5/4)*c^(11/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{x^{5/2}(A+Bx^2)}{(b+cx^2)^3}\,dx$$

$$= -\frac{(bB-Ac)x^{7/2}}{4bc\,(b+cx^2)^2} + \frac{\left(\frac{7bB}{2}+\frac{Ac}{2}\right)\int \frac{x^{5/2}}{(b+cx^2)^2}\,dx}{4bc}$$

$$= -\frac{(bB-Ac)x^{7/2}}{4bc\,(b+cx^2)^2} - \frac{(7bB+Ac)x^{3/2}}{16bc^2\,(b+cx^2)} + \frac{(3(7bB+Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32bc^2}$$

$$= -\frac{(bB-Ac)x^{7/2}}{4bc\,(b+cx^2)^2} - \frac{(7bB+Ac)x^{3/2}}{16bc^2\,(b+cx^2)} + \frac{(3(7bB+Ac))\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16bc^2}$$

$$= -\frac{(bB-Ac)x^{7/2}}{4bc\,(b+cx^2)^2} - \frac{(7bB+Ac)x^{3/2}}{16bc^2\,(b+cx^2)} - \frac{(3(7bB+Ac))\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32bc^{5/2}}$$
$$+ \frac{(3(7bB+Ac))\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32bc^{5/2}}$$

$$=-\frac{(bB-Ac)x^{7/2}}{4bc\left(b+cx^2\right)^2}-\frac{(7bB+Ac)x^{3/2}}{16bc^2\left(b+cx^2\right)}$$

$$+\frac{(3(7bB+Ac))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64bc^3}$$

$$+\frac{(3(7bB+Ac))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64bc^3}$$

$$+\frac{(3(7bB+Ac))\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{64\sqrt{2}b^{5/4}c^{11/4}}$$

$$+\frac{(3(7bB+Ac))\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{64\sqrt{2}b^{5/4}c^{11/4}}$$

$$=-\frac{(bB-Ac)x^{7/2}}{4bc\left(b+cx^2\right)^2}-\frac{(7bB+Ac)x^{3/2}}{16bc^2\left(b+cx^2\right)}+\frac{3(7bB+Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{11/4}}$$

$$-\frac{3(7bB+Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{11/4}}$$

$$+\frac{(3(7bB+Ac))\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{5/4}c^{11/4}}$$

$$-\frac{(3(7bB+Ac))\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{5/4}c^{11/4}}$$

$$= -\frac{(bB-Ac)x^{7/2}}{4bc\,(b+cx^2)^2} - \frac{(7bB+Ac)x^{3/2}}{16bc^2\,(b+cx^2)} - \frac{3(7bB+Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{5/4}c^{11/4}}$$
$$+ \frac{3(7bB+Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{5/4}c^{11/4}}$$
$$+ \frac{3(7bB+Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{11/4}}$$
$$- \frac{3(7bB+Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{11/4}}$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.58

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{-\frac{4\sqrt[4]{b}c^{3/4}x^{3/2}(7b^2B-3Ac^2x^2+bc(A+11Bx^2))}{(b+cx^2)^2} - 3\sqrt{2}(7bB+Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) - 3\sqrt{}}{64b^{5/4}c^{11/4}}$$

```
[In] Integrate[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*b^(1/4)*c^(3/4)*x^(3/2)*(7*b^2*B - 3*A*c^2*x^2 + b*c*(A + 11*B*x^2)))/
(b + c*x^2)^2 - 3*Sqrt[2]*(7*b*B + A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[
2]*b^(1/4)*c^(1/4)*Sqrt[x])] - 3*Sqrt[2]*(7*b*B + A*c)*ArcTanh[(Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(64*b^(5/4)*c^(11/4))
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.57

method	result
derivativedivides	$\frac{\frac{(3Ac-11Bb)x^{\frac{7}{2}}}{16bc}-\frac{(Ac+7Bb)x^{\frac{3}{2}}}{16c^2}}{(cx^2+b)^2} + \frac{3(Ac+7Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\right.\right.}{128c^3b\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$\frac{\frac{(3Ac-11Bb)x^{\frac{7}{2}}}{16bc}-\frac{(Ac+7Bb)x^{\frac{3}{2}}}{16c^2}}{(cx^2+b)^2} + \frac{3(Ac+7Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}-\right.\right.}{128c^3b\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

```
[In] int(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/32*(3*A*c-11*B*b)/b/c*x^(7/2)-1/32*(A*c+7*B*b)/c^2*x^(3/2))/(c*x^2+b)^
2+3/128*(A*c+7*B*b)/c^3/b/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(1/2
)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2
*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(
1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 871, normalized size of antiderivative = 2.97

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3\left(bc^4x^4+2\,b^2c^3x^2+b^3c^2\right)\left(-\frac{2401\,B^4b^4+1372\,AB^3b^3c+294\,A^2B^2b^2c^2+28\,A^3Bbc^3+A^4c^4}{b^5c^{11}}\right)^{\frac{1}{4}}\log\left(\right.}{}$$

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(3*(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)*(-(2401*B^4*b^4 + 1372*A*B^3*
b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(1/4)*l
og(27*b^4*c^8*(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28
*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(3/4) + 27*(343*B^3*b^3 + 147*A*B^2*b^2
*c + 21*A^2*B*b*c^2 + A^3*c^3)*sqrt(x)) - 3*(I*b*c^4*x^4 + 2*I*b^2*c^3*x^2
+ I*b^3*c^2)*(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*
A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(1/4)*log(27*I*b^4*c^8*(-(2401*B^4*b^4 +
 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^
11))^(3/4) + 27*(343*B^3*b^3 + 147*A*B^2*b^2*c + 21*A^2*B*b*c^2 + A^3*c^3)*
sqrt(x)) - 3*(-I*b*c^4*x^4 - 2*I*b^2*c^3*x^2 - I*b^3*c^2)*(-(2401*B^4*b^4 +
 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^
11))^(1/4)*log(-27*I*b^4*c^8*(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B
^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(3/4) + 27*(343*B^3*b^3
+ 147*A*B^2*b^2*c + 21*A^2*B*b*c^2 + A^3*c^3)*sqrt(x)) - 3*(b*c^4*x^4 + 2*b
^2*c^3*x^2 + b^3*c^2)*(-(2401*B^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*
c^2 + 28*A^3*B*b*c^3 + A^4*c^4)/(b^5*c^11))^(1/4)*log(-27*b^4*c^8*(-(2401*B
^4*b^4 + 1372*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 28*A^3*B*b*c^3 + A^4*c^4)
/(b^5*c^11))^(3/4) + 27*(343*B^3*b^3 + 147*A*B^2*b^2*c + 21*A^2*B*b*c^2 + A
^3*c^3)*sqrt(x)) - 4*((11*B*b*c - 3*A*c^2)*x^3 + (7*B*b^2 + A*b*c)*x)*sqrt(
x))/(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**(17/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.86

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{(11\,Bbc-3\,Ac^2)x^{\frac{7}{2}}+(7\,Bb^2+Abc)x^{\frac{3}{2}}}{16\,(bc^4x^4+2\,b^2c^3x^2+b^3c^2)} + \frac{3\,(7\,Bb+Ac)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}}+\cdots\right.}{128\,bc^2}$$

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((11*B*b*c - 3*A*c^2)*x^(7/2) + (7*B*b^2 + A*b*c)*x^(3/2))/(b*c^4*x^4
 + 2*b^2*c^3*x^2 + b^3*c^2) + 3/128*(7*B*b + A*c)*(2*sqrt(2)*arctan(1/2*sqr
t(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(
sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(
1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt
(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + s
qrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) +
sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/(b*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{11\,Bbcx^{\frac{7}{2}} - 3\,Ac^2x^{\frac{7}{2}} + 7\,Bb^2x^{\frac{3}{2}} + Abcx^{\frac{3}{2}}}{16\,(cx^2+b)^2bc^2} + \frac{3\,\sqrt{2}\Big(7\,(bc^3)^{\frac{3}{4}}\,Bb + (bc^3)^{\frac{3}{4}}\,Ac\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^5} + \frac{3\,\sqrt{2}\Big(7\,(bc^3)^{\frac{3}{4}}\,Bb + (bc^3)^{\frac{3}{4}}\,Ac\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^5} - \frac{3\,\sqrt{2}\Big(7\,(bc^3)^{\frac{3}{4}}\,Bb + (bc^3)^{\frac{3}{4}}\,Ac\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{128\,b^2c^5} + \frac{3\,\sqrt{2}\Big(7\,(bc^3)^{\frac{3}{4}}\,Bb + (bc^3)^{\frac{3}{4}}\,Ac\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{128\,b^2c^5}$$

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/16*(11*B*b*c*x^(7/2) - 3*A*c^2*x^(7/2) + 7*B*b^2*x^(3/2) + A*b*c*x^(3/2)
)/((c*x^2 + b)^2*b*c^2) + 3/64*sqrt(2)*(7*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)
*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^
2*c^5) + 3/64*sqrt(2)*(7*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*arctan(-1/2
*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^5) - 3/128*s
qrt(2)*(7*(b*c^3)^(3/4)*B*b + (b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^
(1/4) + x + sqrt(b/c))/(b^2*c^5) + 3/128*sqrt(2)*(7*(b*c^3)^(3/4)*B*b + (b*
c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^5)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.42

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(A\,c+7\,B\,b)}{32\,(-b)^{5/4}\,c^{11/4}} - \frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(A\,c+7\,B\,b)}{32\,(-b)^{5/4}\,c^{11/4}} - \frac{\frac{x^{3/2}\,(A\,c+7\,B\,b)}{16\,c^2} - \frac{x^{7/2}\,(3\,A\,c-11\,B\,b)}{16\,b\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}$$

```
[In] int((x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (3*atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c + 7*B*b))/(32*(-b)^(5/4)*c^(11/
4)) - (3*atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(A*c + 7*B*b))/(32*(-b)^(5/4)*c
^(11/4)) - ((x^(3/2)*(A*c + 7*B*b))/(16*c^2) - (x^(7/2)*(3*A*c - 11*B*b))/(
16*b*c))/(b^2 + c^2*x^4 + 2*b*c*x^2)
```

3.211 $\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 298

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{(bB-Ac)x^{5/2}}{4bc\,(b+cx^2)^2}-\frac{(5bB+3Ac)\sqrt{x}}{16bc^2\,(b+cx^2)} - \frac{(5bB+3Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{7/4}c^{9/4}} + \frac{(5bB+3Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{7/4}c^{9/4}} - \frac{(5bB+3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{9/4}} + \frac{(5bB+3Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{9/4}}$$

```
[Out] -1/4*(-A*c+B*b)*x^(5/2)/b/c/(c*x^2+b)^2-1/64*(3*A*c+5*B*b)*arctan(1-c^(1/4)
*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)/c^(9/4)*2^(1/2)+1/64*(3*A*c+5*B*b)*arctan
(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(7/4)/c^(9/4)*2^(1/2)-1/128*(3*A*c+5*
B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/c^(9/4)*
2^(1/2)+1/128*(3*A*c+5*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^
(1/2))/b^(7/4)/c^(9/4)*2^(1/2)-1/16*(3*A*c+5*B*b)*x^(1/2)/b/c^2/(c*x^2+b)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1598, 468, 294, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx=-\frac{(3Ac+5bB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{7/4}c^{9/4}}+\frac{(3Ac+5bB)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{7/4}c^{9/4}}-\frac{(3Ac+5bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{9/4}}+\frac{(3Ac+5bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{9/4}}-\frac{\sqrt{x}(3Ac+5bB)}{16bc^2(b+cx^2)}-\frac{x^{5/2}(bB-Ac)}{4bc(b+cx^2)^2}$$

```
[In] Int[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*((b*B - A*c)*x^(5/2))/(b*c*(b + c*x^2)^2) - ((5*b*B + 3*A*c)*Sqrt[x])/
(16*b*c^2*(b + c*x^2)) - ((5*b*B + 3*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[
x])/b^(1/4)])/(32*Sqrt[2]*b^(7/4)*c^(9/4)) + ((5*b*B + 3*A*c)*ArcTan[1 + (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(7/4)*c^(9/4)) - ((5*b*B +
3*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt
[2]*b^(7/4)*c^(9/4)) + ((5*b*B + 3*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/
4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(7/4)*c^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{x^{3/2}(A+Bx^2)}{(b+cx^2)^3}\,dx$$

$$= -\frac{(bB-Ac)x^{5/2}}{4bc\,(b+cx^2)^2} + \frac{\left(\frac{5bB}{2}+\frac{3Ac}{2}\right)\int \frac{x^{3/2}}{(b+cx^2)^2}\,dx}{4bc}$$

$$= -\frac{(bB-Ac)x^{5/2}}{4bc\,(b+cx^2)^2} - \frac{(5bB+3Ac)\sqrt{x}}{16bc^2\,(b+cx^2)} + \frac{(5bB+3Ac)\int \frac{1}{\sqrt{x}(b+cx^2)}\,dx}{32bc^2}$$

$$= -\frac{(bB-Ac)x^{5/2}}{4bc\,(b+cx^2)^2} - \frac{(5bB+3Ac)\sqrt{x}}{16bc^2\,(b+cx^2)} + \frac{(5bB+3Ac)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16bc^2}$$

$$= -\frac{(bB-Ac)x^{5/2}}{4bc\,(b+cx^2)^2} - \frac{(5bB+3Ac)\sqrt{x}}{16bc^2\,(b+cx^2)} + \frac{(5bB+3Ac)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^{3/2}c^2}$$
$$+ \frac{(5bB+3Ac)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^{3/2}c^2}$$

$$= -\frac{(bB-Ac)x^{5/2}}{4bc\left(b+cx^2\right)^2} - \frac{(5bB+3Ac)\sqrt{x}}{16bc^2\left(b+cx^2\right)} + \frac{(5bB+3Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64b^{3/2}c^{5/2}} + \frac{(5bB+3Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64b^{3/2}c^{5/2}} - \frac{(5bB+3Ac)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{64\sqrt{2}b^{7/4}c^{9/4}} - \frac{(5bB+3Ac)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{64\sqrt{2}b^{7/4}c^{9/4}}$$

$$= -\frac{(bB-Ac)x^{5/2}}{4bc\left(b+cx^2\right)^2} - \frac{(5bB+3Ac)\sqrt{x}}{16bc^2\left(b+cx^2\right)} - \frac{(5bB+3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{9/4}} + \frac{(5bB+3Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{9/4}} + \frac{(5bB+3Ac)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{7/4}c^{9/4}} - \frac{(5bB+3Ac)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{7/4}c^{9/4}}$$

$$= -\frac{(bB-Ac)x^{5/2}}{4bc\left(b+cx^2\right)^2} - \frac{(5bB+3Ac)\sqrt{x}}{16bc^2\left(b+cx^2\right)} - \frac{(5bB+3Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{7/4}c^{9/4}}$$
$$+ \frac{(5bB+3Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{7/4}c^{9/4}}$$
$$- \frac{(5bB+3Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{9/4}}$$
$$+ \frac{(5bB+3Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{9/4}}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.58

$$\int \frac{x^{15/2}(A+Bx^2)}{\left(bx^2+cx^4\right)^3}\,dx = \frac{-\frac{4b^{3/4}\sqrt[4]{c}\sqrt{x}\left(5b^2B-Ac^2x^2+3bc(A+3Bx^2)\right)}{\left(b+cx^2\right)^2} - \sqrt{2}(5bB+3Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) + \sqrt{2}}{64b^{7/4}c^{9/4}}$$

```
[In] Integrate[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*b^(3/4)*c^(1/4)*Sqrt[x]*(5*b^2*B - A*c^2*x^2 + 3*b*c*(A + 3*B*x^2)))/(
b + c*x^2)^2 - Sqrt[2]*(5*b*B + 3*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x])] + Sqrt[2]*(5*b*B + 3*A*c)*ArcTanh[(Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(64*b^(7/4)*c^(9/4))
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.56

method	result
derivativedivides	$\frac{\frac{(Ac-9Bb)x^{\frac{5}{2}}}{16bc}-\frac{(3Ac+5Bb)\sqrt{x}}{16c^2}}{\left(c\,x^2+b\right)^2}+\frac{(3Ac+5Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{128c^2b^2}$
default	$\frac{\frac{(Ac-9Bb)x^{\frac{5}{2}}}{16bc}-\frac{(3Ac+5Bb)\sqrt{x}}{16c^2}}{\left(c\,x^2+b\right)^2}+\frac{(3Ac+5Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{128c^2b^2}$

```
[In] int(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/32*(A*c-9*B*b)/b/c*x^(5/2)-1/32*(3*A*c+5*B*b)/c^2*x^(1/2))/(c*x^2+b)^2
+1/128*(3*A*c+5*B*b)/c^2/b^2*(1/c*b)^(1/4)*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(
```

```
1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))
)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*
x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 763, normalized size of antiderivative = 2.56

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(bc^4x^4+2\,b^2c^3x^2+b^3c^2)\left(-\frac{625\,B^4b^4+1500\,AB^3b^3c+1350\,A^2B^2b^2c^2+540\,A^3Bbc^3+81\,A^4c^4}{b^7c^9}\right)^{\frac{1}{4}}\log\Big(b$$

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*((b*c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)*(-(625*B^4*b^4 + 1500*A*B^3*b^3
*c + 1350*A^2*B^2*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(1/4)*
log(b^2*c^2*(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 540*
A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(1/4) + (5*B*b + 3*A*c)*sqrt(x)) - (-I
*b*c^4*x^4 - 2*I*b^2*c^3*x^2 - I*b^3*c^2)*(-(625*B^4*b^4 + 1500*A*B^3*b^3*c
 + 1350*A^2*B^2*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(1/4)*lo
g(I*b^2*c^2*(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 540*
A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(1/4) + (5*B*b + 3*A*c)*sqrt(x)) - (I*
b*c^4*x^4 + 2*I*b^2*c^3*x^2 + I*b^3*c^2)*(-(625*B^4*b^4 + 1500*A*B^3*b^3*c
+ 1350*A^2*B^2*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(1/4)*log
(-I*b^2*c^2*(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 540*
A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(1/4) + (5*B*b + 3*A*c)*sqrt(x)) - (b*
c^4*x^4 + 2*b^2*c^3*x^2 + b^3*c^2)*(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350
*A^2*B^2*b^2*c^2 + 540*A^3*B*b*c^3 + 81*A^4*c^4)/(b^7*c^9))^(1/4)*log(-b^2*
c^2*(-(625*B^4*b^4 + 1500*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 540*A^3*B*b*
c^3 + 81*A^4*c^4)/(b^7*c^9))^(1/4) + (5*B*b + 3*A*c)*sqrt(x)) - 4*(5*B*b^2
+ 3*A*b*c + (9*B*b*c - A*c^2)*x^2)*sqrt(x))/(b*c^4*x^4 + 2*b^2*c^3*x^2 + b^
3*c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**(15/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.94

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{(9\,Bbc-Ac^2)x^{\frac{5}{2}}+(5\,Bb^2+3\,Abc)\sqrt{x}}{16\,(bc^4x^4+2\,b^2c^3x^2+b^3c^2)} + \frac{\frac{2\sqrt{2}(5\,Bb+3\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(5\,Bb+3\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}(5\,Bb+3\,Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}}{128\,bc^2}$$

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((9*B*b*c - A*c^2)*x^(5/2) + (5*B*b^2 + 3*A*b*c)*sqrt(x))/(b*c^4*x^4
+ 2*b^2*c^3*x^2 + b^3*c^2) + 1/128*(2*sqrt(2)*(5*B*b + 3*A*c)*arctan(1/2*sq
rt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/
(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(5*B*b + 3*A*c)*arctan(-1/2*sqr
t(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(
sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(5*B*b + 3*A*c)*log(sqrt(2)*b^(1/4
)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(5*B*b
 + 3*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3
/4)*c^(1/4)))/(b*c^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb+3\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^3} + \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb+3\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^3} + \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb+3\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^2c^3} - \frac{\sqrt{2}\left(5\,(bc^3)^{\frac{1}{4}}\,Bb+3\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^2c^3} - \frac{9\,Bbcx^{\frac{5}{2}}-Ac^2x^{\frac{5}{2}}+5\,Bb^2\sqrt{x}+3\,Abc\sqrt{x}}{16\,(cx^2+b)^2bc^2}$$

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*(5*(b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) + 1/64*sqrt(2)*(5
*(b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c
)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) + 1/128*sqrt(2)*(5*(b*c^3)^(1/4
)*B*b + 3*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c
))/(b^2*c^3) - 1/128*sqrt(2)*(5*(b*c^3)^(1/4)*B*b + 3*(b*c^3)^(1/4)*A*c)*lo
g(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^3) - 1/16*(9*B*b*c*x
^(5/2) - A*c^2*x^(5/2) + 5*B*b^2*sqrt(x) + 3*A*b*c*sqrt(x))/((c*x^2 + b)^2*
b*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 799, normalized size of antiderivative = 2.68

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{\frac{\sqrt{x}\,(3\,A\,c+5\,B\,b)}{16\,c^2}-\frac{x^{5/2}\,(A\,c-9\,B\,b)}{16\,b\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4} + \frac{\mathrm{atan}\left(\frac{\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2\right)}{64\,b^2\,c}-\frac{\left(3\,A\,c^2+5\,B\,b\,c\right)(3\,A\,c+5\,B\,b)}{64\,(-b)^{7/4}\,c^{9/4}}\right)1i}{64\,(-b)^{7/4}\,c^{9/4}}+\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2\right)}{64\,b^2\,c}+\frac{\left(3\,A\,c^2+\cdots\right.}{64\cdots}\right.}{64\,(-b)^{7/4}\,c^{9/4}}}{\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2\right)}{64\,b^2\,c}-\frac{\left(3\,A\,c^2+5\,B\,b\,c\right)(3\,A\,c+5\,B\,b)}{64\,(-b)^{7/4}\,c^{9/4}}\right)}{64\,(-b)^{7/4}\,c^{9/4}}-\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2\right)}{64\,b^2\,c}+\frac{\left(3\,A\,c^2+\cdots\right.}{64\cdots}\right.}{64\,(-b)^{7/4}\,c^{9/4}}}\right.\cdots}{32\,(-b)^{7/4}\,c^{9/4}} + \frac{\mathrm{atan}\left(\frac{\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2\right)}{64\,b^2\,c}-\frac{\left(3\,A\,c^2+5\,B\,b\,c\right)(3\,A\,c+5\,B\,b)\,1i}{64\,(-b)^{7/4}\,c^{9/4}}\right)}{64\,(-b)^{7/4}\,c^{9/4}}+\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2\right)}{64\,b^2\,c}+\frac{\left(3\,A\,c^2\cdots\right.}{\cdots}\right.}{64\,(-b)^{7/4}\,c^{9/4}}}{\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2\right)}{64\,b^2\,c}-\frac{\left(3\,A\,c^2+5\,B\,b\,c\right)(3\,A\,c+5\,B\,b)\,1i}{64\,(-b)^{7/4}\,c^{9/4}}\right)1i}{64\,(-b)^{7/4}\,c^{9/4}}-\frac{(3\,A\,c+5\,B\,b)\left(\frac{\sqrt{x}\left(9\,A^2\,c^2+30\,A\,B\,b\,c+25\,B^2\,b^2\right)}{64\,b^2\,c}+\frac{\left(3\,A\,c^2\cdots\right.}{\cdots}\right.}{64\,(-b)^{7/4}\,c^{9/4}}}\right.\cdots}{32\,(-b)^{7/4}\,c^{9/4}}$$

```
[In] int((x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (atan((((3*A*c + 5*B*b)*((x^(1/2)*(9*A^2*c^2 + 25*B^2*b^2 + 30*A*B*b*c))/(6
4*b^2*c) - ((3*A*c^2 + 5*B*b*c)*(3*A*c + 5*B*b))/(64*(-b)^(7/4)*c^(9/4)))*1
i)/(64*(-b)^(7/4)*c^(9/4)) + ((3*A*c + 5*B*b)*((x^(1/2)*(9*A^2*c^2 + 25*B^2
*b^2 + 30*A*B*b*c))/(64*b^2*c) + ((3*A*c^2 + 5*B*b*c)*(3*A*c + 5*B*b))/(64*
(-b)^(7/4)*c^(9/4)))*1i)/(64*(-b)^(7/4)*c^(9/4)))/(((3*A*c + 5*B*b)*((x^(1/
2)*(9*A^2*c^2 + 25*B^2*b^2 + 30*A*B*b*c))/(64*b^2*c) - ((3*A*c^2 + 5*B*b*c)
*(3*A*c + 5*B*b))/(64*(-b)^(7/4)*c^(9/4))))/(64*(-b)^(7/4)*c^(9/4)) - ((3*A
*c + 5*B*b)*((x^(1/2)*(9*A^2*c^2 + 25*B^2*b^2 + 30*A*B*b*c))/(64*b^2*c) + (
(3*A*c^2 + 5*B*b*c)*(3*A*c + 5*B*b))/(64*(-b)^(7/4)*c^(9/4))))/(64*(-b)^(7/
4)*c^(9/4))))*(3*A*c + 5*B*b)*1i)/(32*(-b)^(7/4)*c^(9/4)) - ((x^(1/2)*(3*A*
c + 5*B*b))/(16*c^2) - (x^(5/2)*(A*c - 9*B*b))/(16*b*c))/(b^2 + c^2*x^4 + 2
*b*c*x^2) + (atan((((3*A*c + 5*B*b)*((x^(1/2)*(9*A^2*c^2 + 25*B^2*b^2 + 30*
A*B*b*c))/(64*b^2*c) - ((3*A*c^2 + 5*B*b*c)*(3*A*c + 5*B*b)*1i)/(64*(-b)^(7
/4)*c^(9/4))))/(64*(-b)^(7/4)*c^(9/4)) + ((3*A*c + 5*B*b)*((x^(1/2)*(9*A^2*
c^2 + 25*B^2*b^2 + 30*A*B*b*c))/(64*b^2*c) + ((3*A*c^2 + 5*B*b*c)*(3*A*c +
5*B*b)*1i)/(64*(-b)^(7/4)*c^(9/4))))/(64*(-b)^(7/4)*c^(9/4)))/(((3*A*c + 5*
B*b)*((x^(1/2)*(9*A^2*c^2 + 25*B^2*b^2 + 30*A*B*b*c))/(64*b^2*c) - ((3*A*c^
2 + 5*B*b*c)*(3*A*c + 5*B*b)*1i)/(64*(-b)^(7/4)*c^(9/4)))*1i)/(64*(-b)^(7/4
)*c^(9/4)) - ((3*A*c + 5*B*b)*((x^(1/2)*(9*A^2*c^2 + 25*B^2*b^2 + 30*A*B*b*
c))/(64*b^2*c) + ((3*A*c^2 + 5*B*b*c)*(3*A*c + 5*B*b)*1i)/(64*(-b)^(7/4)*c^
(9/4)))*1i)/(64*(-b)^(7/4)*c^(9/4))))*(3*A*c + 5*B*b))/(32*(-b)^(7/4)*c^(9/
4))
```

3.212 $\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 298

$$\begin{aligned}\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx &= -\frac{(bB-Ac)x^{3/2}}{4bc\,(b+cx^2)^2}+\frac{(3bB+5Ac)x^{3/2}}{16b^2c\,(b+cx^2)}\\
&-\frac{(3bB+5Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{9/4}c^{7/4}}+\frac{(3bB+5Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{9/4}c^{7/4}}\\
&+\frac{(3bB+5Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{7/4}}\\
&-\frac{(3bB+5Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{7/4}}\end{aligned}$$

```
[Out] -1/4*(-A*c+B*b)*x^(3/2)/b/c/(c*x^2+b)^2+1/16*(5*A*c+3*B*b)*x^(3/2)/b^2/c/(c
*x^2+b)-1/64*(5*A*c+3*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(9/4
)/c^(7/4)*2^(1/2)+1/64*(5*A*c+3*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/
4))/b^(9/4)/c^(7/4)*2^(1/2)+1/128*(5*A*c+3*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4
)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)/c^(7/4)*2^(1/2)-1/128*(5*A*c+3*B*b)*ln(b
^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)/c^(7/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1598, 468, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{(5Ac+3bB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{9/4}c^{7/4}} + \frac{(5Ac+3bB)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{9/4}c^{7/4}} + \frac{(5Ac+3bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{7/4}} - \frac{(5Ac+3bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{7/4}} + \frac{x^{3/2}(5Ac+3bB)}{16b^2c\,(b+cx^2)} - \frac{x^{3/2}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

```
[In] Int[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*((b*B - A*c)*x^(3/2))/(b*c*(b + c*x^2)^2) + ((3*b*B + 5*A*c)*x^(3/2))/
(16*b^2*c*(b + c*x^2)) - ((3*b*B + 5*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[
x])/b^(1/4)])/(32*Sqrt[2]*b^(9/4)*c^(7/4)) + ((3*b*B + 5*A*c)*ArcTan[1 + (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(9/4)*c^(7/4)) + ((3*b*B +
5*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt
[2]*b^(9/4)*c^(7/4)) - ((3*b*B + 5*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/
4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(9/4)*c^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{x}(A+Bx^2)}{(b+cx^2)^3}\,dx$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\,(b+cx^2)^2} + \frac{\left(\frac{3bB}{2}+\frac{5Ac}{2}\right)\int \frac{\sqrt{x}}{(b+cx^2)^2}\,dx}{4bc}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\,(b+cx^2)^2} + \frac{(3bB+5Ac)x^{3/2}}{16b^2c\,(b+cx^2)} + \frac{(3bB+5Ac)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32b^2c}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\,(b+cx^2)^2} + \frac{(3bB+5Ac)x^{3/2}}{16b^2c\,(b+cx^2)} + \frac{(3bB+5Ac)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^2c}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\,(b+cx^2)^2} + \frac{(3bB+5Ac)x^{3/2}}{16b^2c\,(b+cx^2)} - \frac{(3bB+5Ac)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^2c^{3/2}} + \frac{(3bB+5Ac)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^2c^{3/2}}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\left(b+cx^2\right)^2}+\frac{(3bB+5Ac)x^{3/2}}{16b^2c\left(b+cx^2\right)}$$
$$+\frac{(3bB+5Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64b^2c^2}$$
$$+\frac{(3bB+5Ac)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64b^2c^2}$$
$$+\frac{(3bB+5Ac)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{64\sqrt{2}b^{9/4}c^{7/4}}$$
$$+\frac{(3bB+5Ac)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{64\sqrt{2}b^{9/4}c^{7/4}}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\left(b+cx^2\right)^2}+\frac{(3bB+5Ac)x^{3/2}}{16b^2c\left(b+cx^2\right)}$$
$$+\frac{(3bB+5Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{7/4}}$$
$$-\frac{(3bB+5Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{7/4}}$$
$$+\frac{(3bB+5Ac)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{9/4}c^{7/4}}$$
$$-\frac{(3bB+5Ac)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{9/4}c^{7/4}}$$

$$= -\frac{(bB-Ac)x^{3/2}}{4bc\,(b+cx^2)^2} + \frac{(3bB+5Ac)x^{3/2}}{16b^2c\,(b+cx^2)} - \frac{(3bB+5Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{9/4}c^{7/4}}$$
$$+ \frac{(3bB+5Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{9/4}c^{7/4}}$$
$$+ \frac{(3bB+5Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{7/4}}$$
$$- \frac{(3bB+5Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{7/4}}$$

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.59

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{4\sqrt[4]{b}c^{3/4}x^{3/2}(-b^2B+9Abc+3bBcx^2+5Ac^2x^2)}{(b+cx^2)^2} - \sqrt{2}(3bB+5Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right) - \sqrt{2}}{64b^{9/4}c^{7/4}}$$

```
[In] Integrate[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*b^(1/4)*c^(3/4)*x^(3/2)*(-(b^2*B) + 9*A*b*c + 3*b*B*c*x^2 + 5*A*c^2*x^2
))/(b + c*x^2)^2 - Sqrt[2]*(3*b*B + 5*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sq
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] - Sqrt[2]*(3*b*B + 5*A*c)*ArcTanh[(Sqrt[2]*
b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(64*b^(9/4)*c^(7/4))
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.56

method	result
derivativedivides	$\frac{\frac{(5Ac+3Bb)x^{\frac{7}{2}}}{16b^2}+\frac{(9Ac-Bb)x^{\frac{3}{2}}}{16bc}}{(c\,x^2+b)^2}+\frac{(5Ac+3Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{128b^2c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}$
default	$\frac{\frac{(5Ac+3Bb)x^{\frac{7}{2}}}{16b^2}+\frac{(9Ac-Bb)x^{\frac{3}{2}}}{16bc}}{(c\,x^2+b)^2}+\frac{(5Ac+3Bb)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right.\right.}{128b^2c^2\left(\frac{b}{c}\right)^{\frac{1}{4}}}$

```
[In] int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/32*(5*A*c+3*B*b)/b^2*x^(7/2)+1/32*(9*A*c-B*b)/b/c*x^(3/2))/(c*x^2+b)^2
+1/128*(5*A*c+3*B*b)/b^2/c^2/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(1/4)*x^(
1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))
)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*
x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 878, normalized size of antiderivative = 2.95

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(b^2c^3x^4+2\,b^3c^2x^2+b^4c)\left(-\frac{81\,B^4b^4+540\,AB^3b^3c+1350\,A^2B^2b^2c^2+1500\,A^3Bbc^3+625\,A^4c^4}{b^9c^7}\right)^{\frac{1}{4}}\log\Big(b}{}$$

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*((b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)*(-(81*B^4*b^4 + 540*A*B^3*b^3*c
 + 1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(1/4)*
log(b^7*c^5*(-(81*B^4*b^4 + 540*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A
^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(3/4) + (27*B^3*b^3 + 135*A*B^2*b^2*c
+ 225*A^2*B*b*c^2 + 125*A^3*c^3)*sqrt(x)) - (I*b^2*c^3*x^4 + 2*I*b^3*c^2*x^
2 + I*b^4*c)*(-(81*B^4*b^4 + 540*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 1500*
A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(1/4)*log(I*b^7*c^5*(-(81*B^4*b^4 + 5
40*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^
9*c^7))^(3/4) + (27*B^3*b^3 + 135*A*B^2*b^2*c + 225*A^2*B*b*c^2 + 125*A^3*c
^3)*sqrt(x)) - (-I*b^2*c^3*x^4 - 2*I*b^3*c^2*x^2 - I*b^4*c)*(-(81*B^4*b^4 +
 540*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(
b^9*c^7))^(1/4)*log(-I*b^7*c^5*(-(81*B^4*b^4 + 540*A*B^3*b^3*c + 1350*A^2*B
^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(3/4) + (27*B^3*b^3
 + 135*A*B^2*b^2*c + 225*A^2*B*b*c^2 + 125*A^3*c^3)*sqrt(x)) - (b^2*c^3*x^4
 + 2*b^3*c^2*x^2 + b^4*c)*(-(81*B^4*b^4 + 540*A*B^3*b^3*c + 1350*A^2*B^2*b^
2*c^2 + 1500*A^3*B*b*c^3 + 625*A^4*c^4)/(b^9*c^7))^(1/4)*log(-b^7*c^5*(-(81
*B^4*b^4 + 540*A*B^3*b^3*c + 1350*A^2*B^2*b^2*c^2 + 1500*A^3*B*b*c^3 + 625*
A^4*c^4)/(b^9*c^7))^(3/4) + (27*B^3*b^3 + 135*A*B^2*b^2*c + 225*A^2*B*b*c^2
 + 125*A^3*c^3)*sqrt(x)) + 4*((3*B*b*c + 5*A*c^2)*x^3 - (B*b^2 - 9*A*b*c)*x
)*sqrt(x))/(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.85

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(3\,Bbc+5\,Ac^2)x^{\frac{7}{2}}-(Bb^2-9\,Abc)x^{\frac{3}{2}}}{16\,(b^2c^3x^4+2\,b^3c^2x^2+b^4c)} + \frac{(3\,Bb+5\,Ac)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \right.}{128\,b^2c}$$

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
[Out] 1/16*((3*B*b*c + 5*A*c^2)*x^(7/2) - (B*b^2 - 9*A*b*c)*x^(3/2))/(b^2*c^3*x^4
 + 2*b^3*c^2*x^2 + b^4*c) + 1/128*(3*B*b + 5*A*c)*(2*sqrt(2)*arctan(1/2*sqr
t(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(
sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(
1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt
(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + s
qrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) +
sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/(b^2*c)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3\,Bbcx^{\frac{7}{2}}+5\,Ac^2x^{\frac{7}{2}}-Bb^2x^{\frac{3}{2}}+9\,Abcx^{\frac{3}{2}}}{16\,(cx^2+b)^2b^2c} + \frac{\sqrt{2}\left(3\,(bc^3)^{\frac{3}{4}}\,Bb+5\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^3c^4} + \frac{\sqrt{2}\left(3\,(bc^3)^{\frac{3}{4}}\,Bb+5\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^3c^4} - \frac{\sqrt{2}\left(3\,(bc^3)^{\frac{3}{4}}\,Bb+5\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^3c^4} + \frac{\sqrt{2}\left(3\,(bc^3)^{\frac{3}{4}}\,Bb+5\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^3c^4}$$

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
[Out] 1/16*(3*B*b*c*x^(7/2) + 5*A*c^2*x^(7/2) - B*b^2*x^(3/2) + 9*A*b*c*x^(3/2))/
((c*x^2 + b)^2*b^2*c) + 1/64*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + 5*(b*c^3)^(3/4)
*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^
3*c^4) + 1/64*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + 5*(b*c^3)^(3/4)*A*c)*arctan(-1
/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^4) - 1/128
*sqrt(2)*(3*(b*c^3)^(3/4)*B*b + 5*(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b
/c)^(1/4) + x + sqrt(b/c))/(b^3*c^4) + 1/128*sqrt(2)*(3*(b*c^3)^(3/4)*B*b +
 5*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^
3*c^4)
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.42

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{x^{7/2}\,(5\,A\,c+3\,B\,b)}{16\,b^2}+\frac{x^{3/2}\,(9\,A\,c-B\,b)}{16\,b\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4} + \frac{\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(5\,A\,c+3\,B\,b)}{32\,(-b)^{9/4}\,c^{7/4}} - \frac{\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(5\,A\,c+3\,B\,b)}{32\,(-b)^{9/4}\,c^{7/4}}$$

```
[In] int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^(7/2)*(5*A*c + 3*B*b))/(16*b^2) + (x^(3/2)*(9*A*c - B*b))/(16*b*c))/(b^
2 + c^2*x^4 + 2*b*c*x^2) + (atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(5*A*c + 3*B
*b))/(32*(-b)^(9/4)*c^(7/4)) - (atanh((c^(1/4)*x^(1/2))/(-b)^(1/4))*(5*A*c
+ 3*B*b))/(32*(-b)^(9/4)*c^(7/4))
```

3.213 $\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 293

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{(bB-Ac)\sqrt{x}}{4bc\,(b+cx^2)^2}+\frac{(bB+7Ac)\sqrt{x}}{16b^2c\,(b+cx^2)} - \frac{3(bB+7Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{11/4}c^{5/4}} + \frac{3(bB+7Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{11/4}c^{5/4}} - \frac{3(bB+7Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}c^{5/4}} + \frac{3(bB+7Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}c^{5/4}}$$

```
[Out] -3/64*(7*A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/4)/c^(5/4
)*2^(1/2)+3/64*(7*A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(11/
4)/c^(5/4)*2^(1/2)-3/128*(7*A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2
^(1/2)*x^(1/2))/b^(11/4)/c^(5/4)*2^(1/2)+3/128*(7*A*c+B*b)*ln(b^(1/2)+x*c^(
1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(11/4)/c^(5/4)*2^(1/2)-1/4*(-A*c+B*
b)*x^(1/2)/b/c/(c*x^2+b)^2+1/16*(7*A*c+B*b)*x^(1/2)/b^2/c/(c*x^2+b)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1598, 468, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{3(7Ac+bB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{11/4}c^{5/4}} + \frac{3(7Ac+bB)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{11/4}c^{5/4}} - \frac{3(7Ac+bB)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}c^{5/4}} + \frac{3(7Ac+bB)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}c^{5/4}} + \frac{\sqrt{x}(7Ac+bB)}{16b^2c\,(b+cx^2)} - \frac{\sqrt{x}(bB-Ac)}{4bc\,(b+cx^2)^2}$$

```
[In] Int[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*((b*B - A*c)*Sqrt[x])/(b*c*(b + c*x^2)^2) + ((b*B + 7*A*c)*Sqrt[x])/(1
6*b^2*c*(b + c*x^2)) - (3*(b*B + 7*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/(32*Sqrt[2]*b^(11/4)*c^(5/4)) + (3*(b*B + 7*A*c)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(11/4)*c^(5/4)) - (3*(b*B +
7*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt
[2]*b^(11/4)*c^(5/4)) + (3*(b*B + 7*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1
/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(11/4)*c^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
```

```
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{\sqrt{x}\left(b+cx^2\right)^3}\,dx \\
&= -\frac{(bB-Ac)\sqrt{x}}{4bc\left(b+cx^2\right)^2}+\frac{\left(\frac{bB}{2}+\frac{7Ac}{2}\right)\int\frac{1}{\sqrt{x}(b+cx^2)^2}\,dx}{4bc} \\
&= -\frac{(bB-Ac)\sqrt{x}}{4bc\left(b+cx^2\right)^2}+\frac{(bB+7Ac)\sqrt{x}}{16b^2c\left(b+cx^2\right)}+\frac{(3(bB+7Ac))\int\frac{1}{\sqrt{x}(b+cx^2)}\,dx}{32b^2c} \\
&= -\frac{(bB-Ac)\sqrt{x}}{4bc\left(b+cx^2\right)^2}+\frac{(bB+7Ac)\sqrt{x}}{16b^2c\left(b+cx^2\right)}+\frac{(3(bB+7Ac))\text{Subst}\left(\int\frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{16b^2c} \\
&= -\frac{(bB-Ac)\sqrt{x}}{4bc\left(b+cx^2\right)^2}+\frac{(bB+7Ac)\sqrt{x}}{16b^2c\left(b+cx^2\right)}+\frac{(3(bB+7Ac))\text{Subst}\left(\int\frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{32b^{5/2}c} \\
&\quad+\frac{(3(bB+7Ac))\text{Subst}\left(\int\frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{32b^{5/2}c}
\end{aligned}$$

$$=-\frac{(bB-Ac)\sqrt{x}}{4bc\left(b+cx^2\right)^2}+\frac{(bB+7Ac)\sqrt{x}}{16b^2c\left(b+cx^2\right)}+\frac{(3(bB+7Ac))\text{Subst}\left(\int\frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64b^{5/2}c^{3/2}}+\frac{(3(bB+7Ac))\text{Subst}\left(\int\frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64b^{5/2}c^{3/2}}-\frac{(3(bB+7Ac))\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{64\sqrt{2}b^{11/4}c^{5/4}}-\frac{(3(bB+7Ac))\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{64\sqrt{2}b^{11/4}c^{5/4}}$$

$$=-\frac{(bB-Ac)\sqrt{x}}{4bc\left(b+cx^2\right)^2}+\frac{(bB+7Ac)\sqrt{x}}{16b^2c\left(b+cx^2\right)}-\frac{3(bB+7Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}c^{5/4}}+\frac{3(bB+7Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}c^{5/4}}+\frac{(3(bB+7Ac))\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{11/4}c^{5/4}}-\frac{(3(bB+7Ac))\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{11/4}c^{5/4}}$$

$$= -\frac{(bB-Ac)\sqrt{x}}{4bc\,(b+cx^2)^2}+\frac{(bB+7Ac)\sqrt{x}}{16b^2c\,(b+cx^2)}-\frac{3(bB+7Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{11/4}c^{5/4}}$$
$$+\frac{3(bB+7Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{11/4}c^{5/4}}$$
$$-\frac{3(bB+7Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}c^{5/4}}$$
$$+\frac{3(bB+7Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}c^{5/4}}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.59

$$\int\frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx=\frac{\frac{4b^{3/4}\sqrt[4]{c}\sqrt{x}(-3b^2B+11Abc+bBcx^2+7Ac^2x^2)}{(b+cx^2)^2}-3\sqrt{2}(bB+7Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)+3\sqrt{}}{64b^{11/4}c^{5/4}}$$

```
[In] Integrate[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*b^(3/4)*c^(1/4)*Sqrt[x]*(-3*b^2*B + 11*A*b*c + b*B*c*x^2 + 7*A*c^2*x^2)
)/(b + c*x^2)^2 - 3*Sqrt[2]*(b*B + 7*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqr
t[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 3*Sqrt[2]*(b*B + 7*A*c)*ArcTanh[(Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(64*b^(11/4)*c^(5/4))
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.57

method	result
derivativedivides	$\frac{\frac{(7Ac+Bb)x^{\frac{5}{2}}}{16b^2}+\frac{(11Ac-3Bb)\sqrt{x}}{16bc}}{(cx^2+b)^2}+\frac{3(7Ac+Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{128b^3c}$
default	$\frac{\frac{(7Ac+Bb)x^{\frac{5}{2}}}{16b^2}+\frac{(11Ac-3Bb)\sqrt{x}}{16bc}}{(cx^2+b)^2}+\frac{3(7Ac+Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{128b^3c}$

```
[In] int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/32*(7*A*c+B*b)/b^2*x^(5/2)+1/32*(11*A*c-3*B*b)/b/c*x^(1/2))/(c*x^2+b)^
2+3/128*(7*A*c+B*b)/b^3/c*(1/c*b)^(1/4)*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2
```

```
)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2
*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(
1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 749, normalized size of antiderivative = 2.56

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3\,(b^2c^3x^4+2\,b^3c^2x^2+b^4c)\left(-\frac{B^4b^4+28\,AB^3b^3c+294\,A^2B^2b^2c^2+1372\,A^3Bbc^3+2401\,A^4c^4}{b^{11}c^5}\right)^{\frac{1}{4}}\log\left(3\right.}{}$$

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(3*(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)*(-(B^4*b^4 + 28*A*B^3*b^3*c +
 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(1/4)*l
og(3*b^3*c*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b
*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(1/4) + 3*(B*b + 7*A*c)*sqrt(x)) - 3*(-I*b
^2*c^3*x^4 - 2*I*b^3*c^2*x^2 - I*b^4*c)*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A
^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(1/4)*log(3*I
*b^3*c*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3
 + 2401*A^4*c^4)/(b^11*c^5))^(1/4) + 3*(B*b + 7*A*c)*sqrt(x)) - 3*(I*b^2*c^
3*x^4 + 2*I*b^3*c^2*x^2 + I*b^4*c)*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^
2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(1/4)*log(-3*I*b^3
*c*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2
401*A^4*c^4)/(b^11*c^5))^(1/4) + 3*(B*b + 7*A*c)*sqrt(x)) - 3*(b^2*c^3*x^4
+ 2*b^3*c^2*x^2 + b^4*c)*(-(B^4*b^4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2
+ 1372*A^3*B*b*c^3 + 2401*A^4*c^4)/(b^11*c^5))^(1/4)*log(-3*b^3*c*(-(B^4*b^
4 + 28*A*B^3*b^3*c + 294*A^2*B^2*b^2*c^2 + 1372*A^3*B*b*c^3 + 2401*A^4*c^4)
/(b^11*c^5))^(1/4) + 3*(B*b + 7*A*c)*sqrt(x)) - 4*(3*B*b^2 - 11*A*b*c - (B*
b*c + 7*A*c^2)*x^2)*sqrt(x))/(b^2*c^3*x^4 + 2*b^3*c^2*x^2 + b^4*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.94

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{(Bbc+7\,Ac^2)x^{\frac{5}{2}}-(3\,Bb^2-11\,Abc)\sqrt{x}}{16\,(b^2c^3x^4+2\,b^3c^2x^2+b^4c)} + \frac{3\left(\frac{2\sqrt{2}(Bb+7\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(Bb+7\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}(Bb+7\,Ac)\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{}\right.}{b^{\frac{3}{4}}c^{\frac{1}{4}}}\right.}{128\,b^2c}$$

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((B*b*c + 7*A*c^2)*x^(5/2) - (3*B*b^2 - 11*A*b*c)*sqrt(x))/(b^2*c^3*x^
4 + 2*b^3*c^2*x^2 + b^4*c) + 3/128*(2*sqrt(2)*(B*b + 7*A*c)*arctan(1/2*sqrt
(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(s
qrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(B*b + 7*A*c)*arctan(-1/2*sqrt(2)
*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt
(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(B*b + 7*A*c)*log(sqrt(2)*b^(1/4)*c^(1
/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(B*b + 7*A*c
)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1
/4)))/(b^2*c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{3\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+7\,(bc^3)^{\frac{1}{4}}Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^3c^2}$$
$$+\frac{3\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+7\,(bc^3)^{\frac{1}{4}}Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^3c^2}$$
$$+\frac{3\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+7\,(bc^3)^{\frac{1}{4}}Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^3c^2}$$
$$-\frac{3\sqrt{2}\left((bc^3)^{\frac{1}{4}}Bb+7\,(bc^3)^{\frac{1}{4}}Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^3c^2}$$
$$+\frac{Bbcx^{\frac{5}{2}}+7\,Ac^2x^{\frac{5}{2}}-3\,Bb^2\sqrt{x}+11\,Abc\sqrt{x}}{16\,(cx^2+b)^2b^2c}$$

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*((b*c^3)^(1/4)*B*b + 7*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2)*(
sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) + 3/64*sqrt(2)*((b*
c^3)^(1/4)*B*b + 7*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1
/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2) + 3/128*sqrt(2)*((b*c^3)^(1/4)*B*b
+ 7*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^
3*c^2) - 3/128*sqrt(2)*((b*c^3)^(1/4)*B*b + 7*(b*c^3)^(1/4)*A*c)*log(-sqrt(
2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2) + 1/16*(B*b*c*x^(5/2) + 7
*A*c^2*x^(5/2) - 3*B*b^2*sqrt(x) + 11*A*b*c*sqrt(x))/((c*x^2 + b)^2*b^2*c)
```

Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 780, normalized size of antiderivative = 2.66

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{x^{5/2}\,(7\,A\,c+B\,b)}{16\,b^2}+\frac{\sqrt{x}\,(11\,A\,c-3\,B\,b)}{16\,b\,c}}{b^2+2\,b\,c\,x^2+c^2\,x^4}$$

$$-\frac{\mathrm{atan}\left(\frac{\frac{(7\,A\,c+B\,b)\left(\frac{9\sqrt{x}\left(49\,A^2\,c^3+14\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{64\,b^4}-\frac{9\,(7\,A\,c+B\,b)\left(7\,A\,c^3+B\,b\,c^2\right)}{64\,(-b)^{15/4}\,c^{5/4}}\right)3\mathrm{i}}{64\,(-b)^{11/4}\,c^{5/4}}+\frac{(7\,A\,c+B\,b)\left(\frac{9\sqrt{x}\left(49\,A^2\,c^3+14\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{64\,b^4}+\frac{9\,(7\,A \cdots}{\cdots}\right)\cdots}{64\,(-b)^{11/4}\,c^{5/4}}}{\frac{3\,(7\,A\,c+B\,b)\left(\frac{9\sqrt{x}\left(49\,A^2\,c^3+14\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{64\,b^4}-\frac{9\,(7\,A\,c+B\,b)\left(7\,A\,c^3+B\,b\,c^2\right)}{64\,(-b)^{15/4}\,c^{5/4}}\right)}{64\,(-b)^{11/4}\,c^{5/4}}-\frac{3\,(7\,A\,c+B\,b)\left(\frac{9\sqrt{x}\left(49\,A^2\,c^3+14\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{64\,b^4}+\frac{9\,(7 \cdots}{\cdots}\right)\cdots}{64\,(-b)^{11/4}\,c^{5/4}}}\right)\cdots}{32\,(-b)^{11/4}\,c^{5/4}}$$

$$-\frac{3\,\mathrm{atan}\left(\frac{\frac{3\,(7\,A\,c+B\,b)\left(\frac{9\sqrt{x}\left(49\,A^2\,c^3+14\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{64\,b^4}-\frac{(7\,A\,c+B\,b)\left(7\,A\,c^3+B\,b\,c^2\right)9\mathrm{i}}{64\,(-b)^{15/4}\,c^{5/4}}\right)}{64\,(-b)^{11/4}\,c^{5/4}}+\frac{3\,(7\,A\,c+B\,b)\left(\frac{9\sqrt{x}\left(49\,A^2\,c^3+14\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{64\,b^4}+\frac{(7 \cdots}{\cdots}\right)\cdots}{64\,(-b)^{11/4}\,c^{5/4}}}{\frac{(7\,A\,c+B\,b)\left(\frac{9\sqrt{x}\left(49\,A^2\,c^3+14\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{64\,b^4}-\frac{(7\,A\,c+B\,b)\left(7\,A\,c^3+B\,b\,c^2\right)9\mathrm{i}}{64\,(-b)^{15/4}\,c^{5/4}}\right)3\mathrm{i}}{64\,(-b)^{11/4}\,c^{5/4}}-\frac{(7\,A\,c+B\,b)\left(\frac{9\sqrt{x}\left(49\,A^2\,c^3+14\,A\,B\,b\,c^2+B^2\,b^2\,c\right)}{64\,b^4}+\frac{(7 \cdots}{\cdots}\right)\cdots}{64\,(-b)^{11/4}\,c^{5/4}}}\right)\cdots}{32\,(-b)^{11/4}\,c^{5/4}}$$

```
[In] int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^(5/2)*(7*A*c + B*b))/(16*b^2) + (x^(1/2)*(11*A*c - 3*B*b))/(16*b*c))/(b
^2 + c^2*x^4 + 2*b*c*x^2) - (atan((((7*A*c + B*b)*((9*x^(1/2)*(49*A^2*c^3 +
 B^2*b^2*c + 14*A*B*b*c^2))/(64*b^4) - (9*(7*A*c + B*b)*(7*A*c^3 + B*b*c^2)
)/(64*(-b)^(15/4)*c^(5/4)))*3i)/(64*(-b)^(11/4)*c^(5/4)) + ((7*A*c + B*b)*(
(9*x^(1/2)*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2))/(64*b^4) + (9*(7*A*c +
B*b)*(7*A*c^3 + B*b*c^2))/(64*(-b)^(15/4)*c^(5/4)))*3i)/(64*(-b)^(11/4)*c^(
5/4)))/((3*(7*A*c + B*b)*((9*x^(1/2)*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2
))/(64*b^4) - (9*(7*A*c + B*b)*(7*A*c^3 + B*b*c^2))/(64*(-b)^(15/4)*c^(5/4)
)))/(64*(-b)^(11/4)*c^(5/4)) - (3*(7*A*c + B*b)*((9*x^(1/2)*(49*A^2*c^3 + B
^2*b^2*c + 14*A*B*b*c^2))/(64*b^4) + (9*(7*A*c + B*b)*(7*A*c^3 + B*b*c^2))/
(64*(-b)^(15/4)*c^(5/4))))/(64*(-b)^(11/4)*c^(5/4))))*(7*A*c + B*b)*3i)/(32
*(-b)^(11/4)*c^(5/4)) - (3*atan(((3*(7*A*c + B*b)*((9*x^(1/2)*(49*A^2*c^3 +
 B^2*b^2*c + 14*A*B*b*c^2))/(64*b^4) - ((7*A*c + B*b)*(7*A*c^3 + B*b*c^2)*9
i)/(64*(-b)^(15/4)*c^(5/4))))/(64*(-b)^(11/4)*c^(5/4)) + (3*(7*A*c + B*b)*(
(9*x^(1/2)*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2))/(64*b^4) + ((7*A*c + B*
b)*(7*A*c^3 + B*b*c^2)*9i)/(64*(-b)^(15/4)*c^(5/4))))/(64*(-b)^(11/4)*c^(5/
4)))/(((7*A*c + B*b)*((9*x^(1/2)*(49*A^2*c^3 + B^2*b^2*c + 14*A*B*b*c^2))/(
64*b^4) - ((7*A*c + B*b)*(7*A*c^3 + B*b*c^2)*9i)/(64*(-b)^(15/4)*c^(5/4)))*
3i)/(64*(-b)^(11/4)*c^(5/4)) - ((7*A*c + B*b)*((9*x^(1/2)*(49*A^2*c^3 + B^2
*b^2*c + 14*A*B*b*c^2))/(64*b^4) + ((7*A*c + B*b)*(7*A*c^3 + B*b*c^2)*9i)/(
64*(-b)^(15/4)*c^(5/4)))*3i)/(64*(-b)^(11/4)*c^(5/4))))*(7*A*c + B*b))/(32*
(-b)^(11/4)*c^(5/4))
```

3.214 $\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 316

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{5(bB-9Ac)}{16b^3c\sqrt{x}} - \frac{bB-Ac}{4bc\sqrt{x}\,(b+cx^2)^2} - \frac{bB-9Ac}{16b^2c\sqrt{x}\,(b+cx^2)} - \frac{5(bB-9Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{13/4}c^{3/4}} + \frac{5(bB-9Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{13/4}c^{3/4}} + \frac{5(bB-9Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{13/4}c^{3/4}} - \frac{5(bB-9Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{13/4}c^{3/4}}$$

```
[Out] -5/64*(-9*A*c+B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(13/4)/c^(3/
4)*2^(1/2)+5/64*(-9*A*c+B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(1
3/4)/c^(3/4)*2^(1/2)+5/128*(-9*A*c+B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4
)*2^(1/2)*x^(1/2))/b^(13/4)/c^(3/4)*2^(1/2)-5/128*(-9*A*c+B*b)*ln(b^(1/2)+x
*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)/c^(3/4)*2^(1/2)+5/16*(-9
*A*c+B*b)/b^3/c/x^(1/2)+1/4*(A*c-B*b)/b/c/(c*x^2+b)^2/x^(1/2)+1/16*(9*A*c-B
*b)/b^2/c/(c*x^2+b)/x^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1598, 468, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{5(bB-9Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{13/4}c^{3/4}} + \frac{5(bB-9Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{13/4}c^{3/4}} + \frac{5(bB-9Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{13/4}c^{3/4}} - \frac{5(bB-9Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{13/4}c^{3/4}} + \frac{5(bB-9Ac)}{16b^3c\sqrt{x}} - \frac{bB-9Ac}{16b^2c\sqrt{x}\,(b+cx^2)} - \frac{bB-Ac}{4bc\sqrt{x}\,(b+cx^2)^2}$$

```
[In] Int[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (5*(b*B - 9*A*c))/(16*b^3*c*Sqrt[x]) - (b*B - A*c)/(4*b*c*Sqrt[x]*(b + c*x^
2)^2) - (b*B - 9*A*c)/(16*b^2*c*Sqrt[x]*(b + c*x^2)) - (5*(b*B - 9*A*c)*Arc
Tan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(13/4)*c^(3/4)) +
 (5*(b*B - 9*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2
]*b^(13/4)*c^(3/4)) + (5*(b*B - 9*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(13/4)*c^(3/4)) - (5*(b*B - 9*A*c)*Lo
g[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(13
/4)*c^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{A+Bx^2}{x^{3/2}\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2} + \frac{\left(-\frac{bB}{2}+\frac{9Ac}{2}\right)\int \frac{1}{x^{3/2}\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$= -\frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2} - \frac{bB-9Ac}{16b^2c\sqrt{x}\left(b+cx^2\right)} - \frac{\left(5(bB-9Ac)\right)\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{32b^2c}$$

$$= \frac{5(bB-9Ac)}{16b^3c\sqrt{x}} - \frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2} - \frac{bB-9Ac}{16b^2c\sqrt{x}\left(b+cx^2\right)} + \frac{\left(5(bB-9Ac)\right)\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32b^3}$$

$$= \frac{5(bB-9Ac)}{16b^3c\sqrt{x}} - \frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2} - \frac{bB-9Ac}{16b^2c\sqrt{x}\left(b+cx^2\right)} + \frac{\left(5(bB-9Ac)\right)\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^3}$$

$$= \frac{5(bB-9Ac)}{16b^3c\sqrt{x}} - \frac{bB-Ac}{4bc\sqrt{x}\left(b+cx^2\right)^2} - \frac{bB-9Ac}{16b^2c\sqrt{x}\left(b+cx^2\right)} - \frac{\left(5(bB-9Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^3\sqrt{c}} + \frac{\left(5(bB-9Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^3\sqrt{c}}$$

$$= \frac{5(bB-9Ac)}{16b^3c\sqrt{x}} - \frac{bB-Ac}{4bc\sqrt{x}\,(b+cx^2)^2} - \frac{bB-9Ac}{16b^2c\sqrt{x}\,(b+cx^2)} + \frac{(5(bB-9Ac))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64b^3c} + \frac{(5(bB-9Ac))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64b^3c} + \frac{(5(bB-9Ac))\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{64\sqrt{2}b^{13/4}c^{3/4}} + \frac{(5(bB-9Ac))\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{64\sqrt{2}b^{13/4}c^{3/4}}$$

$$= \frac{5(bB-9Ac)}{16b^3c\sqrt{x}} - \frac{bB-Ac}{4bc\sqrt{x}\,(b+cx^2)^2} - \frac{bB-9Ac}{16b^2c\sqrt{x}\,(b+cx^2)} + \frac{5(bB-9Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{13/4}c^{3/4}} - \frac{5(bB-9Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{13/4}c^{3/4}} + \frac{(5(bB-9Ac))\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{13/4}c^{3/4}} - \frac{(5(bB-9Ac))\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{13/4}c^{3/4}}$$

$$= \frac{5(bB-9Ac)}{16b^3c\sqrt{x}} - \frac{bB-Ac}{4bc\sqrt{x}\,(b+cx^2)^2} - \frac{bB-9Ac}{16b^2c\sqrt{x}\,(b+cx^2)} - \frac{5(bB-9Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{13/4}c^{3/4}} + \frac{5(bB-9Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{13/4}c^{3/4}} + \frac{5(bB-9Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{13/4}c^{3/4}} - \frac{5(bB-9Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{13/4}c^{3/4}}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.59

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{-\frac{4\sqrt[4]{b}(-bBx^2(9b+5cx^2)+A(32b^2+81bcx^2+45c^2x^4))}{\sqrt{x}(b+cx^2)^2} + \frac{5\sqrt{2}(-bB+9Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{c^{3/4}} + \frac{5\sqrt{2}(-\cdots}{\cdots}}{64b^{13/4}}$$

```
[In] Integrate[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*b^(1/4)*(-(b*B*x^2*(9*b + 5*c*x^2)) + A*(32*b^2 + 81*b*c*x^2 + 45*c^2*
x^4)))/(Sqrt[x]*(b + c*x^2)^2) + (5*Sqrt[2]*(-(b*B) + 9*A*c)*ArcTan[(Sqrt[b
] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/c^(3/4) + (5*Sqrt[2]*(-(
b*B) + 9*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*
x)])/c^(3/4))/(64*b^(13/4))
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.55

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{13}{32}A\,c^2-\frac{5}{32}Bbc\right)x^{\frac{7}{2}}+\frac{b(17Ac-9Bb)x^{\frac{3}{2}}}{32}}{\left(c\,x^2+b\right)^2}+\frac{\left(\frac{45Ac}{32}-\frac{5Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{b^3}$
default	$-\frac{2\left(\frac{\left(\frac{13}{32}A\,c^2-\frac{5}{32}Bbc\right)x^{\frac{7}{2}}+\frac{b(17Ac-9Bb)x^{\frac{3}{2}}}{32}}{\left(c\,x^2+b\right)^2}+\frac{\left(\frac{45Ac}{32}-\frac{5Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{b^3}$
risch	$-\frac{2A}{b^3\sqrt{x}}-\frac{\frac{2\left(\frac{13}{32}A\,c^2-\frac{5}{32}Bbc\right)x^{\frac{7}{2}}+\frac{b(17Ac-9Bb)x^{\frac{3}{2}}}{16}}{\left(c\,x^2+b\right)^2}+\frac{\left(\frac{45Ac}{32}-\frac{5Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\right.}{4c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{b^3}$

```
[In] int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^3*(((13/32*A*c^2-5/32*B*b*c)*x^(7/2)+1/32*b*(17*A*c-9*B*b)*x^(3/2))/(c
*x^2+b)^2+1/8*(45/32*A*c-5/32*B*b)/c/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c*b)^(
1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b
)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b
)^(1/4)*x^(1/2)-1)))-2*A/b^3/x^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 870, normalized size of antiderivative = 2.75

$$\int \frac{x^{9/2}(A+Bx^2)}{\left(bx^2+cx^4\right)^3}\,dx =$$

$$-\frac{5\left(b^3c^2x^5+2\,b^4cx^3+b^5x\right)\left(-\frac{B^4b^4-36\,AB^3b^3c+486\,A^2B^2b^2c^2-2916\,A^3Bbc^3+6561\,A^4c^4}{b^{13}c^3}\right)^{\frac{1}{4}}\log\left(125\,b^{10}c^2\left(-\frac{B^4b^4-36\,AB^3b^3c\right.\right.}{}$$

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(5*(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)*(-(B^4*b^4 - 36*A*B^3*b^3*c +
486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(1/4)*lo
g(125*b^10*c^2*(-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3
```

```
*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(3/4) - 125*(B^3*b^3 - 27*A*B^2*b^2*c
+ 243*A^2*B*b*c^2 - 729*A^3*c^3)*sqrt(x)) + 5*(-I*b^3*c^2*x^5 - 2*I*b^4*c*x
^3 - I*b^5*x)*(-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*
B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(1/4)*log(125*I*b^10*c^2*(-(B^4*b^4 - 3
6*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^1
3*c^3))^(3/4) - 125*(B^3*b^3 - 27*A*B^2*b^2*c + 243*A^2*B*b*c^2 - 729*A^3*c
^3)*sqrt(x)) + 5*(I*b^3*c^2*x^5 + 2*I*b^4*c*x^3 + I*b^5*x)*(-(B^4*b^4 - 36*
A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*
c^3))^(1/4)*log(-125*I*b^10*c^2*(-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b
^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(3/4) - 125*(B^3*b^3
- 27*A*B^2*b^2*c + 243*A^2*B*b*c^2 - 729*A^3*c^3)*sqrt(x)) - 5*(b^3*c^2*x^5
 + 2*b^4*c*x^3 + b^5*x)*(-(B^4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 -
 2916*A^3*B*b*c^3 + 6561*A^4*c^4)/(b^13*c^3))^(1/4)*log(-125*b^10*c^2*(-(B^
4*b^4 - 36*A*B^3*b^3*c + 486*A^2*B^2*b^2*c^2 - 2916*A^3*B*b*c^3 + 6561*A^4*
c^4)/(b^13*c^3))^(3/4) - 125*(B^3*b^3 - 27*A*B^2*b^2*c + 243*A^2*B*b*c^2 -
729*A^3*c^3)*sqrt(x)) - 4*(5*(B*b*c - 9*A*c^2)*x^4 - 32*A*b^2 + 9*(B*b^2 -
9*A*b*c)*x^2)*sqrt(x))/(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.81

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{5\,(Bbc-9\,Ac^2)x^4-32\,Ab^2+9\,(Bb^2-9\,Abc)x^2}{16\left(b^3c^2x^{\frac{9}{2}}+2\,b^4cx^{\frac{5}{2}}+b^5\sqrt{x}\right)} + \frac{5\,(Bb-9\,Ac)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} + \frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\sqrt{b}\right)}{b^{\frac{1}{4}}c^{\frac{3}{4}}} + \right.}{128\,b^3}$$

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(5*(B*b*c - 9*A*c^2)*x^4 - 32*A*b^2 + 9*(B*b^2 - 9*A*b*c)*x^2)/(b^3*c^
2*x^(9/2) + 2*b^4*c*x^(5/2) + b^5*sqrt(x)) + 5/128*(B*b - 9*A*c)*(2*sqrt(2)
*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt
(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(
2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sq
rt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x)
+ sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(
1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.95

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{2\,A}{b^3\sqrt{x}} + \frac{5\,Bbcx^{\frac{7}{2}} - 13\,Ac^2x^{\frac{7}{2}} + 9\,Bb^2x^{\frac{3}{2}} - 17\,Abcx^{\frac{3}{2}}}{16\,(cx^2+b)^2b^3}$$
$$+ \frac{5\,\sqrt{2}\Big((bc^3)^{\frac{3}{4}}\,Bb - 9\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4c^3}$$
$$+ \frac{5\,\sqrt{2}\Big((bc^3)^{\frac{3}{4}}\,Bb - 9\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4c^3}$$
$$- \frac{5\,\sqrt{2}\Big((bc^3)^{\frac{3}{4}}\,Bb - 9\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{128\,b^4c^3}$$
$$+ \frac{5\,\sqrt{2}\Big((bc^3)^{\frac{3}{4}}\,Bb - 9\,(bc^3)^{\frac{3}{4}}\,Ac\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{128\,b^4c^3}$$

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -2*A/(b^3*sqrt(x)) + 1/16*(5*B*b*c*x^(7/2) - 13*A*c^2*x^(7/2) + 9*B*b^2*x^(
3/2) - 17*A*b*c*x^(3/2))/((c*x^2 + b)^2*b^3) + 5/64*sqrt(2)*((b*c^3)^(3/4)*
B*b - 9*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/(b^4*c^3) + 5/64*sqrt(2)*((b*c^3)^(3/4)*B*b - 9*(b*c^3)^(
3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4)
)/(b^4*c^3) - 5/128*sqrt(2)*((b*c^3)^(3/4)*B*b - 9*(b*c^3)^(3/4)*A*c)*log(s
qrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c^3) + 5/128*sqrt(2)*((b*c
^3)^(3/4)*B*b - 9*(b*c^3)^(3/4)*A*c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
 sqrt(b/c))/(b^4*c^3)
```

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.42

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{5\,\mathrm{atan}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(9\,A\,c-B\,b)}{32\,(-b)^{13/4}\,c^{3/4}} - \frac{\frac{2\,A}{b}+\frac{9\,x^2\,(9\,A\,c-B\,b)}{16\,b^2}+\frac{5\,c\,x^4\,(9\,A\,c-B\,b)}{16\,b^3}}{b^2\,\sqrt{x}+c^2\,x^{9/2}+2\,b\,c\,x^{5/2}} - \frac{5\,\mathrm{atanh}\left(\frac{c^{1/4}\,\sqrt{x}}{(-b)^{1/4}}\right)\,(9\,A\,c-B\,b)}{32\,(-b)^{13/4}\,c^{3/4}}$$

```
[In] int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (5*atan((c^(1/4)*x^(1/2))/(-b)^(1/4))*(9*A*c - B*b))/(32*(-b)^(13/4)*c^(3/4
)) - ((2*A)/b + (9*x^2*(9*A*c - B*b))/(16*b^2) + (5*c*x^4*(9*A*c - B*b))/(1
6*b^3))/(b^2*x^(1/2) + c^2*x^(9/2) + 2*b*c*x^(5/2)) - (5*atanh((c^(1/4)*x^(
1/2))/(-b)^(1/4))*(9*A*c - B*b))/(32*(-b)^(13/4)*c^(3/4))
```

3.215 $\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 322

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}(b+cx^2)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}(b+cx^2)} - \frac{7(3bB-11Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{15/4}\sqrt[4]{c}} + \frac{7(3bB-11Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{15/4}\sqrt[4]{c}} - \frac{7(3bB-11Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{15/4}\sqrt[4]{c}} + \frac{7(3bB-11Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{15/4}\sqrt[4]{c}}$$

```
[Out] 7/48*(-11*A*c+3*B*b)/b^3/c/x^(3/2)+1/4*(A*c-B*b)/b/c/x^(3/2)/(c*x^2+b)^2+1/
16*(11*A*c-3*B*b)/b^2/c/x^(3/2)/(c*x^2+b)-7/64*(-11*A*c+3*B*b)*arctan(1-c^(
1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)/c^(1/4)*2^(1/2)+7/64*(-11*A*c+3*B*b)
*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(15/4)/c^(1/4)*2^(1/2)-7/128*(
-11*A*c+3*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(15/
4)/c^(1/4)*2^(1/2)+7/128*(-11*A*c+3*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/
4)*2^(1/2)*x^(1/2))/b^(15/4)/c^(1/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1598, 468, 296, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{7(3bB-11Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{15/4}\sqrt[4]{c}} + \frac{7(3bB-11Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{15/4}\sqrt[4]{c}} - \frac{7(3bB-11Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{15/4}\sqrt[4]{c}} + \frac{7(3bB-11Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{15/4}\sqrt[4]{c}} + \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{3bB-11Ac}{16b^2cx^{3/2}(b+cx^2)} - \frac{bB-Ac}{4bcx^{3/2}(b+cx^2)^2}$$

```
[In] Int[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (7*(3*b*B - 11*A*c))/(48*b^3*c*x^(3/2)) - (b*B - A*c)/(4*b*c*x^(3/2)*(b + c
*x^2)^2) - (3*b*B - 11*A*c)/(16*b^2*c*x^(3/2)*(b + c*x^2)) - (7*(3*b*B - 11
*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(15/4)*c
^(1/4)) + (7*(3*b*B - 11*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
)/(32*Sqrt[2]*b^(15/4)*c^(1/4)) - (7*(3*b*B - 11*A*c)*Log[Sqrt[b] - Sqrt[2]
*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(15/4)*c^(1/4)) + (7*(
3*b*B - 11*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])
/(64*Sqrt[2]*b^(15/4)*c^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{A+Bx^2}{x^{5/2}\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} + \frac{\left(-\frac{3bB}{2}+\frac{11Ac}{2}\right)\int \frac{1}{x^{5/2}\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$= -\frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)} - \frac{(7(3bB-11Ac))\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{32b^2c}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)} + \frac{(7(3bB-11Ac))\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32b^3}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)} + \frac{(7(3bB-11Ac))\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^3}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)} + \frac{(7(3bB-11Ac))\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^{7/2}} + \frac{(7(3bB-11Ac))\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^{7/2}}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)} + \frac{(7(3bB-11Ac))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{64b^{7/2}\sqrt{c}} + \frac{(7(3bB-11Ac))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} + x^2}\,dx, x, \sqrt{x}\right)}{64b^{7/2}\sqrt{c}} - \frac{(7(3bB-11Ac))\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{b}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} - x^2}\,dx, x, \sqrt{x}\right)}{64\sqrt{2}b^{15/4}\sqrt[4]{c}} - \frac{(7(3bB-11Ac))\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}} - 2x}{-\frac{\sqrt{b}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}} - x^2}\,dx, x, \sqrt{x}\right)}{64\sqrt{2}b^{15/4}\sqrt[4]{c}}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)} - \frac{7(3bB-11Ac)\log\left(\sqrt{b} - \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{64\sqrt{2}b^{15/4}\sqrt[4]{c}} + \frac{7(3bB-11Ac)\log\left(\sqrt{b} + \sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{c}x\right)}{64\sqrt{2}b^{15/4}\sqrt[4]{c}} + \frac{(7(3bB-11Ac))\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{15/4}\sqrt[4]{c}} - \frac{(7(3bB-11Ac))\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{15/4}\sqrt[4]{c}}$$

$$= \frac{7(3bB-11Ac)}{48b^3cx^{3/2}} - \frac{bB-Ac}{4bcx^{3/2}\left(b+cx^2\right)^2} - \frac{3bB-11Ac}{16b^2cx^{3/2}\left(b+cx^2\right)} - \frac{7(3bB-11Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{15/4}\sqrt[4]{c}} + \frac{7(3bB-11Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{15/4}\sqrt[4]{c}} - \frac{7(3bB-11Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{15/4}\sqrt[4]{c}} + \frac{7(3bB-11Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{15/4}\sqrt[4]{c}}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.58

$$\int \frac{x^{7/2}(A+Bx^2)}{\left(bx^2+cx^4\right)^3}\,dx = \frac{-\frac{4b^{3/4}\left(-3bBx^2(11b+7cx^2)+A\left(32b^2+121bcx^2+77c^2x^4\right)\right)}{x^{3/2}\left(b+cx^2\right)^2} + \frac{21\sqrt{2}(-3bB+11Ac)\arctan\left(\frac{\sqrt{b}-\sqrt{c}x}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}}\right)}{\sqrt[4]{c}} +}{192b^{15/4}}$$

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*b^(3/4)*(-3*b*B*x^2*(11*b + 7*c*x^2) + A*(32*b^2 + 121*b*c*x^2 + 77*c^
2*x^4)))/(x^(3/2)*(b + c*x^2)^2) + (21*Sqrt[2]*(-3*b*B + 11*A*c)*ArcTan[(Sq
rt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])])/c^(1/4) + (21*Sqrt[2
]*(3*b*B - 11*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqr
t[c]*x)])/c^(1/4))/(192*b^(15/4))
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.54

method	result
derivativedivides	$-\frac{2A}{3b^{3}x^{\frac{3}{2}}}-\frac{2\left(\frac{\left(\frac{15}{32}A\,c^{2}-\frac{7}{32}Bbc\right)x^{\frac{5}{2}}+\frac{b(19Ac-11Bb)\sqrt{x}}{32}}{\left(c\,x^{2}+b\right)^{2}}+\frac{7(11Ac-3Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cdots}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\cdots\right.\right.}{256b}\cdots\right.}{b^{3}}$
default	$-\frac{2A}{3b^{3}x^{\frac{3}{2}}}-\frac{2\left(\frac{\left(\frac{15}{32}A\,c^{2}-\frac{7}{32}Bbc\right)x^{\frac{5}{2}}+\frac{b(19Ac-11Bb)\sqrt{x}}{32}}{\left(c\,x^{2}+b\right)^{2}}+\frac{7(11Ac-3Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cdots}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}\cdots\right.\right.}{256b}\cdots\right.}{b^{3}}$
risch	$-\frac{2A}{3b^{3}x^{\frac{3}{2}}}-\frac{\frac{2\left(\frac{15}{32}A\,c^{2}-\frac{7}{32}Bbc\right)x^{\frac{5}{2}}+\frac{b(19Ac-11Bb)\sqrt{x}}{16}}{\left(c\,x^{2}+b\right)^{2}}+\frac{7(11Ac-3Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+\cdots\right.\right.}{128b}\cdots}{b^{3}}$

```
[In] int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*A/b^3/x^(3/2)-2/b^3*(((15/32*A*c^2-7/32*B*b*c)*x^(5/2)+1/32*b*(19*A*c-
11*B*b)*x^(1/2))/(c*x^2+b)^2+7/256*(11*A*c-3*B*b)*(1/c*b)^(1/4)/b*2^(1/2)*(
ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)
*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan
(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 770, normalized size of antiderivative = 2.39

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx =$$

$$-\frac{21\left(b^{3}c^{2}x^{6}+2\,b^{4}cx^{4}+b^{5}x^{2}\right)\left(-\frac{81\,B^{4}b^{4}-1188\,AB^{3}b^{3}c+6534\,A^{2}B^{2}b^{2}c^{2}-15972\,A^{3}Bbc^{3}+14641\,A^{4}c^{4}}{b^{15}c}\right)^{\frac{1}{4}}\log\left(7\,b^{4}\left(-\frac{81\,B^{4}b^{4}-118\cdots}{\cdots}\right.\right.}{\cdots}$$

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(21*(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)*(-(81*B^4*b^4 - 1188*A*B^3
*b^3*c + 6534*A^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c)
)^(1/4)*log(7*b^4*(-(81*B^4*b^4 - 1188*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 -
```

```
 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c))^(1/4) - 7*(3*B*b - 11*A*c)*sq
rt(x)) + 21*(I*b^3*c^2*x^6 + 2*I*b^4*c*x^4 + I*b^5*x^2)*(-(81*B^4*b^4 - 118
8*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(
b^15*c))^(1/4)*log(7*I*b^4*(-(81*B^4*b^4 - 1188*A*B^3*b^3*c + 6534*A^2*B^2*
b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c))^(1/4) - 7*(3*B*b - 1
1*A*c)*sqrt(x)) + 21*(-I*b^3*c^2*x^6 - 2*I*b^4*c*x^4 - I*b^5*x^2)*(-(81*B^4
*b^4 - 1188*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*
A^4*c^4)/(b^15*c))^(1/4)*log(-7*I*b^4*(-(81*B^4*b^4 - 1188*A*B^3*b^3*c + 65
34*A^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c))^(1/4) - 7
*(3*B*b - 11*A*c)*sqrt(x)) - 21*(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)*(-(81
*B^4*b^4 - 1188*A*B^3*b^3*c + 6534*A^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14
641*A^4*c^4)/(b^15*c))^(1/4)*log(-7*b^4*(-(81*B^4*b^4 - 1188*A*B^3*b^3*c +
6534*A^2*B^2*b^2*c^2 - 15972*A^3*B*b*c^3 + 14641*A^4*c^4)/(b^15*c))^(1/4) -
 7*(3*B*b - 11*A*c)*sqrt(x)) - 4*(7*(3*B*b*c - 11*A*c^2)*x^4 - 32*A*b^2 + 1
1*(3*B*b^2 - 11*A*b*c)*x^2)*sqrt(x))/(b^3*c^2*x^6 + 2*b^4*c*x^4 + b^5*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.89

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{7\,(3\,Bbc-11\,Ac^2)x^4-32\,Ab^2+11\,(3\,Bb^2-11\,Abc)x^2}{48\left(b^3c^2x^{\frac{11}{2}}+2\,b^4cx^{\frac{7}{2}}+b^5x^{\frac{3}{2}}\right)} + \frac{7\left(\frac{2\sqrt{2}(3\,Bb-11\,Ac)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(3\,Bb-11\,Ac)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}(3\,Bb-11\,Ac)\log\left(\sqrt{2}\right.}{b^{\frac{3}{4}}c}\right.}{128\,b^3}$$

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
[Out] 1/48*(7*(3*B*b*c - 11*A*c^2)*x^4 - 32*A*b^2 + 11*(3*B*b^2 - 11*A*b*c)*x^2)/
(b^3*c^2*x^(11/2) + 2*b^4*c*x^(7/2) + b^5*x^(3/2)) + 7/128*(2*sqrt(2)*(3*B*
```

```
b - 11*A*c)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x)
)/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(3*B*b
 - 11*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x)
)/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(3*B*b -
 11*A*c)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4
)*c^(1/4)) - sqrt(2)*(3*B*b - 11*A*c)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x)
+ sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)))/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.94

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{7\sqrt{2}\left(3\,(bc^3)^{\frac{1}{4}}\,Bb-11\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4c} + \frac{7\sqrt{2}\left(3\,(bc^3)^{\frac{1}{4}}\,Bb-11\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4c} + \frac{7\sqrt{2}\left(3\,(bc^3)^{\frac{1}{4}}\,Bb-11\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^4c} - \frac{7\sqrt{2}\left(3\,(bc^3)^{\frac{1}{4}}\,Bb-11\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^4c} - \frac{2\,A}{3\,b^3x^{\frac{3}{2}}} + \frac{7\,Bbcx^{\frac{5}{2}}-15\,Ac^2x^{\frac{5}{2}}+11\,Bb^2\sqrt{x}-19\,Abc\sqrt{x}}{16\,(cx^2+b)^2b^3}$$

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 7/64*sqrt(2)*(3*(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt(2
)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c) + 7/64*sqrt(2)*(3*
(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c
)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^4*c) + 7/128*sqrt(2)*(3*(b*c^3)^(1/4)*
B*b - 11*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c)
)/(b^4*c) - 7/128*sqrt(2)*(3*(b*c^3)^(1/4)*B*b - 11*(b*c^3)^(1/4)*A*c)*log(
-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c) - 2/3*A/(b^3*x^(3/2))
 + 1/16*(7*B*b*c*x^(5/2) - 15*A*c^2*x^(5/2) + 11*B*b^2*sqrt(x) - 19*A*b*c*s
qrt(x))/((c*x^2 + b)^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 888, normalized size of antiderivative = 2.76

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Too large to display}$$

```
[In] int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] - ((2*A)/(3*b) + (11*x^2*(11*A*c - 3*B*b))/(48*b^2) + (7*c*x^4*(11*A*c - 3*
B*b))/(48*b^3))/(b^2*x^(3/2) + c^2*x^(11/2) + 2*b*c*x^(7/2)) - (atan((((11*
A*c - 3*B*b)*(x^(1/2)*(97140736*A^2*b^9*c^5 + 7225344*B^2*b^11*c^3 - 529858
56*A*B*b^10*c^4) - (7*(11*A*c - 3*B*b)*(80740352*A*b^13*c^4 - 22020096*B*b^
14*c^3))/(64*(-b)^(15/4)*c^(1/4)))*7i)/(64*(-b)^(15/4)*c^(1/4)) + ((11*A*c
- 3*B*b)*(x^(1/2)*(97140736*A^2*b^9*c^5 + 7225344*B^2*b^11*c^3 - 52985856*A
*B*b^10*c^4) + (7*(11*A*c - 3*B*b)*(80740352*A*b^13*c^4 - 22020096*B*b^14*c
^3))/(64*(-b)^(15/4)*c^(1/4)))*7i)/(64*(-b)^(15/4)*c^(1/4)))/((7*(11*A*c -
3*B*b)*(x^(1/2)*(97140736*A^2*b^9*c^5 + 7225344*B^2*b^11*c^3 - 52985856*A*B
*b^10*c^4) - (7*(11*A*c - 3*B*b)*(80740352*A*b^13*c^4 - 22020096*B*b^14*c^3
))/(64*(-b)^(15/4)*c^(1/4))))/(64*(-b)^(15/4)*c^(1/4)) - (7*(11*A*c - 3*B*b
)*(x^(1/2)*(97140736*A^2*b^9*c^5 + 7225344*B^2*b^11*c^3 - 52985856*A*B*b^10
*c^4) + (7*(11*A*c - 3*B*b)*(80740352*A*b^13*c^4 - 22020096*B*b^14*c^3))/(6
4*(-b)^(15/4)*c^(1/4))))/(64*(-b)^(15/4)*c^(1/4))))*(11*A*c - 3*B*b)*7i)/(3
2*(-b)^(15/4)*c^(1/4)) - (7*atan(((7*(11*A*c - 3*B*b)*(x^(1/2)*(97140736*A^
2*b^9*c^5 + 7225344*B^2*b^11*c^3 - 52985856*A*B*b^10*c^4) - ((11*A*c - 3*B*
b)*(80740352*A*b^13*c^4 - 22020096*B*b^14*c^3)*7i)/(64*(-b)^(15/4)*c^(1/4))
))/(64*(-b)^(15/4)*c^(1/4)) + (7*(11*A*c - 3*B*b)*(x^(1/2)*(97140736*A^2*b^
9*c^5 + 7225344*B^2*b^11*c^3 - 52985856*A*B*b^10*c^4) + ((11*A*c - 3*B*b)*(
80740352*A*b^13*c^4 - 22020096*B*b^14*c^3)*7i)/(64*(-b)^(15/4)*c^(1/4))))/(
64*(-b)^(15/4)*c^(1/4)))/(((11*A*c - 3*B*b)*(x^(1/2)*(97140736*A^2*b^9*c^5
+ 7225344*B^2*b^11*c^3 - 52985856*A*B*b^10*c^4) - ((11*A*c - 3*B*b)*(807403
52*A*b^13*c^4 - 22020096*B*b^14*c^3)*7i)/(64*(-b)^(15/4)*c^(1/4)))*7i)/(64*
(-b)^(15/4)*c^(1/4)) - ((11*A*c - 3*B*b)*(x^(1/2)*(97140736*A^2*b^9*c^5 + 7
225344*B^2*b^11*c^3 - 52985856*A*B*b^10*c^4) + ((11*A*c - 3*B*b)*(80740352*
A*b^13*c^4 - 22020096*B*b^14*c^3)*7i)/(64*(-b)^(15/4)*c^(1/4)))*7i)/(64*(-b
)^(15/4)*c^(1/4))))*(11*A*c - 3*B*b))/(32*(-b)^(15/4)*c^(1/4))
```

3.216 $\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 343

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{9(5bB-13Ac)}{80b^3cx^{5/2}} - \frac{9(5bB-13Ac)}{16b^4\sqrt{x}} - \frac{bB-Ac}{4bcx^{5/2}(b+cx^2)^2} - \frac{5bB-13Ac}{16b^2cx^{5/2}(b+cx^2)} + \frac{9\sqrt[4]{c}(5bB-13Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{17/4}} - \frac{9\sqrt[4]{c}(5bB-13Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{17/4}} - \frac{9\sqrt[4]{c}(5bB-13Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}} + \frac{9\sqrt[4]{c}(5bB-13Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}}$$

```
[Out] 9/80*(-13*A*c+5*B*b)/b^3/c/x^(5/2)+1/4*(A*c-B*b)/b/c/x^(5/2)/(c*x^2+b)^2+1/
16*(13*A*c-5*B*b)/b^2/c/x^(5/2)/(c*x^2+b)+9/64*c^(1/4)*(-13*A*c+5*B*b)*arct
an(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)-9/64*c^(1/4)*(-13*A*
c+5*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(17/4)*2^(1/2)-9/128*c
^(1/4)*(-13*A*c+5*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2)
)/b^(17/4)*2^(1/2)+9/128*c^(1/4)*(-13*A*c+5*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/
4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(17/4)*2^(1/2)-9/16*(-13*A*c+5*B*b)/b^4/x^(1/
2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1598, 468, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{9\sqrt[4]{c}(5bB-13Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{17/4}} - \frac{9\sqrt[4]{c}(5bB-13Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{17/4}} - \frac{9\sqrt[4]{c}(5bB-13Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}} + \frac{9\sqrt[4]{c}(5bB-13Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}} - \frac{9(5bB-13Ac)}{16b^4\sqrt{x}} + \frac{9(5bB-13Ac)}{80b^3cx^{5/2}} - \frac{5bB-13Ac}{16b^2cx^{5/2}(b+cx^2)} - \frac{bB-Ac}{4bcx^{5/2}(b+cx^2)^2}$$

```
[In] Int[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (9*(5*b*B - 13*A*c))/(80*b^3*c*x^(5/2)) - (9*(5*b*B - 13*A*c))/(16*b^4*Sqrt
[x]) - (b*B - A*c)/(4*b*c*x^(5/2)*(b + c*x^2)^2) - (5*b*B - 13*A*c)/(16*b^2
*c*x^(5/2)*(b + c*x^2)) + (9*c^(1/4)*(5*b*B - 13*A*c)*ArcTan[1 - (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(17/4)) - (9*c^(1/4)*(5*b*B - 13*A*
c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(17/4)) - (
9*c^(1/4)*(5*b*B - 13*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
Sqrt[c]*x])/(64*Sqrt[2]*b^(17/4)) + (9*c^(1/4)*(5*b*B - 13*A*c)*Log[Sqrt[b]
 + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(17/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{A+Bx^2}{x^{7/2}\left(b+cx^2\right)^3}\,dx$$

$$= -\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} + \frac{\left(-\frac{5bB}{2}+\frac{13Ac}{2}\right)\int \frac{1}{x^{7/2}\left(b+cx^2\right)^2}\,dx}{4bc}$$

$$= -\frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} - \frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)} - \frac{(9(5bB-13Ac))\int \frac{1}{x^{7/2}\left(b+cx^2\right)}\,dx}{32b^2c}$$

$$= \frac{9(5bB-13Ac)}{80b^3cx^{5/2}} - \frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} - \frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)} + \frac{(9(5bB-13Ac))\int \frac{1}{x^{3/2}\left(b+cx^2\right)}\,dx}{32b^3}$$

$$= \frac{9(5bB-13Ac)}{80b^3cx^{5/2}} - \frac{9(5bB-13Ac)}{16b^4\sqrt{x}} - \frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} - \frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)} - \frac{(9c(5bB-13Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32b^4}$$

$$= \frac{9(5bB-13Ac)}{80b^3cx^{5/2}} - \frac{9(5bB-13Ac)}{16b^4\sqrt{x}} - \frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} - \frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)} - \frac{(9c(5bB-13Ac))\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^4}$$

$$=\frac{9(5bB-13Ac)}{80b^3cx^{5/2}}-\frac{9(5bB-13Ac)}{16b^4\sqrt{x}}-\frac{bB-Ac}{4bcx^{5/2}(b+cx^2)^2}-\frac{5bB-13Ac}{16b^2cx^{5/2}(b+cx^2)}+\frac{\left(9\sqrt{c}(5bB-13Ac)\right)\text{Subst}\left(\int\frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{32b^4}-\frac{\left(9\sqrt{c}(5bB-13Ac)\right)\text{Subst}\left(\int\frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{32b^4}$$

$$=\frac{9(5bB-13Ac)}{80b^3cx^{5/2}}-\frac{9(5bB-13Ac)}{16b^4\sqrt{x}}-\frac{bB-Ac}{4bcx^{5/2}(b+cx^2)^2}-\frac{5bB-13Ac}{16b^2cx^{5/2}(b+cx^2)}-\frac{(9(5bB-13Ac))\text{Subst}\left(\int\frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64b^4}-\frac{(9(5bB-13Ac))\text{Subst}\left(\int\frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64b^4}-\frac{\left(9\sqrt[4]{c}(5bB-13Ac)\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{64\sqrt{2}b^{17/4}}-\frac{\left(9\sqrt[4]{c}(5bB-13Ac)\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{64\sqrt{2}b^{17/4}}$$

$$=\frac{9(5bB-13Ac)}{80b^3cx^{5/2}}-\frac{9(5bB-13Ac)}{16b^4\sqrt{x}}-\frac{bB-Ac}{4bcx^{5/2}(b+cx^2)^2}-\frac{5bB-13Ac}{16b^2cx^{5/2}(b+cx^2)}-\frac{9\sqrt[4]{c}(5bB-13Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}}+\frac{9\sqrt[4]{c}(5bB-13Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}}-\frac{\left(9\sqrt[4]{c}(5bB-13Ac)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{17/4}}+\frac{\left(9\sqrt[4]{c}(5bB-13Ac)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{17/4}}$$

$$= \frac{9(5bB-13Ac)}{80b^3cx^{5/2}} - \frac{9(5bB-13Ac)}{16b^4\sqrt{x}} - \frac{bB-Ac}{4bcx^{5/2}\left(b+cx^2\right)^2} - \frac{5bB-13Ac}{16b^2cx^{5/2}\left(b+cx^2\right)} + \frac{9\sqrt[4]{c}(5bB-13Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{17/4}} - \frac{9\sqrt[4]{c}(5bB-13Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{17/4}} - \frac{9\sqrt[4]{c}(5bB-13Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}} + \frac{9\sqrt[4]{c}(5bB-13Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}}$$

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.61

$$\int \frac{x^{5/2}(A+Bx^2)}{\left(bx^2+cx^4\right)^3}\,dx = \frac{-\frac{4\sqrt[4]{b}\left(5bBx^2\left(32b^2+81bcx^2+45c^2x^4\right)+A\left(32b^3-416b^2cx^2-1053bc^2x^4-585c^3x^6\right)\right)}{x^{5/2}\left(b+cx^2\right)^2} + 45\sqrt{2}\sqrt[4]{c}(5bB-13$$
$$320b^{17/4}$$

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*b^(1/4)*(5*b*B*x^2*(32*b^2 + 81*b*c*x^2 + 45*c^2*x^4) + A*(32*b^3 - 41
6*b^2*c*x^2 - 1053*b*c^2*x^4 - 585*c^3*x^6)))/(x^(5/2)*(b + c*x^2)^2) + 45*
Sqrt[2]*c^(1/4)*(5*b*B - 13*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x])] + 45*Sqrt[2]*c^(1/4)*(5*b*B - 13*A*c)*ArcTanh[(Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(320*b^(17/4))
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.55

method	result
derivativedivides	$\frac{2c\left(\frac{\left(\frac{21}{32}Ac^2-\frac{13}{32}Bbc\right)x^{\frac{7}{2}}+\frac{b(25Ac-17Bb)x^{\frac{3}{2}}}{32}}{(cx^2+b)^2}+\frac{\left(\frac{117Ac}{32}-\frac{45Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{b^4}$
default	$\frac{2c\left(\frac{\left(\frac{21}{32}Ac^2-\frac{13}{32}Bbc\right)x^{\frac{7}{2}}+\frac{b(25Ac-17Bb)x^{\frac{3}{2}}}{32}}{(cx^2+b)^2}+\frac{\left(\frac{117Ac}{32}-\frac{45Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{b^4}$
risch	$-\frac{2(-15Acx^2+5bBx^2+Ab)}{5b^4x^{\frac{5}{2}}}+\frac{c\left(\frac{2\left(\frac{21}{32}Ac^2-\frac{13}{32}Bbc\right)x^{\frac{7}{2}}+\frac{b(25Ac-17Bb)x^{\frac{3}{2}}}{16}}{(cx^2+b)^2}+\frac{\left(\frac{117Ac}{32}-\frac{45Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right.\right.}{4c}\right.}{b^4}$

```
[In] int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*c*(((21/32*A*c^2-13/32*B*b*c)*x^(7/2)+1/32*b*(25*A*c-17*B*b)*x^(3/2))
/(c*x^2+b)^2+1/8*(117/32*A*c-45/32*B*b)/c/(1/c*b)^(1/4)*2^(1/2)*(ln((x-(1/c
*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(
1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(
1/c*b)^(1/4)*x^(1/2)-1)))-2/5*A/b^3/x^(5/2)-2*(-3*A*c+B*b)/b^4/x^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 918, normalized size of antiderivative = 2.68

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{45\left(b^4c^2x^7+2\,b^5cx^5+b^6x^3\right)\left(-\frac{625\,B^4b^4c-6500\,AB^3b^3c^2+25350\,A^2B^2b^2c^3-43940\,A^3Bbc^4+28561\,A^4c^5}{b^{17}}\right.}{}$$

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/320*(45*(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-(625*B^4*b^4*c - 6500*A*B
^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^1
7)^(1/4)*log(729*b^13*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2
*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(3/4) - 729*(125*B^3*b^
```

```
3*c - 975*A*B^2*b^2*c^2 + 2535*A^2*B*b*c^3 - 2197*A^3*c^4)*sqrt(x)) - 45*(I
*b^4*c^2*x^7 + 2*I*b^5*c*x^5 + I*b^6*x^3)*(-(625*B^4*b^4*c - 6500*A*B^3*b^3
*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(1/
4)*log(729*I*b^13*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2
*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(3/4) - 729*(125*B^3*b^3*c
- 975*A*B^2*b^2*c^2 + 2535*A^2*B*b*c^3 - 2197*A^3*c^4)*sqrt(x)) - 45*(-I*b^
4*c^2*x^7 - 2*I*b^5*c*x^5 - I*b^6*x^3)*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^
2 + 25350*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(1/4)*
log(-729*I*b^13*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c
^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(3/4) - 729*(125*B^3*b^3*c -
975*A*B^2*b^2*c^2 + 2535*A^2*B*b*c^3 - 2197*A^3*c^4)*sqrt(x)) - 45*(b^4*c^2
*x^7 + 2*b^5*c*x^5 + b^6*x^3)*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350
*A^2*B^2*b^2*c^3 - 43940*A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(1/4)*log(-729*
b^13*(-(625*B^4*b^4*c - 6500*A*B^3*b^3*c^2 + 25350*A^2*B^2*b^2*c^3 - 43940*
A^3*B*b*c^4 + 28561*A^4*c^5)/b^17)^(3/4) - 729*(125*B^3*b^3*c - 975*A*B^2*b
^2*c^2 + 2535*A^2*B*b*c^3 - 2197*A^3*c^4)*sqrt(x)) - 4*(45*(5*B*b*c^2 - 13*
A*c^3)*x^6 + 81*(5*B*b^2*c - 13*A*b*c^2)*x^4 + 32*A*b^3 + 32*(5*B*b^3 - 13*
A*b^2*c)*x^2)*sqrt(x))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.83

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx =$$

$$-\frac{45\,(5\,Bbc^2-13\,Ac^3)x^6+81\,(5\,Bb^2c-13\,Abc^2)x^4+32\,Ab^3+32\,(5\,Bb^3-13\,Ab^2c)x^2}{80\left(b^4c^2x^{\frac{13}{2}}+2\,b^5cx^{\frac{9}{2}}+b^6x^{\frac{5}{2}}\right)}$$

$$-\frac{9\,(5\,Bbc-13\,Ac^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{c}x+\right.}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right.}{128\,b^4}$$

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/80*(45*(5*B*b*c^2 - 13*A*c^3)*x^6 + 81*(5*B*b^2*c - 13*A*b*c^2)*x^4 + 32
*A*b^3 + 32*(5*B*b^3 - 13*A*b^2*c)*x^2)/(b^4*c^2*x^(13/2) + 2*b^5*c*x^(9/2)
 + b^6*x^(5/2)) - 9/128*(5*B*b*c - 13*A*c^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*
(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(
sqrt(b)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*
c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*
sqrt(c)) - sqrt(2)*log(sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b
))/(b^(1/4)*c^(3/4)) + sqrt(2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(
c)*x + sqrt(b))/(b^(1/4)*c^(3/4)))/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.95

$$
\begin{aligned}
&\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \\
&-\frac{9\sqrt{2}\left(5\,(bc^3)^{\frac{3}{4}}\,Bb-13\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5c^2} \\
&-\frac{9\sqrt{2}\left(5\,(bc^3)^{\frac{3}{4}}\,Bb-13\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5c^2} \\
&+\frac{9\sqrt{2}\left(5\,(bc^3)^{\frac{3}{4}}\,Bb-13\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^5c^2} \\
&-\frac{9\sqrt{2}\left(5\,(bc^3)^{\frac{3}{4}}\,Bb-13\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^5c^2} \\
&-\frac{13\,Bbc^2x^{\frac{7}{2}}-21\,Ac^3x^{\frac{7}{2}}+17\,Bb^2cx^{\frac{3}{2}}-25\,Abc^2x^{\frac{3}{2}}}{16\,(cx^2+b)^2b^4}-\frac{2\,(5\,Bbx^2-15\,Acx^2+Ab)}{5\,b^4x^{\frac{5}{2}}}
\end{aligned}
$$

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -9/64*sqrt(2)*(5*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(
2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^5*c^2) - 9/64*sqrt(2)*
(5*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^5*c^2) + 9/128*sqrt(2)*(5*(b*c^3)^(
3/4)*B*b - 13*(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt
(b/c))/(b^5*c^2) - 9/128*sqrt(2)*(5*(b*c^3)^(3/4)*B*b - 13*(b*c^3)^(3/4)*A*
```

```
c)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^5*c^2) - 1/16*(13*B
*b*c^2*x^(7/2) - 21*A*c^3*x^(7/2) + 17*B*b^2*c*x^(3/2) - 25*A*b*c^2*x^(3/2)
)/((c*x^2 + b)^2*b^4) - 2/5*(5*B*b*x^2 - 15*A*c*x^2 + A*b)/(b^4*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.44

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{2\,x^2\,(13\,A\,c-5\,B\,b)}{5\,b^2}-\frac{2\,A}{5\,b}+\frac{9\,c^2\,x^6\,(13\,A\,c-5\,B\,b)}{16\,b^4}+\frac{81\,c\,x^4\,(13\,A\,c-5\,B\,b)}{80\,b^3}}{b^2\,x^{5/2}+c^2\,x^{13/2}+2\,b\,c\,x^{9/2}} + \frac{9\,(-c)^{1/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(13\,A\,c-5\,B\,b)}{32\,b^{17/4}} - \frac{9\,(-c)^{1/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(13\,A\,c-5\,B\,b)}{32\,b^{17/4}}$$

```
[In] int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((2*x^2*(13*A*c - 5*B*b))/(5*b^2) - (2*A)/(5*b) + (9*c^2*x^6*(13*A*c - 5*B*
b))/(16*b^4) + (81*c*x^4*(13*A*c - 5*B*b))/(80*b^3))/(b^2*x^(5/2) + c^2*x^(
13/2) + 2*b*c*x^(9/2)) + (9*(-c)^(1/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4))*(
13*A*c - 5*B*b))/(32*b^(17/4)) - (9*(-c)^(1/4)*atanh(((-c)^(1/4)*x^(1/2))/b
^(1/4))*(13*A*c - 5*B*b))/(32*b^(17/4))
```

3.217 $\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3} dx$

Optimal result

Integrand size = 26, antiderivative size = 343

$$\begin{aligned}\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx &= \frac{11(7bB-15Ac)}{112b^3cx^{7/2}} - \frac{11(7bB-15Ac)}{48b^4x^{3/2}} - \frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2} \\ &- \frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)} + \frac{11c^{3/4}(7bB-15Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{19/4}} \\ &- \frac{11c^{3/4}(7bB-15Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{19/4}} \\ &+ \frac{11c^{3/4}(7bB-15Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}} \\ &- \frac{11c^{3/4}(7bB-15Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}}\end{aligned}$$

```
[Out] 11/112*(-15*A*c+7*B*b)/b^3/c/x^(7/2)-11/48*(-15*A*c+7*B*b)/b^4/x^(3/2)+1/4*
(A*c-B*b)/b/c/x^(7/2)/(c*x^2+b)^2+1/16*(15*A*c-7*B*b)/b^2/c/x^(7/2)/(c*x^2+
b)+11/64*c^(3/4)*(-15*A*c+7*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/
b^(19/4)*2^(1/2)-11/64*c^(3/4)*(-15*A*c+7*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(
1/2)/b^(1/4))/b^(19/4)*2^(1/2)+11/128*c^(3/4)*(-15*A*c+7*B*b)*ln(b^(1/2)+x*
c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(19/4)*2^(1/2)-11/128*c^(3/4)*(-
15*A*c+7*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(19/4
)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1598, 468, 296, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{11c^{3/4}(7bB-15Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{19/4}} - \frac{11c^{3/4}(7bB-15Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{19/4}} + \frac{11c^{3/4}(7bB-15Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}} - \frac{11c^{3/4}(7bB-15Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}} - \frac{11(7bB-15Ac)}{48b^4x^{3/2}} + \frac{11(7bB-15Ac)}{112b^3cx^{7/2}} - \frac{7bB-15Ac}{16b^2cx^{7/2}(b+cx^2)} - \frac{bB-Ac}{4bcx^{7/2}(b+cx^2)^2}$$

```
[In] Int[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (11*(7*b*B - 15*A*c))/(112*b^3*c*x^(7/2)) - (11*(7*b*B - 15*A*c))/(48*b^4*x
^(3/2)) - (b*B - A*c)/(4*b*c*x^(7/2)*(b + c*x^2)^2) - (7*b*B - 15*A*c)/(16*
b^2*c*x^(7/2)*(b + c*x^2)) + (11*c^(3/4)*(7*b*B - 15*A*c)*ArcTan[1 - (Sqrt[
2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(19/4)) - (11*c^(3/4)*(7*b*B -
15*A*c)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(19/4)
) + (11*c^(3/4)*(7*b*B - 15*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt
[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(19/4)) - (11*c^(3/4)*(7*b*B - 15*A*c)*Log[
Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(19/4
))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{x^{9/2}\left(b+cx^2\right)^3}\,dx \\
&= -\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}+\frac{\left(-\frac{7bB}{2}+\frac{15Ac}{2}\right)\int \frac{1}{x^{9/2}\left(b+cx^2\right)^2}\,dx}{4bc} \\
&= -\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}-\frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)}-\frac{(11(7bB-15Ac))\int \frac{1}{x^{9/2}\left(b+cx^2\right)}\,dx}{32b^2c} \\
&= \frac{11(7bB-15Ac)}{112b^3cx^{7/2}}-\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2} \\
&\quad -\frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)}+\frac{(11(7bB-15Ac))\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{32b^3} \\
&= \frac{11(7bB-15Ac)}{112b^3cx^{7/2}}-\frac{11(7bB-15Ac)}{48b^4x^{3/2}}-\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2} \\
&\quad -\frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)}-\frac{(11c(7bB-15Ac))\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32b^4} \\
&= \frac{11(7bB-15Ac)}{112b^3cx^{7/2}}-\frac{11(7bB-15Ac)}{48b^4x^{3/2}}-\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2} \\
&\quad -\frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)}-\frac{(11c(7bB-15Ac))\,\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{16b^4}
\end{aligned}$$

$$=\frac{11(7bB-15Ac)}{112b^3cx^{7/2}}-\frac{11(7bB-15Ac)}{48b^4x^{3/2}}-\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}-\frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)}-\frac{(11c(7bB-15Ac))\text{Subst}\left(\int\frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{32b^{9/2}}-\frac{(11c(7bB-15Ac))\text{Subst}\left(\int\frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{32b^{9/2}}$$

$$=\frac{11(7bB-15Ac)}{112b^3cx^{7/2}}-\frac{11(7bB-15Ac)}{48b^4x^{3/2}}-\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}-\frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)}-\frac{\left(11\sqrt{c}(7bB-15Ac)\right)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64b^{9/2}}-\frac{\left(11\sqrt{c}(7bB-15Ac)\right)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64b^{9/2}}+\frac{\left(11c^{3/4}(7bB-15Ac)\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{64\sqrt{2}b^{19/4}}+\frac{\left(11c^{3/4}(7bB-15Ac)\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{64\sqrt{2}b^{19/4}}$$

$$=\frac{11(7bB-15Ac)}{112b^3cx^{7/2}}-\frac{11(7bB-15Ac)}{48b^4x^{3/2}}-\frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}-\frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)}+\frac{11c^{3/4}(7bB-15Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}}-\frac{11c^{3/4}(7bB-15Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}}-\frac{\left(11c^{3/4}(7bB-15Ac)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{19/4}}+\frac{\left(11c^{3/4}(7bB-15Ac)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{19/4}}$$

$$= \frac{11(7bB-15Ac)}{112b^3cx^{7/2}} - \frac{11(7bB-15Ac)}{48b^4x^{3/2}} - \frac{bB-Ac}{4bcx^{7/2}\left(b+cx^2\right)^2}$$
$$- \frac{7bB-15Ac}{16b^2cx^{7/2}\left(b+cx^2\right)} + \frac{11c^{3/4}(7bB-15Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{19/4}}$$
$$- \frac{11c^{3/4}(7bB-15Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{19/4}}$$
$$+ \frac{11c^{3/4}(7bB-15Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}}$$
$$- \frac{11c^{3/4}(7bB-15Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}}$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.61

$$\int \frac{x^{3/2}(A+Bx^2)}{\left(bx^2+cx^4\right)^3}\,dx = \frac{-\frac{4b^{3/4}\left(7bBx^2\left(32b^2+121bcx^2+77c^2x^4\right)+3A\left(32b^3-160b^2cx^2-605bc^2x^4-385c^3x^6\right)\right)}{x^{7/2}\left(b+cx^2\right)^2}+231\sqrt{2}c^{3/4}(7bB-}{1344b}$$

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*b^(3/4)*(7*b*B*x^2*(32*b^2 + 121*b*c*x^2 + 77*c^2*x^4) + 3*A*(32*b^3 -
 160*b^2*c*x^2 - 605*b*c^2*x^4 - 385*c^3*x^6)))/(x^(7/2)*(b + c*x^2)^2) + 2
31*Sqrt[2]*c^(3/4)*(7*b*B - 15*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x])] + 231*Sqrt[2]*c^(3/4)*(-7*b*B + 15*A*c)*ArcTanh[(S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])/(Sqrt[b] + Sqrt[c]*x)])/(1344*b^(19/4))
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.55

method	result
derivativedivides	$-\frac{2A}{7b^3x^{\frac{7}{2}}}-\frac{2(-3Ac+Bb)}{3b^4x^{\frac{3}{2}}}+\frac{2c\left(\frac{\left(\frac{23}{32}A\,c^2-\frac{15}{32}Bbc\right)x^{\frac{5}{2}}+\frac{b(27Ac-19Bb)\sqrt{x}}{32}}{\left(c\,x^2+b\right)^2}+\frac{11(15Ac-7Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{}}\right.\right.}{}\right.}{b^4}$
default	$-\frac{2A}{7b^3x^{\frac{7}{2}}}-\frac{2(-3Ac+Bb)}{3b^4x^{\frac{3}{2}}}+\frac{2c\left(\frac{\left(\frac{23}{32}A\,c^2-\frac{15}{32}Bbc\right)x^{\frac{5}{2}}+\frac{b(27Ac-19Bb)\sqrt{x}}{32}}{\left(c\,x^2+b\right)^2}+\frac{11(15Ac-7Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{}}\right.\right.}{}\right.}{b^4}$
risch	$-\frac{2(-21Ac\,x^2+7bB\,x^2+3Ab)}{21b^4x^{\frac{7}{2}}}+\frac{c\left(\frac{2\left(\frac{23}{32}A\,c^2-\frac{15}{32}Bbc\right)x^{\frac{5}{2}}+\frac{b(27Ac-19Bb)\sqrt{x}}{16}}{\left(c\,x^2+b\right)^2}+\frac{11(15Ac-7Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{}}\right.\right.}{}\right.}{b^4}$

```
[In] int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/7*A/b^3/x^(7/2)-2/3*(-3*A*c+B*b)/b^4/x^(3/2)+2/b^4*c*(((23/32*A*c^2-15/3
2*B*b*c)*x^(5/2)+1/32*b*(27*A*c-19*B*b)*x^(1/2))/(c*x^2+b)^2+11/256*(15*A*c
-7*B*b)*(1/c*b)^(1/4)/b*2^(1/2)*(ln((x+(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b
)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/
(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 822, normalized size of antiderivative = 2.40

$$\int \frac{x^{3/2}(A+Bx^2)}{\left(bx^2+cx^4\right)^3}\,dx=\frac{231\left(b^4c^2x^8+2\,b^5cx^6+b^6x^4\right)\left(-\frac{2401\,B^4b^4c^3-20580\,AB^3b^3c^4+66150\,A^2B^2b^2c^5-94500\,A^3Bbc^6+50625\,}{b^{19}}\right.}{}$$

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/1344*(231*(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-(2401*B^4*b^4*c^3 - 205
80*A*B^3*b^3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^
7)/b^19)^(1/4)*log(11*b^5*(-(2401*B^4*b^4*c^3 - 20580*A*B^3*b^3*c^4 + 66150
*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^7)/b^19)^(1/4) - 11*(7*B
```

```
*b*c - 15*A*c^2)*sqrt(x)) - 231*(-I*b^4*c^2*x^8 - 2*I*b^5*c*x^6 - I*b^6*x^4
)*(-(2401*B^4*b^4*c^3 - 20580*A*B^3*b^3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500
*A^3*B*b*c^6 + 50625*A^4*c^7)/b^19)^(1/4)*log(11*I*b^5*(-(2401*B^4*b^4*c^3
- 20580*A*B^3*b^3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A
^4*c^7)/b^19)^(1/4) - 11*(7*B*b*c - 15*A*c^2)*sqrt(x)) - 231*(I*b^4*c^2*x^8
 + 2*I*b^5*c*x^6 + I*b^6*x^4)*(-(2401*B^4*b^4*c^3 - 20580*A*B^3*b^3*c^4 + 6
6150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^7)/b^19)^(1/4)*log(-
11*I*b^5*(-(2401*B^4*b^4*c^3 - 20580*A*B^3*b^3*c^4 + 66150*A^2*B^2*b^2*c^5
- 94500*A^3*B*b*c^6 + 50625*A^4*c^7)/b^19)^(1/4) - 11*(7*B*b*c - 15*A*c^2)*
sqrt(x)) - 231*(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)*(-(2401*B^4*b^4*c^3 -
20580*A*B^3*b^3*c^4 + 66150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4
*c^7)/b^19)^(1/4)*log(-11*b^5*(-(2401*B^4*b^4*c^3 - 20580*A*B^3*b^3*c^4 + 6
6150*A^2*B^2*b^2*c^5 - 94500*A^3*B*b*c^6 + 50625*A^4*c^7)/b^19)^(1/4) - 11*
(7*B*b*c - 15*A*c^2)*sqrt(x)) - 4*(77*(7*B*b*c^2 - 15*A*c^3)*x^6 + 121*(7*B
*b^2*c - 15*A*b*c^2)*x^4 + 96*A*b^3 + 32*(7*B*b^3 - 15*A*b^2*c)*x^2)*sqrt(x
))/(b^4*c^2*x^8 + 2*b^5*c*x^6 + b^6*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.94

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{77\,(7\,Bbc^2-15\,Ac^3)x^6+121\,(7\,Bb^2c-15\,Abc^2)x^4+96\,Ab^3+32\,(7\,Bb^3-15\,Ab^2c)x^2}{336\left(b^4c^2x^{\frac{15}{2}}+2\,b^5cx^{\frac{11}{2}}+b^6x^{\frac{7}{2}}\right)} - \frac{11\left(\frac{2\sqrt{2}(7\,Bbc-15\,Ac^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(7\,Bbc-15\,Ac^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}(7\,Bbc-15\,Ac}{}\right.}{128\,b^4}$$

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/336*(77*(7*B*b*c^2 - 15*A*c^3)*x^6 + 121*(7*B*b^2*c - 15*A*b*c^2)*x^4 +
96*A*b^3 + 32*(7*B*b^3 - 15*A*b^2*c)*x^2)/(b^4*c^2*x^(15/2) + 2*b^5*c*x^(11
/2) + b^6*x^(7/2)) - 11/128*(2*sqrt(2)*(7*B*b*c - 15*A*c^2)*arctan(1/2*sqrt
(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(s
qrt(b)*sqrt(sqrt(b)*sqrt(c))) + 2*sqrt(2)*(7*B*b*c - 15*A*c^2)*arctan(-1/2*
sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c))
)/(sqrt(b)*sqrt(sqrt(b)*sqrt(c))) + sqrt(2)*(7*B*b*c - 15*A*c^2)*log(sqrt(2
)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2
)*(7*B*b*c - 15*A*c^2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + s
qrt(b))/(b^(3/4)*c^(1/4)))/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.92

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{11\sqrt{2}\left(7\,(bc^3)^{\frac{1}{4}}\,Bb-15\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5} - \frac{11\sqrt{2}\left(7\,(bc^3)^{\frac{1}{4}}\,Bb-15\,(bc^3)^{\frac{1}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5} - \frac{11\sqrt{2}\left(7\,(bc^3)^{\frac{1}{4}}\,Bb-15\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^5} + \frac{11\sqrt{2}\left(7\,(bc^3)^{\frac{1}{4}}\,Bb-15\,(bc^3)^{\frac{1}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^5} - \frac{15\,Bbc^2x^{\frac{5}{2}}-23\,Ac^3x^{\frac{5}{2}}+19\,Bb^2c\sqrt{x}-27\,Abc^2\sqrt{x}}{16\,(cx^2+b)^2b^4} - \frac{2\,(7\,Bbx^2-21\,Acx^2+3\,Ab)}{21\,b^4x^{\frac{7}{2}}}$$

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] -11/64*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 15*(b*c^3)^(1/4)*A*c)*arctan(1/2*sqrt
(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^5 - 11/64*sqrt(2)*(7*(
b*c^3)^(1/4)*B*b - 15*(b*c^3)^(1/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)
```

```
^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^5 - 11/128*sqrt(2)*(7*(b*c^3)^(1/4)*B*b
- 15*(b*c^3)^(1/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^
5 + 11/128*sqrt(2)*(7*(b*c^3)^(1/4)*B*b - 15*(b*c^3)^(1/4)*A*c)*log(-sqrt(2
)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^5 - 1/16*(15*B*b*c^2*x^(5/2) - 23*
A*c^3*x^(5/2) + 19*B*b^2*c*sqrt(x) - 27*A*b*c^2*sqrt(x))/((c*x^2 + b)^2*b^4
) - 2/21*(7*B*b*x^2 - 21*A*c*x^2 + 3*A*b)/(b^4*x^(7/2))
```

Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 626, normalized size of antiderivative = 1.83

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{\frac{2\,x^2\,(15\,A\,c-7\,B\,b)}{21\,b^2}-\frac{2\,A}{7\,b}+\frac{11\,c^2\,x^6\,(15\,A\,c-7\,B\,b)}{48\,b^4}+\frac{121\,c\,x^4\,(15\,A\,c-7\,B\,b)}{336\,b^3}}{b^2\,x^{7/2}+c^2\,x^{15/2}+2\,b\,c\,x^{11/2}}$$

$$+\frac{11\,(-c)^{3/4}\,\mathrm{atan}\left(\frac{\frac{11\,(-c)^{3/4}\,(15\,A\,c-7\,B\,b)\left(\sqrt{x}\left(446054400\,A^2\,b^{12}\,c^7-416317440\,A\,B\,b^{13}\,c^6+97140736\,B^2\,b^{14}\,c^5\right)-\frac{(-c)^{3/4}\,(15\,A\,c-7\,B\,b)\left(173015040\right.}{64\,b^{19}}\right.}{64\,b^{19/4}}}{\frac{(-c)^{3/4}\,(15\,A\,c-7\,B\,b)\left(\sqrt{x}\left(446054400\,A^2\,b^{12}\,c^7-416317440\,A\,B\,b^{13}\,c^6+97140736\,B^2\,b^{14}\,c^5\right)-\frac{(-c)^{3/4}\,(15\,A\,c-7\,B\,b)\left(173015040\,A\,b\right.}{64\,b^{19/4}}\right.}{64\,b^{19/4}}}\right.}{}$$

$$-\frac{(-c)^{3/4}\,\mathrm{atan}\left(\frac{A^3\,c^8\,\sqrt{x}\,3375\mathrm{i}-B^3\,b^3\,c^5\,\sqrt{x}\,343\mathrm{i}-A^2\,B\,b\,c^7\,\sqrt{x}\,4725\mathrm{i}+A\,B^2\,b^2\,c^6\,\sqrt{x}\,2205\mathrm{i}}{b^{1/4}\,(-c)^{19/4}\,(c\,(c\,(3375\,A^3\,c-4725\,A^2\,B\,b)+2205\,A\,B^2\,b^2)-343\,B^3\,b^3)}\right)(15\,A\,c-7\,B\,b)\,11\mathrm{i}}{32\,b^{19/4}}$$

```
[In] int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] ((2*x^2*(15*A*c - 7*B*b))/(21*b^2) - (2*A)/(7*b) + (11*c^2*x^6*(15*A*c - 7*
B*b))/(48*b^4) + (121*c*x^4*(15*A*c - 7*B*b))/(336*b^3))/(b^2*x^(7/2) + c^2
*x^(15/2) + 2*b*c*x^(11/2)) + (11*(-c)^(3/4)*atan(((11*(-c)^(3/4)*(15*A*c -
 7*B*b)*(x^(1/2)*(446054400*A^2*b^12*c^7 + 97140736*B^2*b^14*c^5 - 41631744
0*A*B*b^13*c^6) - ((-c)^(3/4)*(15*A*c - 7*B*b)*(173015040*A*b^17*c^5 - 8074
0352*B*b^18*c^4)*11i)/(64*b^(19/4))))/(64*b^(19/4)) + (11*(-c)^(3/4)*(15*A*
c - 7*B*b)*(x^(1/2)*(446054400*A^2*b^12*c^7 + 97140736*B^2*b^14*c^5 - 41631
7440*A*B*b^13*c^6) + ((-c)^(3/4)*(15*A*c - 7*B*b)*(173015040*A*b^17*c^5 - 8
0740352*B*b^18*c^4)*11i)/(64*b^(19/4))))/(64*b^(19/4)))/(((-c)^(3/4)*(15*A*
c - 7*B*b)*(x^(1/2)*(446054400*A^2*b^12*c^7 + 97140736*B^2*b^14*c^5 - 41631
7440*A*B*b^13*c^6) - ((-c)^(3/4)*(15*A*c - 7*B*b)*(173015040*A*b^17*c^5 - 8
0740352*B*b^18*c^4)*11i)/(64*b^(19/4)))*11i)/(64*b^(19/4)) - ((-c)^(3/4)*(1
5*A*c - 7*B*b)*(x^(1/2)*(446054400*A^2*b^12*c^7 + 97140736*B^2*b^14*c^5 - 4
16317440*A*B*b^13*c^6) + ((-c)^(3/4)*(15*A*c - 7*B*b)*(173015040*A*b^17*c^5
 - 80740352*B*b^18*c^4)*11i)/(64*b^(19/4)))*11i)/(64*b^(19/4))))*(15*A*c -
7*B*b))/(32*b^(19/4)) - ((-c)^(3/4)*atan((A^3*c^8*x^(1/2)*3375i - B^3*b^3*c
^5*x^(1/2)*343i - A^2*B*b*c^7*x^(1/2)*4725i + A*B^2*b^2*c^6*x^(1/2)*2205i)/
(b^(1/4)*(-c)^(19/4)*(c*(c*(3375*A^3*c - 4725*A^2*B*b) + 2205*A*B^2*b^2) -
343*B^3*b^3)))*(15*A*c - 7*B*b)*11i)/(32*b^(19/4))
```

3.218 $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3} dx$

Optimal result

Integrand size = 26, antiderivative size = 365

$$\begin{aligned}\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3} dx &= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} \\ &- \frac{bB-Ac}{4bcx^{9/2}(b+cx^2)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}(b+cx^2)} \\ &- \frac{13c^{5/4}(9bB-17Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{21/4}} \\ &+ \frac{13c^{5/4}(9bB-17Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{21/4}} \\ &+ \frac{13c^{5/4}(9bB-17Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}} \\ &- \frac{13c^{5/4}(9bB-17Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}}\end{aligned}$$

```
[Out] 13/144*(-17*A*c+9*B*b)/b^3/c/x^(9/2)-13/80*(-17*A*c+9*B*b)/b^4/x^(5/2)+1/4*
(A*c-B*b)/b/c/x^(9/2)/(c*x^2+b)^2+1/16*(17*A*c-9*B*b)/b^2/c/x^(9/2)/(c*x^2+
b)-13/64*c^(5/4)*(-17*A*c+9*B*b)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/
b^(21/4)*2^(1/2)+13/64*c^(5/4)*(-17*A*c+9*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(
1/2)/b^(1/4))/b^(21/4)*2^(1/2)+13/128*c^(5/4)*(-17*A*c+9*B*b)*ln(b^(1/2)+x*
c^(1/2)-b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(21/4)*2^(1/2)-13/128*c^(5/4)*(-
17*A*c+9*B*b)*ln(b^(1/2)+x*c^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(21/4
)*2^(1/2)+13/16*c*(-17*A*c+9*B*b)/b^5/x^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1598, 468, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = -\frac{13c^{5/4}(9bB-17Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{21/4}} + \frac{13c^{5/4}(9bB-17Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{21/4}} + \frac{13c^{5/4}(9bB-17Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}} - \frac{13c^{5/4}(9bB-17Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{9bB-17Ac}{16b^2cx^{9/2}(b+cx^2)} - \frac{bB-Ac}{4bcx^{9/2}(b+cx^2)^2}$$

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (13*(9*b*B - 17*A*c))/(144*b^3*c*x^(9/2)) - (13*(9*b*B - 17*A*c))/(80*b^4*x
^(5/2)) + (13*c*(9*b*B - 17*A*c))/(16*b^5*Sqrt[x]) - (b*B - A*c)/(4*b*c*x^(
9/2)*(b + c*x^2)^2) - (9*b*B - 17*A*c)/(16*b^2*c*x^(9/2)*(b + c*x^2)) - (13
*c^(5/4)*(9*b*B - 17*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(3
2*Sqrt[2]*b^(21/4)) + (13*c^(5/4)*(9*b*B - 17*A*c)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(21/4)) + (13*c^(5/4)*(9*b*B - 17*A*c)
*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^
(21/4)) - (13*c^(5/4)*(9*b*B - 17*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(21/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
```

```
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{A+Bx^2}{x^{11/2}(b+cx^2)^3}\,dx$$

$$= -\frac{bB-Ac}{4bcx^{9/2}(b+cx^2)^2} + \frac{\left(-\frac{9bB}{2}+\frac{17Ac}{2}\right)\int \frac{1}{x^{11/2}(b+cx^2)^2}\,dx}{4bc}$$

$$= -\frac{bB-Ac}{4bcx^{9/2}(b+cx^2)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}(b+cx^2)} - \frac{(13(9bB-17Ac))\int \frac{1}{x^{11/2}(b+cx^2)}\,dx}{32b^2c}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{bB-Ac}{4bcx^{9/2}(b+cx^2)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}(b+cx^2)} + \frac{(13(9bB-17Ac))\int \frac{1}{x^{7/2}(b+cx^2)}\,dx}{32b^3}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} - \frac{bB-Ac}{4bcx^{9/2}(b+cx^2)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}(b+cx^2)} - \frac{(13c(9bB-17Ac))\int \frac{1}{x^{3/2}(b+cx^2)}\,dx}{32b^4}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}(b+cx^2)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}(b+cx^2)} + \frac{(13c^2(9bB-17Ac))\int \frac{\sqrt{x}}{b+cx^2}\,dx}{32b^5}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}(b+cx^2)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}(b+cx^2)} + \frac{(13c^2(9bB-17Ac))\,\text{Subst}\left(\int \frac{x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{16b^5}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}(b+cx^2)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}(b+cx^2)} - \frac{(13c^{3/2}(9bB-17Ac))\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^5} + \frac{(13c^{3/2}(9bB-17Ac))\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx, x, \sqrt{x}\right)}{32b^5}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}(b+cx^2)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}(b+cx^2)} + \frac{(13c(9bB-17Ac))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64b^5} + \frac{(13c(9bB-17Ac))\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx, x, \sqrt{x}\right)}{64b^5} + \frac{(13c^{5/4}(9bB-17Ac))\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{64\sqrt{2}b^{21/4}} + \frac{(13c^{5/4}(9bB-17Ac))\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx, x, \sqrt{x}\right)}{64\sqrt{2}b^{21/4}}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}(b+cx^2)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}(b+cx^2)} + \frac{13c^{5/4}(9bB-17Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}} - \frac{13c^{5/4}(9bB-17Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}} + \frac{(13c^{5/4}(9bB-17Ac))\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{21/4}} - \frac{(13c^{5/4}(9bB-17Ac))\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{21/4}}$$

$$= \frac{13(9bB-17Ac)}{144b^3cx^{9/2}} - \frac{13(9bB-17Ac)}{80b^4x^{5/2}} + \frac{13c(9bB-17Ac)}{16b^5\sqrt{x}} - \frac{bB-Ac}{4bcx^{9/2}(b+cx^2)^2} - \frac{9bB-17Ac}{16b^2cx^{9/2}(b+cx^2)} - \frac{13c^{5/4}(9bB-17Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{21/4}} + \frac{13c^{5/4}(9bB-17Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{21/4}} + \frac{13c^{5/4}(9bB-17Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}} - \frac{13c^{5/4}(9bB-17Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}}$$

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{-\frac{4\sqrt[4]{b}(-9bBx^2(-32b^3+416b^2cx^2+1053bc^2x^4+585c^3x^6)+A(160b^4-544b^3cx^2+7072b^2c^2x^4+17901bc^3x^6+9945c^4x^8))}{x^{9/2}(b+cx^2)^2} + 585\sqrt{2}c^{5/4}(-9}{2880b^{21/4}}$$

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*b^(1/4)*(-9*b*B*x^2*(-32*b^3 + 416*b^2*c*x^2 + 1053*b*c^2*x^4 + 585*c^
3*x^6) + A*(160*b^4 - 544*b^3*c*x^2 + 7072*b^2*c^2*x^4 + 17901*b*c^3*x^6 +
9945*c^4*x^8)))/(x^(9/2)*(b + c*x^2)^2) + 585*Sqrt[2]*c^(5/4)*(-9*b*B + 17*
```

```
A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 585*
Sqrt[2]*c^(5/4)*(-9*b*B + 17*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])
/(Sqrt[b] + Sqrt[c]*x)])/(2880*b^(21/4))
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.58

method	result
derivativedivides	$-\frac{2c^2\left(\frac{\frac{c(29Ac-21Bb)x^{\frac{7}{2}}}{32}+\left(\frac{33}{32}Abc-\frac{25}{32}B\,b^2\right)x^{\frac{3}{2}}}{\left(c\,x^2+b\right)^2}+\frac{\left(\frac{221Ac}{32}-\frac{117Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{ar}\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{b^5}$
default	$-\frac{2c^2\left(\frac{\frac{c(29Ac-21Bb)x^{\frac{7}{2}}}{32}+\left(\frac{33}{32}Abc-\frac{25}{32}B\,b^2\right)x^{\frac{3}{2}}}{\left(c\,x^2+b\right)^2}+\frac{\left(\frac{221Ac}{32}-\frac{117Bb}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{ar}\right.}{8c\left(\frac{b}{c}\right)^{\frac{1}{4}}}}{b^5}$
risch	$-\frac{2\left(270A\,c^2x^4-135x^4Bbc-27Abc\,x^2+9b^2B\,x^2+5b^2A\right)}{45b^5x^{\frac{9}{2}}}-\frac{c^2\left(\frac{\frac{c(29Ac-21Bb)x^{\frac{7}{2}}}{16}+2\left(\frac{33}{32}Abc-\frac{25}{32}B\,b^2\right)x^{\frac{3}{2}}}{\left(c\,x^2+b\right)^2}+\frac{\left(\frac{221Ac}{32}-\frac{117Bb}{32}\right.}{}\right.}{}$

```
[In] int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^5*c^2*((1/32*c*(29*A*c-21*B*b)*x^(7/2)+(33/32*A*b*c-25/32*B*b^2)*x^(3/
2))/(c*x^2+b)^2+1/8*(221/32*A*c-117/32*B*b)/c/(1/c*b)^(1/4)*2^(1/2)*(ln((x-
(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x+(1/c*b)^(1/4)*x^(1/2)*2^(1/
2)+(1/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/
2)/(1/c*b)^(1/4)*x^(1/2)-1)))-2/9*A/b^3/x^(9/2)-2/5*(-3*A*c+B*b)/b^4/x^(5/2
)-6*c*(2*A*c-B*b)/b^5/x^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 966, normalized size of antiderivative = 2.65

$$\int \frac{\sqrt{x}(A+Bx^2)}{\left(bx^2+cx^4\right)^3}\,dx =$$

$$-\frac{585\left(b^5c^2x^9+2\,b^6cx^7+b^7x^5\right)\left(-\frac{6561\,B^4b^4c^5-49572\,AB^3b^3c^6+140454\,A^2B^2b^2c^7-176868\,A^3Bbc^8+83521\,A^4c^9}{b^{21}}\right)^{\frac{1}{4}}\log\left(2197\right.}{}$$

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2880*(585*(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x^5)*(-(6561*B^4*b^4*c^5 - 49
572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4
*c^9)/b^21)^(1/4)*log(2197*b^16*(-(6561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 +
 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^21)^(3/4) -
 2197*(729*B^3*b^3*c^4 - 4131*A*B^2*b^2*c^5 + 7803*A^2*B*b*c^6 - 4913*A^3*c
^7)*sqrt(x)) + 585*(-I*b^5*c^2*x^9 - 2*I*b^6*c*x^7 - I*b^7*x^5)*(-(6561*B^4
*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^
8 + 83521*A^4*c^9)/b^21)^(1/4)*log(2197*I*b^16*(-(6561*B^4*b^4*c^5 - 49572*
A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9
)/b^21)^(3/4) - 2197*(729*B^3*b^3*c^4 - 4131*A*B^2*b^2*c^5 + 7803*A^2*B*b*c
^6 - 4913*A^3*c^7)*sqrt(x)) + 585*(I*b^5*c^2*x^9 + 2*I*b^6*c*x^7 + I*b^7*x^
5)*(-(6561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176
868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^21)^(1/4)*log(-2197*I*b^16*(-(6561*B^4*b
^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B*b*c^8
+ 83521*A^4*c^9)/b^21)^(3/4) - 2197*(729*B^3*b^3*c^4 - 4131*A*B^2*b^2*c^5 +
 7803*A^2*B*b*c^6 - 4913*A^3*c^7)*sqrt(x)) - 585*(b^5*c^2*x^9 + 2*b^6*c*x^7
 + b^7*x^5)*(-(6561*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*
c^7 - 176868*A^3*B*b*c^8 + 83521*A^4*c^9)/b^21)^(1/4)*log(-2197*b^16*(-(656
1*B^4*b^4*c^5 - 49572*A*B^3*b^3*c^6 + 140454*A^2*B^2*b^2*c^7 - 176868*A^3*B
*b*c^8 + 83521*A^4*c^9)/b^21)^(3/4) - 2197*(729*B^3*b^3*c^4 - 4131*A*B^2*b^
2*c^5 + 7803*A^2*B*b*c^6 - 4913*A^3*c^7)*sqrt(x)) - 4*(585*(9*B*b*c^3 - 17*
A*c^4)*x^8 + 1053*(9*B*b^2*c^2 - 17*A*b*c^3)*x^6 - 160*A*b^4 + 416*(9*B*b^3
*c - 17*A*b^2*c^2)*x^4 - 32*(9*B*b^4 - 17*A*b^3*c)*x^2)*sqrt(x))/(b^5*c^2*x
^9 + 2*b^6*c*x^7 + b^7*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{585\,(9\,Bbc^3-17\,Ac^4)x^8+1053\,(9\,Bb^2c^2-17\,Abc^3)x^6-160\,Ab^4+416\,(9\,Bb^3c-17\,Ab^2c^2)x^4-32\,(9\,Bb^4}{720\left(b^5c^2x^{\frac{17}{2}}+2\,b^6cx^{\frac{13}{2}}+b^7x^{\frac{9}{2}}\right)} + \frac{13\,(9\,Bbc^2-17\,Ac^3)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\,\sqrt{c}\sqrt{x}\right)}{2\,\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{\sqrt{b}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{x}+\sqrt{}\right.}{b^{\frac{1}{4}}c^{\frac{3}{4}}}\right.}{128\,b^5}$$

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/720*(585*(9*B*b*c^3 - 17*A*c^4)*x^8 + 1053*(9*B*b^2*c^2 - 17*A*b*c^3)*x^6
 - 160*A*b^4 + 416*(9*B*b^3*c - 17*A*b^2*c^2)*x^4 - 32*(9*B*b^4 - 17*A*b^3*
c)*x^2)/(b^5*c^2*x^(17/2) + 2*b^6*c*x^(13/2) + b^7*x^(9/2)) + 13/128*(9*B*b
*c^2 - 17*A*c^3)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) + 2
*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) +
2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/4) - 2*sqrt(c)*sqrt(x))
/sqrt(sqrt(b)*sqrt(c)))/(sqrt(sqrt(b)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(
2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(3/4)) + sqrt(
2)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(1/4)*c^(
3/4)))/b^5
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx = \frac{13\sqrt{2}\left(9\,(bc^3)^{\frac{3}{4}}\,Bb-17\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6c} + \frac{13\sqrt{2}\left(9\,(bc^3)^{\frac{3}{4}}\,Bb-17\,(bc^3)^{\frac{3}{4}}\,Ac\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6c} - \frac{13\sqrt{2}\left(9\,(bc^3)^{\frac{3}{4}}\,Bb-17\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^6c} + \frac{13\sqrt{2}\left(9\,(bc^3)^{\frac{3}{4}}\,Bb-17\,(bc^3)^{\frac{3}{4}}\,Ac\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^6c} + \frac{21\,Bbc^3x^{\frac{7}{2}}-29\,Ac^4x^{\frac{7}{2}}+25\,Bb^2c^2x^{\frac{3}{2}}-33\,Abc^3x^{\frac{3}{2}}}{16\,(cx^2+b)^2b^5} + \frac{2\,(135\,Bbcx^4-270\,Ac^2x^4-9\,Bb^2x^2+27\,Abcx^2-5\,Ab^2)}{45\,b^5x^{\frac{9}{2}}}$$

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^3,x, algorithm="giac")
```

```
[Out] 13/64*sqrt(2)*(9*(b*c^3)^(3/4)*B*b - 17*(b*c^3)^(3/4)*A*c)*arctan(1/2*sqrt(
2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^6*c) + 13/64*sqrt(2)*(
9*(b*c^3)^(3/4)*B*b - 17*(b*c^3)^(3/4)*A*c)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b
/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^6*c) - 13/128*sqrt(2)*(9*(b*c^3)^(3/
4)*B*b - 17*(b*c^3)^(3/4)*A*c)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b
/c))/(b^6*c) + 13/128*sqrt(2)*(9*(b*c^3)^(3/4)*B*b - 17*(b*c^3)^(3/4)*A*c)*
log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^6*c) + 1/16*(21*B*b*c^
3*x^(7/2) - 29*A*c^4*x^(7/2) + 25*B*b^2*c^2*x^(3/2) - 33*A*b*c^3*x^(3/2))/(
(c*x^2 + b)^2*b^5) + 2/45*(135*B*b*c*x^4 - 270*A*c^2*x^4 - 9*B*b^2*x^2 + 27
*A*b*c*x^2 - 5*A*b^2)/(b^5*x^(9/2))
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^3}\,dx$$
$$= \frac{13\,(-c)^{5/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(17\,A\,c-9\,B\,b)}{32\,b^{21/4}} - \frac{\frac{2\,A}{9\,b}-\frac{2\,x^2\,(17\,A\,c-9\,B\,b)}{45\,b^2}+\frac{117\,c^2\,x^6\,(17\,A\,c-9\,B\,b)}{80\,b^4}+\frac{13\,c^3\,x^8\,(17\,A\,c-9\,B\,b)}{16\,b^5}+\frac{26\,c\,x^4\,(17\,A\,c-9\,B\,b)}{45\,b^3}}{b^2\,x^{9/2}+c^2\,x^{17/2}+2\,b\,c\,x^{13/2}} - \frac{13\,(-c)^{5/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,\sqrt{x}}{b^{1/4}}\right)\,(17\,A\,c-9\,B\,b)}{32\,b^{21/4}}$$

```
[In] int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^3,x)
```

```
[Out] (13*(-c)^(5/4)*atan(((-c)^(1/4)*x^(1/2))/b^(1/4))*(17*A*c - 9*B*b))/(32*b^(
21/4)) - ((2*A)/(9*b) - (2*x^2*(17*A*c - 9*B*b))/(45*b^2) + (117*c^2*x^6*(1
7*A*c - 9*B*b))/(80*b^4) + (13*c^3*x^8*(17*A*c - 9*B*b))/(16*b^5) + (26*c*x
^4*(17*A*c - 9*B*b))/(45*b^3))/(b^2*x^(9/2) + c^2*x^(17/2) + 2*b*c*x^(13/2)
) - (13*(-c)^(5/4)*atanh(((-c)^(1/4)*x^(1/2))/b^(1/4))*(17*A*c - 9*B*b))/(3
2*b^(21/4))
```

3.219 $\int \frac{A+Bx^2}{\sqrt{x}(bx^2+cx^4)^3} dx$

Optimal result

Integrand size = 26, antiderivative size = 365

$$\begin{aligned}\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx &= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} \\ &\quad - \frac{bB-Ac}{4bcx^{11/2}\,(b+cx^2)^2} - \frac{11bB-19Ac}{16b^2cx^{11/2}\,(b+cx^2)} \\ &\quad - \frac{15c^{7/4}(11bB-19Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{23/4}} \\ &\quad + \frac{15c^{7/4}(11bB-19Ac)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{23/4}} \\ &\quad - \frac{15c^{7/4}(11bB-19Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}} \\ &\quad + \frac{15c^{7/4}(11bB-19Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}}\end{aligned}$$

```
[Out] 15/176*(-19*A*c+11*B*b)/b^3/c/x^(11/2)-15/112*(-19*A*c+11*B*b)/b^4/x^(7/2)+
5/16*c*(-19*A*c+11*B*b)/b^5/x^(3/2)+1/4*(A*c-B*b)/b/c/x^(11/2)/(c*x^2+b)^2+
1/16*(19*A*c-11*B*b)/b^2/c/x^(11/2)/(c*x^2+b)-15/64*c^(7/4)*(-19*A*c+11*B*b
)*arctan(1-c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(23/4)*2^(1/2)+15/64*c^(7/4)*
(-19*A*c+11*B*b)*arctan(1+c^(1/4)*2^(1/2)*x^(1/2)/b^(1/4))/b^(23/4)*2^(1/2)
-15/128*c^(7/4)*(-19*A*c+11*B*b)*ln(b^(1/2)+x*c^(1/2)-b^(1/4)*c^(1/4)*2^(1/
2)*x^(1/2))/b^(23/4)*2^(1/2)+15/128*c^(7/4)*(-19*A*c+11*B*b)*ln(b^(1/2)+x*c
^(1/2)+b^(1/4)*c^(1/4)*2^(1/2)*x^(1/2))/b^(23/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1598, 468, 296, 331, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx = -\frac{15c^{7/4}(11bB-19Ac)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{23/4}} + \frac{15c^{7/4}(11bB-19Ac)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{23/4}} - \frac{15c^{7/4}(11bB-19Ac)\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}} + \frac{15c^{7/4}(11bB-19Ac)\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{11bB-19Ac}{16b^2cx^{11/2}(b+cx^2)} - \frac{bB-Ac}{4bcx^{11/2}(b+cx^2)^2}$$

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^3),x]
```

```
[Out] (15*(11*b*B - 19*A*c))/(176*b^3*c*x^(11/2)) - (15*(11*b*B - 19*A*c))/(112*b
^4*x^(7/2)) + (5*c*(11*b*B - 19*A*c))/(16*b^5*x^(3/2)) - (b*B - A*c)/(4*b*c
*x^(11/2)*(b + c*x^2)^2) - (11*b*B - 19*A*c)/(16*b^2*c*x^(11/2)*(b + c*x^2)
) - (15*c^(7/4)*(11*b*B - 19*A*c)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/(32*Sqrt[2]*b^(23/4)) + (15*c^(7/4)*(11*b*B - 19*A*c)*ArcTan[1 + (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(23/4)) - (15*c^(7/4)*(11*b*B
 - 19*A*c)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*
Sqrt[2]*b^(23/4)) + (15*c^(7/4)*(11*b*B - 19*A*c)*Log[Sqrt[b] + Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(23/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{A+Bx^2}{x^{13/2}\left(b+cx^2\right)^3}\,dx \\
&= -\frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2}+\frac{\left(-\frac{11bB}{2}+\frac{19Ac}{2}\right)\int \frac{1}{x^{13/2}\left(b+cx^2\right)^2}\,dx}{4bc} \\
&= -\frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2}-\frac{11bB-19Ac}{16b^2cx^{11/2}\left(b+cx^2\right)}-\frac{(15(11bB-19Ac))\int \frac{1}{x^{13/2}\left(b+cx^2\right)}\,dx}{32b^2c} \\
&= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}}-\frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} \\
&\quad -\frac{11bB-19Ac}{16b^2cx^{11/2}\left(b+cx^2\right)}+\frac{(15(11bB-19Ac))\int \frac{1}{x^{9/2}\left(b+cx^2\right)}\,dx}{32b^3} \\
&= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}}-\frac{15(11bB-19Ac)}{112b^4x^{7/2}}-\frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} \\
&\quad -\frac{11bB-19Ac}{16b^2cx^{11/2}\left(b+cx^2\right)}-\frac{(15c(11bB-19Ac))\int \frac{1}{x^{5/2}\left(b+cx^2\right)}\,dx}{32b^4}
\end{aligned}
$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \frac{11bB-19Ac}{16b^2cx^{11/2}\left(b+cx^2\right)} + \frac{\left(15c^2(11bB-19Ac)\right)\int \frac{1}{\sqrt{x}\left(b+cx^2\right)}\,dx}{32b^5}$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \frac{11bB-19Ac}{16b^2cx^{11/2}\left(b+cx^2\right)} + \frac{\left(15c^2(11bB-19Ac)\right)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx,x,\sqrt{x}\right)}{16b^5}$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \frac{11bB-19Ac}{16b^2cx^{11/2}\left(b+cx^2\right)} + \frac{\left(15c^2(11bB-19Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{32b^{11/2}} + \frac{\left(15c^2(11bB-19Ac)\right)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}x^2}{b+cx^4}\,dx,x,\sqrt{x}\right)}{32b^{11/2}}$$

$$= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}\left(b+cx^2\right)^2} - \frac{11bB-19Ac}{16b^2cx^{11/2}\left(b+cx^2\right)} + \frac{\left(15c^{3/2}(11bB-19Ac)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64b^{11/2}} + \frac{\left(15c^{3/2}(11bB-19Ac)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}+x^2}\,dx,x,\sqrt{x}\right)}{64b^{11/2}} - \frac{\left(15c^{7/4}(11bB-19Ac)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{64\sqrt{2}b^{23/4}} - \frac{\left(15c^{7/4}(11bB-19Ac)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{b}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{c}}-x^2}\,dx,x,\sqrt{x}\right)}{64\sqrt{2}b^{23/4}}$$

$$
\begin{aligned}
&= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}(b+cx^2)^2} \\
&\quad - \frac{11bB-19Ac}{16b^2cx^{11/2}(b+cx^2)} - \frac{15c^{7/4}(11bB-19Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}} \\
&\quad + \frac{15c^{7/4}(11bB-19Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}} \\
&\quad + \frac{(15c^{7/4}(11bB-19Ac))\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{23/4}} \\
&\quad - \frac{(15c^{7/4}(11bB-19Ac))\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{23/4}} \\
&= \frac{15(11bB-19Ac)}{176b^3cx^{11/2}} - \frac{15(11bB-19Ac)}{112b^4x^{7/2}} + \frac{5c(11bB-19Ac)}{16b^5x^{3/2}} - \frac{bB-Ac}{4bcx^{11/2}(b+cx^2)^2} \\
&\quad - \frac{11bB-19Ac}{16b^2cx^{11/2}(b+cx^2)} - \frac{15c^{7/4}(11bB-19Ac)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{23/4}} \\
&\quad + \frac{15c^{7/4}(11bB-19Ac)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{23/4}} \\
&\quad - \frac{15c^{7/4}(11bB-19Ac)\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}} \\
&\quad + \frac{15c^{7/4}(11bB-19Ac)\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.63

$$
\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx = \frac{-\frac{4b^{3/4}(-11bBx^2(-32b^3+160b^2cx^2+605bc^2x^4+385c^3x^6)+A(224b^4-608b^3cx^2+3040b^2c^2x^4+11495bc^3x^6+7315c^4x^8))}{x^{11/2}(b+cx^2)^2} + 1155\sqrt{2}c^{7/4}(-1}{4928b^{23/4}}
$$

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-4*b^(3/4)*(-11*b*B*x^2*(-32*b^3 + 160*b^2*c*x^2 + 605*b*c^2*x^4 + 385*c^
3*x^6) + A*(224*b^4 - 608*b^3*c*x^2 + 3040*b^2*c^2*x^4 + 11495*b*c^3*x^6 +
7315*c^4*x^8)))/(x^(11/2)*(b + c*x^2)^2) + 1155*Sqrt[2]*c^(7/4)*(-11*b*B +
```

```
19*A*c)*ArcTan[(Sqrt[b] - Sqrt[c]*x)/(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x])] + 1
155*Sqrt[2]*c^(7/4)*(11*b*B - 19*A*c)*ArcTanh[(Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt
[x])/(Sqrt[b] + Sqrt[c]*x)])/(4928*b^(23/4))
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.58

method	result
derivativedivides	$-\frac{2c^2\left(\frac{\left(\frac{31}{32}Ac^2-\frac{23}{32}Bbc\right)x^{\frac{5}{2}}+\frac{b(35Ac-27Bb)\sqrt{x}}{32}}{\left(cx^2+b\right)^2}+\frac{15(19Ac-11Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)\right.}{256b}\right.}{b^5}$
default	$-\frac{2c^2\left(\frac{\left(\frac{31}{32}Ac^2-\frac{23}{32}Bbc\right)x^{\frac{5}{2}}+\frac{b(35Ac-27Bb)\sqrt{x}}{32}}{\left(cx^2+b\right)^2}+\frac{15(19Ac-11Bb)\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}{x-\left(\frac{b}{c}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{b}{c}\right)^{\frac{1}{4}}}+1\right)\right.}{256b}\right.}{b^5}$
risch	$-\frac{2(154Ac^2x^4-77x^4Bbc-33Abcx^2+11b^2Bx^2+7b^2A)}{77b^5x^{\frac{11}{2}}}-\frac{c^2\left(\frac{2\left(\frac{31}{32}Ac^2-\frac{23}{32}Bbc\right)x^{\frac{5}{2}}+\frac{b(35Ac-27Bb)\sqrt{x}}{16}}{\left(cx^2+b\right)^2}+\frac{15(19Ac-11B}{}\right.}{}$

```
[In] int((B*x^2+A)/(c*x^4+b*x^2)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^5*c^2*(((31/32*A*c^2-23/32*B*b*c)*x^(5/2)+1/32*b*(35*A*c-27*B*b)*x^(1/
2))/(c*x^2+b)^2+15/256*(19*A*c-11*B*b)*(1/c*b)^(1/4)/b*2^(1/2)*(ln((x+(1/c*
b)^(1/4)*x^(1/2)*2^(1/2)+(1/c*b)^(1/2))/(x-(1/c*b)^(1/4)*x^(1/2)*2^(1/2)+(1
/c*b)^(1/2)))+2*arctan(2^(1/2)/(1/c*b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(1
/c*b)^(1/4)*x^(1/2)-1)))-2/11*A/b^3/x^(11/2)-2/7*(-3*A*c+B*b)/b^4/x^(7/2)-2
*c*(2*A*c-B*b)/b^5/x^(3/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 854, normalized size of antiderivative = 2.34

$$\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^3}\,dx =$$

$$-\frac{1155\left(b^5c^2x^{10}+2\,b^6cx^8+b^7x^6\right)\left(-\frac{14641\,B^4b^4c^7-101156\,AB^3b^3c^8+262086\,A^2B^2b^2c^9-301796\,A^3Bbc^{10}+130321\,A^4c^{11}}{b^{23}}\right)^{\frac{1}{4}}\log}{}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4928*(1155*(b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6)*(-(14641*B^4*b^4*c^7 -
 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 1303
21*A^4*c^11)/b^23)^(1/4)*log(15*b^6*(-(14641*B^4*b^4*c^7 - 101156*A*B^3*b^3
*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 130321*A^4*c^11)/b^23
)^(1/4) - 15*(11*B*b*c^2 - 19*A*c^3)*sqrt(x)) + 1155*(I*b^5*c^2*x^10 + 2*I*
b^6*c*x^8 + I*b^7*x^6)*(-(14641*B^4*b^4*c^7 - 101156*A*B^3*b^3*c^8 + 262086
*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 130321*A^4*c^11)/b^23)^(1/4)*log(1
5*I*b^6*(-(14641*B^4*b^4*c^7 - 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^
9 - 301796*A^3*B*b*c^10 + 130321*A^4*c^11)/b^23)^(1/4) - 15*(11*B*b*c^2 - 1
9*A*c^3)*sqrt(x)) + 1155*(-I*b^5*c^2*x^10 - 2*I*b^6*c*x^8 - I*b^7*x^6)*(-(1
4641*B^4*b^4*c^7 - 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A
^3*B*b*c^10 + 130321*A^4*c^11)/b^23)^(1/4)*log(-15*I*b^6*(-(14641*B^4*b^4*c
^7 - 101156*A*B^3*b^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 +
130321*A^4*c^11)/b^23)^(1/4) - 15*(11*B*b*c^2 - 19*A*c^3)*sqrt(x)) - 1155*(
b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6)*(-(14641*B^4*b^4*c^7 - 101156*A*B^3*b
^3*c^8 + 262086*A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 130321*A^4*c^11)/b^
23)^(1/4)*log(-15*b^6*(-(14641*B^4*b^4*c^7 - 101156*A*B^3*b^3*c^8 + 262086*
A^2*B^2*b^2*c^9 - 301796*A^3*B*b*c^10 + 130321*A^4*c^11)/b^23)^(1/4) - 15*(
11*B*b*c^2 - 19*A*c^3)*sqrt(x)) - 4*(385*(11*B*b*c^3 - 19*A*c^4)*x^8 + 605*
(11*B*b^2*c^2 - 19*A*b*c^3)*x^6 - 224*A*b^4 + 160*(11*B*b^3*c - 19*A*b^2*c^
2)*x^4 - 32*(11*B*b^4 - 19*A*b^3*c)*x^2)*sqrt(x))/(b^5*c^2*x^10 + 2*b^6*c*x
^8 + b^7*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^2}{\sqrt{x}\,(bx^2 + cx^4)^3}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**3/x**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^3}\,dx = \frac{385\,(11\,Bbc^3-19\,Ac^4)x^8+605\,(11\,Bb^2c^2-19\,Abc^3)x^6-224\,Ab^4+160\,(11\,Bb^3c-19\,Ab^2c^2)x^4-32\,(11}{1232\left(b^5c^2x^{\frac{19}{2}}+2\,b^6cx^{\frac{15}{2}}+b^7x^{\frac{11}{2}}\right)} + \frac{15\left(\frac{2\sqrt{2}(11\,Bbc^2-19\,Ac^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}+2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{2\sqrt{2}(11\,Bbc^2-19\,Ac^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}c^{\frac{1}{4}}-2\sqrt{c}\sqrt{x}\right)}{2\sqrt{\sqrt{b}\sqrt{c}}}\right)}{\sqrt{b}\sqrt{\sqrt{b}\sqrt{c}}}+\frac{\sqrt{2}(11\,B}{}\right.}{128\,b^5}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/1232*(385*(11*B*b*c^3 - 19*A*c^4)*x^8 + 605*(11*B*b^2*c^2 - 19*A*b*c^3)*x
^6 - 224*A*b^4 + 160*(11*B*b^3*c - 19*A*b^2*c^2)*x^4 - 32*(11*B*b^4 - 19*A*
b^3*c)*x^2)/(b^5*c^2*x^(19/2) + 2*b^6*c*x^(15/2) + b^7*x^(11/2)) + 15/128*(
2*sqrt(2)*(11*B*b*c^2 - 19*A*c^3)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4)*c^(1/
4) + 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)*sqrt(c
))) + 2*sqrt(2)*(11*B*b*c^2 - 19*A*c^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4
)*c^(1/4) - 2*sqrt(c)*sqrt(x))/sqrt(sqrt(b)*sqrt(c)))/(sqrt(b)*sqrt(sqrt(b)
*sqrt(c))) + sqrt(2)*(11*B*b*c^2 - 19*A*c^3)*log(sqrt(2)*b^(1/4)*c^(1/4)*sq
rt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*c^(1/4)) - sqrt(2)*(11*B*b*c^2 - 19*A
*c^3)*log(-sqrt(2)*b^(1/4)*c^(1/4)*sqrt(x) + sqrt(c)*x + sqrt(b))/(b^(3/4)*
c^(1/4)))/b^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx$$

$$= \frac{15\sqrt{2}\left(11\,(bc^3)^{\frac{1}{4}}\,Bbc-19\,(bc^3)^{\frac{1}{4}}\,Ac^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6}$$

$$+ \frac{15\sqrt{2}\left(11\,(bc^3)^{\frac{1}{4}}\,Bbc-19\,(bc^3)^{\frac{1}{4}}\,Ac^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^6}$$

$$+ \frac{15\sqrt{2}\left(11\,(bc^3)^{\frac{1}{4}}\,Bbc-19\,(bc^3)^{\frac{1}{4}}\,Ac^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^6}$$

$$- \frac{15\sqrt{2}\left(11\,(bc^3)^{\frac{1}{4}}\,Bbc-19\,(bc^3)^{\frac{1}{4}}\,Ac^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^6}$$

$$+ \frac{23\,Bbc^3x^{\frac{5}{2}}-31\,Ac^4x^{\frac{5}{2}}+27\,Bb^2c^2\sqrt{x}-35\,Abc^3\sqrt{x}}{16\,(cx^2+b)^2b^5}$$

$$+ \frac{2\,(77\,Bbcx^4-154\,Ac^2x^4-11\,Bb^2x^2+33\,Abcx^2-7\,Ab^2)}{77\,b^5x^{\frac{11}{2}}}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 15/64*sqrt(2)*(11*(b*c^3)^(1/4)*B*b*c - 19*(b*c^3)^(1/4)*A*c^2)*arctan(1/2*
sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^6 + 15/64*sqrt(2)*
(11*(b*c^3)^(1/4)*B*b*c - 19*(b*c^3)^(1/4)*A*c^2)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^6 + 15/128*sqrt(2)*(11*(b*c^3)^
(1/4)*B*b*c - 19*(b*c^3)^(1/4)*A*c^2)*log(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
 sqrt(b/c))/b^6 - 15/128*sqrt(2)*(11*(b*c^3)^(1/4)*B*b*c - 19*(b*c^3)^(1/4)
*A*c^2)*log(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^6 + 1/16*(23*B*
b*c^3*x^(5/2) - 31*A*c^4*x^(5/2) + 27*B*b^2*c^2*sqrt(x) - 35*A*b*c^3*sqrt(x
))/((c*x^2 + b)^2*b^5) + 2/77*(77*B*b*c*x^4 - 154*A*c^2*x^4 - 11*B*b^2*x^2
+ 33*A*b*c*x^2 - 7*A*b^2)/(b^5*x^(11/2))
```

Mupad [B] (verification not implemented)

Time = 9.31 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.75

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^3}\,dx$$

$$= \frac{15\,(-c)^{7/4}\,\mathrm{atan}\left(\frac{6859\,A^3\,c^{10}\,\sqrt{x}-1331\,B^3\,b^3\,c^7\,\sqrt{x}-11913\,A^2\,B\,b\,c^9\,\sqrt{x}+6897\,A\,B^2\,b^2\,c^8\,\sqrt{x}}{b^{1/4}\,(-c)^{27/4}\,(c\,(c\,(6859\,A^3\,c-11913\,A^2\,B\,b)+6897\,A\,B^2\,b^2)-1331\,B^3\,b^3)}\right)\,(19\,A\,c-11\,B\,b)}{32\,b^{23/4}}$$

$$-\frac{\frac{2\,A}{11\,b}-\frac{2\,x^2\,(19\,A\,c-11\,B\,b)}{77\,b^2}+\frac{55\,c^2\,x^6\,(19\,A\,c-11\,B\,b)}{112\,b^4}+\frac{5\,c^3\,x^8\,(19\,A\,c-11\,B\,b)}{16\,b^5}+\frac{10\,c\,x^4\,(19\,A\,c-11\,B\,b)}{77\,b^3}}{b^2\,x^{11/2}+c^2\,x^{19/2}+2\,b\,c\,x^{15/2}}$$

$$-\frac{(-c)^{7/4}\,\mathrm{atan}\left(\frac{\frac{(-c)^{7/4}\,(19\,A\,c-11\,B\,b)\,\left(\sqrt{x}\,\left(1330790400\,A^2\,b^{15}\,c^9-1540915200\,A\,B\,b^{16}\,c^8+446054400\,B^2\,b^{17}\,c^7\right)-\frac{15\,(-c)^{7/4}\,(19\,A\,c-11\,B\,b)\,(29884\cdots}{64\cdots}}{64\,b^{23/4}}}{\frac{15\,(-c)^{7/4}\,(19\,A\,c-11\,B\,b)\,\left(\sqrt{x}\,\left(1330790400\,A^2\,b^{15}\,c^9-1540915200\,A\,B\,b^{16}\,c^8+446054400\,B^2\,b^{17}\,c^7\right)-\frac{15\,(-c)^{7/4}\,(19\,A\,c-11\,B\,b)\,(29\cdots}{\cdots}}{64\,b^{23/4}}}\cdots\right.}{\cdots}$$

```
[In] int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^3),x)
```

```
[Out] (15*(-c)^(7/4)*atan((6859*A^3*c^10*x^(1/2) - 1331*B^3*b^3*c^7*x^(1/2) - 119
13*A^2*B*b*c^9*x^(1/2) + 6897*A*B^2*b^2*c^8*x^(1/2))/(b^(1/4)*(-c)^(27/4)*(
c*(c*(6859*A^3*c - 11913*A^2*B*b) + 6897*A*B^2*b^2) - 1331*B^3*b^3)))*(19*A
*c - 11*B*b))/(32*b^(23/4)) - ((2*A)/(11*b) - (2*x^2*(19*A*c - 11*B*b))/(77
*b^2) + (55*c^2*x^6*(19*A*c - 11*B*b))/(112*b^4) + (5*c^3*x^8*(19*A*c - 11*
B*b))/(16*b^5) + (10*c*x^4*(19*A*c - 11*B*b))/(77*b^3))/(b^2*x^(11/2) + c^2
*x^(19/2) + 2*b*c*x^(15/2)) - ((-c)^(7/4)*atan((((-c)^(7/4)*(19*A*c - 11*B*
b)*(x^(1/2)*(1330790400*A^2*b^15*c^9 + 446054400*B^2*b^17*c^7 - 1540915200*
A*B*b^16*c^8) - (15*(-c)^(7/4)*(19*A*c - 11*B*b)*(298844160*A*b^21*c^6 - 17
3015040*B*b^22*c^5))/(64*b^(23/4)))*15i)/(64*b^(23/4)) + ((-c)^(7/4)*(19*A*
c - 11*B*b)*(x^(1/2)*(1330790400*A^2*b^15*c^9 + 446054400*B^2*b^17*c^7 - 15
40915200*A*B*b^16*c^8) + (15*(-c)^(7/4)*(19*A*c - 11*B*b)*(298844160*A*b^21
*c^6 - 173015040*B*b^22*c^5))/(64*b^(23/4)))*15i)/(64*b^(23/4)))/((15*(-c)^
(7/4)*(19*A*c - 11*B*b)*(x^(1/2)*(1330790400*A^2*b^15*c^9 + 446054400*B^2*b
^17*c^7 - 1540915200*A*B*b^16*c^8) - (15*(-c)^(7/4)*(19*A*c - 11*B*b)*(2988
44160*A*b^21*c^6 - 173015040*B*b^22*c^5))/(64*b^(23/4))))/(64*b^(23/4)) - (
15*(-c)^(7/4)*(19*A*c - 11*B*b)*(x^(1/2)*(1330790400*A^2*b^15*c^9 + 4460544
00*B^2*b^17*c^7 - 1540915200*A*B*b^16*c^8) + (15*(-c)^(7/4)*(19*A*c - 11*B*
b)*(298844160*A*b^21*c^6 - 173015040*B*b^22*c^5))/(64*b^(23/4))))/(64*b^(23
/4))))*(19*A*c - 11*B*b)*15i)/(32*b^(23/4))
```

3.220 $\int x^{5/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 243

$$\begin{aligned}\int x^{5/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx &= \frac{4b^2(3bB-5Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}}\\
&-\frac{4b(3bB-5Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^2}\\
&-\frac{2(3bB-5Ac)x^{7/2}\sqrt{bx^2+cx^4}}{55c}+\frac{2Bx^{3/2}(bx^2+cx^4)^{3/2}}{15c}\\
&-\frac{2b^{11/4}(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231c^{13/4}\sqrt{bx^2+cx^4}}\end{aligned}$$

```
[Out] 2/15*B*x^(3/2)*(c*x^4+b*x^2)^(3/2)/c-4/385*b*(-5*A*c+3*B*b)*x^(3/2)*(c*x^4+
b*x^2)^(1/2)/c^2-2/55*(-5*A*c+3*B*b)*x^(7/2)*(c*x^4+b*x^2)^(1/2)/c+4/231*b^
2*(-5*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/c^3/x^(1/2)-2/231*b^(11/4)*(-5*A*c+3*B
*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)
*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(
1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(13/4)/
(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2064, 2046, 2049, 2057, 335, 226}

$$\int x^{5/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = -\frac{2b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-5Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231c^{13/4}\sqrt{bx^2+cx^4}} + \frac{4b^2\sqrt{bx^2+cx^4}(3bB-5Ac)}{231c^3\sqrt{x}} - \frac{4bx^{3/2}\sqrt{bx^2+cx^4}(3bB-5Ac)}{385c^2} - \frac{2x^{7/2}\sqrt{bx^2+cx^4}(3bB-5Ac)}{55c} + \frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{3/2}}{15c}$$

```
[In] Int[x^(5/2)*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (4*b^2*(3*b*B - 5*A*c)*Sqrt[b*x^2 + c*x^4])/(231*c^3*Sqrt[x]) - (4*b*(3*b*B
 - 5*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(385*c^2) - (2*(3*b*B - 5*A*c)*x^(7/
2)*Sqrt[b*x^2 + c*x^4])/(55*c) + (2*B*x^(3/2)*(b*x^2 + c*x^4)^(3/2))/(15*c)
 - (2*b^(11/4)*(3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sq
rt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/
(231*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2Bx^{3/2}(bx^2+cx^4)^{3/2}}{15c} - \frac{\left(2\left(\frac{9bB}{2}-\frac{15Ac}{2}\right)\right)\int x^{5/2}\sqrt{bx^2+cx^4}\,dx}{15c} \\
&= -\frac{2(3bB-5Ac)x^{7/2}\sqrt{bx^2+cx^4}}{55c} + \frac{2Bx^{3/2}(bx^2+cx^4)^{3/2}}{15c} - \frac{(2b(3bB-5Ac))\int\frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx}{55c} \\
&= -\frac{4b(3bB-5Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^2} - \frac{2(3bB-5Ac)x^{7/2}\sqrt{bx^2+cx^4}}{55c} \\
&\quad + \frac{2Bx^{3/2}(bx^2+cx^4)^{3/2}}{15c} + \frac{(2b^2(3bB-5Ac))\int\frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{77c^2} \\
&= \frac{4b^2(3bB-5Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{4b(3bB-5Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^2} \\
&\quad - \frac{2(3bB-5Ac)x^{7/2}\sqrt{bx^2+cx^4}}{55c} + \frac{2Bx^{3/2}(bx^2+cx^4)^{3/2}}{15c} \\
&\quad - \frac{(2b^3(3bB-5Ac))\int\frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{231c^3}
\end{aligned}
$$

$$= \frac{4b^2(3bB-5Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{4b(3bB-5Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^2} - \frac{2(3bB-5Ac)x^{7/2}\sqrt{bx^2+cx^4}}{55c} + \frac{2Bx^{3/2}(bx^2+cx^4)^{3/2}}{15c} - \frac{\left(2b^3(3bB-5Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{231c^3\sqrt{bx^2+cx^4}}$$

$$= \frac{4b^2(3bB-5Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{4b(3bB-5Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^2} - \frac{2(3bB-5Ac)x^{7/2}\sqrt{bx^2+cx^4}}{55c} + \frac{2Bx^{3/2}(bx^2+cx^4)^{3/2}}{15c} - \frac{\left(4b^3(3bB-5Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{231c^3\sqrt{bx^2+cx^4}}$$

$$= \frac{4b^2(3bB-5Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{4b(3bB-5Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^2} - \frac{2(3bB-5Ac)x^{7/2}\sqrt{bx^2+cx^4}}{55c} + \frac{2Bx^{3/2}(bx^2+cx^4)^{3/2}}{15c} - \frac{2b^{11/4}(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{13/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.56

$$\int x^{5/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{2\sqrt{x^2(b+cx^2)}\left((b+cx^2)\sqrt{1+\frac{cx^2}{b}}(45b^2B+7c^2x^2(15A+11Bx^2)-3bc(25A+21\ldots\right.}{1155c^3\sqrt{x}\sqrt{1+\ldots}}$$

```
[In] Integrate[x^(5/2)*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)*Sqrt[1 + (c*x^2)/b]*(45*b^2*B + 7*c^2
*x^2*(15*A + 11*B*x^2) - 3*b*c*(25*A + 21*B*x^2)) + 15*b^2*(-3*b*B + 5*A*c)
*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)]))/(1155*c^3*Sqrt[x]*Sqrt[1
 + (c*x^2)/b])
```

Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.99

method	result
risch	$-\frac{2(-77B\,c^3x^6-105A\,c^3x^4-14Bb\,c^2x^4-30Ab\,c^2x^2+18B\,b^2c\,x^2+50b^2Ac-30B\,b^3)\sqrt{x^2(c\,x^2+b)}}{1155c^3\sqrt{x}}+\frac{2b^3(5Ac-3Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-b}}{c}\right.}{\sqrt{-bc}}}}{}$
default	$\frac{2\sqrt{x^4c+b\,x^2}\left(77B\,c^5x^9+105A\,c^5x^7+91Bb\,c^4x^7+25A\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b^3c+135Ab\,c\right.}{1155}$

```
[In] int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/1155*(-77*B*c^3*x^6-105*A*c^3*x^4-14*B*b*c^2*x^4-30*A*b*c^2*x^2+18*B*b^2
*c*x^2+50*A*b^2*c-30*B*b^3)/c^3/x^(1/2)*(x^2*(c*x^2+b))^(1/2)+2/231*b^3*(5*
A*c-3*B*b)/c^4*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2
*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^
3+b*x)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.51

$$\int x^{5/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = -\frac{2\left(10\,(3\,Bb^4-5\,Ab^3c)\sqrt{c}x\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-(77\,Bc^4x^6+30\,Bb^3c-50\,Ab^2c^2+7\,(2\,Bbc^3+\right.}{1155\,c^4x}$$

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/1155*(10*(3*B*b^4 - 5*A*b^3*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0,
x) - (77*B*c^4*x^6 + 30*B*b^3*c - 50*A*b^2*c^2 + 7*(2*B*b*c^3 + 15*A*c^4)*x
^4 - 6*(3*B*b^2*c^2 - 5*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^4*x)
```

Sympy [F]

$$\int x^{5/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx=\int x^{\frac{5}{2}}\sqrt{x^2\,(b+cx^2)}\,(A+Bx^2)\,\,dx$$

```
[In] integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x**(5/2)*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Maxima [F]

$$\int x^{5/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2}(Bx^2+A)x^{\frac{5}{2}}\,dx$$

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*x^(5/2), x)
```

Giac [F]

$$\int x^{5/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2}(Bx^2+A)x^{\frac{5}{2}}\,dx$$

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx=\int x^{5/2}\,\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}\,dx$$

```
[In] int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
[Out] int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2), x)
```

3.221 $\int x^{3/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 369

$$\int x^{3/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{4b^2(7bB-13Ac)x^{3/2}(b+cx^2)}{195c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{4b(7bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2} - \frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c} + \frac{2B\sqrt{x}(bx^2+cx^4)^{3/2}}{13c} - \frac{4b^{9/4}(7bB-13Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}} + \frac{2b^{9/4}(7bB-13Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/13*B*(c*x^4+b*x^2)^(3/2)*x^(1/2)/c+4/195*b^2*(-13*A*c+7*B*b)*x^(3/2)*(c*x
^2+b)/c^(5/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2/117*(-13*A*c+7*B*b)
*x^(5/2)*(c*x^4+b*x^2)^(1/2)/c-4/585*b*(-13*A*c+7*B*b)*x^(1/2)*(c*x^4+b*x^2
)^(1/2)/c^2-4/195*b^(9/4)*(-13*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*ar
ctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/
(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)+2/195*b^(9/4)*(-1
3*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))
),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/
c^(11/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2064, 2046, 2049, 2057, 335, 311, 226, 1210}

$$\int x^{3/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{2b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-13Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}} - \frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-13Ac)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}} + \frac{4b^2x^{3/2}(b+cx^2)(7bB-13Ac)}{195c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{4b\sqrt{x}\sqrt{bx^2+cx^4}(7bB-13Ac)}{585c^2} - \frac{2x^{5/2}\sqrt{bx^2+cx^4}(7bB-13Ac)}{117c} + \frac{2B\sqrt{x}(bx^2+cx^4)^{3/2}}{13c}$$

```
[In] Int[x^(3/2)*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (4*b^2*(7*b*B - 13*A*c)*x^(3/2)*(b + c*x^2))/(195*c^(5/2)*(Sqrt[b] + Sqrt[c
]*x)*Sqrt[b*x^2 + c*x^4]) - (4*b*(7*b*B - 13*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^
4])/(585*c^2) - (2*(7*b*B - 13*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(117*c) +
(2*B*Sqrt[x]*(b*x^2 + c*x^4)^(3/2))/(13*c) - (4*b^(9/4)*(7*b*B - 13*A*c)*x*
(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2
*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(195*c^(11/4)*Sqrt[b*x^2 + c*x^4]
) + (2*b^(9/4)*(7*b*B - 13*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(S
qrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])
/(195*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
```

```
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\text{integral} = \frac{2B\sqrt{x}(bx^2+cx^4)^{3/2}}{13c} - \frac{\left(2\left(\frac{7bB}{2}-\frac{13Ac}{2}\right)\right)\int x^{3/2}\sqrt{bx^2+cx^4}\,dx}{13c}$$

$$= -\frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c} + \frac{2B\sqrt{x}(bx^2+cx^4)^{3/2}}{13c} - \frac{(2b(7bB-13Ac))\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{117c}$$

$$= -\frac{4b(7bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2} - \frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c} + \frac{2B\sqrt{x}(bx^2+cx^4)^{3/2}}{13c} + \frac{(2b^2(7bB-13Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{195c^2}$$

$$= -\frac{4b(7bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2} - \frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c} + \frac{2B\sqrt{x}(bx^2+cx^4)^{3/2}}{13c} + \frac{\left(2b^2(7bB-13Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{195c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{4b(7bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2} - \frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c} + \frac{2B\sqrt{x}(bx^2+cx^4)^{3/2}}{13c} + \frac{\left(4b^2(7bB-13Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{195c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{4b(7bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2} - \frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c} + \frac{2B\sqrt{x}(bx^2+cx^4)^{3/2}}{13c} + \frac{\left(4b^{5/2}(7bB-13Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{195c^{5/2}\sqrt{bx^2+cx^4}} - \frac{\left(4b^{5/2}(7bB-13Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{195c^{5/2}\sqrt{bx^2+cx^4}}$$

$$= \frac{4b^2(7bB-13Ac)x^{3/2}(b+cx^2)}{195c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{4b(7bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2} - \frac{2(7bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c} + \frac{2B\sqrt{x}(bx^2+cx^4)^{3/2}}{13c} - \frac{4b^{9/4}(7bB-13Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}} + \frac{2b^{9/4}(7bB-13Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.30

$$\int x^{3/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{2\sqrt{x}\sqrt{x^2(b+cx^2)}\left(-\left((b+cx^2)\sqrt{1+\frac{cx^2}{b}}(7bB-13Ac-9Bcx^2)\right)+b(7bB-13A\right.}{117c^2\sqrt{1+\frac{cx^2}{b}}}$$

```
[In] Integrate[x^(3/2)*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)*Sqrt[1 + (c*x^2)/b]*(7*b*B
- 13*A*c - 9*B*c*x^2)) + b*(7*b*B - 13*A*c)*Hypergeometric2F1[-1/2, 3/4, 7/
4, -((c*x^2)/b)]))/(117*c^2*Sqrt[1 + (c*x^2)/b])
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.72

method	result
risch	$\frac{2\sqrt{x}\,(45Bc^2x^4+65Ac^2x^2+10Bbc\,x^2+26Abc-14Bb^2)\sqrt{x^2(c\,x^2+b)}}{585c^2} - \frac{2b^2(13Ac-7Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{}}{}$
default	$-\frac{2\sqrt{x^4c+b\,x^2}\left(-45B\,x^8c^4-65A\,x^6c^4-55B\,x^6b\,c^3+78A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b^3c-39A\sqrt{}\right.}{}$

[In] int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/585*x^(1/2)*(45*B*c^2*x^4+65*A*c^2*x^2+10*B*b*c*x^2+26*A*b*c-14*B*b^2)/c^2*(x^2*(c*x^2+b))^(1/2)-2/195*b^2*(13*A*c-7*B*b)/c^3*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c)^(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.28

$$\int x^{3/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = -\frac{2\left(6\left(7\,Bb^3-13\,Ab^2c\right)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\left(45\,Bc^3x^4-14\,Bb^2c\right.\right.}{585\,c^3}$$

[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")

[Out] -2/585*(6*(7*B*b^3 - 13*A*b^2*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x)) - (45*B*c^3*x^4 - 14*B*b^2*c + 26*A*b*c^2 + 5*(2*B*b*c^2 + 13*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/c^3

Sympy [F]

$$\int x^{3/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \int x^{\frac{3}{2}}\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)\,dx$$

```
[In] integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x**(3/2)*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Maxima [F]

$$\int x^{3/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2}(Bx^2+A)x^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*x^(3/2), x)
```

Giac [F]

$$\int x^{3/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2}(Bx^2+A)x^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \int x^{3/2}\,\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}\,dx$$

```
[In] int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
[Out] int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2), x)
```

3.222 $\int \sqrt{x}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 204

$$\int \sqrt{x}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = -\frac{4b(5bB-11Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}-\frac{2(5bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c}+\frac{2B(bx^2+cx^4)^{3/2}}{11c\sqrt{x}}+\frac{2b^{7/4}(5bB-11Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/11*B*(c*x^4+b*x^2)^(3/2)/c/x^(1/2)-2/77*(-11*A*c+5*B*b)*x^(3/2)*(c*x^4+b*
x^2)^(1/2)/c-4/231*b*(-11*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/c^2/x^(1/2)+2/231*
b^(7/4)*(-11*A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/
cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2)
)^2)^(1/2)/c^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used

$= \{2064, 2046, 2049, 2057, 335, 226\}$

$$\int \sqrt{x}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$
$$= \frac{2b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(5bB-11Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}$$
$$-\frac{4b\sqrt{bx^2+cx^4}(5bB-11Ac)}{231c^2\sqrt{x}}-\frac{2x^{3/2}\sqrt{bx^2+cx^4}(5bB-11Ac)}{77c}+\frac{2B(bx^2+cx^4)^{3/2}}{11c\sqrt{x}}$$

```
[In] Int[Sqrt[x]*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-4*b*(5*b*B - 11*A*c)*Sqrt[b*x^2 + c*x^4])/(231*c^2*Sqrt[x]) - (2*(5*b*B -
 11*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(77*c) + (2*B*(b*x^2 + c*x^4)^(3/2))/
(11*c*Sqrt[x]) + (2*b^(7/4)*(5*b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(
b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(231*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
```

```
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2B(bx^2+cx^4)^{3/2}}{11c\sqrt{x}} - \frac{\left(2\left(\frac{5bB}{2}-\frac{11Ac}{2}\right)\right)\int\sqrt{x}\sqrt{bx^2+cx^4}\,dx}{11c} \\
&= -\frac{2(5bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} + \frac{2B(bx^2+cx^4)^{3/2}}{11c\sqrt{x}} - \frac{(2b(5bB-11Ac))\int\frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{77c} \\
&= -\frac{4b(5bB-11Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{2(5bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} \\
&\quad + \frac{2B(bx^2+cx^4)^{3/2}}{11c\sqrt{x}} + \frac{(2b^2(5bB-11Ac))\int\frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{231c^2} \\
&= -\frac{4b(5bB-11Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{2(5bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} \\
&\quad + \frac{2B(bx^2+cx^4)^{3/2}}{11c\sqrt{x}} + \frac{\left(2b^2(5bB-11Ac)x\sqrt{b+cx^2}\right)\int\frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{231c^2\sqrt{bx^2+cx^4}} \\
&= -\frac{4b(5bB-11Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{2(5bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} \\
&\quad + \frac{2B(bx^2+cx^4)^{3/2}}{11c\sqrt{x}} + \frac{\left(4b^2(5bB-11Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{231c^2\sqrt{bx^2+cx^4}}
\end{aligned}$$

$$= -\frac{4b(5bB-11Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{2(5bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} + \frac{2B(bx^2+cx^4)^{3/2}}{11c\sqrt{x}} + \frac{2b^{7/4}(5bB-11Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.54

$$\int \sqrt{x}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx$$

$$= \frac{2\sqrt{x^2(b+cx^2)}\left(-\left((b+cx^2)\sqrt{1+\frac{cx^2}{b}}(5bB-11Ac-7Bcx^2)\right)+b(5bB-11Ac)\,\text{Hypergeometric2F1}\left(-\right.\right.}{77c^2\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

```
[In] Integrate[Sqrt[x]*(A + B*x^2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)*Sqrt[1 + (c*x^2)/b]*(5*b*B - 11*A*c
 - 7*B*c*x^2)) + b*(5*b*B - 11*A*c)*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*
x^2)/b)]))/(77*c^2*Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.06

method	result
risch	$\frac{2(21Bc^2x^4+33Ac^2x^2+6Bbcx^2+22Abc-10Bb^2)\sqrt{x^2(cx^2+b)}}{231c^2\sqrt{x}} - \frac{2b^2(11Ac-5Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}}{231c^3\sqrt{cx^3+bx}\,x^{\frac{3}{2}}(}$
default	$-\frac{2\sqrt{x^4c+bx^2}\left(-21Bc^4x^7+11A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b^2c-33Ac^4x^5-5B\sqrt{-bc}\sqrt{\frac{cx+}{}}\right.}{231x^{\frac{3}{2}}(cx^2+b)c}$

```
[In] int((B*x^2+A)*x^(1/2)*(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/231*(21*B*c^2*x^4+33*A*c^2*x^2+6*B*b*c*x^2+22*A*b*c-10*B*b^2)/c^2/x^(1/2)
*(x^2*(c*x^2+b))^(1/2)-2/231*b^2*(11*A*c-5*B*b)/c^3*(-b*c)^(1/2)*((x+1/c*(-
b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^
(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*EllipticF(((x+1/c*(-b*c)^
(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*
x^2+b)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.48

$$\int \sqrt{x}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \frac{2\left(2\left(5\,Bb^3-11\,Ab^2c\right)\sqrt{c}x\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(21\,Bc^3x^4-10\,Bb^2c+22\,Abc^2+3\left(2\,Bbc^2+\right.\right.\right.}{231\,c^3x}$$

```
[In] integrate((B*x^2+A)*x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/231*(2*(5*B*b^3 - 11*A*b^2*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x)
 + (21*B*c^3*x^4 - 10*B*b^2*c + 22*A*b*c^2 + 3*(2*B*b*c^2 + 11*A*c^3)*x^2)*
sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^3*x)
```

Sympy [F]

$$\int \sqrt{x}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \int \sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\left(A+Bx^2\right)\,dx$$

```
[In] integrate((B*x**2+A)*x**(1/2)*(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(sqrt(x)*sqrt(x**2*(b + c*x**2))*(A + B*x**2), x)
```

Maxima [F]

$$\int \sqrt{x}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}\,dx$$

```
[In] integrate((B*x^2+A)*x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x), x)
```

Giac [F]

$$\int \sqrt{x}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2}\left(Bx^2+A\right)\sqrt{x}\,dx$$

```
[In] integrate((B*x^2+A)*x^(1/2)*(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(A+Bx^2)\sqrt{bx^2+cx^4}\,dx=\int \sqrt{x}\,\left(B\,x^2+A\right)\,\sqrt{c\,x^4+b\,x^2}\,dx$$

```
[In] int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2),x)
[Out] int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(1/2), x)
```

3.223 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 326

$$\begin{aligned}\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx &= -\frac{4b(bB-3Ac)x^{3/2}(b+cx^2)}{15c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{2(bB-3Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{15c}+\frac{2B(bx^2+cx^4)^{3/2}}{9cx^{3/2}}\\ &+\frac{4b^{5/4}(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}\\ &-\frac{2b^{5/4}(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}\end{aligned}$$

```
[Out] 2/9*B*(c*x^4+b*x^2)^(3/2)/c/x^(3/2)-4/15*b*(-3*A*c+B*b)*x^(3/2)*(c*x^2+b)/c
^(3/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2/15*(-3*A*c+B*b)*x^(1/2)*(c
*x^4+b*x^2)^(1/2)/c+4/15*b^(5/4)*(-3*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2
+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)-2/15*b^(5/4)*(
-3*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan
(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))
,1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c
^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2064, 2046, 2057, 335, 311, 226, 1210}

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$$
$$= -\frac{2b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-3Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}$$
$$+\frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-3Ac)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}$$
$$-\frac{4bx^{3/2}(b+cx^2)(bB-3Ac)}{15c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt{x}\sqrt{bx^2+cx^4}(bB-3Ac)}{15c}+\frac{2B(bx^2+cx^4)^{3/2}}{9cx^{3/2}}$$

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/Sqrt[x],x]
```

```
[Out] (-4*b*(b*B - 3*A*c)*x^(3/2)*(b + c*x^2))/(15*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*
Sqrt[b*x^2 + c*x^4]) - (2*(b*B - 3*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(15*c)
 + (2*B*(b*x^2 + c*x^4)^(3/2))/(9*c*x^(3/2)) + (4*b^(5/4)*(b*B - 3*A*c)*x*(
Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*
ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(7/4)*Sqrt[b*x^2 + c*x^4]) -
 (2*b^(5/4)*(b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
 + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c
^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\text{integral} = \frac{2B(bx^2+cx^4)^{3/2}}{9cx^{3/2}} - \frac{\left(2\left(\frac{3bB}{2}-\frac{9Ac}{2}\right)\right)\int\frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx}{9c}$$

$$= -\frac{2(bB-3Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{15c} + \frac{2B(bx^2+cx^4)^{3/2}}{9cx^{3/2}} - \frac{(2b(bB-3Ac))\int\frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{15c}$$

$$= -\frac{2(bB-3Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{15c} + \frac{2B(bx^2+cx^4)^{3/2}}{9cx^{3/2}} - \frac{\left(2b(bB-3Ac)x\sqrt{b+cx^2}\right)\int\frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{15c\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(bB-3Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{15c} + \frac{2B(bx^2+cx^4)^{3/2}}{9cx^{3/2}} - \frac{\left(4b(bB-3Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15c\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(bB-3Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{15c} + \frac{2B(bx^2+cx^4)^{3/2}}{9cx^{3/2}} - \frac{\left(4b^{3/2}(bB-3Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15c^{3/2}\sqrt{bx^2+cx^4}} + \frac{\left(4b^{3/2}(bB-3Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15c^{3/2}\sqrt{bx^2+cx^4}}$$

$$= -\frac{4b(bB-3Ac)x^{3/2}(b+cx^2)}{15c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2(bB-3Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{15c} + \frac{2B(bx^2+cx^4)^{3/2}}{9cx^{3/2}} + \frac{4b^{5/4}(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}} - \frac{2b^{5/4}(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.29

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\sqrt{x^2(b+cx^2)}\left(B(b+cx^2)\sqrt{1+\frac{cx^2}{b}} + (-bB+3Ac)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^2}{b}\right)\right)}{9c\sqrt{1+\frac{cx^2}{b}}}$$

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(B*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (-(b*
B) + 3*A*c)*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^2)/b)]))/(9*c*Sqrt[1 +
 (c*x^2)/b])
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.75

method	result
risch	$\frac{2\sqrt{x}\,(5Bc\,x^2+9Ac+2Bb)\sqrt{x^2(c\,x^2+b)}}{45c}+\frac{2b(3Ac-Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-b}}{c}\right.}{\sqrt{-bc}}}\right.}{c}\right.}{15c^2\sqrt{c\,x^3+bx}\,x^{\frac{3}{2}}\,(c\,x^2}$
default	$\frac{2\sqrt{x^4c+b\,x^2}\left(5B\,c^3x^6+18A\,b^2c\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-9A\,b^2c\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.}{}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45*x^(1/2)*(5*B*c*x^2+9*A*c+2*B*b)/c*(x^2*(c*x^2+b))^(1/2)+2/15*b*(3*A*c-
B*b)/c^2*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/
c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)
^(1/2)*(-2/c*(-b*c)^(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(
1/2),1/2*2^(1/2))+1/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)
^(1/2))^(1/2),1/2*2^(1/2)))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x
^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.24

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$$
$$=\frac{2\left(6\,(Bb^2-3\,Abc)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\left(5\,Bc^2x^2+2\,Bbc+9\,Ac\right.\right.}{45\,c^2}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(6*(B*b^2 - 3*A*b*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrassPIn
verse(-4*b/c, 0, x)) + (5*B*c^2*x^2 + 2*B*b*c + 9*A*c^2)*sqrt(c*x^4 + b*x^2
)*sqrt(x))/c^2
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{\sqrt{x}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(1/2),x)
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/sqrt(x), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{\sqrt{x}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/sqrt(x), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{\sqrt{x}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{\sqrt{x}}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(1/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(1/2), x)
```

3.224 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 165

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = -\frac{2(bB-7Ac)\sqrt{bx^2+cx^4}}{21c\sqrt{x}}+\frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}} - \frac{2b^{3/4}(bB-7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21c^{5/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/7*B*(c*x^4+b*x^2)^(3/2)/c/x^(5/2)-2/21*(-7*A*c+B*b)*(c*x^4+b*x^2)^(1/2)/c
/x^(1/2)-2/21*b^(3/4)*(-7*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))
)^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^
(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2
)+x*c^(1/2))^2)^(1/2)/c^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2064, 2046, 2057, 335, 226}

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = -\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-7Ac)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21c^{5/4}\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}(bB-7Ac)}{21c\sqrt{x}}+\frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}}$$

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(3/2),x]
```

```
[Out] (-2*(b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(21*c*Sqrt[x]) + (2*B*(b*x^2 + c*x^4
)^(3/2))/(7*c*x^(5/2)) - (2*b^(3/4)*(b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(21*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\text{integral} = \frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}} - \frac{\left(2\left(\frac{bB}{2}-\frac{7Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx}{7c}$$

$$= -\frac{2(bB-7Ac)\sqrt{bx^2+cx^4}}{21c\sqrt{x}} + \frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}} - \frac{(2b(bB-7Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{21c}$$

$$= -\frac{2(bB-7Ac)\sqrt{bx^2+cx^4}}{21c\sqrt{x}} + \frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}} - \frac{\left(2b(bB-7Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{21c\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(bB-7Ac)\sqrt{bx^2+cx^4}}{21c\sqrt{x}} + \frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}} - \frac{\left(4b(bB-7Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{21c\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(bB-7Ac)\sqrt{bx^2+cx^4}}{21c\sqrt{x}} + \frac{2B(bx^2+cx^4)^{3/2}}{7cx^{5/2}} - \frac{2b^{3/4}(bB-7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{21c^{5/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.57

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = \frac{2\sqrt{x^2(b+cx^2)}\left(B(b+cx^2)\sqrt{1+\frac{cx^2}{b}} + (-bB+7Ac)\,\text{Hypergeometric2F1}\right.}{7c\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(B*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (-(b*B) + 7*A
*c)*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)]))/(7*c*Sqrt[x]*Sqrt[1 +
 (c*x^2)/b])
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.17

method	result
risch	$\frac{2(3Bc\,x^2+7Ac+2Bb)\sqrt{x^2(c\,x^2+b)}}{21c\sqrt{x}}+\frac{2b(7Ac-Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{x^2(}}{21c^2\sqrt{c\,x^3+bx}\,x^{\frac{3}{2}}(c\,x^2+b)}$
default	$\frac{2\sqrt{x^4c+b\,x^2}\left(7A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc-B\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-}}}\right.}{21x^{\frac{3}{2}}(c\,x^2+b)c^2}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*(3*B*c*x^2+7*A*c+2*B*b)/c/x^(1/2)*(x^2*(c*x^2+b))^(1/2)+2/21*b*(7*A*c-
B*b)/c^2*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/
c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)
^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(
x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.45

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = -\frac{2\left(2\left(Bb^2-7\,Abc\right)\sqrt{c}x\,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\left(3\,Bc^2x^2+2\,Bbc+7\,Ac^2\right)\sqrt{cx^4+bx^2}\sqrt{x}\right)}{21\,c^2x}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] -2/21*(2*(B*b^2 - 7*A*b*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) - (3
*B*c^2*x^2 + 2*B*b*c + 7*A*c^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^2*x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(3/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^{3/2}}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(3/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(3/2), x)
```

3.225 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 323

$$
\begin{aligned}
\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx &= \frac{4(bB+5Ac)x^{3/2}(b+cx^2)}{5\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} \\
&+ \frac{2(bB+5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2A(bx^2+cx^4)^{3/2}}{bx^{7/2}} \\
&- \frac{4\sqrt[4]{b}(bB+5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}} \\
&+ \frac{2\sqrt[4]{b}(bB+5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}}
\end{aligned}
$$

```
[Out] -2*A*(c*x^4+b*x^2)^(3/2)/b/x^(7/2)+4/5*(5*A*c+B*b)*x^(3/2)*(c*x^2+b)/c^(1/2
)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+2/5*(5*A*c+B*b)*x^(1/2)*(c*x^4+b*
x^2)^(1/2)/b-4/5*b^(1/4)*(5*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1
/2)+x*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2)^(1/2)+2/5*b^(1/4)*(5*A*c+B*b)
*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^
(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2
))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x
^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2063, 2046, 2057, 335, 311, 226, 1210}

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx = \frac{2\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(5Ac+bB)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}}$$
$$-\frac{4\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(5Ac+bB)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}}$$
$$+\frac{2\sqrt{x}\sqrt{bx^2+cx^4}(5Ac+bB)}{5b}+\frac{4x^{3/2}(b+cx^2)\,(5Ac+bB)}{5\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{2A(bx^2+cx^4)^{3/2}}{bx^{7/2}}$$

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(5/2),x]
```

```
[Out] (4*(b*B + 5*A*c)*x^(3/2)*(b + c*x^2))/(5*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[b*x^2 + c*x^4]) + (2*(b*B + 5*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*b) - (2
*A*(b*x^2 + c*x^4)^(3/2))/(b*x^(7/2)) - (4*b^(1/4)*(b*B + 5*A*c)*x*(Sqrt[b]
 + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[
(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(3/4)*Sqrt[b*x^2 + c*x^4]) + (2*b^(1
/4)*(b*B + 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[
c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(3/4)*Sq
rt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\text{integral} = -\frac{2A(bx^2+cx^4)^{3/2}}{bx^{7/2}} - \frac{\left(2\left(-\frac{bB}{2}-\frac{5Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx}{b}$$

$$= \frac{2(bB+5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2A(bx^2+cx^4)^{3/2}}{bx^{7/2}} + \frac{1}{5}(2(bB+5Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{2(bB+5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2A(bx^2+cx^4)^{3/2}}{bx^{7/2}} + \frac{\left(2(bB+5Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5\sqrt{bx^2+cx^4}}$$

$$= \frac{2(bB+5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2A(bx^2+cx^4)^{3/2}}{bx^{7/2}} + \frac{\left(4(bB+5Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{bx^2+cx^4}}$$

$$= \frac{2(bB+5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2A(bx^2+cx^4)^{3/2}}{bx^{7/2}} + \frac{\left(4\sqrt{b}(bB+5Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{c}\sqrt{bx^2+cx^4}} - \frac{\left(4\sqrt{b}(bB+5Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{c}\sqrt{bx^2+cx^4}}$$

$$= \frac{4(bB+5Ac)x^{3/2}(b+cx^2)}{5\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{2(bB+5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2A(bx^2+cx^4)^{3/2}}{bx^{7/2}} - \frac{4\sqrt[4]{b}(bB+5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}} + \frac{2\sqrt[4]{b}(bB+5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.30

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx = \frac{2\sqrt{x^2(b+cx^2)}\left(-3A(b+cx^2)\sqrt{1+\frac{cx^2}{b}}+(bB+5Ac)x^2\,\text{Hypergeometric2}\right.}{3bx^{3/2}\sqrt{1+\frac{cx^2}{b}}}$$

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(5/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-3*A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (b*B + 5*A
*c)*x^2*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^2)/b)]))/(3*b*x^(3/2)*Sqrt
[1 + (c*x^2)/b])
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{2\left(-x^{2} B+5 A\right) \sqrt{x^{2}\left(c\,x^{2}+b\right)}}{5 x^{\frac{3}{2}}}+\frac{\left(2 A c+\frac{2 B b}{5}\right) \sqrt{-b c}\, \sqrt{\frac{\left(x+\frac{\sqrt{-b c}}{c}\right) c}{\sqrt{-b c}}}\, \sqrt{-\frac{2\left(x-\frac{\sqrt{-b c}}{c}\right) c}{\sqrt{-b c}}}\, \sqrt{-\frac{x c}{\sqrt{-b c}}}\, \left(-\frac{2 \sqrt{-b c}\, E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-b c}}{c}\right) c}{\sqrt{-b c}}}, \frac{\sqrt{2}}{2}\right)}{c}\right.}{c \sqrt{c\,x^{3}+b x}\, x^{\frac{3}{2}}\left(c\,x^{2}+b\right)}$
default	$\frac{2 \sqrt{x^{4} c+b\,x^{2}}\, \left(10 A \sqrt{\frac{c x+\sqrt{-b c}}{\sqrt{-b c}}}\, \sqrt{2}\, \sqrt{\frac{-c x+\sqrt{-b c}}{\sqrt{-b c}}}\, \sqrt{-\frac{x c}{\sqrt{-b c}}}\, E\left(\sqrt{\frac{c x+\sqrt{-b c}}{\sqrt{-b c}}}, \frac{\sqrt{2}}{2}\right) b c-5 A \sqrt{\frac{c x+\sqrt{-b c}}{\sqrt{-b c}}}\, \sqrt{2}\, \sqrt{\frac{-c x+\sqrt{-b c}}{\sqrt{-b c}}}\, \sqrt{-\frac{x c}{\sqrt{-b c}}}\, F\left(\sqrt{\frac{c x}{}}\right.\right.}{}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(-B*x^2+5*A)*(x^2*(c*x^2+b))^(1/2)/x^(3/2)+(2*A*c+2/5*B*b)/c*(-b*c)^(1
/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/
(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c
)^(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+
1/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*
2^(1/2)))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.22

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx =$$
$$-\frac{2\left(2\left(Bb+5\,Ac\right)\sqrt{c}x^2\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\sqrt{cx^4+bx^2}(Bcx^2-5\,Ac)\right.}{5\,cx^2}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/5*(2*(B*b + 5*A*c)*sqrt(c)*x^2*weierstrassZeta(-4*b/c, 0, weierstrassPIn
verse(-4*b/c, 0, x)) - sqrt(c*x^4 + b*x^2)*(B*c*x^2 - 5*A*c)*sqrt(x))/(c*x^
2)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx = \int \frac{\sqrt{x^2\left(b+cx^2\right)}(A+Bx^2)}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(5/2),x)
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(5/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(5/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^{5/2}}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(5/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(5/2), x)
```

3.226 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 163

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx = \frac{2(bB+Ac)\sqrt{bx^2+cx^4}}{3b\sqrt{x}} - \frac{2A(bx^2+cx^4)^{3/2}}{3bx^{9/2}} + \frac{2(bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/3*A*(c*x^4+b*x^2)^(3/2)/b/x^(9/2)+2/3*(A*c+B*b)*(c*x^4+b*x^2)^(1/2)/b/x^
(1/2)+2/3*(A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)
^(1/2)/b^(1/4)/c^(1/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2063, 2046, 2057, 335, 226}

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx = \frac{2x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+bB)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt[4]{c}\sqrt{bx^2+cx^4}} + \frac{2\sqrt{bx^2+cx^4}(Ac+bB)}{3b\sqrt{x}} - \frac{2A(bx^2+cx^4)^{3/2}}{3bx^{9/2}}$$

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(7/2),x]
```

```
[Out] (2*(b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(3*b*Sqrt[x]) - (2*A*(b*x^2 + c*x^4)^(3
/2))/(3*b*x^(9/2)) + (2*(b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2
)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1
/2])/(3*b^(1/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\text{integral} = -\frac{2A(bx^2+cx^4)^{3/2}}{3bx^{9/2}} + -\frac{\left(2\left(-\frac{3bB}{2}-\frac{3Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx}{3b}$$

$$= \frac{2(bB+Ac)\sqrt{bx^2+cx^4}}{3b\sqrt{x}} - \frac{2A(bx^2+cx^4)^{3/2}}{3bx^{9/2}} + \frac{1}{3}(2(bB+Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{2(bB+Ac)\sqrt{bx^2+cx^4}}{3b\sqrt{x}} - \frac{2A(bx^2+cx^4)^{3/2}}{3bx^{9/2}} + \frac{\left(2(bB+Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{3\sqrt{bx^2+cx^4}}$$

$$= \frac{2(bB+Ac)\sqrt{bx^2+cx^4}}{3b\sqrt{x}} - \frac{2A(bx^2+cx^4)^{3/2}}{3bx^{9/2}} + \frac{\left(4(bB+Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{3\sqrt{bx^2+cx^4}}$$

$$= \frac{2(bB+Ac)\sqrt{bx^2+cx^4}}{3b\sqrt{x}} - \frac{2A(bx^2+cx^4)^{3/2}}{3bx^{9/2}} + \frac{2(bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.60

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx = \frac{2\sqrt{x^2(b+cx^2)}\left(-A(b+cx^2)\sqrt{1+\frac{cx^2}{b}} + 3(bB+Ac)x^2\,\text{Hypergeometric2F1}\right.}{3bx^{5/2}\sqrt{1+\frac{cx^2}{b}}}$$

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(7/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-(A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b]) + 3*(b*B + A
*c)*x^2*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^2)/b)]))/(3*b*x^(5/2)*Sqrt
[1 + (c*x^2)/b])
```

Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.10

method	result
risch	$-\frac{2(-x^2B+A)\sqrt{x^2(cx^2+b)}}{3x^{\frac{5}{2}}}+\frac{\left(\frac{2Ac}{3}+\frac{2Bb}{3}\right)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{x^2(cx^2}}{c\sqrt{cx^3+bx}\,x^{\frac{3}{2}}(cx^2+b)}$
default	$\frac{2\sqrt{x^4c+bx^2}\left(A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)cx+B\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{x}{\sqrt{-}}}\right.}{3x^{\frac{5}{2}}(cx^2+b)c}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-B*x^2+A)/x^(5/2)*(x^2*(c*x^2+b))^(1/2)+(2/3*A*c+2/3*B*b)/c*(-b*c)^(1
/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/
(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*EllipticF((
(x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(x^2*(c*x^2+b))^(1/
2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx=\frac{2\left(2\,(Bb+Ac)\sqrt{c}x^3\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\sqrt{cx^4+bx^2}(Bcx^2-A\right.}{3\,cx^3}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*(B*b + A*c)*sqrt(c)*x^3*weierstrassPInverse(-4*b/c, 0, x) + sqrt(c*x
^4 + b*x^2)*(B*c*x^2 - A*c)*sqrt(x))/(c*x^3)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx=\int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{x^{\frac{7}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(7/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(7/2), x)
```

Maxima [F]

$$\int \frac{(A + Bx^2)\sqrt{bx^2 + cx^4}}{x^{7/2}}\,dx = \int \frac{\sqrt{cx^4 + bx^2}(Bx^2 + A)}{x^{\frac{7}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(7/2), x)
```

Giac [F]

$$\int \frac{(A + Bx^2)\sqrt{bx^2 + cx^4}}{x^{7/2}}\,dx = \int \frac{\sqrt{cx^4 + bx^2}(Bx^2 + A)}{x^{\frac{7}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(7/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + Bx^2)\sqrt{bx^2 + cx^4}}{x^{7/2}}\,dx = \int \frac{(B\,x^2 + A)\,\sqrt{c\,x^4 + b\,x^2}}{x^{7/2}}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(7/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(7/2), x)
```

3.227 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 328

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx = \frac{4\sqrt{c}(5bB+Ac)x^{3/2}(b+cx^2)}{5b\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} - \frac{4\sqrt[4]{c}(5bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}} + \frac{2\sqrt[4]{c}(5bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/5*A*(c*x^4+b*x^2)^(3/2)/b/x^(11/2)+4/5*(A*c+5*B*b)*x^(3/2)*(c*x^2+b)*c^(
1/2)/b/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2/5*(A*c+5*B*b)*(c*x^4+b*x^2
)^(1/2)/b/x^(3/2)-4/5*c^(1/4)*(A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*ar
ctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/
(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/(c*x^4+b*x^2)^(1/2)+2/5*c^(1/4)*(A*c+5
*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/
4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2
^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)
/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2063, 2045, 2057, 335, 311, 226, 1210}

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx = \frac{2\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+5bB)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}} - \frac{4\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+5bB)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}} + \frac{4\sqrt{c}x^{3/2}(b+cx^2)(Ac+5bB)}{5b\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}(Ac+5bB)}{5bx^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}}$$

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(9/2),x]
```

```
[Out] (4*Sqrt[c]*(5*b*B + A*c)*x^(3/2)*(b + c*x^2))/(5*b*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[b*x^2 + c*x^4]) - (2*(5*b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(5*b*x^(3/2)) -
(2*A*(b*x^2 + c*x^4)^(3/2))/(5*b*x^(11/2)) - (4*c^(1/4)*(5*b*B + A*c)*x*(Sq
rt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*Ar
cTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[b*x^2 + c*x^4]) + (2
*c^(1/4)*(5*b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(3/
4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} - \frac{\left(2\left(-\frac{5bB}{2}-\frac{Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx}{5b} \\
&= -\frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} + \frac{(2c(5bB+Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{5b} \\
&= -\frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} + \frac{\left(2c(5bB+Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5b\sqrt{bx^2+cx^4}}
\end{aligned}
$$

$$= -\frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} + \frac{(4c(5bB+Ac)x\sqrt{b+cx^2})\,\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} + \frac{(4\sqrt{c}(5bB+Ac)x\sqrt{b+cx^2})\,\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{b}\sqrt{bx^2+cx^4}} - \frac{(4\sqrt{c}(5bB+Ac)x\sqrt{b+cx^2})\,\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{b}\sqrt{bx^2+cx^4}}$$

$$= \frac{4\sqrt{c}(5bB+Ac)x^{3/2}(b+cx^2)}{5b\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2(5bB+Ac)\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{5bx^{11/2}} - \frac{4\sqrt[4]{c}(5bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}} + \frac{2\sqrt[4]{c}(5bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.29

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx = -\frac{2\sqrt{x^2(b+cx^2)}\left(A(b+cx^2)\sqrt{1+\frac{cx^2}{b}} + (5bB+Ac)x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{3}{4},-\frac{cx^2}{b}\right)\right)}{5bx^{7/2}\sqrt{1+\frac{cx^2}{b}}}$$

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(9/2),x]
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (5*b*B + A*c
)*x^2*Hypergeometric2F1[-1/2, -1/4, 3/4, -((c*x^2)/b)]))/(5*b*x^(7/2)*Sqrt[
1 + (c*x^2)/b])
```

Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{2(2Acx^2+5bBx^2+Ab)\sqrt{x^2(cx^2+b)}}{5x^{\frac{7}{2}}b}+\frac{2(Ac+5Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-b}}{c}\right.}{\sqrt{-bc}}}\right.}{c}\right.}{5b\sqrt{cx^3+bx}\,x^{\frac{3}{2}}(cx^2-}$
default	$\frac{2\sqrt{x^4c+bx^2}\left(2A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bcx^2-A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\right.\right.}{}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(2*A*c*x^2+5*B*b*x^2+A*b)/x^(7/2)/b*(x^2*(c*x^2+b))^(1/2)+2/5*(A*c+5*B
*b)/b*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(
-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1
/2)*(-2/c*(-b*c)^(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2
),1/2*2^(1/2))+1/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1
/2))^(1/2),1/2*2^(1/2)))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+
b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.23

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx =$$
$$-\frac{2\left(2\left(5\,Bb+Ac\right)\sqrt{c}x^4\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}((5\,Bb+2\,Ac\right.}{5\,bx^4}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] -2/5*(2*(5*B*b + A*c)*sqrt(c)*x^4*weierstrassZeta(-4*b/c, 0, weierstrassPIn
verse(-4*b/c, 0, x)) + sqrt(c*x^4 + b*x^2)*((5*B*b + 2*A*c)*x^2 + A*b)*sqrt
(x))/(b*x^4)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx = \int \frac{\sqrt{x^2\,(b+cx^2)}(A+Bx^2)}{x^{\frac{9}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(9/2),x)
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(9/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{9}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(9/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{9}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(9/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^{9/2}}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(9/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(9/2), x)
```

3.228 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 167

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx=-\frac{2(7bB-Ac)\sqrt{bx^2+cx^4}}{21bx^{5/2}}-\frac{2A(bx^2+cx^4)^{3/2}}{7bx^{13/2}}+\frac{2c^{3/4}(7bB-Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21b^{5/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/7*A*(c*x^4+b*x^2)^(3/2)/b/x^(13/2)-2/21*(-A*c+7*B*b)*(c*x^4+b*x^2)^(1/2)
/b/x^(5/2)+2/21*c^(3/4)*(-A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1
/2)+x*c^(1/2))^2)^(1/2)/b^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2063, 2045, 2057, 335, 226}

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx=\frac{2c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}(7bB-Ac)}{21bx^{5/2}}-\frac{2A(bx^2+cx^4)^{3/2}}{7bx^{13/2}}$$

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(11/2),x]
```

```
[Out] (-2*(7*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(21*b*x^(5/2)) - (2*A*(b*x^2 + c*x^4
)^(3/2))/(7*b*x^(13/2)) + (2*c^(3/4)*(7*b*B - A*c)*x*(Sqrt[b] + Sqrt[c]*x)*
Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[
x])/b^(1/4)], 1/2])/(21*b^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\text{integral} = -\frac{2A(bx^2+cx^4)^{3/2}}{7bx^{13/2}} - \frac{\left(2\left(-\frac{7bB}{2}+\frac{Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx}{7b}$$

$$= -\frac{2(7bB-Ac)\sqrt{bx^2+cx^4}}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{7bx^{13/2}} + \frac{(2c(7bB-Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{21b}$$

$$= -\frac{2(7bB-Ac)\sqrt{bx^2+cx^4}}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{7bx^{13/2}} + \frac{\left(2c(7bB-Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{21b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(7bB-Ac)\sqrt{bx^2+cx^4}}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{7bx^{13/2}} + \frac{\left(4c(7bB-Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{21b\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(7bB-Ac)\sqrt{bx^2+cx^4}}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{7bx^{13/2}} + \frac{2c^{3/4}(7bB-Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.59

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx = -\frac{2\sqrt{x^2(b+cx^2)}\left(3A(b+cx^2)\sqrt{1+\frac{cx^2}{b}}+(7bB-Ac)x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{4},-\frac{cx^2}{b}\right)\right)}{21bx^{9/2}\sqrt{1+\frac{cx^2}{b}}}$$

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(11/2),x]
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(3*A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (7*b*B - A
*c)*x^2*Hypergeometric2F1[-3/4, -1/2, 1/4, -((c*x^2)/b)]))/(21*b*x^(9/2)*Sq
rt[1 + (c*x^2)/b])
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{2\left(2Ac\,x^{2}+7bB\,x^{2}+3Ab\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{21x^{\frac{9}{2}}b}-\frac{2(Ac-7Bb)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\cdots}{21b\sqrt{c\,x^{3}+bx}\,x^{\frac{3}{2}}\left(c\,x^{2}+b\right)}$
default	$-\frac{2\sqrt{x^{4}c+b\,x^{2}}\,\left(A\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)c\,x^{3}-7B\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\cdots\right)}{21x^{\frac{9}{2}}\left(c\,x^{2}+b\right)b}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(2*A*c*x^2+7*B*b*x^2+3*A*b)/x^(9/2)/b*(x^2*(c*x^2+b))^(1/2)-2/21*(A*c
-7*B*b)/b*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1
/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x
)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*
(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.43

$$\int \frac{\left(A+Bx^{2}\right)\sqrt{bx^{2}+cx^{4}}}{x^{11/2}}\,dx=\frac{2\left(2\left(7\,Bb-Ac\right)\sqrt{c}x^{5}\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\sqrt{cx^{4}+bx^{2}}((7\,Bb+2\cdots\right)}{21\,bx^{5}}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] 2/21*(2*(7*B*b - A*c)*sqrt(c)*x^5*weierstrassPInverse(-4*b/c, 0, x) - sqrt(
c*x^4 + b*x^2)*((7*B*b + 2*A*c)*x^2 + 3*A*b)*sqrt(x))/(b*x^5)
```

Sympy [F]

$$\int \frac{\left(A+Bx^{2}\right)\sqrt{bx^{2}+cx^{4}}}{x^{11/2}}\,dx=\int \frac{\sqrt{x^{2}\left(b+cx^{2}\right)}\left(A+Bx^{2}\right)}{x^{\frac{11}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(11/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(11/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{11}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(11/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{11}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(11/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^{11/2}}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(11/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(11/2), x)
```

3.229 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 369

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = \frac{4c^{3/2}(3bB-Ac)x^{3/2}(b+cx^2)}{15b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{4c(3bB-Ac)\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{9bx^{15/2}} - \frac{4c^{5/4}(3bB-Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}} + \frac{2c^{5/4}(3bB-Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/9*A*(c*x^4+b*x^2)^(3/2)/b/x^(15/2)+4/15*c^(3/2)*(-A*c+3*B*b)*x^(3/2)*(c*
x^2+b)/b^2/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2/15*(-A*c+3*B*b)*(c*x^4
+b*x^2)^(1/2)/b/x^(7/2)-4/15*c*(-A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^(3/2)
-4/15*c^(5/4)*(-A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/
2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^
(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1
/2))^2)^(1/2)/b^(7/4)/(c*x^4+b*x^2)^(1/2)+2/15*c^(5/4)*(-A*c+3*B*b)*x*(cos(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1
/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(7/4)/(c*x^4+b*x^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2063, 2045, 2050, 2057, 335, 311, 226, 1210}

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = \frac{2c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\right.}{15b^{7/4}\sqrt{bx^2+cx^4}}$$
$$-\frac{4c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-Ac)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}}$$
$$+\frac{4c^{3/2}x^{3/2}(b+cx^2)(3bB-Ac)}{15b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}(3bB-Ac)}{15b^2x^{3/2}}$$
$$-\frac{2\sqrt{bx^2+cx^4}(3bB-Ac)}{15bx^{7/2}}-\frac{2A(bx^2+cx^4)^{3/2}}{9bx^{15/2}}$$

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(13/2),x]
```

```
[Out] (4*c^(3/2)*(3*b*B - A*c)*x^(3/2)*(b + c*x^2))/(15*b^2*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[b*x^2 + c*x^4]) - (2*(3*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(15*b*x^(7/2)
) - (4*c*(3*b*B - A*c)*Sqrt[b*x^2 + c*x^4])/(15*b^2*x^(3/2)) - (2*A*(b*x^2
+ c*x^4)^(3/2))/(9*b*x^(15/2)) - (4*c^(5/4)*(3*b*B - A*c)*x*(Sqrt[b] + Sqrt
[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4
)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[b*x^2 + c*x^4]) + (2*c^(5/4)*(3
*b*B - A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^
2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[b*
x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
```

0]

Rubi steps

$$\text{integral} = -\frac{2A(bx^2+cx^4)^{3/2}}{9bx^{15/2}} - \frac{\left(2\left(-\frac{9bB}{2}+\frac{3Ac}{2}\right)\right)\int\frac{\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx}{9b}$$

$$= -\frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{9bx^{15/2}} + \frac{(2c(3bB-Ac))\int\frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{15b}$$

$$= -\frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{4c(3bB-Ac)\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{9bx^{15/2}} + \frac{(2c^2(3bB-Ac))\int\frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{15b^2}$$

$$= -\frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{4c(3bB-Ac)\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{9bx^{15/2}} + \frac{\left(2c^2(3bB-Ac)x\sqrt{b+cx^2}\right)\int\frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{15b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{4c(3bB-Ac)\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{9bx^{15/2}} + \frac{\left(4c^2(3bB-Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{15b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{4c(3bB-Ac)\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{9bx^{15/2}} + \frac{\left(4c^{3/2}(3bB-Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{15b^{3/2}\sqrt{bx^2+cx^4}} - \frac{\left(4c^{3/2}(3bB-Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{15b^{3/2}\sqrt{bx^2+cx^4}}$$

$$= \frac{4c^{3/2}(3bB-Ac)x^{3/2}(b+cx^2)}{15b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2(3bB-Ac)\sqrt{bx^2+cx^4}}{15bx^{7/2}} - \frac{4c(3bB-Ac)\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{9bx^{15/2}} - \frac{4c^{5/4}(3bB-Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}} + \frac{2c^{5/4}(3bB-Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.27

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx =$$
$$-\frac{2\sqrt{x^2(b+cx^2)}\left(5A(b+cx^2)\sqrt{1+\frac{cx^2}{b}}+3(3bB-Ac)x^2\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-\frac{cx^2}{b}\right)\right)}{45bx^{11/2}\sqrt{1+\frac{cx^2}{b}}}$$

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(13/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(5*A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + 3*(3*b*B -
 A*c)*x^2*Hypergeometric2F1[-5/4, -1/2, -1/4, -((c*x^2)/b)]))/(45*b*x^(11/2
)*Sqrt[1 + (c*x^2)/b])
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{2(-6A\,c^2x^4+18x^4Bbc+2Abc\,x^2+9b^2B\,x^2+5b^2A)\sqrt{x^2(c\,x^2+b)}}{45x^{\frac{11}{2}}b^2} - \frac{2c(Ac-3Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}}{}$
default	$-\frac{2\sqrt{x^4c+b\,x^2}\left(6A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b\,c^2x^4-3A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F(}{}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(-6*A*c^2*x^4+18*B*b*c*x^4+2*A*b*c*x^2+9*B*b^2*x^2+5*A*b^2)/x^(11/2)/
b^2*(x^2*(c*x^2+b))^(1/2)-2/15*c*(A*c-3*B*b)/b^2*(-b*c)^(1/2)*((x+1/c*(-b*c
)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/
2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c)^(1/2)*EllipticE
(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1/c*(-b*c)^(1/2)*
EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)))*(x^2*(c
*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.28

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = -\frac{2\left(6\left(3\,Bbc-Ac^2\right)\sqrt{c}x^6\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\left(6\left(3\,Bbc-Ac^2\right)x^4+5\right.\right.}{45\,b^2x^6}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="fricas")
[Out] -2/45*(6*(3*B*b*c - A*c^2)*sqrt(c)*x^6*weierstrassZeta(-4*b/c, 0, weierstra
ssPInverse(-4*b/c, 0, x)) + (6*(3*B*b*c - A*c^2)*x^4 + 5*A*b^2 + (9*B*b^2 +
 2*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^2*x^6)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = \int \frac{\sqrt{x^2\left(b+cx^2\right)}(A+Bx^2)}{x^{\frac{13}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(13/2),x)
[Out] Integral(sqrt(x**2*(b + c*x**2))*(A + B*x**2)/x**(13/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{13}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(13/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{13}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(13/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(13/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^{13/2}}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(13/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(13/2), x)
```

3.230 $\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 204

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx = -\frac{2(11bB-5Ac)\sqrt{bx^2+cx^4}}{77bx^{9/2}} - \frac{4c(11bB-5Ac)\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}} - \frac{2c^{7/4}(11bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/11*A*(c*x^4+b*x^2)^(3/2)/b/x^(17/2)-2/77*(-5*A*c+11*B*b)*(c*x^4+b*x^2)^(
1/2)/b/x^(9/2)-4/231*c*(-5*A*c+11*B*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^(5/2)-2/23
1*c^(7/4)*(-5*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2
)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(
1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/
2))^2)^(1/2)/b^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used

= {2063, 2045, 2050, 2057, 335, 226}

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx =$$
$$-\frac{2c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(11bB-5Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}$$
$$-\frac{4c\sqrt{bx^2+cx^4}(11bB-5Ac)}{231b^2x^{5/2}}-\frac{2\sqrt{bx^2+cx^4}(11bB-5Ac)}{77bx^{9/2}}-\frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}}$$

```
[In] Int[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(15/2),x]
```

```
[Out] (-2*(11*b*B - 5*A*c)*Sqrt[b*x^2 + c*x^4])/(77*b*x^(9/2)) - (4*c*(11*b*B - 5
*A*c)*Sqrt[b*x^2 + c*x^4])/(231*b^2*x^(5/2)) - (2*A*(b*x^2 + c*x^4)^(3/2))/
(11*b*x^(17/2)) - (2*c^(7/4)*(11*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[
(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b
^(1/4)], 1/2])/(231*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
```

```
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}} - \frac{\left(2\left(-\frac{11bB}{2}+\frac{5Ac}{2}\right)\right)\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx}{11b}\\
&= -\frac{2(11bB-5Ac)\sqrt{bx^2+cx^4}}{77bx^{9/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}} + \frac{(2c(11bB-5Ac))\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{77b}\\
&= -\frac{2(11bB-5Ac)\sqrt{bx^2+cx^4}}{77bx^{9/2}} - \frac{4c(11bB-5Ac)\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}\\
&\quad - \frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}} - \frac{(2c^2(11bB-5Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{231b^2}\\
&= -\frac{2(11bB-5Ac)\sqrt{bx^2+cx^4}}{77bx^{9/2}} - \frac{4c(11bB-5Ac)\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}\\
&\quad - \frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}} - \frac{\left(2c^2(11bB-5Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{231b^2\sqrt{bx^2+cx^4}}\\
&= -\frac{2(11bB-5Ac)\sqrt{bx^2+cx^4}}{77bx^{9/2}} - \frac{4c(11bB-5Ac)\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} - \frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}}\\
&\quad - \frac{\left(4c^2(11bB-5Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{231b^2\sqrt{bx^2+cx^4}}
\end{aligned}
$$

$$=-\frac{2(11bB-5Ac)\sqrt{bx^2+cx^4}}{77bx^{9/2}}-\frac{4c(11bB-5Ac)\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}-\frac{2A(bx^2+cx^4)^{3/2}}{11bx^{17/2}}-\frac{2c^{7/4}(11bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.48

$$\int\frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx=-\frac{2\sqrt{x^2(b+cx^2)}\left(7A(b+cx^2)\sqrt{1+\frac{cx^2}{b}}+(11bB-5Ac)x^2\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{1}{2},-\frac{3}{4},-\frac{cx^2}{b}\right)\right)}{77bx^{13/2}\sqrt{1+\frac{cx^2}{b}}}$$

```
[In] Integrate[((A + B*x^2)*Sqrt[b*x^2 + c*x^4])/x^(15/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(7*A*(b + c*x^2)*Sqrt[1 + (c*x^2)/b] + (11*b*B -
5*A*c)*x^2*Hypergeometric2F1[-7/4, -1/2, -3/4, -((c*x^2)/b)]))/(77*b*x^(13/
2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{2\left(-10A\,c^2x^4+22x^4Bbc+6Abc\,x^2+33b^2B\,x^2+21b^2A\right)\sqrt{x^2(c\,x^2+b)}}{231x^{\frac{13}{2}}b^2}+\frac{2c(5Ac-11Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\cdots}}{231b^2\sqrt{c\,x^3+b\cdots}}$
default	$\frac{2\sqrt{x^4c+b\,x^2}\left(5A\sqrt{-bc}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\,c^2x^5-11B\sqrt{-bc}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx\cdots}{\cdots}}\right.\right.}{231x^{\frac{13}{2}}(c\,x^2+b)b^2}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(15/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/231*(-10*A*c^2*x^4+22*B*b*c*x^4+6*A*b*c*x^2+33*B*b^2*x^2+21*A*b^2)/x^(13
/2)/b^2*(x^2*(c*x^2+b))^(1/2)+2/231*c*(5*A*c-11*B*b)/b^2*(-b*c)^(1/2)*((x+1
/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1
/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*EllipticF(((x+1/c*(-
b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(x^2*(c*x^2+b))^(1/2)/x^(3/2
)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.47

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx = -\frac{2\left(2\left(11\,Bbc-5\,Ac^2\right)\sqrt{c}x^7\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(2\left(11\,Bbc-5\,Ac^2\right)x^4+21\,Ab^2+3\left(11\,Bb^2+\right.\right.\right.}{231\,b^2x^7}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="fricas")
```

```
[Out] -2/231*(2*(11*B*b*c - 5*A*c^2)*sqrt(c)*x^7*weierstrassPInverse(-4*b/c, 0, x
) + (2*(11*B*b*c - 5*A*c^2)*x^4 + 21*A*b^2 + 3*(11*B*b^2 + 2*A*b*c)*x^2)*sq
rt(c*x^4 + b*x^2)*sqrt(x))/(b^2*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(1/2)/x**(15/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{15}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(15/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx = \int \frac{\sqrt{cx^4+bx^2}(Bx^2+A)}{x^{\frac{15}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(1/2)/x^(15/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2)*(B*x^2 + A)/x^(15/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{c\,x^4+b\,x^2}}{x^{15/2}}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(15/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(1/2))/x^(15/2), x)
```

3.231 $\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 486

$$\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{88b^5(3bB-5Ac)x^{3/2}(b+cx^2)}{16575c^{9/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}$$
$$-\frac{88b^4(3bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{49725c^4}+\frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+cx^4}}{69615c^3}$$
$$-\frac{8b^2(3bB-5Ac)x^{9/2}\sqrt{bx^2+cx^4}}{7735c^2}-\frac{4b(3bB-5Ac)x^{13/2}\sqrt{bx^2+cx^4}}{595c}$$
$$-\frac{2(3bB-5Ac)x^{9/2}\left(bx^2+cx^4\right)^{3/2}}{105c}+\frac{2Bx^{5/2}\left(bx^2+cx^4\right)^{5/2}}{25c}$$
$$-\frac{88b^{21/4}(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{16575c^{19/4}\sqrt{bx^2+cx^4}}$$
$$+\frac{44b^{21/4}(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{16575c^{19/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/105*(-5*A*c+3*B*b)*x^(9/2)*(c*x^4+b*x^2)^(3/2)/c+2/25*B*x^(5/2)*(c*x^4+b
*x^2)^(5/2)/c+88/16575*b^5*(-5*A*c+3*B*b)*x^(3/2)*(c*x^2+b)/c^(9/2)/(b^(1/2
)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+88/69615*b^3*(-5*A*c+3*B*b)*x^(5/2)*(c*x^4
+b*x^2)^(1/2)/c^3-8/7735*b^2*(-5*A*c+3*B*b)*x^(9/2)*(c*x^4+b*x^2)^(1/2)/c^2
-4/595*b*(-5*A*c+3*B*b)*x^(13/2)*(c*x^4+b*x^2)^(1/2)/c-88/49725*b^4*(-5*A*c
+3*B*b)*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c^4-88/16575*b^(21/4)*(-5*A*c+3*B*b)*x*
(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*
(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(19/4)/(c*x^4
+b*x^2)^(1/2)+44/16575*b^(21/4)*(-5*A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1
```

```
/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin
(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^
2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(19/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2064, 2046, 2049, 2057, 335, 311, 226, 1210}

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}dx = \frac{44b^{21/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-5Ac)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{16575c^{19/4}\sqrt{bx^2+cx^4}} - \frac{88b^{21/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-5Ac)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{16575c^{19/4}\sqrt{bx^2+cx^4}} + \frac{88b^5x^{3/2}(b+cx^2)(3bB-5Ac)}{16575c^{9/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{88b^4\sqrt{x}\sqrt{bx^2+cx^4}(3bB-5Ac)}{49725c^4} + \frac{88b^3x^{5/2}\sqrt{bx^2+cx^4}(3bB-5Ac)}{69615c^3} - \frac{8b^2x^{9/2}\sqrt{bx^2+cx^4}(3bB-5Ac)}{7735c^2} - \frac{4bx^{13/2}\sqrt{bx^2+cx^4}(3bB-5Ac)}{595c} - \frac{2x^{9/2}(bx^2+cx^4)^{3/2}(3bB-5Ac)}{105c} + \frac{2Bx^{5/2}(bx^2+cx^4)^{5/2}}{25c}$$

```
[In] Int[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (88*b^5*(3*b*B - 5*A*c)*x^(3/2)*(b + c*x^2))/(16575*c^(9/2)*(Sqrt[b] + Sqrt
[c]*x)*Sqrt[b*x^2 + c*x^4]) - (88*b^4*(3*b*B - 5*A*c)*Sqrt[x]*Sqrt[b*x^2 +
c*x^4])/(49725*c^4) + (88*b^3*(3*b*B - 5*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/
(69615*c^3) - (8*b^2*(3*b*B - 5*A*c)*x^(9/2)*Sqrt[b*x^2 + c*x^4])/(7735*c^2
) - (4*b*(3*b*B - 5*A*c)*x^(13/2)*Sqrt[b*x^2 + c*x^4])/(595*c) - (2*(3*b*B
- 5*A*c)*x^(9/2)*(b*x^2 + c*x^4)^(3/2))/(105*c) + (2*B*x^(5/2)*(b*x^2 + c*x
^4)^(5/2))/(25*c) - (88*b^(21/4)*(3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x]
)/b^(1/4)], 1/2])/(16575*c^(19/4)*Sqrt[b*x^2 + c*x^4]) + (44*b^(21/4)*(3*b*
B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(16575*c^(19/4)*Sqrt
[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2Bx^{5/2}(bx^2+cx^4)^{5/2}}{25c} - \frac{\left(2\left(\frac{15bB}{2}-\frac{25Ac}{2}\right)\right)\int x^{7/2}(bx^2+cx^4)^{3/2}\,dx}{25c}\\
&= -\frac{2(3bB-5Ac)x^{9/2}(bx^2+cx^4)^{3/2}}{105c} + \frac{2Bx^{5/2}(bx^2+cx^4)^{5/2}}{25c}\\
&\quad - \frac{(2b(3bB-5Ac))\int x^{11/2}\sqrt{bx^2+cx^4}\,dx}{35c}\\
&= -\frac{4b(3bB-5Ac)x^{13/2}\sqrt{bx^2+cx^4}}{595c} - \frac{2(3bB-5Ac)x^{9/2}(bx^2+cx^4)^{3/2}}{105c}\\
&\quad + \frac{2Bx^{5/2}(bx^2+cx^4)^{5/2}}{25c} - \frac{(4b^2(3bB-5Ac))\int \frac{x^{15/2}}{\sqrt{bx^2+cx^4}}\,dx}{595c}\\
&= -\frac{8b^2(3bB-5Ac)x^{9/2}\sqrt{bx^2+cx^4}}{7735c^2} - \frac{4b(3bB-5Ac)x^{13/2}\sqrt{bx^2+cx^4}}{595c}\\
&\quad - \frac{2(3bB-5Ac)x^{9/2}(bx^2+cx^4)^{3/2}}{105c} + \frac{2Bx^{5/2}(bx^2+cx^4)^{5/2}}{25c}\\
&\quad + \frac{(44b^3(3bB-5Ac))\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx}{7735c^2}\\
&= \frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+cx^4}}{69615c^3} - \frac{8b^2(3bB-5Ac)x^{9/2}\sqrt{bx^2+cx^4}}{7735c^2}\\
&\quad - \frac{4b(3bB-5Ac)x^{13/2}\sqrt{bx^2+cx^4}}{595c} - \frac{2(3bB-5Ac)x^{9/2}(bx^2+cx^4)^{3/2}}{105c}\\
&\quad + \frac{2Bx^{5/2}(bx^2+cx^4)^{5/2}}{25c} - \frac{(44b^4(3bB-5Ac))\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{9945c^3}
\end{aligned}
$$

$$= -\frac{88b^4(3bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{49725c^4} + \frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+cx^4}}{69615c^3} - \frac{8b^2(3bB-5Ac)x^{9/2}\sqrt{bx^2+cx^4}}{7735c^2} - \frac{4b(3bB-5Ac)x^{13/2}\sqrt{bx^2+cx^4}}{595c} - \frac{2(3bB-5Ac)x^{9/2}(bx^2+cx^4)^{3/2}}{105c} + \frac{2Bx^{5/2}(bx^2+cx^4)^{5/2}}{25c} + \frac{(44b^5(3bB-5Ac))\int\frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{16575c^4}$$

$$= -\frac{88b^4(3bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{49725c^4} + \frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+cx^4}}{69615c^3} - \frac{8b^2(3bB-5Ac)x^{9/2}\sqrt{bx^2+cx^4}}{7735c^2} - \frac{4b(3bB-5Ac)x^{13/2}\sqrt{bx^2+cx^4}}{595c} - \frac{2(3bB-5Ac)x^{9/2}(bx^2+cx^4)^{3/2}}{105c} + \frac{2Bx^{5/2}(bx^2+cx^4)^{5/2}}{25c} + \frac{\left(44b^5(3bB-5Ac)x\sqrt{b+cx^2}\right)\int\frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{16575c^4\sqrt{bx^2+cx^4}}$$

$$= -\frac{88b^4(3bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{49725c^4} + \frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+cx^4}}{69615c^3} - \frac{8b^2(3bB-5Ac)x^{9/2}\sqrt{bx^2+cx^4}}{7735c^2} - \frac{4b(3bB-5Ac)x^{13/2}\sqrt{bx^2+cx^4}}{595c} - \frac{2(3bB-5Ac)x^{9/2}(bx^2+cx^4)^{3/2}}{105c} + \frac{2Bx^{5/2}(bx^2+cx^4)^{5/2}}{25c} + \frac{\left(88b^5(3bB-5Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{16575c^4\sqrt{bx^2+cx^4}}$$

$$= -\frac{88b^4(3bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{49725c^4} + \frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+cx^4}}{69615c^3} - \frac{8b^2(3bB-5Ac)x^{9/2}\sqrt{bx^2+cx^4}}{7735c^2} - \frac{4b(3bB-5Ac)x^{13/2}\sqrt{bx^2+cx^4}}{595c} - \frac{2(3bB-5Ac)x^{9/2}(bx^2+cx^4)^{3/2}}{105c} + \frac{2Bx^{5/2}(bx^2+cx^4)^{5/2}}{25c} + \frac{\left(88b^{11/2}(3bB-5Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{16575c^{9/2}\sqrt{bx^2+cx^4}} - \frac{\left(88b^{11/2}(3bB-5Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{16575c^{9/2}\sqrt{bx^2+cx^4}}$$

$$= \frac{88b^5(3bB-5Ac)x^{3/2}(b+cx^2)}{16575c^{9/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{88b^4(3bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{49725c^4} + \frac{88b^3(3bB-5Ac)x^{5/2}\sqrt{bx^2+cx^4}}{69615c^3} - \frac{8b^2(3bB-5Ac)x^{9/2}\sqrt{bx^2+cx^4}}{7735c^2} - \frac{4b(3bB-5Ac)x^{13/2}\sqrt{bx^2+cx^4}}{595c} - \frac{2(3bB-5Ac)x^{9/2}(bx^2+cx^4)^{3/2}}{105c} + \frac{2Bx^{5/2}(bx^2+cx^4)^{5/2}}{25c} - \frac{88b^{21/4}(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{16575c^{19/4}\sqrt{bx^2+cx^4}} + \frac{44b^{21/4}(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{16575c^{19/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.24 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.33

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{2\sqrt{x}\sqrt{x^2(b+cx^2)}\left(-(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}(1155b^3B-221c^3x^4(25A+21Bx^2)-55b^2c(35A+\ldots\right.}{116025c^4\ldots}$$

```
[In] Integrate[x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(1155
*b^3*B - 221*c^3*x^4*(25*A + 21*B*x^2) - 55*b^2*c*(35*A + 39*B*x^2) + 65*b*
c^2*x^2*(55*A + 51*B*x^2))) + 385*b^4*(3*b*B - 5*A*c)*Hypergeometric2F1[-3/
2, 3/4, 7/4, -((c*x^2)/b)]))/(116025*c^4*Sqrt[1 + (c*x^2)/b])
```

Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.70

method	result
risch	$\frac{2\sqrt{x}\,(13923B\,c^5x^{10}+16575A\,c^5x^8+17901Bb\,c^4x^8+22425Ab\,c^4x^6+468B\,b^2c^3x^6+900A\,b^2c^3x^4-540B\,b^3c^2x^4-1100A\,b^3c^2x^2+660B}{348075c^4}$
default	$-\frac{2(x^4c+b\,x^2)^{\frac{3}{2}}\left(-13923B\,c^7x^{14}-16575A\,c^7x^{12}-31824Bb\,c^6x^{12}-39000Ab\,c^6x^{10}-18369B\,b^2c^5x^{10}-23325A\,b^2c^5x^8+72B\,b^3c^4x^8+\right.}{}$

```
[In] int(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/348075/c^4*x^(1/2)*(13923*B*c^5*x^10+16575*A*c^5*x^8+17901*B*b*c^4*x^8+22
425*A*b*c^4*x^6+468*B*b^2*c^3*x^6+900*A*b^2*c^3*x^4-540*B*b^3*c^2*x^4-1100*
A*b^3*c^2*x^2+660*B*b^4*c*x^2+1540*A*b^4*c-924*B*b^5)*(x^2*(c*x^2+b))^(1/2)
-44/16575*b^5/c^5*(5*A*c-3*B*b)*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)
^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(
1/2))^(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c)^(1/2)*EllipticE(((x+1/c*(-b*c)^(
1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1/c*(-b*c)^(1/2)*EllipticF(((x+1/c
*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)))*(x^2*(c*x^2+b))^(1/2)/x^
(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.36

$$\int x^{7/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx =$$
$$-\frac{2\left(924\left(3\,Bb^6-5\,Ab^5c\right)\sqrt{c}\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\left(13923\,Bc^6x^{10}+66\right.\right.}{}$$

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/348075*(924*(3*B*b^6 - 5*A*b^5*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, wei
erstrassPInverse(-4*b/c, 0, x)) - (13923*B*c^6*x^10 + 663*(27*B*b*c^5 + 25*
A*c^6)*x^8 - 924*B*b^5*c + 1540*A*b^4*c^2 + 39*(12*B*b^2*c^4 + 575*A*b*c^5)
*x^6 - 180*(3*B*b^3*c^3 - 5*A*b^2*c^4)*x^4 + 220*(3*B*b^4*c^2 - 5*A*b^3*c^3
)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/c^5
```

Sympy [F(-1)]

Timed out.

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate(x**(7/2)*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2\right)^{\frac{3}{2}}(Bx^2+A)x^{\frac{7}{2}}\,dx$$

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(7/2), x)
```

Giac [F]

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2\right)^{\frac{3}{2}}(Bx^2+A)x^{\frac{7}{2}}\,dx$$

```
[In] integrate(x^(7/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{7/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\int x^{7/2}\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}\,dx$$

```
[In] int(x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
[Out] int(x^(7/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

3.232 $\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 321

$$\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx = -\frac{24b^4(13bB-23Ac)\sqrt{bx^2+cx^4}}{33649c^4\sqrt{x}}+\frac{72b^3(13bB-23Ac)x^{3/2}\sqrt{bx^2+cx^4}}{168245c^3} -\frac{8b^2(13bB-23Ac)x^{7/2}\sqrt{bx^2+cx^4}}{24035c^2}-\frac{4b(13bB-23Ac)x^{11/2}\sqrt{bx^2+cx^4}}{2185c} -\frac{2(13bB-23Ac)x^{7/2}\left(bx^2+cx^4\right)^{3/2}}{437c}+\frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{5/2}}{23c} +\frac{12b^{19/4}(13bB-23Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{33649c^{17/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/437*(-23*A*c+13*B*b)*x^(7/2)*(c*x^4+b*x^2)^(3/2)/c+2/23*B*x^(3/2)*(c*x^4
+b*x^2)^(5/2)/c+72/168245*b^3*(-23*A*c+13*B*b)*x^(3/2)*(c*x^4+b*x^2)^(1/2)/
c^3-8/24035*b^2*(-23*A*c+13*B*b)*x^(7/2)*(c*x^4+b*x^2)^(1/2)/c^2-4/2185*b*(
-23*A*c+13*B*b)*x^(11/2)*(c*x^4+b*x^2)^(1/2)/c-24/33649*b^4*(-23*A*c+13*B*b
)*(c*x^4+b*x^2)^(1/2)/c^4/x^(1/2)+12/33649*b^(19/4)*(-23*A*c+13*B*b)*x*(cos
(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(
1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(17/4)/(c*x^4+b*x
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2064, 2046, 2049, 2057, 335, 226}

$$\int x^{5/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{12b^{19/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(13bB-23Ac)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{33649c^{17/4}\sqrt{bx^2+cx^4}}-\frac{24b^4\sqrt{bx^2+cx^4}(13bB-23Ac)}{33649c^4\sqrt{x}}+\frac{72b^3x^{3/2}\sqrt{bx^2+cx^4}(13bB-23Ac)}{168245c^3}-\frac{8b^2x^{7/2}\sqrt{bx^2+cx^4}(13bB-23Ac)}{24035c^2}-\frac{4bx^{11/2}\sqrt{bx^2+cx^4}(13bB-23Ac)}{2185c}-\frac{2x^{7/2}\left(bx^2+cx^4\right)^{3/2}(13bB-23Ac)}{437c}+\frac{2Bx^{3/2}\left(bx^2+cx^4\right)^{5/2}}{23c}$$

```
[In] Int[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-24*b^4*(13*b*B - 23*A*c)*Sqrt[b*x^2 + c*x^4])/(33649*c^4*Sqrt[x]) + (72*b
^3*(13*b*B - 23*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(168245*c^3) - (8*b^2*(13
*b*B - 23*A*c)*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(24035*c^2) - (4*b*(13*b*B - 23
*A*c)*x^(11/2)*Sqrt[b*x^2 + c*x^4])/(2185*c) - (2*(13*b*B - 23*A*c)*x^(7/2)
*(b*x^2 + c*x^4)^(3/2))/(437*c) + (2*B*x^(3/2)*(b*x^2 + c*x^4)^(5/2))/(23*c
) + (12*b^(19/4)*(13*b*B - 23*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)
/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/
2])/(33649*c^(17/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
```

```
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2Bx^{3/2}(bx^2+cx^4)^{5/2}}{23c} - \frac{\left(2\left(\frac{13bB}{2}-\frac{23Ac}{2}\right)\right)\int x^{5/2}(bx^2+cx^4)^{3/2}\,dx}{23c} \\
&= -\frac{2(13bB-23Ac)x^{7/2}(bx^2+cx^4)^{3/2}}{437c} + \frac{2Bx^{3/2}(bx^2+cx^4)^{5/2}}{23c} \\
&\quad - \frac{(6b(13bB-23Ac))\int x^{9/2}\sqrt{bx^2+cx^4}\,dx}{437c} \\
&= -\frac{4b(13bB-23Ac)x^{11/2}\sqrt{bx^2+cx^4}}{2185c} - \frac{2(13bB-23Ac)x^{7/2}(bx^2+cx^4)^{3/2}}{437c} \\
&\quad + \frac{2Bx^{3/2}(bx^2+cx^4)^{5/2}}{23c} - \frac{(4b^2(13bB-23Ac))\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx}{2185c}
\end{aligned}$$

$$= -\frac{8b^2(13bB-23Ac)x^{7/2}\sqrt{bx^2+cx^4}}{24035c^2} - \frac{4b(13bB-23Ac)x^{11/2}\sqrt{bx^2+cx^4}}{2185c} - \frac{2(13bB-23Ac)x^{7/2}(bx^2+cx^4)^{3/2}}{437c} + \frac{2Bx^{3/2}(bx^2+cx^4)^{5/2}}{23c} + \frac{(36b^3(13bB-23Ac))\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx}{24035c^2}$$

$$= \frac{72b^3(13bB-23Ac)x^{3/2}\sqrt{bx^2+cx^4}}{168245c^3} - \frac{8b^2(13bB-23Ac)x^{7/2}\sqrt{bx^2+cx^4}}{24035c^2} - \frac{4b(13bB-23Ac)x^{11/2}\sqrt{bx^2+cx^4}}{2185c} - \frac{2(13bB-23Ac)x^{7/2}(bx^2+cx^4)^{3/2}}{437c} + \frac{2Bx^{3/2}(bx^2+cx^4)^{5/2}}{23c} - \frac{(36b^4(13bB-23Ac))\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{33649c^3}$$

$$= -\frac{24b^4(13bB-23Ac)\sqrt{bx^2+cx^4}}{33649c^4\sqrt{x}} + \frac{72b^3(13bB-23Ac)x^{3/2}\sqrt{bx^2+cx^4}}{168245c^3} - \frac{8b^2(13bB-23Ac)x^{7/2}\sqrt{bx^2+cx^4}}{24035c^2} - \frac{4b(13bB-23Ac)x^{11/2}\sqrt{bx^2+cx^4}}{2185c} - \frac{2(13bB-23Ac)x^{7/2}(bx^2+cx^4)^{3/2}}{437c} + \frac{2Bx^{3/2}(bx^2+cx^4)^{5/2}}{23c} + \frac{(12b^5(13bB-23Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{33649c^4}$$

$$= -\frac{24b^4(13bB-23Ac)\sqrt{bx^2+cx^4}}{33649c^4\sqrt{x}} + \frac{72b^3(13bB-23Ac)x^{3/2}\sqrt{bx^2+cx^4}}{168245c^3} - \frac{8b^2(13bB-23Ac)x^{7/2}\sqrt{bx^2+cx^4}}{24035c^2} - \frac{4b(13bB-23Ac)x^{11/2}\sqrt{bx^2+cx^4}}{2185c} - \frac{2(13bB-23Ac)x^{7/2}(bx^2+cx^4)^{3/2}}{437c} + \frac{2Bx^{3/2}(bx^2+cx^4)^{5/2}}{23c} + \frac{\left(12b^5(13bB-23Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{33649c^4\sqrt{bx^2+cx^4}}$$

$$= -\frac{24b^4(13bB-23Ac)\sqrt{bx^2+cx^4}}{33649c^4\sqrt{x}} + \frac{72b^3(13bB-23Ac)x^{3/2}\sqrt{bx^2+cx^4}}{168245c^3} - \frac{8b^2(13bB-23Ac)x^{7/2}\sqrt{bx^2+cx^4}}{24035c^2} - \frac{4b(13bB-23Ac)x^{11/2}\sqrt{bx^2+cx^4}}{2185c} - \frac{2(13bB-23Ac)x^{7/2}(bx^2+cx^4)^{3/2}}{437c} + \frac{2Bx^{3/2}(bx^2+cx^4)^{5/2}}{23c} + \frac{\left(24b^5(13bB-23Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{33649c^4\sqrt{bx^2+cx^4}}$$

$$= -\frac{24b^4(13bB-23Ac)\sqrt{bx^2+cx^4}}{33649c^4\sqrt{x}} + \frac{72b^3(13bB-23Ac)x^{3/2}\sqrt{bx^2+cx^4}}{168245c^3} - \frac{8b^2(13bB-23Ac)x^{7/2}\sqrt{bx^2+cx^4}}{24035c^2} - \frac{4b(13bB-23Ac)x^{11/2}\sqrt{bx^2+cx^4}}{2185c} - \frac{2(13bB-23Ac)x^{7/2}(bx^2+cx^4)^{3/2}}{437c} + \frac{2Bx^{3/2}(bx^2+cx^4)^{5/2}}{23c} + \frac{12b^{19/4}(13bB-23Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{33649c^{17/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.23 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.50

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{2\sqrt{x^2(b+cx^2)}\left(-(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}(195b^3B-55c^3x^4(23A+19Bx^2)+11bc^2x^2(69A+65\ldots\right.}{24035c^4\sqrt{\ldots}}$$

```
[In] Integrate[x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(195*b^3*B -
55*c^3*x^4*(23*A + 19*B*x^2) + 11*b*c^2*x^2*(69*A + 65*B*x^2) - 3*b^2*c*(11
5*A + 143*B*x^2))) + 15*b^4*(13*b*B - 23*A*c)*Hypergeometric2F1[-3/2, 1/4,
5/4, -((c*x^2)/b)]))/(24035*c^4*Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.90

method	result
risch	$\frac{2(7315Bc^5x^{10}+8855Ac^5x^8+9625Bbc^4x^8+12397Abc^4x^6+308Bb^2c^3x^6+644Ab^2c^3x^4-364Bb^3c^2x^4-828Ab^3c^2x^2+468Bb^4cx^2+\ldots}{168245c^4\sqrt{x}}$
default	$-\frac{2(x^4c+bx^2)^{\frac{3}{2}}\left(-7315Bc^7x^{13}-8855Ac^7x^{11}-16940Bbc^6x^{11}-21252Abc^6x^9-9933Bb^2c^5x^9-13041Ab^2c^5x^7+56Bb^3c^4x^7+690A\ldots\right.}{\ldots}$

```
[In] int(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/168245/c^4*(7315*B*c^5*x^10+8855*A*c^5*x^8+9625*B*b*c^4*x^8+12397*A*b*c^4
*x^6+308*B*b^2*c^3*x^6+644*A*b^2*c^3*x^4-364*B*b^3*c^2*x^4-828*A*b^3*c^2*x^
2+468*B*b^4*c*x^2+1380*A*b^4*c-780*B*b^5)/x^(1/2)*(x^2*(c*x^2+b))^(1/2)-12/
33649*b^5/c^5*(23*A*c-13*B*b)*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(
1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/
2))^(1/2)/(c*x^3+b*x)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))
^(1/2),1/2*2^(1/2))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(
1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.53

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{2\left(60\left(13\,Bb^6-23\,Ab^5c\right)\sqrt{c}x\,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(7315\,Bc^6x^{10}+385\left(25\,Bbc^5+\right.\right.\right.}{}$$

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/168245*(60*(13*B*b^6 - 23*A*b^5*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c,
0, x) + (7315*B*c^6*x^10 + 385*(25*B*b*c^5 + 23*A*c^6)*x^8 - 780*B*b^5*c +
1380*A*b^4*c^2 + 77*(4*B*b^2*c^4 + 161*A*b*c^5)*x^6 - 28*(13*B*b^3*c^3 - 23
*A*b^2*c^4)*x^4 + 36*(13*B*b^4*c^2 - 23*A*b^3*c^3)*x^2)*sqrt(c*x^4 + b*x^2)
*sqrt(x))/(c^5*x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(x**(5/2)*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2\right)^{\frac{3}{2}}(Bx^2+A)x^{\frac{5}{2}}\,dx$$

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(5/2), x)
```

Giac [F]

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2\right)^{\frac{3}{2}}(Bx^2+A)x^{\frac{5}{2}}\,dx$$

```
[In] integrate(x^(5/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\int x^{5/2}\,\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}\,dx$$

```
[In] int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
[Out] int(x^(5/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

3.233 $\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 447

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = -\frac{8b^4(11bB-21Ac)x^{3/2}(b+cx^2)}{3315c^{7/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{8b^3(11bB-21Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{9945c^3} - \frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+cx^4}}{13923c^2}-\frac{4b(11bB-21Ac)x^{9/2}\sqrt{bx^2+cx^4}}{1547c} - \frac{2(11bB-21Ac)x^{5/2}(bx^2+cx^4)^{3/2}}{357c}+\frac{2B\sqrt{x}(bx^2+cx^4)^{5/2}}{21c} + \frac{8b^{17/4}(11bB-21Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3315c^{15/4}\sqrt{bx^2+cx^4}} - \frac{4b^{17/4}(11bB-21Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3315c^{15/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/357*(-21*A*c+11*B*b)*x^(5/2)*(c*x^4+b*x^2)^(3/2)/c+2/21*B*(c*x^4+b*x^2)^
(5/2)*x^(1/2)/c-8/3315*b^4*(-21*A*c+11*B*b)*x^(3/2)*(c*x^2+b)/c^(7/2)/(b^(1
/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-8/13923*b^2*(-21*A*c+11*B*b)*x^(5/2)*(c*
x^4+b*x^2)^(1/2)/c^2-4/1547*b*(-21*A*c+11*B*b)*x^(9/2)*(c*x^4+b*x^2)^(1/2)/
c+8/9945*b^3*(-21*A*c+11*B*b)*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c^3+8/3315*b^(17/
4)*(-21*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)
^(1/2)/c^(15/4)/(c*x^4+b*x^2)^(1/2)-4/3315*b^(17/4)*(-21*A*c+11*B*b)*x*(cos
(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b
```

```
^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(
1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(15/4)/(c*x^4+b*x
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2064, 2046, 2049, 2057, 335, 311, 226, 1210}

$$\int x^{3/2}\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}\,dx=$$
$$-\frac{4b^{17/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(11bB-21Ac)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3315c^{15/4}\sqrt{bx^2+cx^4}}$$
$$+\frac{8b^{17/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(11bB-21Ac)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3315c^{15/4}\sqrt{bx^2+cx^4}}$$
$$-\frac{8b^4x^{3/2}(b+cx^2)(11bB-21Ac)}{3315c^{7/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{8b^3\sqrt{x}\sqrt{bx^2+cx^4}(11bB-21Ac)}{9945c^3}$$
$$-\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}(11bB-21Ac)}{13923c^2}-\frac{4bx^{9/2}\sqrt{bx^2+cx^4}(11bB-21Ac)}{1547c}$$
$$-\frac{2x^{5/2}\left(bx^2+cx^4\right)^{3/2}(11bB-21Ac)}{357c}+\frac{2B\sqrt{x}\left(bx^2+cx^4\right)^{5/2}}{21c}$$

```
[In] Int[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-8*b^4*(11*b*B - 21*A*c)*x^(3/2)*(b + c*x^2))/(3315*c^(7/2)*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[b*x^2 + c*x^4]) + (8*b^3*(11*b*B - 21*A*c)*Sqrt[x]*Sqrt[b*x^2
+ c*x^4])/(9945*c^3) - (8*b^2*(11*b*B - 21*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4]
)/(13923*c^2) - (4*b*(11*b*B - 21*A*c)*x^(9/2)*Sqrt[b*x^2 + c*x^4])/(1547*c
) - (2*(11*b*B - 21*A*c)*x^(5/2)*(b*x^2 + c*x^4)^(3/2))/(357*c) + (2*B*Sqrt
[x]*(b*x^2 + c*x^4)^(5/2))/(21*c) + (8*b^(17/4)*(11*b*B - 21*A*c)*x*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3315*c^(15/4)*Sqrt[b*x^2 + c*x^4]) - (4
*b^(17/4)*(11*b*B - 21*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[
b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(33
15*c^(15/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2B\sqrt{x}(bx^2+cx^4)^{5/2}}{21c} - \frac{\left(2\left(\frac{11bB}{2}-\frac{21Ac}{2}\right)\right)\int x^{3/2}(bx^2+cx^4)^{3/2}\,dx}{21c}\\
&= -\frac{2(11bB-21Ac)x^{5/2}(bx^2+cx^4)^{3/2}}{357c} + \frac{2B\sqrt{x}(bx^2+cx^4)^{5/2}}{21c}\\
&\quad - \frac{(2b(11bB-21Ac))\int x^{7/2}\sqrt{bx^2+cx^4}\,dx}{119c}\\
&= -\frac{4b(11bB-21Ac)x^{9/2}\sqrt{bx^2+cx^4}}{1547c} - \frac{2(11bB-21Ac)x^{5/2}(bx^2+cx^4)^{3/2}}{357c}\\
&\quad + \frac{2B\sqrt{x}(bx^2+cx^4)^{5/2}}{21c} - \frac{(4b^2(11bB-21Ac))\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx}{1547c}\\
&= -\frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+cx^4}}{13923c^2} - \frac{4b(11bB-21Ac)x^{9/2}\sqrt{bx^2+cx^4}}{1547c}\\
&\quad - \frac{2(11bB-21Ac)x^{5/2}(bx^2+cx^4)^{3/2}}{357c}\\
&\quad + \frac{2B\sqrt{x}(bx^2+cx^4)^{5/2}}{21c} + \frac{(4b^3(11bB-21Ac))\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{1989c^2}\\
&= \frac{8b^3(11bB-21Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{9945c^3} - \frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+cx^4}}{13923c^2}\\
&\quad - \frac{4b(11bB-21Ac)x^{9/2}\sqrt{bx^2+cx^4}}{1547c} - \frac{2(11bB-21Ac)x^{5/2}(bx^2+cx^4)^{3/2}}{357c}\\
&\quad + \frac{2B\sqrt{x}(bx^2+cx^4)^{5/2}}{21c} - \frac{(4b^4(11bB-21Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{3315c^3}
\end{aligned}$$

$$= \frac{8b^3(11bB-21Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{9945c^3} - \frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+cx^4}}{13923c^2} - \frac{4b(11bB-21Ac)x^{9/2}\sqrt{bx^2+cx^4}}{1547c} - \frac{2(11bB-21Ac)x^{5/2}(bx^2+cx^4)^{3/2}}{357c} + \frac{2B\sqrt{x}(bx^2+cx^4)^{5/2}}{21c} - \frac{\left(4b^4(11bB-21Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{3315c^3\sqrt{bx^2+cx^4}}$$

$$= \frac{8b^3(11bB-21Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{9945c^3} - \frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+cx^4}}{13923c^2} - \frac{4b(11bB-21Ac)x^{9/2}\sqrt{bx^2+cx^4}}{1547c} - \frac{2(11bB-21Ac)x^{5/2}(bx^2+cx^4)^{3/2}}{357c} + \frac{2B\sqrt{x}(bx^2+cx^4)^{5/2}}{21c} - \frac{\left(8b^4(11bB-21Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{3315c^3\sqrt{bx^2+cx^4}}$$

$$= \frac{8b^3(11bB-21Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{9945c^3} - \frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+cx^4}}{13923c^2} - \frac{4b(11bB-21Ac)x^{9/2}\sqrt{bx^2+cx^4}}{1547c} - \frac{2(11bB-21Ac)x^{5/2}(bx^2+cx^4)^{3/2}}{357c} + \frac{2B\sqrt{x}(bx^2+cx^4)^{5/2}}{21c} - \frac{\left(8b^{9/2}(11bB-21Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{3315c^{7/2}\sqrt{bx^2+cx^4}} + \frac{\left(8b^{9/2}(11bB-21Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{3315c^{7/2}\sqrt{bx^2+cx^4}}$$

$$= -\frac{8b^4(11bB-21Ac)x^{3/2}(b+cx^2)}{3315c^{7/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{8b^3(11bB-21Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{9945c^3} - \frac{8b^2(11bB-21Ac)x^{5/2}\sqrt{bx^2+cx^4}}{13923c^2} - \frac{4b(11bB-21Ac)x^{9/2}\sqrt{bx^2+cx^4}}{1547c} - \frac{2(11bB-21Ac)x^{5/2}(bx^2+cx^4)^{3/2}}{357c} + \frac{2B\sqrt{x}(bx^2+cx^4)^{5/2}}{21c} + \frac{8b^{17/4}(11bB-21Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3315c^{15/4}\sqrt{bx^2+cx^4}} - \frac{4b^{17/4}(11bB-21Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3315c^{15/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.31

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{2\sqrt{x}\sqrt{x^2(b+cx^2)}\left((b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}(77b^2B+13c^2x^2(21A+17Bx^2)-bc(147A+143Bx\right.}{4641c^3\sqrt{1+\frac{cx^2}{b}}}$$

```
[In] Integrate[x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(77*b^2
*B + 13*c^2*x^2*(21*A + 17*B*x^2) - b*c*(147*A + 143*B*x^2)) + 7*b^3*(-11*b
*B + 21*A*c)*Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*x^2)/b)]))/(4641*c^3*Sq
rt[1 + (c*x^2)/b])
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{2\sqrt{x}\,(-3315B\,x^8c^4-4095A\,x^6c^4-4485B\,x^6b\,c^3-5985A\,x^4b\,c^3-180B\,x^4b^2c^2-420A\,x^2b^2c^2+220B\,x^2b^3c+588A\,b^3c-308B\,b^4)\sqrt{x}}{69615c^3}$
default	$\frac{2(x^4c+b\,x^2)^{\frac{3}{2}}\left(3315B\,c^6x^{12}+4095A\,c^6x^{10}+7800Bb\,c^5x^{10}+10080Ab\,c^5x^8+4665B\,b^2c^4x^8+6405A\,b^2c^4x^6-40B\,b^3c^3x^6+1764A\,b^5c\sqrt{}\right.}{}$

```
[In] int(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/69615/c^3*x^(1/2)*(-3315*B*c^4*x^8-4095*A*c^4*x^6-4485*B*b*c^3*x^6-5985*
A*b*c^3*x^4-180*B*b^2*c^2*x^4-420*A*b^2*c^2*x^2+220*B*b^3*c*x^2+588*A*b^3*c
-308*B*b^4)*(x^2*(c*x^2+b))^(1/2)+4/3315*b^4/c^4*(21*A*c-11*B*b)*(-b*c)^(1/
2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(
-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c)
^(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1
/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2
^(1/2)))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.34

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{2\left(84\left(11\,Bb^5-21\,Ab^4c\right)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4b}{c},0,\text{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)+(3315\,B\right.}{}$$

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/69615*(84*(11*B*b^5 - 21*A*b^4*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weie
rstrassPInverse(-4*b/c, 0, x)) + (3315*B*c^5*x^8 + 195*(23*B*b*c^4 + 21*A*c
^5)*x^6 + 308*B*b^4*c - 588*A*b^3*c^2 + 45*(4*B*b^2*c^3 + 133*A*b*c^4)*x^4
- 20*(11*B*b^3*c^2 - 21*A*b^2*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/c^4
```

Sympy [F(-1)]

Timed out.

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(x**(3/2)*(B*x**2+A)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \int \left(cx^4+bx^2\right)^{\frac{3}{2}}(Bx^2+A)x^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(3/2), x)
```

Giac [F]

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\int\left(cx^4+bx^2\right)^{\frac{3}{2}}(Bx^2+A)x^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^(3/2)*(B*x^2+A)*(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\int x^{3/2}\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}\,dx$$

```
[In] int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
[Out] int(x^(3/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

3.234 $\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 282

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx=\frac{8b^3(9bB-19Ac)\sqrt{bx^2+cx^4}}{4389c^3\sqrt{x}}-\frac{8b^2(9bB-19Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7315c^2}-\frac{4b(9bB-19Ac)x^{7/2}\sqrt{bx^2+cx^4}}{1045c}-\frac{2(9bB-19Ac)x^{3/2}\left(bx^2+cx^4\right)^{3/2}}{285c}+\frac{2B\left(bx^2+cx^4\right)^{5/2}}{19c\sqrt{x}}-\frac{4b^{15/4}(9bB-19Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{4389c^{13/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/285*(-19*A*c+9*B*b)*x^(3/2)*(c*x^4+b*x^2)^(3/2)/c+2/19*B*(c*x^4+b*x^2)^(
5/2)/c/x^(1/2)-8/7315*b^2*(-19*A*c+9*B*b)*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c^2-4
/1045*b*(-19*A*c+9*B*b)*x^(7/2)*(c*x^4+b*x^2)^(1/2)/c+8/4389*b^3*(-19*A*c+9
*B*b)*(c*x^4+b*x^2)^(1/2)/c^3/x^(1/2)-4/4389*b^(15/4)*(-19*A*c+9*B*b)*x*(co
s(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/
b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^
(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(13/4)/(c*x^4+b*
x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2064, 2046, 2049, 2057, 335, 226}

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = -\frac{4b^{15/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9bB-19Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{4389c^{13/4}\sqrt{bx^2+cx^4}} + \frac{8b^3\sqrt{bx^2+cx^4}(9bB-19Ac)}{4389c^3\sqrt{x}} - \frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}(9bB-19Ac)}{7315c^2} - \frac{4bx^{7/2}\sqrt{bx^2+cx^4}(9bB-19Ac)}{1045c} - \frac{2x^{3/2}(bx^2+cx^4)^{3/2}(9bB-19Ac)}{285c} + \frac{2B(bx^2+cx^4)^{5/2}}{19c\sqrt{x}}$$

```
[In] Int[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (8*b^3*(9*b*B - 19*A*c)*Sqrt[b*x^2 + c*x^4])/(4389*c^3*Sqrt[x]) - (8*b^2*(9
*b*B - 19*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(7315*c^2) - (4*b*(9*b*B - 19*A
*c)*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(1045*c) - (2*(9*b*B - 19*A*c)*x^(3/2)*(b*
x^2 + c*x^4)^(3/2))/(285*c) + (2*B*(b*x^2 + c*x^4)^(5/2))/(19*c*Sqrt[x]) -
(4*b^(15/4)*(9*b*B - 19*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt
[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(4
389*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
```

```
x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2B(bx^2+cx^4)^{5/2}}{19c\sqrt{x}} - \frac{\left(2\left(\frac{9bB}{2}-\frac{19Ac}{2}\right)\right)\int \sqrt{x}(bx^2+cx^4)^{3/2}\,dx}{19c} \\
&= -\frac{2(9bB-19Ac)x^{3/2}(bx^2+cx^4)^{3/2}}{285c} + \frac{2B(bx^2+cx^4)^{5/2}}{19c\sqrt{x}} \\
&\quad - \frac{(2b(9bB-19Ac))\int x^{5/2}\sqrt{bx^2+cx^4}\,dx}{95c} \\
&= -\frac{4b(9bB-19Ac)x^{7/2}\sqrt{bx^2+cx^4}}{1045c} - \frac{2(9bB-19Ac)x^{3/2}(bx^2+cx^4)^{3/2}}{285c} \\
&\quad + \frac{2B(bx^2+cx^4)^{5/2}}{19c\sqrt{x}} - \frac{(4b^2(9bB-19Ac))\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx}{1045c}
\end{aligned}$$

$$= -\frac{8b^2(9bB-19Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7315c^2} - \frac{4b(9bB-19Ac)x^{7/2}\sqrt{bx^2+cx^4}}{1045c} - \frac{2(9bB-19Ac)x^{3/2}(bx^2+cx^4)^{3/2}}{285c} + \frac{2B(bx^2+cx^4)^{5/2}}{19c\sqrt{x}} + \frac{\left(4b^3(9bB-19Ac)\right)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{1463c^2}$$

$$= \frac{8b^3(9bB-19Ac)\sqrt{bx^2+cx^4}}{4389c^3\sqrt{x}} - \frac{8b^2(9bB-19Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7315c^2} - \frac{4b(9bB-19Ac)x^{7/2}\sqrt{bx^2+cx^4}}{1045c} - \frac{2(9bB-19Ac)x^{3/2}(bx^2+cx^4)^{3/2}}{285c} + \frac{2B(bx^2+cx^4)^{5/2}}{19c\sqrt{x}} - \frac{\left(4b^4(9bB-19Ac)\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{4389c^3}$$

$$= \frac{8b^3(9bB-19Ac)\sqrt{bx^2+cx^4}}{4389c^3\sqrt{x}} - \frac{8b^2(9bB-19Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7315c^2} - \frac{4b(9bB-19Ac)x^{7/2}\sqrt{bx^2+cx^4}}{1045c} - \frac{2(9bB-19Ac)x^{3/2}(bx^2+cx^4)^{3/2}}{285c} + \frac{2B(bx^2+cx^4)^{5/2}}{19c\sqrt{x}} - \frac{\left(4b^4(9bB-19Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{4389c^3\sqrt{bx^2+cx^4}}$$

$$= \frac{8b^3(9bB-19Ac)\sqrt{bx^2+cx^4}}{4389c^3\sqrt{x}} - \frac{8b^2(9bB-19Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7315c^2} - \frac{4b(9bB-19Ac)x^{7/2}\sqrt{bx^2+cx^4}}{1045c} - \frac{2(9bB-19Ac)x^{3/2}(bx^2+cx^4)^{3/2}}{285c} + \frac{2B(bx^2+cx^4)^{5/2}}{19c\sqrt{x}} - \frac{\left(8b^4(9bB-19Ac)x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{4389c^3\sqrt{bx^2+cx^4}}$$

$$= \frac{8b^3(9bB-19Ac)\sqrt{bx^2+cx^4}}{4389c^3\sqrt{x}} - \frac{8b^2(9bB-19Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7315c^2} - \frac{4b(9bB-19Ac)x^{7/2}\sqrt{bx^2+cx^4}}{1045c} - \frac{2(9bB-19Ac)x^{3/2}(bx^2+cx^4)^{3/2}}{285c} + \frac{2B(bx^2+cx^4)^{5/2}}{19c\sqrt{x}} - \frac{4b^{15/4}(9bB-19Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{4389c^{13/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.49

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \frac{2\sqrt{x^2(b+cx^2)}\left((b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}(45b^2B+11c^2x^2(19A+15Bx^2)-bc(95A+99Bx^2))+5\right.}{3135c^3\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

```
[In] Integrate[Sqrt[x]*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(45*b^2*B + 11*
c^2*x^2*(19*A + 15*B*x^2) - b*c*(95*A + 99*B*x^2)) + 5*b^3*(-9*b*B + 19*A*c
)*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]))/(3135*c^3*Sqrt[x]*Sqrt[
1 + (c*x^2)/b])
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{2\left(-1155B\,x^8c^4-1463A\,x^6c^4-1617B\,x^6b\,c^3-2261A\,x^4b\,c^3-84B\,x^4b^2c^2-228A\,x^2b^2c^2+108B\,x^2b^3c+380A\,b^3c-180B\,b^4\right)\sqrt{x^2(c\,x^2}}{21945c^3\sqrt{x}}$
default	$\frac{2\left(x^4c+b\,x^2\right)^{\frac{3}{2}}\left(1155B\,c^6x^{11}+1463A\,c^6x^9+2772Bb\,c^5x^9+3724Ab\,c^5x^7+1701B\,b^2c^4x^7+190A\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-}\right.}{}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21945/c^3*(-1155*B*c^4*x^8-1463*A*c^4*x^6-1617*B*b*c^3*x^6-2261*A*b*c^3*
x^4-84*B*b^2*c^2*x^4-228*A*b^2*c^2*x^2+108*B*b^3*c*x^2+380*A*b^3*c-180*B*b^
4)/x^(1/2)*(x^2*(c*x^2+b))^(1/2)+4/4389*b^4/c^4*(19*A*c-9*B*b)*(-b*c)^(1/2)
*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b
*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*EllipticF(((x+
1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(x^2*(c*x^2+b))^(1/2)/
x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.52

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = -\frac{2\left(20\left(9\,Bb^5-19\,Ab^4c\right)\sqrt{c}x\,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\left(1155\,Bc^5x^8+77\left(21\,Bbc^4+19\,Ac^5\right)x^6+18\right.\right.}{21945\,c^4x}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] -2/21945*(20*(9*B*b^5 - 19*A*b^4*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0
, x) - (1155*B*c^5*x^8 + 77*(21*B*b*c^4 + 19*A*c^5)*x^6 + 180*B*b^4*c - 380
*A*b^3*c^2 + 7*(12*B*b^2*c^3 + 323*A*b*c^4)*x^4 - 12*(9*B*b^3*c^2 - 19*A*b^
2*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^4*x)
```

Sympy [F]

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \int \sqrt{x}\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\left(A+Bx^2\right)\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*(x**2*(b + c*x**2))**(3/2)*(A + B*x**2), x)
```

Maxima [F]

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \int \left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)\sqrt{x}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*sqrt(x), x)
```

Giac [F]

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \int \left(cx^4+bx^2\right)^{\frac{3}{2}}(Bx^2+A)\sqrt{x}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)*x^(1/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}\,dx = \int \sqrt{x}\,\left(B\,x^2+A\right)\,\left(c\,x^4+b\,x^2\right)^{3/2}\,dx$$

```
[In] int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2),x)
[Out] int(x^(1/2)*(A + B*x^2)*(b*x^2 + c*x^4)^(3/2), x)
```

3.235 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 408

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx = \frac{8b^3(7bB-17Ac)x^{3/2}(b+cx^2)}{1105c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{8b^2(7bB-17Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2} - \frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c} - \frac{2(7bB-17Ac)\sqrt{x}(bx^2+cx^4)^{3/2}}{221c} + \frac{2B(bx^2+cx^4)^{5/2}}{17cx^{3/2}} - \frac{8b^{13/4}(7bB-17Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105c^{11/4}\sqrt{bx^2+cx^4}} + \frac{4b^{13/4}(7bB-17Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{1105c^{11/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/17*B*(c*x^4+b*x^2)^(5/2)/c/x^(3/2)-2/221*(-17*A*c+7*B*b)*(c*x^4+b*x^2)^(3
/2)*x^(1/2)/c+8/1105*b^3*(-17*A*c+7*B*b)*x^(3/2)*(c*x^2+b)/c^(5/2)/(b^(1/2)
+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-4/663*b*(-17*A*c+7*B*b)*x^(5/2)*(c*x^4+b*x^
2)^(1/2)/c-8/3315*b^2*(-17*A*c+7*B*b)*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c^2-8/110
5*b^(13/4)*(-17*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/
2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^
(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1
/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)+4/1105*b^(13/4)*(-17*A*c+7*B*b)*
x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(
1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2)
)*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x
^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2064, 2046, 2049, 2057, 335, 311, 226, 1210}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{\sqrt{x}}dx = \frac{4b^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-17Ac)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{1105c^{11/4}\sqrt{bx^2+cx^4}} - \frac{8b^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-17Ac)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105c^{11/4}\sqrt{bx^2+cx^4}} + \frac{8b^3x^{3/2}(b+cx^2)(7bB-17Ac)}{1105c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{8b^2\sqrt{x}\sqrt{bx^2+cx^4}(7bB-17Ac)}{3315c^2} - \frac{2\sqrt{x}(bx^2+cx^4)^{3/2}(7bB-17Ac)}{221c} - \frac{4bx^{5/2}\sqrt{bx^2+cx^4}(7bB-17Ac)}{663c} + \frac{2B(bx^2+cx^4)^{5/2}}{17cx^{3/2}}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/Sqrt[x],x]
```

```
[Out] (8*b^3*(7*b*B - 17*A*c)*x^(3/2)*(b + c*x^2))/(1105*c^(5/2)*(Sqrt[b] + Sqrt[
c]*x)*Sqrt[b*x^2 + c*x^4]) - (8*b^2*(7*b*B - 17*A*c)*Sqrt[x]*Sqrt[b*x^2 + c
*x^4])/(3315*c^2) - (4*b*(7*b*B - 17*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(663
*c) - (2*(7*b*B - 17*A*c)*Sqrt[x]*(b*x^2 + c*x^4)^(3/2))/(221*c) + (2*B*(b*
x^2 + c*x^4)^(5/2))/(17*c*x^(3/2)) - (8*b^(13/4)*(7*b*B - 17*A*c)*x*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(1105*c^(11/4)*Sqrt[b*x^2 + c*x^4]) + (4
*b^(13/4)*(7*b*B - 17*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b
] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(110
5*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\text{integral} = \frac{2B(bx^2+cx^4)^{5/2}}{17cx^{3/2}} - \frac{\left(2\left(\frac{7bB}{2}-\frac{17Ac}{2}\right)\right)\int \frac{(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx}{17c}$$

$$= -\frac{2(7bB-17Ac)\sqrt{x}(bx^2+cx^4)^{3/2}}{221c} + \frac{2B(bx^2+cx^4)^{5/2}}{17cx^{3/2}} - \frac{(6b(7bB-17Ac))\int x^{3/2}\sqrt{bx^2+cx^4}\,dx}{221c}$$

$$= -\frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c} - \frac{2(7bB-17Ac)\sqrt{x}(bx^2+cx^4)^{3/2}}{221c} + \frac{2B(bx^2+cx^4)^{5/2}}{17cx^{3/2}} - \frac{(4b^2(7bB-17Ac))\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{663c}$$

$$= -\frac{8b^2(7bB-17Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2} - \frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c} - \frac{2(7bB-17Ac)\sqrt{x}(bx^2+cx^4)^{3/2}}{221c} + \frac{2B(bx^2+cx^4)^{5/2}}{17cx^{3/2}} + \frac{(4b^3(7bB-17Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{1105c^2}$$

$$= -\frac{8b^2(7bB-17Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2} - \frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c} - \frac{2(7bB-17Ac)\sqrt{x}(bx^2+cx^4)^{3/2}}{221c} + \frac{2B(bx^2+cx^4)^{5/2}}{17cx^{3/2}} + \frac{\left(4b^3(7bB-17Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{1105c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{8b^2(7bB-17Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2} - \frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c} - \frac{2(7bB-17Ac)\sqrt{x}(bx^2+cx^4)^{3/2}}{221c} + \frac{2B(bx^2+cx^4)^{5/2}}{17cx^{3/2}} + \frac{\left(8b^3(7bB-17Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{1105c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{8b^2(7bB-17Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2} - \frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c}$$
$$- \frac{2(7bB-17Ac)\sqrt{x}(bx^2+cx^4)^{3/2}}{221c} + \frac{2B(bx^2+cx^4)^{5/2}}{17cx^{3/2}}$$
$$+ \frac{\left(8b^{7/2}(7bB-17Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{1105c^{5/2}\sqrt{bx^2+cx^4}}$$
$$- \frac{\left(8b^{7/2}(7bB-17Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{1105c^{5/2}\sqrt{bx^2+cx^4}}$$
$$= \frac{8b^3(7bB-17Ac)x^{3/2}(b+cx^2)}{1105c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{8b^2(7bB-17Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2}$$
$$- \frac{4b(7bB-17Ac)x^{5/2}\sqrt{bx^2+cx^4}}{663c}$$
$$- \frac{2(7bB-17Ac)\sqrt{x}(bx^2+cx^4)^{3/2}}{221c} + \frac{2B(bx^2+cx^4)^{5/2}}{17cx^{3/2}}$$
$$- \frac{8b^{13/4}(7bB-17Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105c^{11/4}\sqrt{bx^2+cx^4}}$$
$$+ \frac{4b^{13/4}(7bB-17Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105c^{11/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.28

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\sqrt{x^2(b+cx^2)}\left(-(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}(7bB-17Ac-13Bcx^2)+b^2(7\right.}{221c^2\sqrt{1+\frac{cx^2}{b}}}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(7*b*
B - 17*A*c - 13*B*c*x^2)) + b^2*(7*b*B - 17*A*c)*Hypergeometric2F1[-3/2, 3/
4, 7/4, -((c*x^2)/b)]))/(221*c^2*Sqrt[1 + (c*x^2)/b])
```

Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.71

method	result
risch	$\frac{2\sqrt{x}\,(195B\,c^3x^6+255A\,c^3x^4+285Bb\,c^2x^4+425Ab\,c^2x^2+20B\,b^2c\,x^2+68b^2Ac-28B\,b^3)\sqrt{x^2(c\,x^2+b)}}{3315c^2} - \frac{4b^3(17Ac-7Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\right.}{}}}{}$
default	$-\frac{2(x^4c+b\,x^2)^{\frac{3}{2}}\left(-195B\,c^5x^{10}-255A\,c^5x^8-480Bb\,c^4x^8-680Ab\,c^4x^6-305B\,b^2c^3x^6+204A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}E\right.}{}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3315/c^2*x^(1/2)*(195*B*c^3*x^6+255*A*c^3*x^4+285*B*b*c^2*x^4+425*A*b*c^2
*x^2+20*B*b^2*c*x^2+68*A*b^2*c-28*B*b^3)*(x^2*(c*x^2+b))^(1/2)-4/1105*b^3/c
^3*(17*A*c-7*B*b)*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*
(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c
*x^3+b*x)^(1/2)*(-2/c*(-b*c)^(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)
^(1/2))^(1/2),1/2*2^(1/2))+1/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))
*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b
)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.31

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx =$$
$$-\frac{2\left(12\left(7\,Bb^4-17\,Ab^3c\right)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\left(195\,Bc^4x^6-28\,Bb^3c\right.\right.}{3315\,c^3}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3315*(12*(7*B*b^4 - 17*A*b^3*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weier
strassPInverse(-4*b/c, 0, x)) - (195*B*c^4*x^6 - 28*B*b^3*c + 68*A*b^2*c^2
+ 15*(19*B*b*c^3 + 17*A*c^4)*x^4 + 5*(4*B*b^2*c^2 + 85*A*b*c^3)*x^2)*sqrt(c
*x^4 + b*x^2)*sqrt(x))/c^3
```

Sympy [F]

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{\sqrt{x}}\,dx = \int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}(A+Bx^2)}{\sqrt{x}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(1/2),x)
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/sqrt(x), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{\sqrt{x}}\,dx = \int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}(Bx^2+A)}{\sqrt{x}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/sqrt(x), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{\sqrt{x}}\,dx = \int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}(Bx^2+A)}{\sqrt{x}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{\sqrt{x}}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{\sqrt{x}}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(1/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(1/2), x)
```

3.236 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 239

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx = -\frac{8b^2(bB-3Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{4b(bB-3Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} - \frac{2(bB-3Ac)(bx^2+cx^4)^{3/2}}{33c\sqrt{x}} + \frac{2B(bx^2+cx^4)^{5/2}}{15cx^{5/2}} + \frac{4b^{11/4}(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/15*B*(c*x^4+b*x^2)^(5/2)/c/x^(5/2)-2/33*(-3*A*c+B*b)*(c*x^4+b*x^2)^(3/2)/
c/x^(1/2)-4/77*b*(-3*A*c+B*b)*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c-8/231*b^2*(-3*A
*c+B*b)*(c*x^4+b*x^2)^(1/2)/c^2/x^(1/2)+4/231*b^(11/4)*(-3*A*c+B*b)*x*(cos(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1
/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(9/4)/(c*x^4+b*x^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used

= {2064, 2046, 2049, 2057, 335, 226}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx = \frac{4b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-3Ac)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.\right.}{231c^{9/4}\sqrt{bx^2+cx^4}}$$
$$-\frac{8b^2\sqrt{bx^2+cx^4}(bB-3Ac)}{231c^2\sqrt{x}}-\frac{2(bx^2+cx^4)^{3/2}(bB-3Ac)}{33c\sqrt{x}}$$
$$-\frac{4bx^{3/2}\sqrt{bx^2+cx^4}(bB-3Ac)}{77c}+\frac{2B(bx^2+cx^4)^{5/2}}{15cx^{5/2}}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(3/2),x]
```

```
[Out] (-8*b^2*(b*B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(231*c^2*Sqrt[x]) - (4*b*(b*B -
3*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(77*c) - (2*(b*B - 3*A*c)*(b*x^2 + c*x^
4)^(3/2))/(33*c*Sqrt[x]) + (2*B*(b*x^2 + c*x^4)^(5/2))/(15*c*x^(5/2)) + (4*
b^(11/4)*(b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(231*c^(
9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
```

```
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\text{integral} = \frac{2B(bx^2+cx^4)^{5/2}}{15cx^{5/2}} - \frac{\left(2\left(\frac{5bB}{2}-\frac{15Ac}{2}\right)\right)\int \frac{(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx}{15c}$$

$$= -\frac{2(bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{33c\sqrt{x}} + \frac{2B(bx^2+cx^4)^{5/2}}{15cx^{5/2}} - \frac{(2b(bB-3Ac))\int \sqrt{x}\sqrt{bx^2+cx^4}\,dx}{11c}$$

$$= -\frac{4b(bB-3Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} - \frac{2(bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{33c\sqrt{x}} + \frac{2B(bx^2+cx^4)^{5/2}}{15cx^{5/2}} - \frac{(4b^2(bB-3Ac))\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{77c}$$

$$= -\frac{8b^2(bB-3Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{4b(bB-3Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} - \frac{2(bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{33c\sqrt{x}} + \frac{2B(bx^2+cx^4)^{5/2}}{15cx^{5/2}} + \frac{(4b^3(bB-3Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{231c^2}$$

$$= -\frac{8b^2(bB-3Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{4b(bB-3Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} - \frac{2(bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{33c\sqrt{x}} + \frac{2B(bx^2+cx^4)^{5/2}}{15cx^{5/2}} + \frac{\left(4b^3(bB-3Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{231c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{8b^2(bB-3Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{4b(bB-3Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} - \frac{2(bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{33c\sqrt{x}} + \frac{2B\left(bx^2+cx^4\right)^{5/2}}{15cx^{5/2}} + \frac{\left(8b^3(bB-3Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{231c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{8b^2(bB-3Ac)\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} - \frac{4b(bB-3Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c} - \frac{2(bB-3Ac)\left(bx^2+cx^4\right)^{3/2}}{33c\sqrt{x}} + \frac{2B\left(bx^2+cx^4\right)^{5/2}}{15cx^{5/2}} + \frac{4b^{11/4}(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.48

$$\int\frac{\left(A+Bx^2\right)\left(bx^2+cx^4\right)^{3/2}}{x^{3/2}}\,dx = \frac{2\sqrt{x^2\left(b+cx^2\right)}\left(-\left(b+cx^2\right)^2\sqrt{1+\frac{cx^2}{b}}\left(5bB-15Ac-11Bcx^2\right)+5b^2(bB\right.}{165c^2\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-((b + c*x^2)^2*Sqrt[1 + (c*x^2)/b]*(5*b*B - 15*A
*c - 11*B*c*x^2)) + 5*b^2*(b*B - 3*A*c)*Hypergeometric2F1[-3/2, 1/4, 5/4, -
((c*x^2)/b)]))/(165*c^2*Sqrt[x]*Sqrt[1 + (c*x^2)/b])
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.01

method	result
risch	$\frac{2\left(77B\,c^3x^6+105A\,c^3x^4+119Bb\,c^2x^4+195Ab\,c^2x^2+12B\,b^2c\,x^2+60b^2Ac-20B\,b^3\right)\sqrt{x^2\left(c\,x^2+b\right)}}{1155c^2\sqrt{x}} - \frac{4b^3(3Ac-Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right.}{\sqrt{-bc}}}}{}$
default	$-\frac{2\left(x^4c+b\,x^2\right)^{\frac{3}{2}}\left(-77B\,c^5x^9-105A\,c^5x^7-196Bb\,c^4x^7+30A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b^3c\right.}{}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1155/c^2*(77*B*c^3*x^6+105*A*c^3*x^4+119*B*b*c^2*x^4+195*A*b*c^2*x^2+12*B
*b^2*c*x^2+60*A*b^2*c-20*B*b^3)/x^(1/2)*(x^2*(c*x^2+b))^(1/2)-4/231*b^3/c^3
*(3*A*c-B*b)*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(
x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+
b*x)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.51

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx = \frac{2\left(20\left(Bb^4-3\,Ab^3c\right)\sqrt{c}x\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(77\,Bc^4x^6-20\right.\right.}{}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(20*(B*b^4 - 3*A*b^3*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x)
+ (77*B*c^4*x^6 - 20*B*b^3*c + 60*A*b^2*c^2 + 7*(17*B*b*c^3 + 15*A*c^4)*x^4
 + 3*(4*B*b^2*c^2 + 65*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^3*x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}\,(A+Bx^2)}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(3/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{3/2}}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(3/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(3/2), x)
```

3.237 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 369

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx = -\frac{8b^2(3bB-13Ac)x^{3/2}(b+cx^2)}{195c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{4b(3bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{195c} - \frac{2(3bB-13Ac)(bx^2+cx^4)^{3/2}}{117cx^{3/2}} + \frac{2B(bx^2+cx^4)^{5/2}}{13cx^{7/2}} + \frac{8b^{9/4}(3bB-13Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{7/4}\sqrt{bx^2+cx^4}} - \frac{4b^{9/4}(3bB-13Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{195c^{7/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/117*(-13*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/c/x^(3/2)+2/13*B*(c*x^4+b*x^2)^(
5/2)/c/x^(7/2)-8/195*b^2*(-13*A*c+3*B*b)*x^(3/2)*(c*x^2+b)/c^(3/2)/(b^(1/2)
+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-4/195*b*(-13*A*c+3*B*b)*x^(1/2)*(c*x^4+b*x^
2)^(1/2)/c+8/195*b^(9/4)*(-13*A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(
b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)-4/195*b^(9/4)*(-13*
A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),
1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^
(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2064, 2046, 2057, 335, 311, 226, 1210}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx = -\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-13Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{195c^{7/4}\sqrt{bx^2+cx^4}} + \frac{8b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-13Ac)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{7/4}\sqrt{bx^2+cx^4}} - \frac{8b^2x^{3/2}(b+cx^2)(3bB-13Ac)}{195c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{4b\sqrt{x}\sqrt{bx^2+cx^4}(3bB-13Ac)}{195c} - \frac{2(bx^2+cx^4)^{3/2}(3bB-13Ac)}{117cx^{3/2}} + \frac{2B(bx^2+cx^4)^{5/2}}{13cx^{7/2}}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(5/2),x]
```

```
[Out] (-8*b^2*(3*b*B - 13*A*c)*x^(3/2)*(b + c*x^2))/(195*c^(3/2)*(Sqrt[b] + Sqrt[
c]*x)*Sqrt[b*x^2 + c*x^4]) - (4*b*(3*b*B - 13*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x
^4])/(195*c) - (2*(3*b*B - 13*A*c)*(b*x^2 + c*x^4)^(3/2))/(117*c*x^(3/2)) +
 (2*B*(b*x^2 + c*x^4)^(5/2))/(13*c*x^(7/2)) + (8*b^(9/4)*(3*b*B - 13*A*c)*x
*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[
2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(195*c^(7/4)*Sqrt[b*x^2 + c*x^4]
) - (4*b^(9/4)*(3*b*B - 13*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(S
qrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])
/(195*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\text{integral} = \frac{2B(bx^2+cx^4)^{5/2}}{13cx^{7/2}} - \frac{\left(2\left(\frac{3bB}{2}-\frac{13Ac}{2}\right)\right)\int\frac{(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx}{13c}$$

$$= -\frac{2(3bB-13Ac)\,(bx^2+cx^4)^{3/2}}{117cx^{3/2}} + \frac{2B(bx^2+cx^4)^{5/2}}{13cx^{7/2}} - \frac{(2b(3bB-13Ac))\int\frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx}{39c}$$

$$= -\frac{4b(3bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{195c} - \frac{2(3bB-13Ac)\left(bx^2+cx^4\right)^{3/2}}{117cx^{3/2}} + \frac{2B\left(bx^2+cx^4\right)^{5/2}}{13cx^{7/2}} - \frac{\left(4b^2(3bB-13Ac)\right)\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{195c}$$

$$= -\frac{4b(3bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{195c} - \frac{2(3bB-13Ac)\left(bx^2+cx^4\right)^{3/2}}{117cx^{3/2}} + \frac{2B\left(bx^2+cx^4\right)^{5/2}}{13cx^{7/2}} - \frac{\left(4b^2(3bB-13Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{195c\sqrt{bx^2+cx^4}}$$

$$= -\frac{4b(3bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{195c} - \frac{2(3bB-13Ac)\left(bx^2+cx^4\right)^{3/2}}{117cx^{3/2}} + \frac{2B\left(bx^2+cx^4\right)^{5/2}}{13cx^{7/2}} - \frac{\left(8b^2(3bB-13Ac)x\sqrt{b+cx^2}\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{195c\sqrt{bx^2+cx^4}}$$

$$= -\frac{4b(3bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{195c} - \frac{2(3bB-13Ac)\left(bx^2+cx^4\right)^{3/2}}{117cx^{3/2}} + \frac{2B\left(bx^2+cx^4\right)^{5/2}}{13cx^{7/2}} - \frac{\left(8b^{5/2}(3bB-13Ac)x\sqrt{b+cx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{195c^{3/2}\sqrt{bx^2+cx^4}} + \frac{\left(8b^{5/2}(3bB-13Ac)x\sqrt{b+cx^2}\right)\operatorname{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{195c^{3/2}\sqrt{bx^2+cx^4}}$$

$$= -\frac{8b^2(3bB-13Ac)x^{3/2}(b+cx^2)}{195c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{4b(3bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{195c} - \frac{2(3bB-13Ac)\left(bx^2+cx^4\right)^{3/2}}{117cx^{3/2}} + \frac{2B\left(bx^2+cx^4\right)^{5/2}}{13cx^{7/2}} + \frac{8b^{9/4}(3bB-13Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{7/4}\sqrt{bx^2+cx^4}} - \frac{4b^{9/4}(3bB-13Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{7/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.27

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx = \frac{2\sqrt{x}\sqrt{x^2(b+cx^2)}\left(3B(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}+b(-3bB+13Ac)\,\text{Hypergeom}\right.}{39c\sqrt{1+\frac{cx^2}{b}}}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(5/2),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)]*(3*B*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + b
*(-3*b*B + 13*A*c)*Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*x^2)/b)]))/(39*c*
Sqrt[1 + (c*x^2)/b])
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.72

method	result
risch	$\frac{2\sqrt{x}\left(45B\,c^2x^4+65A\,c^2x^2+75Bbc\,x^2+143Abc+12B\,b^2\right)\sqrt{x^2(c\,x^2+b)}}{585c}+\frac{4b^2(13Ac-3Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-}}{}$
default	$\frac{2\left(x^4c+b\,x^2\right)^{\frac{3}{2}}\left(45B\,x^8c^4+65A\,x^6c^4+120B\,x^6b\,c^3+156A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b^3c-78A\sqrt{\frac{cx+}{\sqrt{-}}}\right.}{}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/585/c*x^(1/2)*(45*B*c^2*x^4+65*A*c^2*x^2+75*B*b*c*x^2+143*A*b*c+12*B*b^2)
*(x^2*(c*x^2+b))^(1/2)+4/195*b^2/c^2*(13*A*c-3*B*b)*(-b*c)^(1/2)*((x+1/c*(-
b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^
(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c)^(1/2)*Ellipt
icE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1/c*(-b*c)^(1/
2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)))*(x^2
*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.28

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx = \frac{2\left(12\left(3\,Bb^3-13\,Ab^2c\right)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\right.\right.}{}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(12*(3*B*b^3 - 13*A*b^2*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierst
rassPInverse(-4*b/c, 0, x)) + (45*B*c^3*x^4 + 12*B*b^2*c + 143*A*b*c^2 + 5*
(15*B*b*c^2 + 13*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/c^2
```

Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}\,(A+Bx^2)}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(5/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(5/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(5/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{5/2}}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(5/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(5/2), x)
```

3.238 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 201

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx = -\frac{4b(bB-11Ac)\sqrt{bx^2+cx^4}}{77c\sqrt{x}} - \frac{2(bB-11Ac)(bx^2+cx^4)^{3/2}}{77cx^{5/2}} + \frac{2B(bx^2+cx^4)^{5/2}}{11cx^{9/2}} - \frac{4b^{7/4}(bB-11Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77c^{5/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/77*(-11*A*c+B*b)*(c*x^4+b*x^2)^(3/2)/c/x^(5/2)+2/11*B*(c*x^4+b*x^2)^(5/2
)/c/x^(9/2)-4/77*b*(-11*A*c+B*b)*(c*x^4+b*x^2)^(1/2)/c/x^(1/2)-4/77*b^(7/4)
*(-11*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2
)/c^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used

$= \{2064, 2046, 2057, 335, 226\}$

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}\,dx = -\frac{4b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-11Ac)\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77c^{5/4}\sqrt{bx^2+cx^4}} - \frac{4b\sqrt{bx^2+cx^4}(bB-11Ac)}{77c\sqrt{x}} - \frac{2(bx^2+cx^4)^{3/2}(bB-11Ac)}{77cx^{5/2}} + \frac{2B(bx^2+cx^4)^{5/2}}{11cx^{9/2}}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(7/2),x]
```

```
[Out] (-4*b*(b*B - 11*A*c)*Sqrt[b*x^2 + c*x^4])/(77*c*Sqrt[x]) - (2*(b*B - 11*A*c
)*(b*x^2 + c*x^4)^(3/2))/(77*c*x^(5/2)) + (2*B*(b*x^2 + c*x^4)^(5/2))/(11*c
*x^(9/2)) - (4*b^(7/4)*(b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x
^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
 1/2])/(77*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2B(bx^2+cx^4)^{5/2}}{11cx^{9/2}} - \frac{\left(2\left(\frac{bB}{2}-\frac{11Ac}{2}\right)\right)\int \frac{(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx}{11c} \\
&= -\frac{2(bB-11Ac)\left(bx^2+cx^4\right)^{3/2}}{77cx^{5/2}} + \frac{2B(bx^2+cx^4)^{5/2}}{11cx^{9/2}} - \frac{(6b(bB-11Ac))\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx}{77c} \\
&= -\frac{4b(bB-11Ac)\sqrt{bx^2+cx^4}}{77c\sqrt{x}} - \frac{2(bB-11Ac)\left(bx^2+cx^4\right)^{3/2}}{77cx^{5/2}} \\
&\quad + \frac{2B(bx^2+cx^4)^{5/2}}{11cx^{9/2}} - \frac{(4b^2(bB-11Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{77c} \\
&= -\frac{4b(bB-11Ac)\sqrt{bx^2+cx^4}}{77c\sqrt{x}} - \frac{2(bB-11Ac)\left(bx^2+cx^4\right)^{3/2}}{77cx^{5/2}} \\
&\quad + \frac{2B(bx^2+cx^4)^{5/2}}{11cx^{9/2}} - \frac{\left(4b^2(bB-11Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{77c\sqrt{bx^2+cx^4}} \\
&= -\frac{4b(bB-11Ac)\sqrt{bx^2+cx^4}}{77c\sqrt{x}} - \frac{2(bB-11Ac)\left(bx^2+cx^4\right)^{3/2}}{77cx^{5/2}} \\
&\quad + \frac{2B(bx^2+cx^4)^{5/2}}{11cx^{9/2}} - \frac{\left(8b^2(bB-11Ac)x\sqrt{b+cx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{77c\sqrt{bx^2+cx^4}} \\
&= -\frac{4b(bB-11Ac)\sqrt{bx^2+cx^4}}{77c\sqrt{x}} - \frac{2(bB-11Ac)\left(bx^2+cx^4\right)^{3/2}}{77cx^{5/2}} + \frac{2B(bx^2+cx^4)^{5/2}}{11cx^{9/2}} \\
&\quad - \frac{4b^{7/4}(bB-11Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77c^{5/4}\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.48

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx = \frac{2\sqrt{x^2(b+cx^2)}\left(B(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}+b(-bB+11Ac)\,\text{Hypergeometric2}\right.}{11c\sqrt{x}\sqrt{1+\frac{cx^2}{b}}}$$

[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(7/2),x]

[Out] (2*Sqrt[x^2*(b + c*x^2)]*(B*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + b*(-(b*B) + 11*A*c)*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]))/(11*c*Sqrt[x]*Sqrt[1 + (c*x^2)/b])

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.08

method	result
risch	$\frac{2(7Bc^2x^4+11Ac^2x^2+13Bbcx^2+33Abc+4Bb^2)\sqrt{x^2(cx^2+b)}}{77c\sqrt{x}} + \frac{4b^2(11Ac-Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F}{77c^2\sqrt{cx^3+bx}\,x^{\frac{3}{2}}(cx}$
default	$\frac{2(x^4c+bx^2)^{\frac{3}{2}}\left(7Bc^4x^7+22A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b^2c+11Ac^4x^5-2B\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.}{77x^{\frac{7}{2}}(cx^2+b)^2c^2}$

[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(7/2),x,method=_RETURNVERBOSE)

[Out] 2/77/c*(7*B*c^2*x^4+11*A*c^2*x^2+13*B*b*c*x^2+33*A*b*c+4*B*b^2)/x^(1/2)*(x^2*(c*x^2+b))^(1/2)+4/77*b^2/c^2*(11*A*c-B*b)*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.48

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx = -\frac{2\left(4\left(Bb^3-11\,Ab^2c\right)\sqrt{c}x\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\left(7\,Bc^3x^4+4\,Bb^2c+33\,Abc^2+\left(13\,Bbc^2+11\,A\right.\right.\right.}{77\,c^2x}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="fricas")
[Out] -2/77*(4*(B*b^3 - 11*A*b^2*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) -
 (7*B*c^3*x^4 + 4*B*b^2*c + 33*A*b*c^2 + (13*B*b*c^2 + 11*A*c^3)*x^2)*sqrt(
c*x^4 + b*x^2)*sqrt(x))/(c^2*x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx = \int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}(A+Bx^2)}{x^{\frac{7}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(7/2),x)
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(7/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{7}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(7/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{7}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(7/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx = \int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2)^{3/2}}{x^{7/2}}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(7/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(7/2), x)
```

3.239 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 356

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx = \frac{8b(bB+9Ac)x^{3/2}(b+cx^2)}{15\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}$$

$$+\frac{4}{15}(bB+9Ac)\sqrt{x}\sqrt{bx^2+cx^4}+\frac{2(bB+9Ac)(bx^2+cx^4)^{3/2}}{9bx^{3/2}}-\frac{2A(bx^2+cx^4)^{5/2}}{bx^{11/2}}-\frac{8b^{5/4}(bB+9Ac)x\left(\sqrt{b}+\right.}{}$$

```
[Out] 2/9*(9*A*c+B*b)*(c*x^4+b*x^2)^(3/2)/b/x^(3/2)-2*A*(c*x^4+b*x^2)^(5/2)/b/x^(
11/2)+8/15*b*(9*A*c+B*b)*x^(3/2)*(c*x^2+b)/c^(1/2)/(b^(1/2)+x*c^(1/2))/(c*x
^4+b*x^2)^(1/2)+4/15*(9*A*c+B*b)*x^(1/2)*(c*x^4+b*x^2)^(1/2)-8/15*b^(5/4)*(
9*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),
1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^
(3/4)/(c*x^4+b*x^2)^(1/2)+4/15*b^(5/4)*(9*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*
x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF
(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((
c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2063, 2046, 2057, 335, 311, 226, 1210}

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx = \frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9Ac+bB)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\right.}{15c^{3/4}\sqrt{bx^2+cx^4}}$$
$$-\frac{8b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9Ac+bB)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}}$$
$$+\frac{4}{15}\sqrt{x}\sqrt{bx^2+cx^4}(9Ac+bB)+\frac{2(bx^2+cx^4)^{3/2}\,(9Ac+bB)}{9bx^{3/2}}+\frac{8bx^{3/2}(b+cx^2)\,(9Ac+bB)}{15\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{2A(bx^2+cx^4}{bx^{11/2}}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(9/2),x]
```

```
[Out] (8*b*(b*B + 9*A*c)*x^(3/2)*(b + c*x^2))/(15*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*S
qrt[b*x^2 + c*x^4]) + (4*(b*B + 9*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/15 + (2
*(b*B + 9*A*c)*(b*x^2 + c*x^4)^(3/2))/(9*b*x^(3/2)) - (2*A*(b*x^2 + c*x^4)^
(5/2))/(b*x^(11/2)) - (8*b^(5/4)*(b*B + 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/
b^(1/4)], 1/2])/(15*c^(3/4)*Sqrt[b*x^2 + c*x^4]) + (4*b^(5/4)*(b*B + 9*A*c)
*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(3/4)*Sqrt[b*x^2 + c*x^4
])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\text{integral} = -\frac{2A(bx^2+cx^4)^{5/2}}{bx^{11/2}} - \frac{\left(2\left(-\frac{bB}{2}-\frac{9Ac}{2}\right)\right)\int\frac{(bx^2+cx^4)^{3/2}}{x^{5/2}}\,dx}{b}$$

$$= \frac{2(bB+9Ac)\left(bx^2+cx^4\right)^{3/2}}{9bx^{3/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{bx^{11/2}} + \frac{1}{3}(2(bB+9Ac))\int\frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$$

$$= \frac{4}{15}(bB+9Ac)\sqrt{x}\sqrt{bx^2+cx^4}+\frac{2(bB+9Ac)\left(bx^2+cx^4\right)^{3/2}}{9bx^{3/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{bx^{11/2}}+\frac{1}{15}(4b(bB+9Ac))\int\frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{4}{15}(bB+9Ac)\sqrt{x}\sqrt{bx^2+cx^4}+\frac{2(bB+9Ac)\left(bx^2+cx^4\right)^{3/2}}{9bx^{3/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{bx^{11/2}}+\frac{\left(4b(bB+9Ac)x\sqrt{b+cx^2}\right)\int\frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{15\sqrt{bx^2+cx^4}}$$

$$= \frac{4}{15}(bB+9Ac)\sqrt{x}\sqrt{bx^2+cx^4}+\frac{2(bB+9Ac)\left(bx^2+cx^4\right)^{3/2}}{9bx^{3/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{bx^{11/2}}+\frac{\left(8b(bB+9Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{15\sqrt{bx^2+cx^4}}$$

$$= \frac{4}{15}(bB+9Ac)\sqrt{x}\sqrt{bx^2+cx^4}+\frac{2(bB+9Ac)\left(bx^2+cx^4\right)^{3/2}}{9bx^{3/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{bx^{11/2}} + \frac{\left(8b^{3/2}(bB+9Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{15\sqrt{c}\sqrt{bx^2+cx^4}} - \frac{\left(8b^{3/2}(bB+9Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{15\sqrt{c}\sqrt{bx^2+cx^4}}$$

$$= \frac{8b(bB+9Ac)x^{3/2}(b+cx^2)}{15\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{4}{15}(bB+9Ac)\sqrt{x}\sqrt{bx^2+cx^4}+\frac{2(bB+9Ac)\left(bx^2+cx^4\right)^{3/2}}{9bx^{3/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{bx^{11/2}} - \frac{8b^{5/4}(bB+9Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}} + \frac{4b^{5/4}(bB+9Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.24

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx = \frac{2\sqrt{x^2\,(b+cx^2)}\left(-\frac{3A(b+cx^2)^2}{b}+\frac{(bB+9Ac)x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^2}{b}\right)}{\sqrt{1+\frac{cx^2}{b}}}\right)}{3x^{3/2}}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(9/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*((-3*A*(b + c*x^2)^2)/b + ((b*B + 9*A*c)*x^2*Hyper
geometric2F1[-3/2, 3/4, 7/4, -((c*x^2)/b)])/Sqrt[1 + (c*x^2)/b]))/(3*x^(3/2
))
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{2(-5Bc\,x^4-9Ac\,x^2-11bB\,x^2+45Ab)\sqrt{x^2(c\,x^2+b)}}{45x^{\frac{3}{2}}}+\frac{4b(9Ac+Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\cdots}{\cdots}\right.}{15c\sqrt{c\cdots}}$
default	$\frac{2(x^4c+b\,x^2)^{\frac{3}{2}}\left(5B\,c^3x^6+108A\,b^2c\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-54A\,b^2c\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-\cdots}}{\sqrt{-bc}}}\cdots\right.}{\cdots}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(-5*B*c*x^4-9*A*c*x^2-11*B*b*x^2+45*A*b)*(x^2*(c*x^2+b))^(1/2)/x^(3/2
)+4/15*b*(9*A*c+B*b)/c*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(
1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/
2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c)^(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(
-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(
1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*
x^2+b)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.26

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{9/2}}\,dx =$$

$$-\frac{2\left(12\left(Bb^2+9\,Abc\right)\sqrt{c}x^2\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\left(5\,Bc^2x^4-45\,Abc+(1\right.\right.}{45\,cx^2}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] -2/45*(12*(B*b^2 + 9*A*b*c)*sqrt(c)*x^2*weierstrassZeta(-4*b/c, 0, weierstr
assPInverse(-4*b/c, 0, x)) - (5*B*c^2*x^4 - 45*A*b*c + (11*B*b*c + 9*A*c^2)
*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c*x^2)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{9/2}}\,dx = \int \frac{\left(x^2(b+cx^2)\right)^{\frac{3}{2}}(A+Bx^2)}{x^{\frac{9}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(9/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(9/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{9/2}}\,dx = \int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{9}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(9/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{9/2}}\,dx = \int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{9}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(9/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{9/2}}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{9/2}}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(9/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(9/2), x)
```

3.240 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 200

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx = \frac{4(3bB+7Ac)\sqrt{bx^2+cx^4}}{21\sqrt{x}} + \frac{2(3bB+7Ac)(bx^2+cx^4)^{3/2}}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}} + \frac{4b^{3/4}(3bB+7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/21*(7*A*c+3*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^(5/2)-2/3*A*(c*x^4+b*x^2)^(5/2)/
b/x^(13/2)+4/21*(7*A*c+3*B*b)*(c*x^4+b*x^2)^(1/2)/x^(1/2)+4/21*b^(3/4)*(7*A
*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c
^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1
/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(
1/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2063, 2046, 2057, 335, 226}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx = \frac{4b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7Ac+3bB)\operatorname{EllipticF}\Big(2\arctan\Big(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\Big)}{21\sqrt[4]{c}\sqrt{bx^2+cx^4}} + \frac{4\sqrt{bx^2+cx^4}(7Ac+3bB)}{21\sqrt{x}} + \frac{2(bx^2+cx^4)^{3/2}(7Ac+3bB)}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(11/2),x]
```

```
[Out] (4*(3*b*B + 7*A*c)*Sqrt[b*x^2 + c*x^4])/(21*Sqrt[x]) + (2*(3*b*B + 7*A*c)*(
b*x^2 + c*x^4)^(3/2))/(21*b*x^(5/2)) - (2*A*(b*x^2 + c*x^4)^(5/2))/(3*b*x^(
13/2)) + (4*b^(3/4)*(3*b*B + 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2
)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1
/2])/(21*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
```

0]

Rubi steps

$$\text{integral} = -\frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}} - \frac{\left(2\left(-\frac{3bB}{2}-\frac{7Ac}{2}\right)\right)\int \frac{(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx}{3b}$$

$$= \frac{2(3bB+7Ac)(bx^2+cx^4)^{3/2}}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}} + \frac{1}{7}(2(3bB+7Ac))\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx$$

$$= \frac{4(3bB+7Ac)\sqrt{bx^2+cx^4}}{21\sqrt{x}} + \frac{2(3bB+7Ac)(bx^2+cx^4)^{3/2}}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}} + \frac{1}{21}(4b(3bB+7Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{4(3bB+7Ac)\sqrt{bx^2+cx^4}}{21\sqrt{x}} + \frac{2(3bB+7Ac)(bx^2+cx^4)^{3/2}}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}} + \frac{\left(4b(3bB+7Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{21\sqrt{bx^2+cx^4}}$$

$$= \frac{4(3bB+7Ac)\sqrt{bx^2+cx^4}}{21\sqrt{x}} + \frac{2(3bB+7Ac)(bx^2+cx^4)^{3/2}}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}} + \frac{\left(8b(3bB+7Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{21\sqrt{bx^2+cx^4}}$$

$$= \frac{4(3bB+7Ac)\sqrt{bx^2+cx^4}}{21\sqrt{x}} + \frac{2(3bB+7Ac)(bx^2+cx^4)^{3/2}}{21bx^{5/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{3bx^{13/2}} + \frac{4b^{3/4}(3bB+7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.50

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx = -\frac{2\sqrt{x^2(b+cx^2)}\left(A(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}} - b(3bB+7Ac)x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{cx^2}{b}\right)\right)}{3bx^{5/2}\sqrt{1+\frac{cx^2}{b}}}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(11/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] - b*(3*b*B +
 7*A*c)*x^2*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^2)/b)]))/(3*b*x^(5/2)*
Sqrt[1 + (c*x^2)/b])
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{2\left(-3Bc\,x^{4}-7Ac\,x^{2}-9bB\,x^{2}+7Ab\right)\sqrt{x^{2}\left(c\,x^{2}+b\right)}}{21x^{\frac{5}{2}}}+\frac{4b(7Ac+3Bb)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{\left(x+\right.}{}}\right.}{21c\sqrt{c\,x^{3}+bx}\,x^{\frac{3}{2}}\left(c\,x^{2}+b\right)}$
default	$\frac{2\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}\left(14A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{-bc}\,bcx+6B\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{x}{\sqrt{-}}}\right.}{21x^{\frac{9}{2}}\left(c\,x^{2}+b\right)^{2}c}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(-3*B*c*x^4-7*A*c*x^2-9*B*b*x^2+7*A*b)/x^(5/2)*(x^2*(c*x^2+b))^(1/2)+
4/21*b*(7*A*c+3*B*b)/c*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(
1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/
2)/(c*x^3+b*x)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),
1/2*2^(1/2))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.43

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx = \frac{2\left(4\,(3\,Bb^2+7\,Abc)\sqrt{c}x^3\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(3\,Bc^2x^4-7\,A\right.\right.}{21\,cx^3}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] 2/21*(4*(3*B*b^2 + 7*A*b*c)*sqrt(c)*x^3*weierstrassPInverse(-4*b/c, 0, x) +
 (3*B*c^2*x^4 - 7*A*b*c + (9*B*b*c + 7*A*c^2)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt
(x))/(c*x^3)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}(A+Bx^2)}{x^{\frac{11}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(11/2),x)
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(11/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{11}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(11/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{11}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(11/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx = \int \frac{(B\,x^2+A)\,(c\,x^4+b\,x^2)^{3/2}}{x^{11/2}}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(11/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(11/2), x)
```

3.241 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 354

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx = \frac{24\sqrt{c}(bB+Ac)x^{3/2}(b+cx^2)}{5\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{12c(bB+Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2(bB+Ac)(bx^2+cx^4)^{3/2}}{bx^{7/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{5bx^{15/2}} - \frac{24\sqrt[4]{b}\sqrt[4]{c}(bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}} + \frac{12\sqrt[4]{b}\sqrt[4]{c}(bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}}$$

```
[Out] -2*(A*c+B*b)*(c*x^4+b*x^2)^(3/2)/b/x^(7/2)-2/5*A*(c*x^4+b*x^2)^(5/2)/b/x^(1
5/2)+24/5*(A*c+B*b)*x^(3/2)*(c*x^2+b)*c^(1/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*
x^2)^(1/2)+12/5*c*(A*c+B*b)*x^(1/2)*(c*x^4+b*x^2)^(1/2)/b-24/5*b^(1/4)*c^(1
/4)*(A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)
)),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)
/(c*x^4+b*x^2)^(1/2)+12/5*b^(1/4)*c^(1/4)*(A*c+B*b)*x*(cos(2*arctan(c^(1/4)
*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Elliptic
F(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*(
(c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2063, 2045, 2046, 2057, 335, 311, 226, 1210}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13/2}}dx = \frac{12\sqrt[4]{b}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+bB)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\ldots\right.}{5\sqrt{bx^2+cx^4}}$$
$$-\frac{24\sqrt[4]{b}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+bB)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}}$$
$$+\frac{12c\sqrt{x}\sqrt{bx^2+cx^4}(Ac+bB)}{5b}-\frac{2(bx^2+cx^4)^{3/2}(Ac+bB)}{bx^{7/2}}$$
$$+\frac{24\sqrt{c}x^{3/2}(b+cx^2)(Ac+bB)}{5\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{2A(bx^2+cx^4)^{5/2}}{5bx^{15/2}}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(13/2),x]
```

```
[Out] (24*Sqrt[c]*(b*B + A*c)*x^(3/2)*(b + c*x^2))/(5*(Sqrt[b] + Sqrt[c]*x)*Sqrt[
b*x^2 + c*x^4]) + (12*c*(b*B + A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*b) - (2
*(b*B + A*c)*(b*x^2 + c*x^4)^(3/2))/(b*x^(7/2)) - (2*A*(b*x^2 + c*x^4)^(5/2
))/(5*b*x^(15/2)) - (24*b^(1/4)*c^(1/4)*(b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4]) + (12*b^(1/4)*c^(1/4)*(b*B + A
*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ellip
ticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\text{integral} = -\frac{2A(bx^2+cx^4)^{5/2}}{5bx^{15/2}} + -\frac{\left(2\left(-\frac{5bB}{2}-\frac{5Ac}{2}\right)\right)\int \frac{(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx}{5b}$$

$$= -\frac{2(bB+Ac)(bx^2+cx^4)^{3/2}}{bx^{7/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{5bx^{15/2}} + \frac{(6c(bB+Ac))\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx}{b}$$

$$= \frac{12c(bB+Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2(bB+Ac)(bx^2+cx^4)^{3/2}}{bx^{7/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{5bx^{15/2}} + \frac{1}{5}(12c(bB+Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{12c(bB+Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2(bB+Ac)(bx^2+cx^4)^{3/2}}{bx^{7/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{5bx^{15/2}} + \frac{\left(12c(bB+Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5\sqrt{bx^2+cx^4}}$$

$$= \frac{12c(bB+Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2(bB+Ac)(bx^2+cx^4)^{3/2}}{bx^{7/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{5bx^{15/2}} + \frac{\left(24c(bB+Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{bx^2+cx^4}}$$

$$= \frac{12c(bB+Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2(bB+Ac)(bx^2+cx^4)^{3/2}}{bx^{7/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{5bx^{15/2}} + \frac{\left(24\sqrt{b}\sqrt{c}(bB+Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{bx^2+cx^4}} - \frac{\left(24\sqrt{b}\sqrt{c}(bB+Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{bx^2+cx^4}}$$

$$= \frac{24\sqrt{c}(bB+Ac)x^{3/2}(b+cx^2)}{5\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{12c(bB+Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5b} - \frac{2(bB+Ac)(bx^2+cx^4)^{3/2}}{bx^{7/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{5bx^{15/2}} - \frac{24\sqrt[4]{b}\sqrt[4]{c}(bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}} + \frac{12\sqrt[4]{b}\sqrt[4]{c}(bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.28

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx = -\frac{2\sqrt{x^2\,(b+cx^2)}\left(A(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}+5b(bB+Ac)x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},\frac{3}{4},-\frac{cx^2}{b}\right)\right)}{5bx^{7/2}\sqrt{1+\frac{cx^2}{b}}}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(13/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + 5*b*(b*B +
 A*c)*x^2*Hypergeometric2F1[-3/2, -1/4, 3/4, -((c*x^2)/b)]))/(5*b*x^(7/2)*S
qrt[1 + (c*x^2)/b])
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{2\left(-Bc\,x^4+7Ac\,x^2+5bB\,x^2+Ab\right)\sqrt{x^2(c\,x^2+b)}}{5x^{\frac{7}{2}}}+\frac{12(Ac+Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,E\left(\sqrt{\right.}\right.}{}\right.}{5\sqrt{c\,x^3+bx}}$
default	$\frac{2\left(x^4c+b\,x^2\right)^{\frac{3}{2}}\left(12A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc\,x^2-6A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\right.}{}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(-B*c*x^4+7*A*c*x^2+5*B*b*x^2+A*b)/x^(7/2)*(x^2*(c*x^2+b))^(1/2)+12/5*
(A*c+B*b)*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1
/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x
)^(1/2)*(-2/c*(-b*c)^(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^
(1/2),1/2*2^(1/2))+1/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c
)^(1/2))^(1/2),1/2*2^(1/2)))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*
x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.23

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx =$$

$$-\frac{2\left(12\,(Bb+Ac)\sqrt{c}x^4\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-(Bcx^4-(5\,Bb+7\,Ac)x^2-\right.}{5\,x^4}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] -2/5*(12*(B*b + A*c)*sqrt(c)*x^4*weierstrassZeta(-4*b/c, 0, weierstrassPInv
erse(-4*b/c, 0, x)) - (B*c*x^4 - (5*B*b + 7*A*c)*x^2 - A*b)*sqrt(c*x^4 + b*
x^2)*sqrt(x))/x^4
```

Sympy [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx = \int \frac{(x^2(b+cx^2))^{\frac{3}{2}}(A+Bx^2)}{x^{\frac{13}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(13/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)*(A + B*x**2)/x**(13/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{13}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(13/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{13}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(13/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(13/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{13/2}}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(13/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(13/2), x)
```

3.242 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 204

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx = \frac{4c(7bB+3Ac)\sqrt{bx^2+cx^4}}{21b\sqrt{x}} - \frac{2(7bB+3Ac)(bx^2+cx^4)^{3/2}}{21bx^{9/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{7bx^{17/2}} + \frac{4c^{3/4}(7bB+3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21\sqrt[4]{b}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/21*(3*A*c+7*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^(9/2)-2/7*A*(c*x^4+b*x^2)^(5/2)
/b/x^(17/2)+4/21*c*(3*A*c+7*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^(1/2)+4/21*c^(3/4)
*(3*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2
)/b^(1/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2063, 2045, 2046, 2057, 335, 226}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx = \frac{4c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3Ac+7bB)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\right.}{21\sqrt[4]{b}\sqrt{bx^2+cx^4}} + \frac{4c\sqrt{bx^2+cx^4}(3Ac+7bB)}{21b\sqrt{x}} - \frac{2(bx^2+cx^4)^{3/2}(3Ac+7bB)}{21bx^{9/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{7bx^{17/2}}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(15/2),x]
```

```
[Out] (4*c*(7*b*B + 3*A*c)*Sqrt[b*x^2 + c*x^4])/(21*b*Sqrt[x]) - (2*(7*b*B + 3*A*
c)*(b*x^2 + c*x^4)^(3/2))/(21*b*x^(9/2)) - (2*A*(b*x^2 + c*x^4)^(5/2))/(7*b
*x^(17/2)) + (4*c^(3/4)*(7*b*B + 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c
*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)
], 1/2])/(21*b^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\text{integral} = -\frac{2A(bx^2+cx^4)^{5/2}}{7bx^{17/2}} - \frac{\left(2\left(-\frac{7bB}{2}-\frac{3Ac}{2}\right)\right)\int\frac{(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx}{7b}$$

$$= -\frac{2(7bB+3Ac)\left(bx^2+cx^4\right)^{3/2}}{21bx^{9/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{7bx^{17/2}} + \frac{(2c(7bB+3Ac))\int\frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx}{7b}$$

$$= \frac{4c(7bB+3Ac)\sqrt{bx^2+cx^4}}{21b\sqrt{x}} - \frac{2(7bB+3Ac)\left(bx^2+cx^4\right)^{3/2}}{21bx^{9/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{7bx^{17/2}} + \frac{1}{21}(4c(7bB+3Ac))\int\frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx$$

$$= \frac{4c(7bB+3Ac)\sqrt{bx^2+cx^4}}{21b\sqrt{x}} - \frac{2(7bB+3Ac)\left(bx^2+cx^4\right)^{3/2}}{21bx^{9/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{7bx^{17/2}} + \frac{\left(4c(7bB+3Ac)x\sqrt{b+cx^2}\right)\int\frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{21\sqrt{bx^2+cx^4}}$$

$$= \frac{4c(7bB+3Ac)\sqrt{bx^2+cx^4}}{21b\sqrt{x}} - \frac{2(7bB+3Ac)\left(bx^2+cx^4\right)^{3/2}}{21bx^{9/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{7bx^{17/2}} + \frac{\left(8c(7bB+3Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{21\sqrt{bx^2+cx^4}}$$

$$= \frac{4c(7bB+3Ac)\sqrt{bx^2+cx^4}}{21b\sqrt{x}} - \frac{2(7bB+3Ac)\left(bx^2+cx^4\right)^{3/2}}{21bx^{9/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{7bx^{17/2}} + \frac{4c^{3/4}(7bB+3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21\sqrt[4]{b}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.50

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx = -\frac{2\sqrt{x^2\,(b+cx^2)}\left(3A(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}+b(7bB+3Ac)x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},\frac{1}{4},-\frac{cx^2}{b}\right)\right)}{21bx^{9/2}\sqrt{1+\frac{cx^2}{b}}}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(15/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(3*A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + b*(7*b*B
 + 3*A*c)*x^2*Hypergeometric2F1[-3/2, -3/4, 1/4, -((c*x^2)/b)]))/(21*b*x^(9
/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{2\left(-7Bc\,x^4+9Ac\,x^2+7bB\,x^2+3Ab\right)\sqrt{x^2(c\,x^2+b)}}{21x^{\frac{9}{2}}}+\frac{4(3Ac+7Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{\left(x+\right.}{\sqrt{}}}\right.}{21\sqrt{c\,x^3+bx}\,x^{\frac{3}{2}}\,(c\,x^2+b)}$
default	$\frac{2\left(x^4c+b\,x^2\right)^{\frac{3}{2}}\left(6A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)c\,x^3+14B\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.}{21x^{\frac{13}{2}}\,(c\,x^2+b)^2}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(15/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(-7*B*c*x^4+9*A*c*x^2+7*B*b*x^2+3*A*b)/x^(9/2)*(x^2*(c*x^2+b))^(1/2)+
4/21*(3*A*c+7*B*b)*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)
*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(
c*x^3+b*x)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*
2^(1/2))*(x^2*(c*x^2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.37

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{15/2}}\,dx = \frac{2\left(4\left(7\,Bb+3\,Ac\right)\sqrt{c}x^5\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(7\,Bcx^4-\left(7\,Bb+\right.\right.\right.}{21\,x^5}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="fricas")
```

```
[Out] 2/21*(4*(7*B*b + 3*A*c)*sqrt(c)*x^5*weierstrassPInverse(-4*b/c, 0, x) + (7*
B*c*x^4 - (7*B*b + 9*A*c)*x^2 - 3*A*b)*sqrt(c*x^4 + b*x^2)*sqrt(x))/x^5
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{15/2}}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(15/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{15/2}}\,dx = \int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^{\frac{15}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(15/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\left(bx^2+cx^4\right)^{3/2}}{x^{15/2}}\,dx = \int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(Bx^2+A\right)}{x^{\frac{15}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(15/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(15/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{15/2}}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(15/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(15/2), x)
```

3.243 $\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 364

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx = \frac{8c^{3/2}(9bB+Ac)x^{3/2}(b+cx^2)}{15b\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{4c(9bB+Ac)\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2(9bB+Ac)(bx^2+cx^4)^{3/2}}{45bx^{11/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}} - \frac{8c^{5/4}(9bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}} + \frac{4c^{5/4}(9bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/45*(A*c+9*B*b)*(c*x^4+b*x^2)^(3/2)/b/x^(11/2)-2/9*A*(c*x^4+b*x^2)^(5/2)/
b/x^(19/2)+8/15*c^(3/2)*(A*c+9*B*b)*x^(3/2)*(c*x^2+b)/b/(b^(1/2)+x*c^(1/2))
/(c*x^4+b*x^2)^(1/2)-4/15*c*(A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/b/x^(3/2)-8/15*
c^(5/4)*(A*c+9*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)
^(1/2)/b^(3/4)/(c*x^4+b*x^2)^(1/2)+4/15*c^(5/4)*(A*c+9*B*b)*x*(cos(2*arctan
(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*
EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^
(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2063, 2045, 2057, 335, 311, 226, 1210}

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx = \frac{4c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+9bB)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\right.}{15b^{3/4}\sqrt{bx^2+cx^4}}$$
$$-\frac{8c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+9bB)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}}$$
$$+\frac{8c^{3/2}x^{3/2}(b+cx^2)(Ac+9bB)}{15b\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{2(bx^2+cx^4)^{3/2}(Ac+9bB)}{45bx^{11/2}}$$
$$-\frac{4c\sqrt{bx^2+cx^4}(Ac+9bB)}{15bx^{3/2}}-\frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}}$$

```
[In] Int[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(17/2),x]
```

```
[Out] (8*c^(3/2)*(9*b*B + A*c)*x^(3/2)*(b + c*x^2))/(15*b*(Sqrt[b] + Sqrt[c]*x)*S
qrt[b*x^2 + c*x^4]) - (4*c*(9*b*B + A*c)*Sqrt[b*x^2 + c*x^4])/(15*b*x^(3/2)
) - (2*(9*b*B + A*c)*(b*x^2 + c*x^4)^(3/2))/(45*b*x^(11/2)) - (2*A*(b*x^2 +
 c*x^4)^(5/2))/(9*b*x^(19/2)) - (8*c^(5/4)*(9*b*B + A*c)*x*(Sqrt[b] + Sqrt[
c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)
*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[b*x^2 + c*x^4]) + (4*c^(5/4)*(9*
b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[b*x
^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\text{integral} = -\frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}} - \frac{\left(2\left(-\frac{9bB}{2}-\frac{Ac}{2}\right)\right)\int\frac{(bx^2+cx^4)^{3/2}}{x^{13/2}}\,dx}{9b}$$

$$= -\frac{2(9bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{45bx^{11/2}} - \frac{2A(bx^2+cx^4)^{5/2}}{9bx^{19/2}} + \frac{(2c(9bB+Ac))\int\frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx}{15b}$$

$$= -\frac{4c(9bB+Ac)\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2(9bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{45bx^{11/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{9bx^{19/2}} + \frac{\left(4c^2(9bB+Ac)\right)\int\frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{15b}$$

$$= -\frac{4c(9bB+Ac)\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2(9bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{45bx^{11/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{9bx^{19/2}} + \frac{\left(4c^2(9bB+Ac)x\sqrt{b+cx^2}\right)\int\frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{15b\sqrt{bx^2+cx^4}}$$

$$= -\frac{4c(9bB+Ac)\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2(9bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{45bx^{11/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{9bx^{19/2}} + \frac{\left(8c^2(9bB+Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{15b\sqrt{bx^2+cx^4}}$$

$$= -\frac{4c(9bB+Ac)\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2(9bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{45bx^{11/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{9bx^{19/2}} + \frac{\left(8c^{3/2}(9bB+Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{15\sqrt{b}\sqrt{bx^2+cx^4}} - \frac{\left(8c^{3/2}(9bB+Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{15\sqrt{b}\sqrt{bx^2+cx^4}}$$

$$= \frac{8c^{3/2}(9bB+Ac)x^{3/2}(b+cx^2)}{15b\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{4c(9bB+Ac)\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2(9bB+Ac)\left(bx^2+cx^4\right)^{3/2}}{45bx^{11/2}} - \frac{2A\left(bx^2+cx^4\right)^{5/2}}{9bx^{19/2}} - \frac{8c^{5/4}(9bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}} + \frac{4c^{5/4}(9bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.27

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx = -\frac{2\sqrt{x^2(b+cx^2)}\left(5A(b+cx^2)^2\sqrt{1+\frac{cx^2}{b}}+b(9bB+Ac)x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{4},-\frac{1}{4},-\frac{cx^2}{b}\right)\right)}{45bx^{11/2}\sqrt{1+\frac{cx^2}{b}}}$$

```
[In] Integrate[((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(17/2),x]
```

```
[Out] (-2*Sqrt[x^2*(b + c*x^2)]*(5*A*(b + c*x^2)^2*Sqrt[1 + (c*x^2)/b] + b*(9*b*B
 + A*c)*x^2*Hypergeometric2F1[-3/2, -5/4, -1/4, -((c*x^2)/b)]))/(45*b*x^(11
/2)*Sqrt[1 + (c*x^2)/b])
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{2\left(12A\,c^2x^4+63x^4Bbc+11Abc\,x^2+9b^2B\,x^2+5b^2A\right)\sqrt{x^2(c\,x^2+b)}}{45x^{\frac{11}{2}}b}+\frac{4c(Ac+9Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}}{}$
default	$\frac{2(x^4c+b\,x^2)^{\frac{3}{2}}\left(12A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b\,c^2x^4-6A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\right.}{}$

```
[In] int((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(17/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(12*A*c^2*x^4+63*B*b*c*x^4+11*A*b*c*x^2+9*B*b^2*x^2+5*A*b^2)/x^(11/2)
/b*(x^2*(c*x^2+b))^(1/2)+4/15*c*(A*c+9*B*b)/b*(-b*c)^(1/2)*((x+1/c*(-b*c)^(
1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*
(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c)^(1/2)*EllipticE(((
x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1/c*(-b*c)^(1/2)*Ell
ipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)))*(x^2*(c*x^
2+b))^(1/2)/x^(3/2)/(c*x^2+b)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.28

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx = -\frac{2\left(12\left(9\,Bbc+Ac^2\right)\sqrt{c}x^6\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\left(3\left(21\,Bbc+4\,Ac^2\right)x^4\right.\right.}{45\,bx^6}$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="fricas")
```

```
[Out] -2/45*(12*(9*B*b*c + A*c^2)*sqrt(c)*x^6*weierstrassZeta(-4*b/c, 0, weierstr
assPInverse(-4*b/c, 0, x)) + (3*(21*B*b*c + 4*A*c^2)*x^4 + 5*A*b^2 + (9*B*b
^2 + 11*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2)**(3/2)/x**(17/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+Bx^2)(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{17}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(17/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx = \int \frac{(cx^4+bx^2)^{\frac{3}{2}}(Bx^2+A)}{x^{\frac{17}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2)^(3/2)/x^(17/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2)^(3/2)*(B*x^2 + A)/x^(17/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx = \int \frac{(B\,x^2+A)\,\left(c\,x^4+b\,x^2\right)^{3/2}}{x^{17/2}}\,dx$$

```
[In] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(17/2),x)
[Out] int(((A + B*x^2)*(b*x^2 + c*x^4)^(3/2))/x^(17/2), x)
```

3.244 $\int \frac{x^{13/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 243

$$\int \frac{x^{13/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{2b^2(13bB-15Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} + \frac{6b(13bB-15Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^3} - \frac{2(13bB-15Ac)x^{7/2}\sqrt{bx^2+cx^4}}{165c^2} + \frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c} + \frac{b^{11/4}(13bB-15Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77c^{17/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 6/385*b*(-15*A*c+13*B*b)*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c^3-2/165*(-15*A*c+13*
B*b)*x^(7/2)*(c*x^4+b*x^2)^(1/2)/c^2+2/15*B*x^(11/2)*(c*x^4+b*x^2)^(1/2)/c-
2/77*b^2*(-15*A*c+13*B*b)*(c*x^4+b*x^2)^(1/2)/c^4/x^(1/2)+1/77*b^(11/4)*(-1
5*A*c+13*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)
)),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)
/c^(17/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2064, 2049, 2057, 335, 226}

$$\int \frac{x^{13/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(13bB-15Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77c^{17/4}\sqrt{bx^2+cx^4}} - \frac{2b^2\sqrt{bx^2+cx^4}(13bB-15Ac)}{77c^4\sqrt{x}} + \frac{6bx^{3/2}\sqrt{bx^2+cx^4}(13bB-15Ac)}{385c^3} - \frac{2x^{7/2}\sqrt{bx^2+cx^4}(13bB-15Ac)}{165c^2} + \frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c}$$

```
[In] Int[(x^(13/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-2*b^2*(13*b*B - 15*A*c)*Sqrt[b*x^2 + c*x^4])/(77*c^4*Sqrt[x]) + (6*b*(13*
b*B - 15*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(385*c^3) - (2*(13*b*B - 15*A*c)
*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(165*c^2) + (2*B*x^(11/2)*Sqrt[b*x^2 + c*x^4]
)/(15*c) + (b^(11/4)*(13*b*B - 15*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*
x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)]
, 1/2])/(77*c^(17/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c} - \frac{\left(2\left(\frac{13bB}{2}-\frac{15Ac}{2}\right)\right)\int\frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx}{15c} \\
&= -\frac{2(13bB-15Ac)x^{7/2}\sqrt{bx^2+cx^4}}{165c^2} + \frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c} + \frac{(3b(13bB-15Ac))\int\frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx}{55c^2} \\
&= \frac{6b(13bB-15Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^3} - \frac{2(13bB-15Ac)x^{7/2}\sqrt{bx^2+cx^4}}{165c^2} \\
&\quad + \frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c} - \frac{\left(3b^2(13bB-15Ac)\right)\int\frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{77c^3} \\
&= -\frac{2b^2(13bB-15Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} + \frac{6b(13bB-15Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^3} \\
&\quad - \frac{2(13bB-15Ac)x^{7/2}\sqrt{bx^2+cx^4}}{165c^2} + \frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c} \\
&\quad + \frac{\left(b^3(13bB-15Ac)\right)\int\frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{77c^4} \\
&= -\frac{2b^2(13bB-15Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} + \frac{6b(13bB-15Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^3} \\
&\quad - \frac{2(13bB-15Ac)x^{7/2}\sqrt{bx^2+cx^4}}{165c^2} + \frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c} \\
&\quad + \frac{\left(b^3(13bB-15Ac)x\sqrt{b+cx^2}\right)\int\frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{77c^4\sqrt{bx^2+cx^4}}
\end{aligned}
$$

$$= -\frac{2b^2(13bB-15Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} + \frac{6b(13bB-15Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^3} - \frac{2(13bB-15Ac)x^{7/2}\sqrt{bx^2+cx^4}}{165c^2} + \frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c} + \frac{\left(2b^3(13bB-15Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{77c^4\sqrt{bx^2+cx^4}}$$

$$= -\frac{2b^2(13bB-15Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} + \frac{6b(13bB-15Ac)x^{3/2}\sqrt{bx^2+cx^4}}{385c^3} - \frac{2(13bB-15Ac)x^{7/2}\sqrt{bx^2+cx^4}}{165c^2} + \frac{2Bx^{11/2}\sqrt{bx^2+cx^4}}{15c} + \frac{b^{11/4}(13bB-15Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77c^{17/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.59

$$\int \frac{x^{13/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{3/2}\left(-\left((b+cx^2)\left(195b^3B-7c^3x^4(15A+11Bx^2)-9b^2c(25A+13Bx^2)+bc^2x^2(1\right.\right.\right.}{1155c^4\sqrt{3}}$$

```
[In] Integrate[(x^(13/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
[Out] (2*x^(3/2)*(-((b + c*x^2)*(195*b^3*B - 7*c^3*x^4*(15*A + 11*B*x^2) - 9*b^2*
c*(25*A + 13*B*x^2) + b*c^2*x^2*(135*A + 91*B*x^2))) + 15*b^3*(13*b*B - 15*
A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(
1155*c^4*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.98

method	result
risch	$\frac{2(77Bc^3x^6+105Ac^3x^4-91Bbc^2x^4-135Abc^2x^2+117Bb^2cx^2+225b^2Ac-195Bb^3)x^{\frac{3}{2}}(cx^2+b)}{1155c^4\sqrt{x^2(cx^2+b)}} - \frac{b^3(15Ac-13Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-b}}{c}\right.}{\sqrt{-b}}}}{}$
default	$-\frac{\sqrt{x}\left(-154Bc^5x^9-210Ac^5x^7+28Bbc^4x^7+225A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b^3c+60Abc^4x^5\right.}{11}$

```
[In] int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1155*(77*B*c^3*x^6+105*A*c^3*x^4-91*B*b*c^2*x^4-135*A*b*c^2*x^2+117*B*b^2
*c*x^2+225*A*b^2*c-195*B*b^3)/c^4*x^(3/2)*(c*x^2+b)/(x^2*(c*x^2+b))^(1/2)-1
/77*b^3*(15*A*c-13*B*b)/c^5*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/
2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2)
)^(1/2)/(c*x^3+b*x)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(
1/2),1/2*2^(1/2))*x^(1/2)/(x^2*(c*x^2+b))^(1/2)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.50

$$\int \frac{x^{13/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2\left(15\left(13\,Bb^4-15\,Ab^3c\right)\sqrt{c}x\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(77\,Bc^4x^6-195\,Bb^3\right.}{115}$$

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(15*(13*B*b^4 - 15*A*b^3*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0,
 x) + (77*B*c^4*x^6 - 195*B*b^3*c + 225*A*b^2*c^2 - 7*(13*B*b*c^3 - 15*A*c^
4)*x^4 + 9*(13*B*b^2*c^2 - 15*A*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c
^5*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{13/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^{\frac{13}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(13/2)/sqrt(c*x^4 + b*x^2), x)
```

Giac [F]

$$\int \frac{x^{13/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^{\frac{13}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^(13/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{13/2}\,(B\,x^2+A)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

```
[In] int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
[Out] int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.245 $\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 369

$$\begin{aligned}\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx &= -\frac{14b^2(11bB-13Ac)x^{3/2}(b+cx^2)}{195c^{7/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}\\
&+\frac{14b(11bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^3}\\
&-\frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c^2}+\frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c}\\
&+\frac{14b^{9/4}(11bB-13Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{15/4}\sqrt{bx^2+cx^4}}\\
&-\frac{7b^{9/4}(11bB-13Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{195c^{15/4}\sqrt{bx^2+cx^4}}\end{aligned}$$

```
[Out] -14/195*b^2*(-13*A*c+11*B*b)*x^(3/2)*(c*x^2+b)/c^(7/2)/(b^(1/2)+x*c^(1/2))/
(c*x^4+b*x^2)^(1/2)-2/117*(-13*A*c+11*B*b)*x^(5/2)*(c*x^4+b*x^2)^(1/2)/c^2+
2/13*B*x^(9/2)*(c*x^4+b*x^2)^(1/2)/c+14/585*b*(-13*A*c+11*B*b)*x^(1/2)*(c*x
^4+b*x^2)^(1/2)/c^3+14/195*b^(9/4)*(-13*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)
*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Elliptic
E(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*(
(c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(15/4)/(c*x^4+b*x^2)^(1/2)-7/195*b
^(9/4)*(-13*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/
cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/
2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2)
)^2)^(1/2)/c^(15/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2064, 2049, 2057, 335, 311, 226, 1210}

$$\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{7b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(11bB-13Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{195c^{15/4}\sqrt{bx^2+cx^4}} + \frac{14b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(11bB-13Ac)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{15/4}\sqrt{bx^2+cx^4}} - \frac{14b^2x^{3/2}(b+cx^2)(11bB-13Ac)}{195c^{7/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{14b\sqrt{x}\sqrt{bx^2+cx^4}(11bB-13Ac)}{585c^3} - \frac{2x^{5/2}\sqrt{bx^2+cx^4}(11bB-13Ac)}{117c^2} + \frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c}$$

```
[In] Int[(x^(11/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-14*b^2*(11*b*B - 13*A*c)*x^(3/2)*(b + c*x^2))/(195*c^(7/2)*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[b*x^2 + c*x^4]) + (14*b*(11*b*B - 13*A*c)*Sqrt[x]*Sqrt[b*x^2 +
 c*x^4])/(585*c^3) - (2*(11*b*B - 13*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(117
*c^2) + (2*B*x^(9/2)*Sqrt[b*x^2 + c*x^4])/(13*c) + (14*b^(9/4)*(11*b*B - 13
*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ell
ipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(195*c^(15/4)*Sqrt[b*x^2
+ c*x^4]) - (7*b^(9/4)*(11*b*B - 13*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4
)], 1/2])/(195*c^(15/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\text{integral} = \frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c} - \frac{\left(2\left(\frac{11bB}{2}-\frac{13Ac}{2}\right)\right)\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx}{13c}$$

$$= -\frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c^2}+\frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c}+\frac{(7b(11bB-13Ac))\int\frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{117c^2}$$

$$= \frac{14b(11bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^3}-\frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c^2} + \frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c}-\frac{(7b^2(11bB-13Ac))\int\frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{195c^3}$$

$$= \frac{14b(11bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^3}-\frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c^2} + \frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c}-\frac{\left(7b^2(11bB-13Ac)x\sqrt{b+cx^2}\right)\int\frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{195c^3\sqrt{bx^2+cx^4}}$$

$$= \frac{14b(11bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^3} - \frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c^2}+\frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c} - \frac{\left(14b^2(11bB-13Ac)x\sqrt{b+cx^2}\right)\operatorname{Subst}\left(\int\frac{x^2}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{195c^3\sqrt{bx^2+cx^4}}$$

$$= \frac{14b(11bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^3} - \frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c^2}+\frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c} - \frac{\left(14b^{5/2}(11bB-13Ac)x\sqrt{b+cx^2}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{195c^{7/2}\sqrt{bx^2+cx^4}} + \frac{\left(14b^{5/2}(11bB-13Ac)x\sqrt{b+cx^2}\right)\operatorname{Subst}\left(\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{195c^{7/2}\sqrt{bx^2+cx^4}}$$

$$= -\frac{14b^2(11bB-13Ac)x^{3/2}(b+cx^2)}{195c^{7/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{14b(11bB-13Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{585c^3} - \frac{2(11bB-13Ac)x^{5/2}\sqrt{bx^2+cx^4}}{117c^2}+\frac{2Bx^{9/2}\sqrt{bx^2+cx^4}}{13c} + \frac{14b^{9/4}(11bB-13Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{15/4}\sqrt{bx^2+cx^4}} - \frac{7b^{9/4}(11bB-13Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{15/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.33

$$\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{5/2}\Big((b+cx^2)\left(77b^2B+5c^2x^2(13A+9Bx^2)-bc(91A+55Bx^2)\right)+7b^2(-11bB\ \cdots}{585c^3\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(x^(11/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(5/2)*((b + c*x^2)*(77*b^2*B + 5*c^2*x^2*(13*A + 9*B*x^2) - b*c*(91*A
+ 55*B*x^2)) + 7*b^2*(-11*b*B + 13*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2
F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(585*c^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{2x^{\frac{5}{2}}\left(-45Bc^2x^4-65Ac^2x^2+55Bbcx^2+91Abc-77Bb^2\right)\left(cx^2+b\right)}{585c^3\sqrt{x^2\left(cx^2+b\right)}}+\frac{7b^2(13Ac-11Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{\cdots}}{\cdots}$
default	$\frac{\sqrt{x}\left(90Bx^8c^4+130Ax^6c^4-20Bx^6bc^3+546A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b^3c-273A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\cdots}}{\cdots}$

```
[In] int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/585*x^(5/2)*(-45*B*c^2*x^4-65*A*c^2*x^2+55*B*b*c*x^2+91*A*b*c-77*B*b^2)/
c^3*(c*x^2+b)/(x^2*(c*x^2+b))^(1/2)+7/195*b^2*(13*A*c-11*B*b)/c^4*(-b*c)^(1
/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/
(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c
)^(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+
1/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*
2^(1/2)))*x^(1/2)/(x^2*(c*x^2+b))^(1/2)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.22 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.28

$$\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2\left(21\left(11\,Bb^3-13\,Ab^2c\right)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)\right.}{585\,c^4}$$

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/585*(21*(11*B*b^3 - 13*A*b^2*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weiers
trassPInverse(-4*b/c, 0, x)) + (45*B*c^3*x^4 + 77*B*b^2*c - 91*A*b*c^2 - 5*
(11*B*b*c^2 - 13*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/c^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^{\frac{11}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(11/2)/sqrt(c*x^4 + b*x^2), x)
```

Giac [F]

$$\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^{\frac{11}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^(11/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{11/2}\,(B\,x^2+A)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

```
[In] int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
[Out] int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.246 $\int \frac{x^{9/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 204

$$\int \frac{x^{9/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{10b(9bB-11Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{2(9bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{5b^{7/4}(9bB-11Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231c^{13/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/77*(-11*A*c+9*B*b)*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c^2+2/11*B*x^(7/2)*(c*x^4
+b*x^2)^(1/2)/c+10/231*b*(-11*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/c^3/x^(1/2)-5/
231*b^(7/4)*(-11*A*c+9*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1
/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x
^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(
1/2))^2)^(1/2)/c^(13/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used

$= \{2064, 2049, 2057, 335, 226\}$

$$\int \frac{x^{9/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{5b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9bB-11Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231c^{13/4}\sqrt{bx^2+cx^4}} + \frac{10b\sqrt{bx^2+cx^4}(9bB-11Ac)}{231c^3\sqrt{x}} - \frac{2x^{3/2}\sqrt{bx^2+cx^4}(9bB-11Ac)}{77c^2} + \frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c}$$

```
[In] Int[(x^(9/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (10*b*(9*b*B - 11*A*c)*Sqrt[b*x^2 + c*x^4])/(231*c^3*Sqrt[x]) - (2*(9*b*B - 
 11*A*c)*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(77*c^2) + (2*B*x^(7/2)*Sqrt[b*x^2 + 
c*x^4])/(11*c) - (5*b^(7/4)*(9*b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(
b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(231*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{\left(2\left(\frac{9bB}{2}-\frac{11Ac}{2}\right)\right)\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx}{11c} \\
&= -\frac{2(9bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^2} + \frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c} + \frac{(5b(9bB-11Ac))\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{77c^2} \\
&= \frac{10b(9bB-11Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{2(9bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^2} \\
&\quad + \frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{(5b^2(9bB-11Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{231c^3} \\
&= \frac{10b(9bB-11Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{2(9bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^2} \\
&\quad + \frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{\left(5b^2(9bB-11Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{231c^3\sqrt{bx^2+cx^4}} \\
&= \frac{10b(9bB-11Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{2(9bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^2} \\
&\quad + \frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{\left(10b^2(9bB-11Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{231c^3\sqrt{bx^2+cx^4}} \\
&= \frac{10b(9bB-11Ac)\sqrt{bx^2+cx^4}}{231c^3\sqrt{x}} - \frac{2(9bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^2} \\
&\quad + \frac{2Bx^{7/2}\sqrt{bx^2+cx^4}}{11c} - \frac{5b^{7/4}(9bB-11Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{13/4}\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.60

$$\int \frac{x^{9/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{3/2}\Big(\left(b+cx^2\right)\left(45b^2B+3c^2x^2(11A+7Bx^2)-bc(55A+27Bx^2)\right)+5b^2(-9bB+ \cdots}{231c^3\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(x^(9/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*((b + c*x^2)*(45*b^2*B + 3*c^2*x^2*(11*A + 7*B*x^2) - b*c*(55*A
+ 27*B*x^2)) + 5*b^2*(-9*b*B + 11*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F
1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(231*c^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{2\left(-21B\,c^2x^4-33A\,c^2x^2+27Bbc\,x^2+55Abc-45B\,b^2\right)x^{\frac{3}{2}}\left(c\,x^2+b\right)}{231c^3\sqrt{x^2\left(c\,x^2+b\right)}}+\frac{5b^2\left(11Ac-9Bb\right)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{\cdots}}{231c^4\sqrt{c\,x^3+bx}\,\sqrt{x^2 \cdots}}$
default	$\frac{\sqrt{x}\left(42B\,c^4x^7+55A\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b^2c+66A\,c^4x^5-45B\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\cdots}\right.}{231\sqrt{x^4c+b\,x^2}\,c^4}$

```
[In] int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/231*(-21*B*c^2*x^4-33*A*c^2*x^2+27*B*b*c*x^2+55*A*b*c-45*B*b^2)/c^3*x^(3
/2)*(c*x^2+b)/(x^2*(c*x^2+b))^(1/2)+5/231*b^2*(11*A*c-9*B*b)/c^4*(-b*c)^(1/
2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(
-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*EllipticF(((
x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^(1/2)/(x^2*(c*x^2+
b))^(1/2)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.49

$$\int \frac{x^{9/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{2\left(5\left(9\,Bb^3-11\,Ab^2c\right)\sqrt{c}x\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\left(21\,Bc^3x^4+45\,Bb^2c-55\,Abc^2-3\left(9\,Bbc^2-1\right.\right.\right.}{231\,c^4x}$$

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/231*(5*(9*B*b^3 - 11*A*b^2*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x
) - (21*B*c^3*x^4 + 45*B*b^2*c - 55*A*b*c^2 - 3*(9*B*b*c^2 - 11*A*c^3)*x^2)
*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^4*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{9/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^{\frac{9}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(9/2)/sqrt(c*x^4 + b*x^2), x)
```

Giac [F]

$$\int \frac{x^{9/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^{\frac{9}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^(9/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{9/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{9/2}\,(B\,x^2+A)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

```
[In] int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
[Out] int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.247 $\int \frac{x^{7/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 330

$$
\begin{aligned}
\int \frac{x^{7/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx =\; & \frac{2b(7bB-9Ac)x^{3/2}(b+cx^2)}{15c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} \\
& - \frac{2(7bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c} \\
& - \frac{2b^{5/4}(7bB-9Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}} \\
& + \frac{b^{5/4}(7bB-9Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}}
\end{aligned}
$$

```
[Out] 2/15*b*(-9*A*c+7*B*b)*x^(3/2)*(c*x^2+b)/c^(5/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+
b*x^2)^(1/2)+2/9*B*x^(5/2)*(c*x^4+b*x^2)^(1/2)/c-2/45*(-9*A*c+7*B*b)*x^(1/2
)*(c*x^4+b*x^2)^(1/2)/c^2-2/15*b^(5/4)*(-9*A*c+7*B*b)*x*(cos(2*arctan(c^(1/
4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ellipt
icE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))
*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)+1/15*
b^(5/4)*(-9*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/c
os(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))
^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2064, 2049, 2057, 335, 311, 226, 1210}

$$\int \frac{x^{7/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-9Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}} - \frac{2b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-9Ac)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}} + \frac{2bx^{3/2}(b+cx^2)(7bB-9Ac)}{15c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{x}\sqrt{bx^2+cx^4}(7bB-9Ac)}{45c^2} + \frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c}$$

```
[In] Int[(x^(7/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*b*(7*b*B - 9*A*c)*x^(3/2)*(b + c*x^2))/(15*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[b*x^2 + c*x^4]) - (2*(7*b*B - 9*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(45
*c^2) + (2*B*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(9*c) - (2*b^(5/4)*(7*b*B - 9*A*c
)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ellipti
cE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(11/4)*Sqrt[b*x^2 + c*x
^4]) + (b^(5/4)*(7*b*B - 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(S
qrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])
/(15*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c} - \frac{\left(2\left(\frac{7bB}{2}-\frac{9Ac}{2}\right)\right)\int\frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{9c} \\
&= -\frac{2(7bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c} + \frac{(b(7bB-9Ac))\int\frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{15c^2} \\
&= -\frac{2(7bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c} \\
&\quad + \frac{\left(b(7bB-9Ac)x\sqrt{b+cx^2}\right)\int\frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{15c^2\sqrt{bx^2+cx^4}}
\end{aligned}
$$

$$= -\frac{2(7bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c} + \frac{\left(2b(7bB-9Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(7bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c} + \frac{\left(2b^{3/2}(7bB-9Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15c^{5/2}\sqrt{bx^2+cx^4}} - \frac{\left(2b^{3/2}(7bB-9Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15c^{5/2}\sqrt{bx^2+cx^4}}$$

$$= \frac{2b(7bB-9Ac)x^{3/2}(b+cx^2)}{15c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2(7bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2Bx^{5/2}\sqrt{bx^2+cx^4}}{9c} - \frac{2b^{5/4}(7bB-9Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}} + \frac{b^{5/4}(7bB-9Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.29

$$\int \frac{x^{7/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{5/2}\left(-\left((b+cx^2)\left(7bB-9Ac-5Bcx^2\right)\right)+b(7bB-9Ac)\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometr}\right.}{45c^2\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(5/2)*(-((b + c*x^2)*(7*b*B - 9*A*c - 5*B*c*x^2)) + b*(7*b*B - 9*A*c)*
Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(45*c^
2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.73

method	result
risch	$\frac{2x^{\frac{5}{2}}\left(5Bc\,x^{2}+9Ac-7Bb\right)\left(c\,x^{2}+b\right)}{45c^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{b(9Ac-7Bb)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}\right.}{15c^{3}\sqrt{c\,x^{3}+bx}\,\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$
default	$-\frac{\sqrt{x}\left(-10B\,c^{3}x^{6}+54A\,b^{2}c\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-27A\,b^{2}c\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\right.}{}$

```
[In] int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45*x^(5/2)*(5*B*c*x^2+9*A*c-7*B*b)/c^2*(c*x^2+b)/(x^2*(c*x^2+b))^(1/2)-1/
15*b*(9*A*c-7*B*b)/c^3*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(
1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/
2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c)^(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(
-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(
1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)))*x^(1/2)/(x^2*(c*x^2+b))^(1/2)*(x*
(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.24

$$\int \frac{x^{7/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{2\left(3\left(7\,Bb^2-9\,Abc\right)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-\left(5\,Bc^2x^2-7\,Bbc+9\,Ac\right.}{45\,c^3}$$

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*(3*(7*B*b^2 - 9*A*b*c)*sqrt(c)*weierstrassZeta(-4*b/c, 0, weierstrass
PInverse(-4*b/c, 0, x)) - (5*B*c^2*x^2 - 7*B*b*c + 9*A*c^2)*sqrt(c*x^4 + b*
x^2)*sqrt(x))/c^3
```

Sympy [F]

$$\int \frac{x^{7/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{\frac{7}{2}}(A+Bx^2)}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

```
[In] integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(x**(7/2)*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x^{7/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^{\frac{7}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*x^(7/2)/sqrt(c*x^4 + b*x^2), x)
```

Giac [F]

$$\int \frac{x^{7/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^{\frac{7}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^(7/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{7/2}\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

```
[In] int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
[Out] int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.248 $\int \frac{x^{5/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$

Optimal result

Integrand size = 28, antiderivative size = 167

$$\int \frac{x^{5/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{2(5bB-7Ac)\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}}+\frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{b^{3/4}(5bB-7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21c^{9/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/7*B*x^(3/2)*(c*x^4+b*x^2)^(1/2)/c-2/21*(-7*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)
/c^2/x^(1/2)+1/21*b^(3/4)*(-7*A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(
b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2064, 2049, 2057, 335, 226}

$$\int \frac{x^{5/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(5bB-7Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21c^{9/4}\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}(5bB-7Ac)}{21c^2\sqrt{x}}+\frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c}$$

```
[In] Int[(x^(5/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-2*(5*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(21*c^2*Sqrt[x]) + (2*B*x^(3/2)*Sq
rt[b*x^2 + c*x^4])/(7*c) + (b^(3/4)*(5*b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(21*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\text{integral} = \frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} - \frac{\left(2\left(\frac{5bB}{2}-\frac{7Ac}{2}\right)\right)\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{7c}$$

$$= -\frac{2(5bB-7Ac)\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{(b(5bB-7Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{21c^2}$$

$$= -\frac{2(5bB-7Ac)\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{\left(b(5bB-7Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{21c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(5bB-7Ac)\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{\left(2b(5bB-7Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{21c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2(5bB-7Ac)\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}} + \frac{2Bx^{3/2}\sqrt{bx^2+cx^4}}{7c} + \frac{b^{3/4}(5bB-7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{9/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.58

$$\int \frac{x^{5/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{3/2}\left(-\left(\left(b+cx^2\right)\left(5bB-7Ac-3Bcx^2\right)\right)+b(5bB-7Ac)\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric}\right.}{21c^2\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(-((b + c*x^2)*(5*b*B - 7*A*c - 3*B*c*x^2)) + b*(5*b*B - 7*A*c)*
Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(21*c^
2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.14

method	result
risch	$\frac{2(3Bcx^2+7Ac-5Bb)x^{\frac{3}{2}}(cx^2+b)}{21c^2\sqrt{x^2(cx^2+b)}} - \frac{b(7Ac-5Bb)\sqrt{-bc}\sqrt{\frac{(x+\frac{\sqrt{-bc}}{c})c}{\sqrt{-bc}}}\sqrt{-\frac{2(x-\frac{\sqrt{-bc}}{c})c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{(x+\frac{\sqrt{-bc}}{c})c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{x}\,\cdots}{21c^3\sqrt{cx^3+bx}\sqrt{x^2(cx^2+b)}}$
default	$-\frac{\sqrt{x}\,(7A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc-5B\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,\cdots}{21\sqrt{x^4c+bx^2}\,c^3}$

```
[In] int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*(3*B*c*x^2+7*A*c-5*B*b)/c^2*x^(3/2)*(c*x^2+b)/(x^2*(c*x^2+b))^(1/2)-1/
21*b*(7*A*c-5*B*b)/c^3*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(
1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/
2)/(c*x^3+b*x)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),
1/2*2^(1/2))*x^(1/2)/(x^2*(c*x^2+b))^(1/2)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.44

$$\int \frac{x^{5/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2\,((5\,Bb^2-7\,Abc)\sqrt{c}x\,\mathrm{weierstrassPInverse}(-\frac{4\,b}{c},0,x)+(3\,Bc^2x^2-5\,Bbc+7\,Ac^2)\cdots}{21\,c^3x}$$

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*((5*B*b^2 - 7*A*b*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) + (3*
B*c^2*x^2 - 5*B*b*c + 7*A*c^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^3*x)
```

Sympy [F]

$$\int \frac{x^{5/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{\frac{5}{2}}(A+Bx^2)}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

```
[In] integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(5/2)*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x^{5/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^{\frac{5}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*x^(5/2)/sqrt(c*x^4 + b*x^2), x)
```

Giac [F]

$$\int \frac{x^{5/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^{\frac{5}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

```
[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^(5/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{5/2}\,(B\,x^2+A)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

```
[In] int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
[Out] int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.249 $\int \frac{x^{3/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 293

$$\int \frac{x^{3/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{2(3bB-5Ac)x^{3/2}(b+cx^2)}{5c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c}$$
$$+\frac{2\sqrt[4]{b}(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}$$
$$-\frac{\sqrt[4]{b}(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/5*(-5*A*c+3*B*b)*x^(3/2)*(c*x^2+b)/c^(3/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*
x^2)^(1/2)+2/5*B*x^(1/2)*(c*x^4+b*x^2)^(1/2)/c+2/5*b^(1/4)*(-5*A*c+3*B*b)*x
*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1
/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))
*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4
+b*x^2)^(1/2)-1/5*b^(1/4)*(-5*A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(
b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2064, 2057, 335, 311, 226, 1210}

$$\int \frac{x^{3/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = -\frac{\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-5Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}} + \frac{2\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-5Ac)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}} - \frac{2x^{3/2}(b+cx^2)(3bB-5Ac)}{5c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c}$$

```
[In] Int[(x^(3/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-2*(3*b*B - 5*A*c)*x^(3/2)*(b + c*x^2))/(5*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*S
qrt[b*x^2 + c*x^4]) + (2*B*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*c) + (2*b^(1/4)*
(3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]
*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(7/4)*Sqrt
[b*x^2 + c*x^4]) - (b^(1/4)*(3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b
 + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(
1/4)], 1/2])/(5*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c} - \frac{\left(2\left(\frac{3bB}{2}-\frac{5Ac}{2}\right)\right)\int\frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{5c} \\
&= \frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c} - \frac{\left(2\left(\frac{3bB}{2}-\frac{5Ac}{2}\right)x\sqrt{b+cx^2}\right)\int\frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5c\sqrt{bx^2+cx^4}} \\
&= \frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c} - \frac{\left(4\left(\frac{3bB}{2}-\frac{5Ac}{2}\right)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{5c\sqrt{bx^2+cx^4}} \\
&= \frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c} - \frac{\left(4\sqrt{b}\left(\frac{3bB}{2}-\frac{5Ac}{2}\right)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{5c^{3/2}\sqrt{bx^2+cx^4}} \\
&\quad + \frac{\left(4\sqrt{b}\left(\frac{3bB}{2}-\frac{5Ac}{2}\right)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{5c^{3/2}\sqrt{bx^2+cx^4}}
\end{aligned}
$$

$$= -\frac{2(3bB-5Ac)x^{3/2}(b+cx^2)}{5c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{2B\sqrt{x}\sqrt{bx^2+cx^4}}{5c}$$
$$+ \frac{2\sqrt[4]{b}(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}$$
$$- \frac{\sqrt[4]{b}(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.28

$$\int \frac{x^{3/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2x^{5/2}\left(3B(b+cx^2)+(-3bB+5Ac)\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^2}{b}\right)\right)}{15c\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(5/2)*(3*B*(b + c*x^2) + (-3*b*B + 5*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeo
metric2F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(15*c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.77

method	result
risch	$\frac{2B\,x^{\frac{5}{2}}\left(c\,x^2+b\right)}{5c\sqrt{x^2\left(c\,x^2+b\right)}}+\frac{(5Ac-3Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}+\frac{\sqrt{-bc}\,F\left(\sqrt{\frac{\left(x-\cdots\right.}{}}\right.}{}\cdots\right.}{5c^2\sqrt{c\,x^3+bx}\,\sqrt{x^2\left(c\,x^2+b\right)}}$
default	$\frac{\sqrt{x}\left(10Abc\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-5Abc\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\cdots\right.\right.}{\cdots}$

```
[In] int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*B/c*x^(5/2)*(c*x^2+b)/(x^2*(c*x^2+b))^(1/2)+1/5*(5*A*c-3*B*b)/c^2*(-b*c
)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2)
)*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(
```

```
-b*c)^(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/
2))+1/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),
1/2*2^(1/2)))*x^(1/2)/(x^2*(c*x^2+b))^(1/2)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.19

$$\int \frac{x^{3/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2\left(\sqrt{cx^4+bx^2}Bc\sqrt{x}+(3\,Bb-5\,Ac)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\right.\right.}{5\,c^2}$$

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(sqrt(c*x^4 + b*x^2)*B*c*sqrt(x) + (3*B*b - 5*A*c)*sqrt(c)*weierstrassZ
eta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x)))/c^2
```

Sympy [F]

$$\int \frac{x^{3/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{\frac{3}{2}}(A+Bx^2)}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

```
[In] integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(3/2)*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x^{3/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^{\frac{3}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(3/2)/sqrt(c*x^4 + b*x^2), x)
```

Giac [F]

$$\int \frac{x^{3/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)x^{\frac{3}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^(3/2)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{x^{3/2}\,(B\,x^2+A)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

```
[In] int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
[Out] int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.250 $\int \frac{\sqrt{x}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 130

$$\int \frac{\sqrt{x}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}c^{5/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/3*B*(c*x^4+b*x^2)^(1/2)/c/x^(1/2)-1/3*(-3*A*c+B*b)*x*(cos(2*arctan(c^(1/4
)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ellipti
cF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*
((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(1/4)/c^(5/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2064, 2057, 335, 226}

$$\int \frac{\sqrt{x}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-3Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}c^{5/4}\sqrt{bx^2+cx^4}}$$

```
[In] Int[(Sqrt[x]*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*B*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - ((b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[
c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)
*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(1/4)*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{\left(2\left(\frac{bB}{2}-\frac{3Ac}{2}\right)\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{3c} \\
&= \frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{\left(2\left(\frac{bB}{2}-\frac{3Ac}{2}\right)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{3c\sqrt{bx^2+cx^4}} \\
&= \frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{\left(4\left(\frac{bB}{2}-\frac{3Ac}{2}\right)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{3c\sqrt{bx^2+cx^4}}
\end{aligned}$$

$$= \frac{2B\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}c^{5/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{x}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$
$$= \frac{2x^{3/2}\left(B(b+cx^2)+(-bB+3Ac)\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^2}{b}\right)\right)}{3c\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(B*(b + c*x^2) + (-(b*B) + 3*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeome
tric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(3*c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.35

method	result
risch	$\frac{2B\,x^{\frac{3}{2}}\left(c\,x^2+b\right)}{3c\sqrt{x^2\left(c\,x^2+b\right)}}+\frac{(3Ac-Bb)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{x}\,\sqrt{x\left(c\,x^2+b\right)}}{3c^2\sqrt{c\,x^3+bx}\,\sqrt{x^2\left(c\,x^2+b\right)}}$
default	$\frac{\sqrt{x}\,(3A\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)c-B\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F(\sqrt{}}{3\sqrt{x^4c+b\,x^2}\,c^2}$

```
[In] int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*B/c*x^(3/2)*(c*x^2+b)/(x^2*(c*x^2+b))^(1/2)+1/3*(3*A*c-B*b)/c^2*(-b*c)^
(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*
c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*EllipticF
(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^(1/2)/(x^2*(c*x
^2+b))^(1/2)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{x}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx$$
$$= -\frac{2\left((Bb-3\,Ac)\sqrt{c}x\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\sqrt{cx^4+bx^2}Bc\sqrt{x}\right)}{3\,c^2x}$$

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] -2/3*((B*b - 3*A*c)*sqrt(c)*x*weierstrassPInverse(-4*b/c, 0, x) - sqrt(c*x^
4 + b*x^2)*B*c*sqrt(x))/(c^2*x)
```

Sympy [F]

$$\int \frac{\sqrt{x}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{\sqrt{x}\,(A+Bx^2)}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

```
[In] integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral(sqrt(x)*(A + B*x**2)/sqrt(x**2*(b + c*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)\sqrt{x}}{\sqrt{cx^4+bx^2}}\,dx$$

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{x}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)\sqrt{x}}{\sqrt{cx^4+bx^2}}\,dx$$

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx^2)}{\sqrt{bx^2+cx^4}}\,dx = \int \frac{\sqrt{x}\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2}}\,dx$$

```
[In] int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2),x)
[Out] int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(1/2), x)
```

3.251 $\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 281

$$\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx = \frac{2(bB+Ac)x^{3/2}(b+cx^2)}{b\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}} - \frac{2(bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}} + \frac{(bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2*(A*c+B*b)*x^(3/2)*(c*x^2+b)/b/c^(1/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(
1/2)-2*A*(c*x^4+b*x^2)^(1/2)/b/x^(3/2)-2*(A*c+B*b)*x*(cos(2*arctan(c^(1/4)*
x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE
(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((
c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/c^(3/4)/(c*x^4+b*x^2)^(1/2)+(
A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^
(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/
2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3
/4)/c^(3/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2063, 2057, 335, 311, 226, 1210}

$$\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx = \frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+bB)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}} - \frac{2x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+bB)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}} + \frac{2x^{3/2}(b+cx^2)(Ac+bB)}{b\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}}$$

```
[In] Int[(A + B*x^2)/(Sqrt[x]*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (2*(b*B + A*c)*x^(3/2)*(b + c*x^2))/(b*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b
*x^2 + c*x^4]) - (2*A*Sqrt[b*x^2 + c*x^4])/(b*x^(3/2)) - (2*(b*B + A*c)*x*(
Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*
ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(3/4)*c^(3/4)*Sqrt[b*x^2 + c*x^
4]) + ((b*B + A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt
[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(3/4)*c^(
3/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}}+\frac{(bB+Ac)\int\frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{b}\\
&= -\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}}+\frac{\left((bB+Ac)x\sqrt{b+cx^2}\right)\int\frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{b\sqrt{bx^2+cx^4}}\\
&= -\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}}+\frac{\left(2(bB+Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{b\sqrt{bx^2+cx^4}}
\end{aligned}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}} + \frac{\left(2(bB+Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{b}\sqrt{c}\sqrt{bx^2+cx^4}} - \frac{\left(2(bB+Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{b}\sqrt{c}\sqrt{bx^2+cx^4}}$$

$$= \frac{2(bB+Ac)x^{3/2}(b+cx^2)}{b\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2A\sqrt{bx^2+cx^4}}{bx^{3/2}} - \frac{2(bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}} + \frac{(bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.29

$$\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx = \frac{2\sqrt{x}\left(-3A(b+cx^2)+(bB+Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^2}{b}\right)\right)}{3b\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*Sqrt[b*x^2 + c*x^4]),x]
[Out] (2*Sqrt[x]*(-3*A*(b + c*x^2) + (b*B + A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeo
metric2F1[1/2, 3/4, 7/4, -((c*x^2)/b)]))/(3*b*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{2A(c\,x^2+b)\sqrt{x}}{b\sqrt{x^2(c\,x^2+b)}}+\frac{(Ac+Bb)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}+\frac{\sqrt{-bc}\,F\left(\sqrt{\frac{\left(x+\right.}{}}\right.}{c}\right.}{bc\sqrt{c\,x^3+bx}\,\sqrt{x^2(c\,x^2+b)}}$
default	$\frac{\sqrt{x}\left(2Abc\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-Abc\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\right.\right.}{}$

```
[In] int((B*x^2+A)/x^(1/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b*A*(c*x^2+b)*x^(1/2)/(x^2*(c*x^2+b))^(1/2)+(A*c+B*b)/b/c*(-b*c)^(1/2)*(
(x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c
)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c)^(1/
2)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1/c*(
-b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/
2)))*x^(1/2)/(x^2*(c*x^2+b))^(1/2)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.22

$$\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\left((Bb+Ac)\sqrt{c}x^2\text{weierstrassZeta}\left(-\frac{4b}{c},0,\text{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}Ac\sqrt{x}\right)}{bcx^2}$$

```
[In] integrate((B*x^2+A)/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*((B*b + A*c)*sqrt(c)*x^2*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(
-4*b/c, 0, x)) + sqrt(c*x^4 + b*x^2)*A*c*sqrt(x))/(b*c*x^2)
```

Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

```
[In] integrate((B*x**2+A)/x**(1/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(sqrt(x)*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [F]

$$\int \frac{A + Bx^2}{\sqrt{x}\sqrt{bx^2 + cx^4}}\, dx = \int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2}\sqrt{x}}\, dx$$

```
[In] integrate((B*x^2+A)/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*sqrt(x)), x)
```

Giac [F]

$$\int \frac{A + Bx^2}{\sqrt{x}\sqrt{bx^2 + cx^4}}\, dx = \int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2}\sqrt{x}}\, dx$$

```
[In] integrate((B*x^2+A)/x^(1/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2}{\sqrt{x}\sqrt{bx^2 + cx^4}}\, dx = \int \frac{B\,x^2 + A}{\sqrt{x}\,\sqrt{c\,x^4 + b\,x^2}}\, dx$$

```
[In] int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^(1/2)),x)
[Out] int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.252 $\int \frac{A+Bx^2}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 131

$$\int \frac{A+Bx^2}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx=-\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}}+\frac{(3bB-Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/3*A*(c*x^4+b*x^2)^(1/2)/b/x^(5/2)+1/3*(-A*c+3*B*b)*x*(cos(2*arctan(c^(1/
4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ellipt
icF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))
*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(5/4)/c^(1/4)/(c*x^4+b*x^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2063, 2057, 335, 226}

$$\int \frac{A+Bx^2}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx=\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-Ac)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}-\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}}$$

```
[In] Int[(A + B*x^2)/(x^(3/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*A*Sqrt[b*x^2 + c*x^4])/(3*b*x^(5/2)) + ((3*b*B - A*c)*x*(Sqrt[b] + Sqrt
[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4
)*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(5/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}} - \frac{\left(2\left(-\frac{3bB}{2}+\frac{Ac}{2}\right)\right)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{3b} \\
&= -\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}} - \frac{\left(2\left(-\frac{3bB}{2}+\frac{Ac}{2}\right)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{3b\sqrt{bx^2+cx^4}} \\
&= -\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}} - \frac{\left(4\left(-\frac{3bB}{2}+\frac{Ac}{2}\right)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{3b\sqrt{bx^2+cx^4}}
\end{aligned}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{3bx^{5/2}} + \frac{(3bB-Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx^2}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\left(A(b+cx^2)+(-3bB+Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^2}{b}\right)\right)}{3b\sqrt{x}\sqrt{x^2(b+cx^2)}}$$

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*(A*(b + c*x^2) + (-3*b*B + A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2
F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(3*b*Sqrt[x]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.35

method	result
risch	$-\frac{2A(c\,x^2+b)}{3b\sqrt{x}\,\sqrt{x^2(c\,x^2+b)}}-\frac{(Ac-3Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{x}\,\sqrt{x(c\,x^2+b)}}{3bc\sqrt{c\,x^3+bx}\,\sqrt{x^2(c\,x^2+b)}}$
default	$-\frac{A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)cx-3B\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\right.}{3\sqrt{x^4c+b\,x^2}\,\sqrt{x}\,cb}$

```
[In] int((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/b*A*(c*x^2+b)/x^(1/2)/(x^2*(c*x^2+b))^(1/2)-1/3*(A*c-3*B*b)/b/c*(-b*c)
^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))
*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*Elliptic
F(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^(1/2)/(x^2*(c*
x^2+b))^(1/2)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.44

$$\int \frac{A+Bx^2}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx = \frac{2\left((3\,Bb-Ac)\sqrt{c}x^3\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\sqrt{cx^4+bx^2}Ac\sqrt{x}\right)}{3\,bcx^3}$$

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/3*((3*B*b - A*c)*sqrt(c)*x^3*weierstrassPInverse(-4*b/c, 0, x) - sqrt(c*x
^4 + b*x^2)*A*c*sqrt(x))/(b*c*x^3)
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^{\frac{3}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

```
[In] integrate((B*x**2+A)/x**(3/2)/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral((A + B*x**2)/(x**(3/2)*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{B\,x^2+A}{x^{3/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

```
[In] int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(1/2)),x)
[Out] int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.253 $\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 332

$$\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx = \frac{2\sqrt{c}(5bB-3Ac)x^{3/2}(b+cx^2)}{5b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{2(5bB-3Ac)\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} - \frac{2\sqrt[4]{c}(5bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}} + \frac{\sqrt[4]{c}(5bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}}$$

```
[Out] 2/5*(-3*A*c+5*B*b)*x^(3/2)*(c*x^2+b)*c^(1/2)/b^2/(b^(1/2)+x*c^(1/2))/(c*x^4
+b*x^2)^(1/2)-2/5*A*(c*x^4+b*x^2)^(1/2)/b/x^(7/2)-2/5*(-3*A*c+5*B*b)*(c*x^4
+b*x^2)^(1/2)/b^2/x^(3/2)-2/5*c^(1/4)*(-3*A*c+5*B*b)*x*(cos(2*arctan(c^(1/4
)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*Ellipti
cE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*
((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(7/4)/(c*x^4+b*x^2)^(1/2)+1/5*c^(
1/4)*(-3*A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(
2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)
^(1/2)/b^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2063, 2050, 2057, 335, 311, 226, 1210}

$$\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx=\frac{\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(5bB-3Ac)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(5bB-3Ac)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}}+\frac{2\sqrt{c}x^{3/2}(b+cx^2)(5bB-3Ac)}{5b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}(5bB-3Ac)}{5b^2x^{3/2}}-\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}}$$

```
[In] Int[(A + B*x^2)/(x^(5/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (2*Sqrt[c]*(5*b*B - 3*A*c)*x^(3/2)*(b + c*x^2))/(5*b^2*(Sqrt[b] + Sqrt[c]*x
)*Sqrt[b*x^2 + c*x^4]) - (2*A*Sqrt[b*x^2 + c*x^4])/(5*b*x^(7/2)) - (2*(5*b*
B - 3*A*c)*Sqrt[b*x^2 + c*x^4])/(5*b^2*x^(3/2)) - (2*c^(1/4)*(5*b*B - 3*A*c
)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ellipti
cE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[b*x^2 + c*x^4
]) + (c^(1/4)*(5*b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqr
t[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(
5*b^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{\left(2\left(-\frac{5bB}{2}+\frac{3Ac}{2}\right)\right)\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{5b} \\
&= -\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{2(5bB-3Ac)\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} + \frac{(c(5bB-3Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{5b^2} \\
&= -\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{2(5bB-3Ac)\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} + \frac{\left(c(5bB-3Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{5b^2\sqrt{bx^2+cx^4}}
\end{aligned}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{2(5bB-3Ac)\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} + \frac{\left(2c(5bB-3Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{2(5bB-3Ac)\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} + \frac{\left(2\sqrt{c}(5bB-3Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5b^{3/2}\sqrt{bx^2+cx^4}} - \frac{\left(2\sqrt{c}(5bB-3Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5b^{3/2}\sqrt{bx^2+cx^4}}$$

$$= \frac{2\sqrt{c}(5bB-3Ac)x^{3/2}(b+cx^2)}{5b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2A\sqrt{bx^2+cx^4}}{5bx^{7/2}} - \frac{2(5bB-3Ac)\sqrt{bx^2+cx^4}}{5b^2x^{3/2}} - \frac{2\sqrt[4]{c}(5bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}} + \frac{\sqrt[4]{c}(5bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.25

$$\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\left(A(b+cx^2)+(5bB-3Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{cx^2}{b}\right)\right)}{5bx^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(A + B*x^2)/(x^(5/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*(A*(b + c*x^2) + (5*b*B - 3*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric
2F1[-1/4, 1/2, 3/4, -((c*x^2)/b)]))/(5*b*x^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{2(cx^2+b)(-3Acx^2+5bBx^2+Ab)}{5b^2x^{\frac{3}{2}}\sqrt{x^2(cx^2+b)}}-\frac{(3Ac-5Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\ldots\right.}{c}\ldots\right.}{5b^2\sqrt{cx^3+bx}\,\sqrt{x^2(cx^2+b)}\ldots}$
default	$-\frac{6A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bcx^2-3A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\ldots\right.}{\ldots}$

```
[In] int((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(c*x^2+b)*(-3*A*c*x^2+5*B*b*x^2+A*b)/b^2/x^(3/2)/(x^2*(c*x^2+b))^(1/2)
-1/5*(3*A*c-5*B*b)/b^2*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(
1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/
2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c)^(1/2)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(
-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1/c*(-b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(
1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)))*x^(1/2)/(x^2*(c*x^2+b))^(1/2)*(x*
(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.23

$$\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx =$$
$$-\frac{2\left((5\,Bb-3\,Ac)\sqrt{c}x^4\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}((5\,Bb-3\,A\ldots\right.}{5\,b^2x^4}$$

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*((5*B*b - 3*A*c)*sqrt(c)*x^4*weierstrassZeta(-4*b/c, 0, weierstrassPIn
verse(-4*b/c, 0, x)) + sqrt(c*x^4 + b*x^2)*((5*B*b - 3*A*c)*x^2 + A*b)*sqrt
(x))/(b^2*x^4)
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx=\int \frac{A+Bx^2}{x^{\frac{5}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

```
[In] integrate((B*x**2+A)/x**(5/2)/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral((A + B*x**2)/(x**(5/2)*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(5/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx=\int \frac{B\,x^2+A}{x^{5/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

```
[In] int((A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(1/2)),x)
[Out] int((A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.254 $\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 167

$$\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}} - \frac{2(7bB-5Ac)\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} - \frac{c^{3/4}(7bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21b^{9/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/7*A*(c*x^4+b*x^2)^(1/2)/b/x^(9/2)-2/21*(-5*A*c+7*B*b)*(c*x^4+b*x^2)^(1/2
)/b^2/x^(5/2)-1/21*c^(3/4)*(-5*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b
^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*ar
ctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/
(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2063, 2050, 2057, 335, 226}

$$\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-5Ac)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21b^{9/4}\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}(7bB-5Ac)}{21b^2x^{5/2}} - \frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}}$$

```
[In] Int[(A + B*x^2)/(x^(7/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*A*Sqrt[b*x^2 + c*x^4])/(7*b*x^(9/2)) - (2*(7*b*B - 5*A*c)*Sqrt[b*x^2 +
c*x^4])/(21*b^2*x^(5/2)) - (c^(3/4)*(7*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(21*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
```

```
0]
```

Rubi steps

$$\text{integral} = -\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}} - \frac{\left(2\left(-\frac{7bB}{2}+\frac{5Ac}{2}\right)\right)\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{7b}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}} - \frac{2(7bB-5Ac)\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} - \frac{(c(7bB-5Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{21b^2}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}} - \frac{2(7bB-5Ac)\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} - \frac{\left(c(7bB-5Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{21b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}} - \frac{2(7bB-5Ac)\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} - \frac{\left(2c(7bB-5Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{21b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{7bx^{9/2}} - \frac{2(7bB-5Ac)\sqrt{bx^2+cx^4}}{21b^2x^{5/2}} - \frac{c^{3/4}(7bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{21b^{9/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.51

$$\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx = \frac{-6A(b+cx^2)+2(-7bB+5Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-\frac{cx^2}{b}\right)}{21bx^{5/2}\sqrt{x^2(b+cx^2)}}$$

```
[In] Integrate[(A + B*x^2)/(x^(7/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-6*A*(b + c*x^2) + 2*(-7*b*B + 5*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometr
ic2F1[-3/4, 1/2, 1/4, -((c*x^2)/b)])/(21*b*x^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{2\left(c\,x^{2}+b\right)\left(-5Ac\,x^{2}+7bB\,x^{2}+3Ab\right)}{21b^{2}x^{\frac{5}{2}}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{\left(5Ac-7Bb\right)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{x}}{21b^{2}\sqrt{c\,x^{3}+bx}\,\sqrt{x^{2}\left(c\,x^{2}+b\right)}}$
default	$\frac{5A\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)c\,x^{3}-7B\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+}{}}\right.}{21\sqrt{x^{4}c+b\,x^{2}}\,x^{\frac{5}{2}}\,b^{2}}$

```
[In] int((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(c*x^2+b)*(-5*A*c*x^2+7*B*b*x^2+3*A*b)/b^2/x^(5/2)/(x^2*(c*x^2+b))^(1
/2)+1/21*(5*A*c-7*B*b)/b^2*(-b*c)^(1/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2
))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))
^(1/2)/(c*x^3+b*x)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))*x^(1/2)/(x^2*(c*x^2+b))^(1/2)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.41

$$\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\left((7\,Bb-5\,Ac)\sqrt{c}x^5\,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\sqrt{cx^4+bx^2}((7\,Bb-5\,Ac)x^2+3\,Ab)\sqrt{x}\right)}{21\,b^2x^5}$$

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/21*((7*B*b - 5*A*c)*sqrt(c)*x^5*weierstrassPInverse(-4*b/c, 0, x) + sqrt
(c*x^4 + b*x^2)*((7*B*b - 5*A*c)*x^2 + 3*A*b)*sqrt(x))/(b^2*x^5)
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^{\frac{7}{2}}\sqrt{x^2\,(b+cx^2)}}\,dx$$

```
[In] integrate((B*x**2+A)/x**(7/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**(7/2)*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^{\frac{7}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(7/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^{\frac{7}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)/x^(7/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx=\int \frac{B\,x^2+A}{x^{7/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

```
[In] int((A + B*x^2)/(x^(7/2)*(b*x^2 + c*x^4)^(1/2)),x)
[Out] int((A + B*x^2)/(x^(7/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.255 $\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 369

$$\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2c^{3/2}(9bB-7Ac)x^{3/2}(b+cx^2)}{15b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}}-\frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}}+\frac{2c(9bB-7Ac)\sqrt{bx^2+cx^4}}{15b^3x^{3/2}}+\frac{2c^{5/4}(9bB-7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}}-\frac{c^{5/4}(9bB-7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/15*c^(3/2)*(-7*A*c+9*B*b)*x^(3/2)*(c*x^2+b)/b^3/(b^(1/2)+x*c^(1/2))/(c*x
^4+b*x^2)^(1/2)-2/9*A*(c*x^4+b*x^2)^(1/2)/b/x^(11/2)-2/45*(-7*A*c+9*B*b)*(c
*x^4+b*x^2)^(1/2)/b^2/x^(7/2)+2/15*c*(-7*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/b^3
/x^(3/2)+2/15*c^(5/4)*(-7*A*c+9*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1
/2)+x*c^(1/2))^2)^(1/2)/b^(11/4)/(c*x^4+b*x^2)^(1/2)-1/15*c^(5/4)*(-7*A*c+9
*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/
4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2
^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(11/4
)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2063, 2050, 2057, 335, 311, 226, 1210}

$$\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9bB-7Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}} + \frac{2c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9bB-7Ac)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}} - \frac{2c^{3/2}x^{3/2}(b+cx^2)(9bB-7Ac)}{15b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{2c\sqrt{bx^2+cx^4}(9bB-7Ac)}{15b^3x^{3/2}} - \frac{2\sqrt{bx^2+cx^4}(9bB-7Ac)}{45b^2x^{7/2}} - \frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}}$$

```
[In] Int[(A + B*x^2)/(x^(9/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*c^(3/2)*(9*b*B - 7*A*c)*x^(3/2)*(b + c*x^2))/(15*b^3*(Sqrt[b] + Sqrt[c]
*x)*Sqrt[b*x^2 + c*x^4]) - (2*A*Sqrt[b*x^2 + c*x^4])/(9*b*x^(11/2)) - (2*(9
*b*B - 7*A*c)*Sqrt[b*x^2 + c*x^4])/(45*b^2*x^(7/2)) + (2*c*(9*b*B - 7*A*c)*
Sqrt[b*x^2 + c*x^4])/(15*b^3*x^(3/2)) + (2*c^(5/4)*(9*b*B - 7*A*c)*x*(Sqrt[
b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTa
n[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(11/4)*Sqrt[b*x^2 + c*x^4]) - (c^
(5/4)*(9*b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + S
qrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(11
/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\text{integral} = -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{\left(2\left(-\frac{9bB}{2}+\frac{7Ac}{2}\right)\right)\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx}{9b}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} - \frac{(c(9bB-7Ac))\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{15b^2}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} + \frac{2c(9bB-7Ac)\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} - \frac{(c^2(9bB-7Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{15b^3}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} + \frac{2c(9bB-7Ac)\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} - \frac{\left(c^2(9bB-7Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{15b^3\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} + \frac{2c(9bB-7Ac)\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} - \frac{\left(2c^2(9bB-7Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15b^3\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} + \frac{2c(9bB-7Ac)\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} - \frac{\left(2c^{3/2}(9bB-7Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15b^{5/2}\sqrt{bx^2+cx^4}} + \frac{\left(2c^{3/2}(9bB-7Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15b^{5/2}\sqrt{bx^2+cx^4}}$$

$$= -\frac{2c^{3/2}(9bB-7Ac)x^{3/2}(b+cx^2)}{15b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2A\sqrt{bx^2+cx^4}}{9bx^{11/2}} - \frac{2(9bB-7Ac)\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} + \frac{2c(9bB-7Ac)\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} + \frac{2c^{5/4}(9bB-7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}} - \frac{c^{5/4}(9bB-7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.23

$$\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\left(5A(b+cx^2)+(9bB-7Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},-\frac{1}{4},-\frac{cx^2}{b}\right)\right)}{45bx^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(A + B*x^2)/(x^(9/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*(5*A*(b + c*x^2) + (9*b*B - 7*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometr
ic2F1[-5/4, 1/2, -1/4, -((c*x^2)/b)]))/(45*b*x^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{2(c\,x^2+b)\left(21A\,c^2x^4-27x^4Bbc-7Abc\,x^2+9b^2B\,x^2+5b^2A\right)}{45b^3x^{\frac{7}{2}}\sqrt{x^2(c\,x^2+b)}}+\frac{c(7Ac-9Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\ldots\right.}{15b^3\sqrt{c\ldots}}$
default	$\frac{42A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b\,c^2x^4-21A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\ldots\right.}{\ldots}$

```
[In] int((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(c*x^2+b)*(21*A*c^2*x^4-27*B*b*c*x^4-7*A*b*c*x^2+9*B*b^2*x^2+5*A*b^2)
/b^3/x^(7/2)/(x^2*(c*x^2+b))^(1/2)+1/15*c*(7*A*c-9*B*b)/b^3*(-b*c)^(1/2)*((
x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/(-b*c)
^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*(-2/c*(-b*c)^(1/2
)*EllipticE(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+1/c*(-
b*c)^(1/2)*EllipticF(((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
)))*x^(1/2)/(x^2*(c*x^2+b))^(1/2)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.28

$$\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = \frac{2\left(3\left(9\,Bbc-7\,Ac^2\right)\sqrt{c}x^6\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)\right.}{45\,b^3x^6}$$

```
[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*(9*B*b*c - 7*A*c^2)*sqrt(c)*x^6*weierstrassZeta(-4*b/c, 0, weierstr
assPInverse(-4*b/c, 0, x)) + (3*(9*B*b*c - 7*A*c^2)*x^4 - 5*A*b^2 - (9*B*b^
2 - 7*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^3*x^6)
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^{\frac{9}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

```
[In] integrate((B*x**2+A)/x**(9/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**(9/2)*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^{\frac{9}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(9/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^{\frac{9}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)/x^(9/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(9/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx=\int \frac{B\,x^2+A}{x^{9/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

```
[In] int((A + B*x^2)/(x^(9/2)*(b*x^2 + c*x^4)^(1/2)),x)
[Out] int((A + B*x^2)/(x^(9/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.256 $\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 204

$$\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx=-\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}}-\frac{2(11bB-9Ac)\sqrt{bx^2+cx^4}}{77b^2x^{9/2}}+\frac{10c(11bB-9Ac)\sqrt{bx^2+cx^4}}{231b^3x^{5/2}}+\frac{5c^{7/4}(11bB-9Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231b^{13/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/11*A*(c*x^4+b*x^2)^(1/2)/b/x^(13/2)-2/77*(-9*A*c+11*B*b)*(c*x^4+b*x^2)^(
1/2)/b^2/x^(9/2)+10/231*c*(-9*A*c+11*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^(5/2)+5
/231*c^(7/4)*(-9*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(
1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*
x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^
(1/2))^2)^(1/2)/b^(13/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2063, 2050, 2057, 335, 226}

$$\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx=\frac{5c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(11bB-9Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{231b^{13/4}\sqrt{bx^2+cx^4}}+\frac{10c\sqrt{bx^2+cx^4}(11bB-9Ac)}{231b^3x^{5/2}}-\frac{2\sqrt{bx^2+cx^4}(11bB-9Ac)}{77b^2x^{9/2}}-\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}}$$

```
[In] Int[(A + B*x^2)/(x^(11/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*A*Sqrt[b*x^2 + c*x^4])/(11*b*x^(13/2)) - (2*(11*b*B - 9*A*c)*Sqrt[b*x^2
 + c*x^4])/(77*b^2*x^(9/2)) + (10*c*(11*b*B - 9*A*c)*Sqrt[b*x^2 + c*x^4])/(
231*b^3*x^(5/2)) + (5*c^(7/4)*(11*b*B - 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/
b^(1/4)], 1/2])/(231*b^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
```

```
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}} - \frac{\left(2\left(-\frac{11bB}{2}+\frac{9Ac}{2}\right)\right)\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx}{11b} \\
&= -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}} - \frac{2(11bB-9Ac)\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} - \frac{(5c(11bB-9Ac))\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{77b^2} \\
&= -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}} - \frac{2(11bB-9Ac)\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} \\
&\quad + \frac{10c(11bB-9Ac)\sqrt{bx^2+cx^4}}{231b^3x^{5/2}} + \frac{(5c^2(11bB-9Ac))\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{231b^3} \\
&= -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}} - \frac{2(11bB-9Ac)\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} + \frac{10c(11bB-9Ac)\sqrt{bx^2+cx^4}}{231b^3x^{5/2}} \\
&\quad + \frac{\left(5c^2(11bB-9Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{231b^3\sqrt{bx^2+cx^4}} \\
&= -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}} - \frac{2(11bB-9Ac)\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} + \frac{10c(11bB-9Ac)\sqrt{bx^2+cx^4}}{231b^3x^{5/2}} \\
&\quad + \frac{\left(10c^2(11bB-9Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{231b^3\sqrt{bx^2+cx^4}} \\
&= -\frac{2A\sqrt{bx^2+cx^4}}{11bx^{13/2}} - \frac{2(11bB-9Ac)\sqrt{bx^2+cx^4}}{77b^2x^{9/2}} + \frac{10c(11bB-9Ac)\sqrt{bx^2+cx^4}}{231b^3x^{5/2}} \\
&\quad + \frac{5c^{7/4}(11bB-9Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231b^{13/4}\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.41

$$\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = -\frac{2\left(7A(b+cx^2)+(11bB-9Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{1}{2},-\frac{3}{4},-\frac{cx^2}{b}\right)\right)}{77bx^{9/2}\sqrt{x^2(b+cx^2)}}$$

```
[In] Integrate[(A + B*x^2)/(x^(11/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*(7*A*(b + c*x^2) + (11*b*B - 9*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeomet
ric2F1[-7/4, 1/2, -3/4, -((c*x^2)/b)]))/(77*b*x^(9/2)*Sqrt[x^2*(b + c*x^2)]
)
```

Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.06

method	result
risch	$-\frac{2(c\,x^{2}+b)\left(45A\,c^{2}x^{4}-55x^{4}Bbc-27Abc\,x^{2}+33b^{2}B\,x^{2}+21b^{2}A\right)}{231b^{3}x^{\frac{9}{2}}\sqrt{x^{2}(c\,x^{2}+b)}}-\frac{5c(9Ac-11Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-b}}}}{231b^{3}\sqrt{c\,x^{3}+bx}\,\sqrt{x^{2}(c\,x^{2}}}$
default	$-\frac{45A\sqrt{-bc}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\,c^{2}x^{5}-55B\sqrt{-bc}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right.}{231\sqrt{x^{4}c+b\,x^{2}}\,x^{\frac{9}{2}}b^{3}}$

```
[In] int((B*x^2+A)/x^(11/2)/(c*x^4+b*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/231*(c*x^2+b)*(45*A*c^2*x^4-55*B*b*c*x^4-27*A*b*c*x^2+33*B*b^2*x^2+21*A*
b^2)/b^3/x^(9/2)/(x^2*(c*x^2+b))^(1/2)-5/231*c*(9*A*c-11*B*b)/b^3*(-b*c)^(1
/2)*((x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2)*(-2*(x-1/c*(-b*c)^(1/2))*c/
(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)/(c*x^3+b*x)^(1/2)*EllipticF((
(x+1/c*(-b*c)^(1/2))*c/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*x^(1/2)/(x^2*(c*x^2
+b))^(1/2)*(x*(c*x^2+b))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.47

$$\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx=\frac{2\left(5\left(11\,Bbc-9\,Ac^2\right)\sqrt{c}x^7\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+\left(5\left(11\,Bbc-9\,Ac^2\right)x^4-\right.}{231\,b^3x^7}$$

```
[In] integrate((B*x^2+A)/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(5*(11*B*b*c - 9*A*c^2)*sqrt(c)*x^7*weierstrassPInverse(-4*b/c, 0, x)
 + (5*(11*B*b*c - 9*A*c^2)*x^4 - 21*A*b^2 - 3*(11*B*b^2 - 9*A*b*c)*x^2)*sqr
t(c*x^4 + b*x^2)*sqrt(x))/(b^3*x^7)
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{x^{\frac{11}{2}}\sqrt{x^2\,(b+cx^2)}}\,dx$$

```
[In] integrate((B*x**2+A)/x**(11/2)/(c*x**4+b*x**2)**(1/2),x)
[Out] Integral((A + B*x**2)/(x**(11/2)*sqrt(x**2*(b + c*x**2))), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^{\frac{11}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(11/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2}x^{\frac{11}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)/x^(11/2)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2)*x^(11/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx = \int \frac{B\,x^2+A}{x^{11/2}\,\sqrt{c\,x^4+b\,x^2}}\,dx$$

```
[In] int((A + B*x^2)/(x^(11/2)*(b*x^2 + c*x^4)^(1/2)),x)
[Out] int((A + B*x^2)/(x^(11/2)*(b*x^2 + c*x^4)^(1/2)), x)
```

3.257 $\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 251

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}} + \frac{15b(13bB-11Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}} - \frac{9(13bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^3} + \frac{(13bB-11Ac)x^{7/2}\sqrt{bx^2+cx^4}}{11bc^2} - \frac{15b^{7/4}(13bB-11Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{154c^{17/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -(-A*c+B*b)*x^(15/2)/b/c/(c*x^4+b*x^2)^(1/2)-9/77*(-11*A*c+13*B*b)*x^(3/2)*
(c*x^4+b*x^2)^(1/2)/c^3+1/11*(-11*A*c+13*B*b)*x^(7/2)*(c*x^4+b*x^2)^(1/2)/b
/c^2+15/77*b*(-11*A*c+13*B*b)*(c*x^4+b*x^2)^(1/2)/c^4/x^(1/2)-15/154*b^(7/4
)*(-11*A*c+13*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2
*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^
(1/2)/c^(17/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used

$= \{2062, 2049, 2057, 335, 226\}$

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{15b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(13bB-11Ac)\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{154c^{17/4}\sqrt{bx^2+cx^4}} + \frac{15b\sqrt{bx^2+cx^4}(13bB-11Ac)}{77c^4\sqrt{x}} - \frac{9x^{3/2}\sqrt{bx^2+cx^4}(13bB-11Ac)}{77c^3} + \frac{x^{7/2}\sqrt{bx^2+cx^4}(13bB-11Ac)}{11bc^2} - \frac{x^{15/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[In] Int[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^(15/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + (15*b*(13*b*B - 11*A*
c)*Sqrt[b*x^2 + c*x^4])/(77*c^4*Sqrt[x]) - (9*(13*b*B - 11*A*c)*x^(3/2)*Sqr
t[b*x^2 + c*x^4])/(77*c^3) + ((13*b*B - 11*A*c)*x^(7/2)*Sqrt[b*x^2 + c*x^4]
)/(11*b*c^2) - (15*b^(7/4)*(13*b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(
b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(154*c^(17/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2062

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Dist[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1))), Int[(e*x)^(m
- j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n
}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\text{integral} = -\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left(\frac{13bB}{2}-\frac{11Ac}{2}\right)\int\frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$=-\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(13bB-11Ac)x^{7/2}\sqrt{bx^2+cx^4}}{11bc^2}-\frac{(9(13bB-11Ac))\int\frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx}{22c^2}$$

$$=-\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}}-\frac{9(13bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^3}+\frac{(13bB-11Ac)x^{7/2}\sqrt{bx^2+cx^4}}{11bc^2}+\frac{(45b(13bB-11Ac))\int\frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{154c^3}$$

$$=-\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}}+\frac{15b(13bB-11Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}}-\frac{9(13bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^3}+\frac{(13bB-11Ac)x^{7/2}\sqrt{bx^2+cx^4}}{11bc^2}-\frac{(15b^2(13bB-11Ac))\int\frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{154c^4}$$

$$=-\frac{(bB-Ac)x^{15/2}}{bc\sqrt{bx^2+cx^4}}+\frac{15b(13bB-11Ac)\sqrt{bx^2+cx^4}}{77c^4\sqrt{x}}-\frac{9(13bB-11Ac)x^{3/2}\sqrt{bx^2+cx^4}}{77c^3}+\frac{(13bB-11Ac)x^{7/2}\sqrt{bx^2+cx^4}}{11bc^2}-\frac{\left(15b^2(13bB-11Ac)x\sqrt{b+cx^2}\right)\int\frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{154c^4\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB - Ac)x^{15/2}}{bc\sqrt{bx^2 + cx^4}} + \frac{15b(13bB - 11Ac)\sqrt{bx^2 + cx^4}}{77c^4\sqrt{x}} - \frac{9(13bB - 11Ac)x^{3/2}\sqrt{bx^2 + cx^4}}{77c^3} + \frac{(13bB - 11Ac)x^{7/2}\sqrt{bx^2 + cx^4}}{11bc^2} - \frac{\left(15b^2(13bB - 11Ac)x\sqrt{b + cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{77c^4\sqrt{bx^2 + cx^4}}$$

$$= -\frac{(bB - Ac)x^{15/2}}{bc\sqrt{bx^2 + cx^4}} + \frac{15b(13bB - 11Ac)\sqrt{bx^2 + cx^4}}{77c^4\sqrt{x}} - \frac{9(13bB - 11Ac)x^{3/2}\sqrt{bx^2 + cx^4}}{77c^3} + \frac{(13bB - 11Ac)x^{7/2}\sqrt{bx^2 + cx^4}}{11bc^2} - \frac{15b^{7/4}(13bB - 11Ac)x\left(\sqrt{b} + \sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{154c^{17/4}\sqrt{bx^2 + cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.53

$$\int \frac{x^{17/2}(A + Bx^2)}{(bx^2 + cx^4)^{3/2}}\,dx = \frac{x^{3/2}\left(195b^3B + 2c^3x^4(11A + 7Bx^2) - 2bc^2x^2(33A + 13Bx^2) + b^2(-165Ac + 78Bc\right.}{77c^4\sqrt{x^2\,(b +}}$$

```
[In] Integrate[(x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(195*b^3*B + 2*c^3*x^4*(11*A + 7*B*x^2) - 2*b*c^2*x^2*(33*A + 13*B
*x^2) + b^2*(-165*A*c + 78*B*c*x^2) + 15*b^2*(-13*b*B + 11*A*c)*Sqrt[1 + (c
*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(77*c^4*Sqrt[x^2*
(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.12

method	result
default	$\frac{x^{\frac{5}{2}}(c\,x^2+b)\left(28B\,c^4x^7+165A\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b^2c+44A\,c^4x^5-195B\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\cdots\right.}{154(x^4c+b\,x^2)^{\frac{3}{2}}\cdots}$
risch	$-\frac{2(-7B\,c^2x^4-11A\,c^2x^2+20Bbc\,x^2+44Abc-59B\,b^2)x^{\frac{3}{2}}(c\,x^2+b)}{77c^4\sqrt{x^2(c\,x^2+b)}}+\frac{b^2\left(\frac{121A\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\cdots}\right.}{\sqrt{c\,x^3+bx}}\cdots\right)}{\cdots}$

[In] int(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/154/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(28*B*c^4*x^7+165*A*(-b*c)^(1/2
)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c+44*A*c^4*x^5-195*B*(-b*c)^(1/2)*((c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2),1/2*2^(1/2))*b^3-52*B*b*c^3*x^5-132*A*b*c^3*x^3+156*B*b^2*c^2*x^3-3
30*A*b^2*c^2*x+390*B*b^3*c*x)/c^5

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.62

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{15\,((13\,Bb^3c-11\,Ab^2c^2)x^3+(13\,Bb^4-11\,Ab^3c)x)\sqrt{c}\,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-(14\,Bc^4x^6+195\,B\cdots}{77\,(c^6x^3+bc^5x)}$$

[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")

[Out] -1/77*(15*((13*B*b^3*c - 11*A*b^2*c^2)*x^3 + (13*B*b^4 - 11*A*b^3*c)*x)*sqr
t(c)*weierstrassPInverse(-4*b/c, 0, x) - (14*B*c^4*x^6 + 195*B*b^3*c - 165*
A*b^2*c^2 - 2*(13*B*b*c^3 - 11*A*c^4)*x^4 + 6*(13*B*b^2*c^2 - 11*A*b*c^3)*x
^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^6*x^3 + b*c^5*x)

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(17/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{17}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*x^(17/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{17}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^(17/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^(17/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{17/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{17/2}\,(B\,x^2+A)}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

```
[In] int((x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
[Out] int((x^(17/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.258 $\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 377

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}} \, dx = -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}} + \frac{7b(11bB-9Ac)x^{3/2}(b+cx^2)}{15c^{7/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{7(11bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^3} + \frac{(11bB-9Ac)x^{5/2}\sqrt{bx^2+cx^4}}{9bc^2} - \frac{7b^{5/4}(11bB-9Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{15/4}\sqrt{bx^2+cx^4}} + \frac{7b^{5/4}(11bB-9Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{30c^{15/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -(-A*c+B*b)*x^(13/2)/b/c/(c*x^4+b*x^2)^(1/2)+7/15*b*(-9*A*c+11*B*b)*x^(3/2)
*(c*x^2+b)/c^(7/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+1/9*(-9*A*c+11*B
*b)*x^(5/2)*(c*x^4+b*x^2)^(1/2)/b/c^2-7/45*(-9*A*c+11*B*b)*x^(1/2)*(c*x^4+b
*x^2)^(1/2)/c^3-7/15*b^(5/4)*(-9*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2
)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2
*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+
b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(15/4)/(c*x^4+b*x^2)^(1/2)+7/30*b^(5/4)*(
-9*A*c+11*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4
))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2
)/c^(15/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2062, 2049, 2057, 335, 311, 226, 1210}

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{7b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(11bB-9Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{30c^{15/4}\sqrt{bx^2+cx^4}}$$
$$-\frac{7b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(11bB-9Ac)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{15/4}\sqrt{bx^2+cx^4}}$$
$$+\frac{7bx^{3/2}(b+cx^2)(11bB-9Ac)}{15c^{7/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{7\sqrt{x}\sqrt{bx^2+cx^4}(11bB-9Ac)}{45c^3}$$
$$+\frac{x^{5/2}\sqrt{bx^2+cx^4}(11bB-9Ac)}{9bc^2}-\frac{x^{13/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[In] Int[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^(13/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + (7*b*(11*b*B - 9*A*c)
*x^(3/2)*(b + c*x^2))/(15*c^(7/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]
) - (7*(11*b*B - 9*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(45*c^3) + ((11*b*B -
9*A*c)*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(9*b*c^2) - (7*b^(5/4)*(11*b*B - 9*A*c)
*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
E[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(15/4)*Sqrt[b*x^2 + c*x^
4]) + (7*b^(5/4)*(11*b*B - 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/
(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2
])/(30*c^(15/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2062

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Dist[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1))), Int[(e*x)^(m
- j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n
}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\text{integral} = -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}} + \frac{\left(\frac{11bB}{2}-\frac{9Ac}{2}\right)\int\frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(11bB-9Ac)x^{5/2}\sqrt{bx^2+cx^4}}{9bc^2} - \frac{(7(11bB-9Ac))\int\frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{18c^2}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}} - \frac{7(11bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^3} + \frac{(11bB-9Ac)x^{5/2}\sqrt{bx^2+cx^4}}{9bc^2} + \frac{(7b(11bB-9Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{30c^3}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}} - \frac{7(11bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^3} + \frac{(11bB-9Ac)x^{5/2}\sqrt{bx^2+cx^4}}{9bc^2} + \frac{\left(7b(11bB-9Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{30c^3\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}} - \frac{7(11bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^3} + \frac{(11bB-9Ac)x^{5/2}\sqrt{bx^2+cx^4}}{9bc^2} + \frac{\left(7b(11bB-9Ac)x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15c^3\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}} - \frac{7(11bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^3} + \frac{(11bB-9Ac)x^{5/2}\sqrt{bx^2+cx^4}}{9bc^2} + \frac{\left(7b^{3/2}(11bB-9Ac)x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15c^{7/2}\sqrt{bx^2+cx^4}} - \frac{\left(7b^{3/2}(11bB-9Ac)x\sqrt{b+cx^2}\right)\mathrm{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15c^{7/2}\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{13/2}}{bc\sqrt{bx^2+cx^4}} + \frac{7b(11bB-9Ac)x^{3/2}(b+cx^2)}{15c^{7/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{7(11bB-9Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{45c^3} + \frac{(11bB-9Ac)x^{5/2}\sqrt{bx^2+cx^4}}{9bc^2} - \frac{7b^{5/4}(11bB-9Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{15/4}\sqrt{bx^2+cx^4}} + \frac{7b^{5/4}(11bB-9Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{30c^{15/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.29

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{2x^{5/2}\left(77b^2B+c^2x^2(9A+5Bx^2)-bc(63A+11Bx^2)+7b(-11bB+9Ac)\sqrt{1+\frac{cx^2}{b}}\right.}{45c^3\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^(5/2)*(77*b^2*B + c^2*x^2*(9*A + 5*B*x^2) - b*c*(63*A + 11*B*x^2) + 7*
b*(-11*b*B + 9*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -(
(c*x^2)/b)]))/(45*c^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.11

method	result
default	$-\frac{x^{\frac{5}{2}}\left(c\,x^{2}+b\right)\left(-20B\,c^{3}x^{6}+378A\,b^{2}c\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-189A\,b^{2}c\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+}{\sqrt{-}}}\right.}{}$
risch	$\frac{2x^{\frac{5}{2}}\left(5Bc\,x^{2}+9Ac-16Bb\right)\left(c\,x^{2}+b\right)}{45c^{3}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}-\frac{b\left(\frac{(24Ac-31Bb)\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\right.}{c}\right.}{c\sqrt{c\,x^{3}+bx}}\right.}{}$

```
[In] int(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/90/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(-20*B*c^3*x^6+378*A*b^2*c*((c*
x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2),1/2*2^(1/2))-189*A*b^2*c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)
*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-462*B*b^3*(
(c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2),1/2*2^(1/2))+231*B*b^3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
```

```
)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2
)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-36*A*c^3*x
^4+44*B*b*c^2*x^4-126*A*b*c^2*x^2+154*B*b^2*c*x^2)/c^4
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.36

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{21\,(11\,Bb^3-9\,Ab^2c+(11\,Bb^2c-9\,Abc^2)x^2)\sqrt{c}\,\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)-}{45\,(c^5x^2+bc^4)}$$

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/45*(21*(11*B*b^3 - 9*A*b^2*c + (11*B*b^2*c - 9*A*b*c^2)*x^2)*sqrt(c)*wei
erstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x)) - (10*B*c^3*x^4
- 77*B*b^2*c + 63*A*b*c^2 - 2*(11*B*b*c^2 - 9*A*c^3)*x^2)*sqrt(c*x^4 + b*x^
2)*sqrt(x))/(c^5*x^2 + b*c^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(15/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{15}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(15/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{15}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^(15/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^(15/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{15/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{15/2}\,(B\,x^2+A)}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

```
[In] int((x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
[Out] int((x^(15/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.259 $\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 214

$$\begin{aligned}\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx &= -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}} \\ &\quad -\frac{5(9bB-7Ac)\sqrt{bx^2+cx^4}}{21c^3\sqrt{x}}+\frac{(9bB-7Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7bc^2} \\ &\quad +\frac{5b^{3/4}(9bB-7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{42c^{13/4}\sqrt{bx^2+cx^4}}\end{aligned}$$

```
[Out] -(-A*c+B*b)*x^(11/2)/b/c/(c*x^4+b*x^2)^(1/2)+1/7*(-7*A*c+9*B*b)*x^(3/2)*(c*
x^4+b*x^2)^(1/2)/b/c^2-5/21*(-7*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/c^3/x^(1/2)+
5/42*b^(3/4)*(-7*A*c+9*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1
/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x
^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(
1/2))^2)^(1/2)/c^(13/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used

$= \{2062, 2049, 2057, 335, 226\}$

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{5b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9bB-7Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{42c^{13/4}\sqrt{bx^2+cx^4}} - \frac{5\sqrt{bx^2+cx^4}(9bB-7Ac)}{21c^3\sqrt{x}} + \frac{x^{3/2}\sqrt{bx^2+cx^4}(9bB-7Ac)}{7bc^2} - \frac{x^{11/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[In] Int[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^(11/2))/(b*c*Sqrt[b*x^2 + c*x^4])) - (5*(9*b*B - 7*A*c)*Sq
rt[b*x^2 + c*x^4])/(21*c^3*Sqrt[x]) + ((9*b*B - 7*A*c)*x^(3/2)*Sqrt[b*x^2 +
 c*x^4])/(7*b*c^2) + (5*b^(3/4)*(9*b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqr
t[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])
/b^(1/4)], 1/2])/(42*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2062

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Dist[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1))), Int[(e*x)^(m
- j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n
}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\text{integral} = -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}} + \frac{\left(\frac{9bB}{2}-\frac{7Ac}{2}\right)\int\frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(9bB-7Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7bc^2} - \frac{(5(9bB-7Ac))\int\frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{14c^2}$$

$$= -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}} - \frac{5(9bB-7Ac)\sqrt{bx^2+cx^4}}{21c^3\sqrt{x}} + \frac{(9bB-7Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7bc^2} + \frac{(5b(9bB-7Ac))\int\frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{42c^3}$$

$$= -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}} - \frac{5(9bB-7Ac)\sqrt{bx^2+cx^4}}{21c^3\sqrt{x}} + \frac{(9bB-7Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7bc^2} + \frac{\left(5b(9bB-7Ac)x\sqrt{b+cx^2}\right)\int\frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{42c^3\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}} - \frac{5(9bB-7Ac)\sqrt{bx^2+cx^4}}{21c^3\sqrt{x}} + \frac{(9bB-7Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7bc^2} + \frac{\left(5b(9bB-7Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{21c^3\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{11/2}}{bc\sqrt{bx^2+cx^4}} - \frac{5(9bB-7Ac)\sqrt{bx^2+cx^4}}{21c^3\sqrt{x}} + \frac{(9bB-7Ac)x^{3/2}\sqrt{bx^2+cx^4}}{7bc^2} + \frac{5b^{3/4}(9bB-7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{42c^{13/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.51

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x^{3/2}\left(-45b^2B+bc(35A-18Bx^2)+2c^2x^2(7A+3Bx^2)+5b(9bB-7Ac)\sqrt{1+\frac{cx^2}{b}}\right.}{21c^3\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(-45*b^2*B + b*c*(35*A - 18*B*x^2) + 2*c^2*x^2*(7*A + 3*B*x^2) + 5
*b*(9*b*B - 7*A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((
c*x^2)/b)]))/(21*c^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.19

method	result
default	$-\frac{x^{\frac{5}{2}}\left(cx^2+b\right)\left(35A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc-45B\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.}{42\left(x^4c+b\,x^2\right)^{\frac{3}{2}}c^4}$
risch	$\frac{2\left(3Bc\,x^2+7Ac-12Bb\right)x^{\frac{3}{2}}\left(c\,x^2+b\right)}{21c^3\sqrt{x^2\left(c\,x^2+b\right)}}-\frac{b\left(\frac{28A\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{\sqrt{c\,x^3+bx}}-\frac{33Bb\sqrt{-bc}}{}\right.}{}$

```
[In] int(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/42/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(35*A*(-b*c)^(1/2)*((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2),1/2*2^(1/2))*b*c-45*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/
2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2-12*B*
c^3*x^5-28*A*c^3*x^3+36*B*b*c^2*x^3-70*A*b*c^2*x+90*B*b^2*c*x)/c^4
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.60

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{5\left((9\,Bb^2c-7\,Abc^2)x^3+(9\,Bb^3-7\,Ab^2c)x\right)\sqrt{c}\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+(6}{21\,(c^5x^3+bc^4x)}$$

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/21*(5*((9*B*b^2*c - 7*A*b*c^2)*x^3 + (9*B*b^3 - 7*A*b^2*c)*x)*sqrt(c)*wei
erstrassPInverse(-4*b/c, 0, x) + (6*B*c^3*x^4 - 45*B*b^2*c + 35*A*b*c^2 - 2
*(9*B*b*c^2 - 7*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^5*x^3 + b*c^4*x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(13/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{13}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(13/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=\int \frac{(Bx^2+A)x^{\frac{13}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^(13/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^(13/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=\int \frac{x^{13/2}\,(B\,x^2+A)}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

```
[In] int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
[Out] int((x^(13/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.260 $\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 340

$$\begin{aligned}\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx &= -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}} \\ &- \frac{3(7bB-5Ac)x^{3/2}(b+cx^2)}{5c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{(7bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5bc^2} \\ &+ \frac{3\sqrt[4]{b}(7bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{11/4}\sqrt{bx^2+cx^4}} \\ &- \frac{3\sqrt[4]{b}(7bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{10c^{11/4}\sqrt{bx^2+cx^4}}\end{aligned}$$

```
[Out] -(-A*c+B*b)*x^(9/2)/b/c/(c*x^4+b*x^2)^(1/2)-3/5*(-5*A*c+7*B*b)*x^(3/2)*(c*x
^2+b)/c^(5/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+1/5*(-5*A*c+7*B*b)*x^
(1/2)*(c*x^4+b*x^2)^(1/2)/b/c^2+3/5*b^(1/4)*(-5*A*c+7*B*b)*x*(cos(2*arctan(
c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*E
llipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(
1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)-
3/10*b^(1/4)*(-5*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1
/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x
^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(
1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2062, 2049, 2057, 335, 311, 226, 1210}

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{3\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-5Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{10c^{11/4}\sqrt{bx^2+cx^4}} + \frac{3\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-5Ac)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{11/4}\sqrt{bx^2+cx^4}} - \frac{3x^{3/2}(b+cx^2)(7bB-5Ac)}{5c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{\sqrt{x}\sqrt{bx^2+cx^4}(7bB-5Ac)}{5bc^2} - \frac{x^{9/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[In] Int[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^(9/2))/(b*c*Sqrt[b*x^2 + c*x^4])) - (3*(7*b*B - 5*A*c)*x^(
3/2)*(b + c*x^2))/(5*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) + (
(7*b*B - 5*A*c)*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*b*c^2) + (3*b^(1/4)*(7*b*B
- 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*
EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(11/4)*Sqrt[b*x^2
 + c*x^4]) - (3*b^(1/4)*(7*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c
*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)
], 1/2])/(10*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2062

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Dist[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1))), Int[(e*x)^(m
- j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n
}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\text{integral} = -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}} + \frac{\left(\frac{7bB}{2}-\frac{5Ac}{2}\right)\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}$$

$$= -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(7bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5bc^2} - \frac{(3(7bB-5Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{10c^2}$$

$$= -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(7bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5bc^2} - \frac{\left(3(7bB-5Ac)x\sqrt{b+cx^2}\right)\int\frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{10c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(7bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5bc^2} - \frac{\left(3(7bB-5Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{5c^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(7bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5bc^2} - \frac{\left(3\sqrt{b}(7bB-5Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{5c^{5/2}\sqrt{bx^2+cx^4}} + \frac{\left(3\sqrt{b}(7bB-5Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{5c^{5/2}\sqrt{bx^2+cx^4}}$$

$$= -\frac{(bB-Ac)x^{9/2}}{bc\sqrt{bx^2+cx^4}} - \frac{3(7bB-5Ac)x^{3/2}(b+cx^2)}{5c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{(7bB-5Ac)\sqrt{x}\sqrt{bx^2+cx^4}}{5bc^2} + \frac{3\sqrt[4]{b}(7bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{11/4}\sqrt{bx^2+cx^4}} - \frac{3\sqrt[4]{b}(7bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10c^{11/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.25

$$\int\frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{2x^{5/2}\left(-7bB+5Ac+Bcx^2+(7bB-5Ac)\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\right.\right.}{5c^2\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
[Out] (2*x^(5/2)*(-7*b*B + 5*A*c + B*c*x^2 + (7*b*B - 5*A*c)*Sqrt[1 + (c*x^2)/b]*
Hypergeometric2F1[3/4, 3/2, 7/4, -((c*x^2)/b)]))/(5*c^2*Sqrt[x^2*(b + c*x^2
)])
```

Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.16

method	result
default	$\frac{x^{\frac{5}{2}}\left(c\,x^{2}+b\right)\left(30Abc\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-15Abc\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\right.\right.}{}$
risch	$\frac{2B\,x^{\frac{5}{2}}\left(c\,x^{2}+b\right)}{5c^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{\left(\frac{(5Ac-8Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}+\frac{\sqrt{-bc}\,F\left(\sqrt{\frac{\left(x\right.}{}}\right.}{}\right)}{c\sqrt{c\,x^{3}+bx}}\right.}{}$

[In] int(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/10/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(30*A*b*c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-15*A*b*c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-42*B*b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+21*B*b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+4*B*c^2*x^4-10*A*c^2*x^2+14*B*b*c*x^2)/c^3

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.31

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{3\left(7\,Bb^2-5\,Abc+(7\,Bbc-5\,Ac^2)x^2\right)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInver}\right.}{5\left(c^4x^2+bc^3\right)}$$

[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")

```
[Out] 1/5*(3*(7*B*b^2 - 5*A*b*c + (7*B*b*c - 5*A*c^2)*x^2)*sqrt(c)*weierstrassZet
a(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x)) + (2*B*c^2*x^2 + 7*B*b*c -
5*A*c^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^4*x^2 + b*c^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(11/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{11}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*x^(11/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{11}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^(11/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^(11/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{11/2}\,(B\,x^2+A)}{{(c\,x^4+b\,x^2)}^{3/2}}\,dx$$

```
[In] int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
[Out] int((x^(11/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.261 $\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 178

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^{7/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(5bB-3Ac)\sqrt{bx^2+cx^4}}{3bc^2\sqrt{x}} - \frac{(5bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{6\sqrt[4]{b}c^{9/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -(-A*c+B*b)*x^(7/2)/b/c/(c*x^4+b*x^2)^(1/2)+1/3*(-3*A*c+5*B*b)*(c*x^4+b*x^2
)^(1/2)/b/c^2/x^(1/2)-1/6*(-3*A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(
b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(1/4)/c^(9/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2062, 2049, 2057, 335, 226}

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(5bB-3Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{6\sqrt[4]{b}c^{9/4}\sqrt{bx^2+cx^4}} + \frac{\sqrt{bx^2+cx^4}(5bB-3Ac)}{3bc^2\sqrt{x}} - \frac{x^{7/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[In] Int[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^(7/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + ((5*b*B - 3*A*c)*Sqrt[
b*x^2 + c*x^4])/(3*b*c^2*Sqrt[x]) - ((5*b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x
)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqr
t[x])/b^(1/4)], 1/2])/(6*b^(1/4)*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2062

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Dist[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1))), Int[(e*x)^(m
- j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n
}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(bB-Ac)x^{7/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left(\frac{5bB}{2}-\frac{3Ac}{2}\right)\int\frac{x^{5/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}\\
&= -\frac{(bB-Ac)x^{7/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(5bB-3Ac)\sqrt{bx^2+cx^4}}{3bc^2\sqrt{x}}-\frac{(5bB-3Ac)\int\frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{6c^2}\\
&= -\frac{(bB-Ac)x^{7/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(5bB-3Ac)\sqrt{bx^2+cx^4}}{3bc^2\sqrt{x}}-\frac{\left((5bB-3Ac)x\sqrt{b+cx^2}\right)\int\frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{6c^2\sqrt{bx^2+cx^4}}\\
&= -\frac{(bB-Ac)x^{7/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(5bB-3Ac)\sqrt{bx^2+cx^4}}{3bc^2\sqrt{x}}\\
&\quad-\frac{\left((5bB-3Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{3c^2\sqrt{bx^2+cx^4}}\\
&= -\frac{(bB-Ac)x^{7/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(5bB-3Ac)\sqrt{bx^2+cx^4}}{3bc^2\sqrt{x}}\\
&\quad-\frac{(5bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{b}c^{9/4}\sqrt{bx^2+cx^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.48

$$\int\frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=\frac{x^{3/2}\left(5bB-3Ac+2Bcx^2+(-5bB+3Ac)\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\right.\right.}{3c^2\sqrt{x^2(b+cx^2)}}$$

```
[In] Integrate[(x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
[Out] (x^(3/2)*(5*b*B - 3*A*c + 2*B*c*x^2 + (-5*b*B + 3*A*c)*Sqrt[1 + (c*x^2)/b]*
Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)]))/(3*c^2*Sqrt[x^2*(b + c*x^2
)])
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.29

method	result
default	$\frac{x^{\frac{5}{2}}(cx^2+b)\left(3A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)c-5B\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-b}}}\right.}{6(x^4c+bx^2)^{\frac{3}{2}}c^3}$
risch	$\frac{2Bx^{\frac{3}{2}}(cx^2+b)}{3c^2\sqrt{x^2(cx^2+b)}}+\frac{\left(\frac{3A\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{\sqrt{cx^3+bx}}-\frac{4Bb\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2(x-}{}}}{c}\right.}{}$

[In] int(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/6/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(3*A*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*c-5*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b+4*B*c^2*x^3-6*A*c^2*x+10*B*b*c*x)/c^3

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.57

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{((5\,Bbc-3\,Ac^2)x^3+(5\,Bb^2-3\,Abc)x)\sqrt{c}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-(2\,Bc^2x^2+5\,Bbc-3\,Ac^2)\sqrt{cx}}{3\,(c^4x^3+bc^3x)}$$

[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")

[Out] -1/3*(((5*B*b*c - 3*A*c^2)*x^3 + (5*B*b^2 - 3*A*b*c)*x)*sqrt(c)*weierstrassPInverse(-4*b/c, 0, x) - (2*B*c^2*x^2 + 5*B*b*c - 3*A*c^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(c^4*x^3 + b*c^3*x)

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(9/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{9}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*x^(9/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{9}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^(9/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^(9/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{9/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{9/2}\,(B\,x^2+A)}{{(c\,x^4+b\,x^2)}^{3/2}}\,dx$$

```
[In] int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
[Out] int((x^(9/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.262 $\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 299

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{(bB-Ac)x^{5/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(3bB-Ac)x^{3/2}(b+cx^2)}{bc^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{(3bB-Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{7/4}\sqrt{bx^2+cx^4}} + \frac{(3bB-Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{3/4}c^{7/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -(-A*c+B*b)*x^(5/2)/b/c/(c*x^4+b*x^2)^(1/2)+(-A*c+3*B*b)*x^(3/2)*(c*x^2+b)/
b/c^(3/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-(-A*c+3*B*b)*x*(cos(2*arc
tan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)
))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x
*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(3/4)/c^(7/4)/(c*x^4+b*
x^2)^(1/2)+1/2*(-A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1
/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x
^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(
1/2))^2)^(1/2)/b^(3/4)/c^(7/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2062, 2057, 335, 311, 226, 1210}

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{3/4}c^{7/4}\sqrt{bx^2+cx^4}}$$
$$-\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-Ac)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{7/4}\sqrt{bx^2+cx^4}}$$
$$+\frac{x^{3/2}(b+cx^2)\,(3bB-Ac)}{bc^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{x^{5/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[In] Int[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^(5/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + ((3*b*B - A*c)*x^(3/2)
*(b + c*x^2))/(b*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - ((3*b
*B - A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]
*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(3/4)*c^(7/4)*Sqrt
[b*x^2 + c*x^4]) + ((3*b*B - A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/
(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2
])/(2*b^(3/4)*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2062

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Dist[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1))), Int[(e*x)^(m
- j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n
}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(bB-Ac)x^{5/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left(\frac{3bB}{2}-\frac{Ac}{2}\right)\int\frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{bc}\\
&= -\frac{(bB-Ac)x^{5/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left(\left(\frac{3bB}{2}-\frac{Ac}{2}\right)x\sqrt{b+cx^2}\right)\int\frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{bc\sqrt{bx^2+cx^4}}\\
&= -\frac{(bB-Ac)x^{5/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left(2\left(\frac{3bB}{2}-\frac{Ac}{2}\right)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{bc\sqrt{bx^2+cx^4}}\\
&= -\frac{(bB-Ac)x^{5/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left(2\left(\frac{3bB}{2}-\frac{Ac}{2}\right)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{b}c^{3/2}\sqrt{bx^2+cx^4}}\\
&\quad-\frac{\left(2\left(\frac{3bB}{2}-\frac{Ac}{2}\right)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{b}c^{3/2}\sqrt{bx^2+cx^4}}
\end{aligned}$$

$$= -\frac{(bB-Ac)x^{5/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(3bB-Ac)x^{3/2}(b+cx^2)}{bc^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{(3bB-Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{7/4}\sqrt{bx^2+cx^4}} + \frac{(3bB-Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{3/4}c^{7/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.26

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{2x^{5/2}\left(-3bB+(3bB-Ac)\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{cx^2}{b}\right)\right)}{3bc\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
[Out] (-2*x^(5/2)*(-3*b*B + (3*b*B - A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[3
/4, 3/2, 7/4, -((c*x^2)/b)]))/(3*b*c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.30

method	result
default	$-\frac{x^{\frac{5}{2}}\left(c\,x^2+b\right)\left(2Abc\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-Abc\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\right.\right.}{}$

```
[In] int(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(2*A*b*c*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b
*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1
/2))-A*b*c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1
/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-6*B*b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(
```

```
1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(
1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+
3*B*b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-2*A*c^2*x^2+2*B*b*c*x^2)/c^2/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.33

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx =$$
$$-\frac{(3\,Bb^2-Abc+(3\,Bbc-Ac^2)x^2)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+\sqrt{cx^4+bx^2}}{bc^3x^2+b^2c^2}$$

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -((3*B*b^2 - A*b*c + (3*B*b*c - A*c^2)*x^2)*sqrt(c)*weierstrassZeta(-4*b/c,
 0, weierstrassPInverse(-4*b/c, 0, x)) + sqrt(c*x^4 + b*x^2)*(B*b*c - A*c^2
)*sqrt(x))/(b*c^3*x^2 + b^2*c^2)
```

Sympy [F]

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{\frac{7}{2}}(A+Bx^2)}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**(7/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**(7/2)*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{7}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^(7/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{7}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^(7/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^(7/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{7/2}\,(B\,x^2+A)}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

```
[In] int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
[Out] int((x^(7/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.263 $\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx$

Optimal result

Integrand size = 28, antiderivative size = 137

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx = -\frac{(bB-Ac)x^{3/2}}{bc\sqrt{bx^2+cx^4}} + \frac{(bB+Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{5/4}c^{5/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -(-A*c+B*b)*x^(3/2)/b/c/(c*x^4+b*x^2)^(1/2)+1/2*(A*c+B*b)*x*(cos(2*arctan(c
^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*El
lipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1
/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(5/4)/c^(5/4)/(c*x^4+b*x^2)^
(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2062, 2057, 335, 226}

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}} dx = \frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(Ac+bB)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{5/4}c^{5/4}\sqrt{bx^2+cx^4}} - \frac{x^{3/2}(bB-Ac)}{bc\sqrt{bx^2+cx^4}}$$

```
[In] Int[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(((b*B - A*c)*x^(3/2))/(b*c*Sqrt[b*x^2 + c*x^4])) + ((b*B + A*c)*x*(Sqrt[b
] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*b^(5/4)*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2062

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Dist[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1))), Int[(e*x)^(m
- j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m, n
}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(bB-Ac)x^{3/2}}{bc\sqrt{bx^2+cx^4}}+\frac{(bB+Ac)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{2bc}\\
&= -\frac{(bB-Ac)x^{3/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left((bB+Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{2bc\sqrt{bx^2+cx^4}}\\
&= -\frac{(bB-Ac)x^{3/2}}{bc\sqrt{bx^2+cx^4}}+\frac{\left((bB+Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{bc\sqrt{bx^2+cx^4}}
\end{aligned}
$$

$$= -\frac{(bB - Ac)x^{3/2}}{bc\sqrt{bx^2 + cx^4}} + \frac{(bB + Ac)x\left(\sqrt{b} + \sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{5/4}c^{5/4}\sqrt{bx^2 + cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.55

$$\int \frac{x^{5/2}(A + Bx^2)}{(bx^2 + cx^4)^{3/2}}\,dx = \frac{x^{3/2}\left(-bB + Ac + (bB + Ac)\sqrt{1 + \frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{cx^2}{b}\right)\right)}{bc\sqrt{x^2\,(b + cx^2)}}$$

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(-(b*B) + A*c + (b*B + A*c)*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[
1/4, 1/2, 5/4, -((c*x^2)/b)]))/(b*c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.62

method	result
default	$\frac{x^{\frac{5}{2}}\left(c\,x^{2}+b\right)\left(A\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)c+B\sqrt{-bc}\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\right.}{2\left(x^{4}c+b\,x^{2}\right)^{\frac{3}{2}}b\,c^{2}}$

```
[In] int(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(A*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x
*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/
2*2^(1/2))*c+B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)
*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Ellipti
cF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b+2*A*c^2*x-2*B*b*c
*x)/b/c^2
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.66

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{((Bbc+Ac^2)x^3+(Bb^2+Abc)x)\sqrt{c}\,\text{weierstrassPInverse}\left(-\frac{4b}{c},0,x\right)-\sqrt{cx^4+bx^2}(B$$

$$bc^3x^3+b^2c^2x$$

[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")

[Out] (((B*b*c + A*c^2)*x^3 + (B*b^2 + A*b*c)*x)*sqrt(c)*weierstrassPInverse(-4*b /c, 0, x) - sqrt(c*x^4 + b*x^2)*(B*b*c - A*c^2)*sqrt(x))/(b*c^3*x^3 + b^2*c ^2*x)

Sympy [F]

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{\frac{5}{2}}(A+Bx^2)}{(x^2(b+cx^2))^{\frac{3}{2}}}\,dx$$

[In] integrate(x**(5/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)

[Out] Integral(x**(5/2)*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)

Maxima [F]

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{5}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")

[Out] integrate((B*x^2 + A)*x^(5/2)/(c*x^4 + b*x^2)^(3/2), x)

Giac [F]

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{5}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

[In] integrate(x^(5/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")

[Out] integrate((B*x^2 + A)*x^(5/2)/(c*x^4 + b*x^2)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{5/2}\,(B\,x^2+A)}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

```
[In] int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
[Out] int((x^(5/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.264 $\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 318

$$\begin{aligned}
\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx &= -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} \\
&+ \frac{(bB-3Ac)x^{5/2}}{b^2\sqrt{bx^2+cx^4}} - \frac{(bB-3Ac)x^{3/2}(b+cx^2)}{b^2\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} \\
&+ \frac{(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{7/4}c^{3/4}\sqrt{bx^2+cx^4}} \\
&- \frac{(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{7/4}c^{3/4}\sqrt{bx^2+cx^4}}
\end{aligned}$$

```
[Out] (-3*A*c+B*b)*x^(5/2)/b^2/(c*x^4+b*x^2)^(1/2)-(-3*A*c+B*b)*x^(3/2)*(c*x^2+b)
/b^2/c^(1/2)/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)-2*A*x^(1/2)/b/(c*x^4+b
*x^2)^(1/2)+(-3*A*c+B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)
/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1
/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2
))^2)^(1/2)/b^(7/4)/c^(3/4)/(c*x^4+b*x^2)^(1/2)-1/2*(-3*A*c+B*b)*x*(cos(2*a
rctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/
4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)
+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(7/4)/c^(3/4)/(c*x^4+
b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2063, 2048, 2057, 335, 311, 226, 1210}

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-3Ac)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{7/4}c^{3/4}\sqrt{bx^2+cx^4}} + \frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(bB-3Ac)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{7/4}c^{3/4}\sqrt{bx^2+cx^4}} + \frac{x^{5/2}(bB-3Ac)}{b^2\sqrt{bx^2+cx^4}} - \frac{x^{3/2}(b+cx^2)\,(bB-3Ac)}{b^2\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}}$$

```
[In] Int[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*A*Sqrt[x])/(b*Sqrt[b*x^2 + c*x^4]) + ((b*B - 3*A*c)*x^(5/2))/(b^2*Sqrt[
b*x^2 + c*x^4]) - ((b*B - 3*A*c)*x^(3/2)*(b + c*x^2))/(b^2*Sqrt[c]*(Sqrt[b]
 + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) + ((b*B - 3*A*c)*x*(Sqrt[b] + Sqrt[c]*x)
*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt
[x])/b^(1/4)], 1/2])/(b^(7/4)*c^(3/4)*Sqrt[b*x^2 + c*x^4]) - ((b*B - 3*A*c)
*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*b^(7/4)*c^(3/4)*Sqrt[b*x^2
+ c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} - \frac{\left(2\left(-\frac{bB}{2}+\frac{3Ac}{2}\right)\right)\int\frac{x^{7/2}}{(bx^2+cx^4)^{3/2}}\,dx}{b}\\ &= -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x^{5/2}}{b^2\sqrt{bx^2+cx^4}} - \frac{(bB-3Ac)\int\frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{2b^2}\end{aligned}$$

$$= -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x^{5/2}}{b^2\sqrt{bx^2+cx^4}} - \frac{\left((bB-3Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{2b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x^{5/2}}{b^2\sqrt{bx^2+cx^4}} - \frac{\left((bB-3Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x^{5/2}}{b^2\sqrt{bx^2+cx^4}} - \frac{\left((bB-3Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{b^{3/2}\sqrt{c}\sqrt{bx^2+cx^4}} + \frac{\left((bB-3Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{b^{3/2}\sqrt{c}\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A\sqrt{x}}{b\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x^{5/2}}{b^2\sqrt{bx^2+cx^4}} - \frac{(bB-3Ac)x^{3/2}(b+cx^2)}{b^2\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{7/4}c^{3/4}\sqrt{bx^2+cx^4}} - \frac{(bB-3Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{7/4}c^{3/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.24

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{2\sqrt{x}\left(-3Ab+(bB-3Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{cx^2}{b}\right)\right)}{3b^2\sqrt{x^2(b+cx^2)}}$$

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
[Out] (2*Sqrt[x]*(-3*A*b + (b*B - 3*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F
1[3/4, 3/2, 7/4, -((c*x^2)/b)]))/(3*b^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.23

method	result
default	$\frac{x^{\frac{5}{2}}\left(c\,x^{2}+b\right)\left(6Abc\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)-3Abc\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\right.\right.}{}$
risch	$-\frac{2A\left(c\,x^{2}+b\right)\sqrt{x}}{b^{2}\sqrt{x^{2}\left(c\,x^{2}+b\right)}}+\frac{\left(\frac{A\sqrt{-bc}\,\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c}+\frac{\sqrt{-bc}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\right.}{c}\right.}{\sqrt{c\,x^{3}+bx}}\right.}{}$

```
[In] int(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(6*A*b*c*((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*
c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/
2))-3*A*b*c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-2*B*b^2*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^
(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))
+B*b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-6*A*c^2*x^2+2*B*b*c*x^2-4*A*b*c)/c/b^2
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.36

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{((Bbc-3\,Ac^2)x^4+(Bb^2-3\,Abc)x^2)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInvers}\right.}{b^2c^2x^4+b^3cx^2}$$

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (((B*b*c - 3*A*c^2)*x^4 + (B*b^2 - 3*A*b*c)*x^2)*sqrt(c)*weierstrassZeta(-4
*b/c, 0, weierstrassPInverse(-4*b/c, 0, x)) - sqrt(c*x^4 + b*x^2)*(2*A*b*c
- (B*b*c - 3*A*c^2)*x^2)*sqrt(x))/(b^2*c^2*x^4 + b^3*c*x^2)
```

Sympy [F]

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}}(A+Bx^2)}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**(3/2)*(B*x**2+A)/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(x**(3/2)*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{3}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*x^(3/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)x^{\frac{3}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^(3/2)*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^(3/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \int \frac{x^{3/2}\,(B\,x^2+A)}{(c\,x^4+b\,x^2)^{3/2}}\,dx$$

```
[In] int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
[Out] int((x^(3/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.265 $\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 167

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = -\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)x^{3/2}}{3b^2\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{6b^{9/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

```
[Out] 1/3*(-5*A*c+3*B*b)*x^(3/2)/b^2/(c*x^4+b*x^2)^(1/2)-2/3*A/b/x^(1/2)/(c*x^4+b
*x^2)^(1/2)+1/6*(-5*A*c+3*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)
^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4
)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*
c^(1/2))^2)^(1/2)/b^(9/4)/c^(1/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2063, 2048, 2057, 335, 226}

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(3bB-5Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{6b^{9/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}} + \frac{x^{3/2}(3bB-5Ac)}{3b^2\sqrt{bx^2+cx^4}} - \frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}}$$

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*A)/(3*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4]) + ((3*b*B - 5*A*c)*x^(3/2))/(3*b^2
*Sqrt[b*x^2 + c*x^4]) + ((3*b*B - 5*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4
)], 1/2])/(6*b^(9/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\text{integral} = -\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{\left(2\left(-\frac{3bB}{2}+\frac{5Ac}{2}\right)\right)\int \frac{x^{5/2}}{(bx^2+cx^4)^{3/2}}\,dx}{3b}$$

$$= -\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)x^{3/2}}{3b^2\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{6b^2}$$

$$= -\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)x^{3/2}}{3b^2\sqrt{bx^2+cx^4}} + \frac{\left((3bB-5Ac)x\sqrt{b+cx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{6b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)x^{3/2}}{3b^2\sqrt{bx^2+cx^4}} + \frac{\left((3bB-5Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{3b^2\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A}{3b\sqrt{x}\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)x^{3/2}}{3b^2\sqrt{bx^2+cx^4}} + \frac{(3bB-5Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx = \frac{-2Ab+3bBx^2-5Acx^2+(3bB-5Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{c}{}\right.}{3b^2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*A*b + 3*b*B*x^2 - 5*A*c*x^2 + (3*b*B - 5*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*H
ypergeometric2F1[1/4, 1/2, 5/4, -((c*x^2)/b)])/(3*b^2*Sqrt[x]*Sqrt[x^2*(b +
 c*x^2)])
```

Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.41

method	result
default	$-\frac{x^{\frac{3}{2}}(cx^2+b)\left(5A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)cx-3B\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-}\right.}{6(x^4c+bx^2)^{\frac{3}{2}}cb^2}$
risch	$-\frac{2A(cx^2+b)}{3b^2\sqrt{x}\sqrt{x^2(cx^2+b)}}-\frac{\left(\frac{A\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{\sqrt{cx^3+bx}}+3b(Ac-Bb)\left(\frac{x}{b\sqrt{\left(x^2+\frac{b}{c}\right)cx}}+\right.\right.}{3b^2\sqrt{x^2(cx^2+}}$

[In] int((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/6/(c*x^4+b*x^2)^(3/2)*x^(3/2)*(c*x^2+b)*(5*A*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*c*x-3*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*x+10*A*c^2*x^2-6*B*b*c*x^2+4*A*b*c)/c/b^2

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{x}(A+Bx^2)}{(bx^2+cx^4)^{3/2}}\,dx=\frac{((3\,Bbc-5\,Ac^2)x^5+(3\,Bb^2-5\,Abc)x^3)\sqrt{c}\text{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)-\sqrt{cx^4+}}{3\,(b^2c^2x^5+b^3cx^3)}$$

[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")

[Out] 1/3*(((3*B*b*c - 5*A*c^2)*x^5 + (3*B*b^2 - 5*A*b*c)*x^3)*sqrt(c)*weierstrassPInverse(-4*b/c, 0, x) - sqrt(c*x^4 + b*x^2)*(2*A*b*c - (3*B*b*c - 5*A*c^2)*x^2)*sqrt(x))/(b^2*c^2*x^5 + b^3*c*x^3)

Sympy [F]

$$\int \frac{\sqrt{x}(A+Bx^2)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{\sqrt{x}(A+Bx^2)}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)*x**(1/2)/(c*x**4+b*x**2)**(3/2),x)
[Out] Integral(sqrt(x)*(A + B*x**2)/(x**2*(b + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}(A+Bx^2)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{x}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*sqrt(x)/(c*x^4 + b*x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{x}(A+Bx^2)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{x}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*x^(1/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*sqrt(x)/(c*x^4 + b*x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx^2)}{\left(bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{\sqrt{x}\,\left(B\,x^2+A\right)}{\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

```
[In] int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2),x)
[Out] int((x^(1/2)*(A + B*x^2))/(b*x^2 + c*x^4)^(3/2), x)
```

3.266 $\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 368

$$\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}}+\frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} + \frac{3\sqrt{c}(5bB-7Ac)x^{3/2}(b+cx^2)}{5b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{3(5bB-7Ac)\sqrt{bx^2+cx^4}}{5b^3x^{3/2}} - \frac{3\sqrt[4]{c}(5bB-7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{11/4}\sqrt{bx^2+cx^4}} + \frac{3\sqrt[4]{c}(5bB-7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{10b^{11/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/5*A/b/x^(3/2)/(c*x^4+b*x^2)^(1/2)+3/5*(-7*A*c+5*B*b)*x^(3/2)*(c*x^2+b)*c
^(1/2)/b^3/(b^(1/2)+x*c^(1/2))/(c*x^4+b*x^2)^(1/2)+1/5*(-7*A*c+5*B*b)*x^(1/
2)/b^2/(c*x^4+b*x^2)^(1/2)-3/5*(-7*A*c+5*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^(3/
2)-3/5*c^(1/4)*(-7*A*c+5*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^
(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)
*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c
^(1/2))^2)^(1/2)/b^(11/4)/(c*x^4+b*x^2)^(1/2)+3/10*c^(1/4)*(-7*A*c+5*B*b)*x
*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1
/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))
*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(11/4)/(c*x^
4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2063, 2048, 2050, 2057, 335, 311, 226, 1210}

$$\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{3\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(5bB-7Ac)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{10b^{11/4}\sqrt{bx^2+cx^4}}$$
$$-\frac{3\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(5bB-7Ac)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{11/4}\sqrt{bx^2+cx^4}}$$
$$+\frac{3\sqrt{c}x^{3/2}(b+cx^2)(5bB-7Ac)}{5b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}(5bB-7Ac)}{5b^3x^{3/2}}$$
$$+\frac{\sqrt{x}(5bB-7Ac)}{5b^2\sqrt{bx^2+cx^4}}-\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}}$$

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*A)/(5*b*x^(3/2)*Sqrt[b*x^2 + c*x^4]) + ((5*b*B - 7*A*c)*Sqrt[x])/(5*b^2
*Sqrt[b*x^2 + c*x^4]) + (3*Sqrt[c]*(5*b*B - 7*A*c)*x^(3/2)*(b + c*x^2))/(5*
b^3*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (3*(5*b*B - 7*A*c)*Sqrt[b*
x^2 + c*x^4])/(5*b^3*x^(3/2)) - (3*c^(1/4)*(5*b*B - 7*A*c)*x*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/
4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(11/4)*Sqrt[b*x^2 + c*x^4]) + (3*c^(1/4)*(
5*b*B - 7*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*
x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(10*b^(11/4)*Sqr
t[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
```

```
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\text{integral} = -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} - \frac{\left(2\left(-\frac{5bB}{2}+\frac{7Ac}{2}\right)\right)\int \frac{x^{3/2}}{(bx^2+cx^4)^{3/2}}\,dx}{5b}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} + \frac{(3(5bB-7Ac))\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{10b^2}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} - \frac{3(5bB-7Ac)\sqrt{bx^2+cx^4}}{5b^3x^{3/2}} + \frac{(3c(5bB-7Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{10b^3}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} - \frac{3(5bB-7Ac)\sqrt{bx^2+cx^4}}{5b^3x^{3/2}} + \frac{\left(3c(5bB-7Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{10b^3\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} - \frac{3(5bB-7Ac)\sqrt{bx^2+cx^4}}{5b^3x^{3/2}} + \frac{\left(3c(5bB-7Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5b^3\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} - \frac{3(5bB-7Ac)\sqrt{bx^2+cx^4}}{5b^3x^{3/2}} + \frac{\left(3\sqrt{c}(5bB-7Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5b^{5/2}\sqrt{bx^2+cx^4}} - \frac{\left(3\sqrt{c}(5bB-7Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{5b^{5/2}\sqrt{bx^2+cx^4}}$$

$$= -\frac{2A}{5bx^{3/2}\sqrt{bx^2+cx^4}} + \frac{(5bB-7Ac)\sqrt{x}}{5b^2\sqrt{bx^2+cx^4}} + \frac{3\sqrt{c}(5bB-7Ac)x^{3/2}(b+cx^2)}{5b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{3(5bB-7Ac)\sqrt{bx^2+cx^4}}{5b^3x^{3/2}} - \frac{3\sqrt[4]{c}(5bB-7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{11/4}\sqrt{bx^2+cx^4}} + \frac{3\sqrt[4]{c}(5bB-7Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10b^{11/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.21

$$\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{-2Ab+2(-5bB+7Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{4},-\frac{cx^2}{b}\right)}{5b^2x^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(b*x^2 + c*x^4)^(3/2)),x]
[Out] (-2*A*b + 2*(-5*b*B + 7*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-1/4
, 3/2, 3/4, -((c*x^2)/b)])/(5*b^2*x^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.14

method	result
default	$-\frac{\sqrt{x}\,(c\,x^2+b)\left(42A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)bc\,x^2-21A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\right.}{}$
risch	$-\frac{2(c\,x^2+b)\left(-8Ac\,x^2+5bB\,x^2+Ab\right)}{5b^3x^{\frac{3}{2}}\sqrt{x^2(c\,x^2+b)}}-\frac{c^2\left(\frac{(8Ac-5Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\left(-\frac{2\sqrt{-bc}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\right.}{c}\right.}{c^2\sqrt{c\,x^3+bx}}\right.}{}$

[In] int((B*x^2+A)/(c*x^4+b*x^2)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/10/(c*x^4+b*x^2)^(3/2)*x^(1/2)*(c*x^2+b)*(42*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c*x^2-21*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c*x^2-30*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*x^2+15*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*x^2-42*A*c^2*x^4+30*x^4*B*b*c-28*A*b*c*x^2+20*b^2*B*x^2+4*b^2*A)/b^3

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.36

$$\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{3\left((5\,Bbc-7\,Ac^2)x^6+(5\,Bb^2-7\,Abc)x^4\right)\sqrt{c}\,\mathrm{weierstrassZeta}\left(-\frac{4\,b}{c},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)\right)+(3}{5\left(b^3cx^6+b^4x^4\right)}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*(3*((5*B*b*c - 7*A*c^2)*x^6 + (5*B*b^2 - 7*A*b*c)*x^4)*sqrt(c)*weierst
rassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x)) + (3*(5*B*b*c - 7*A*
c^2)*x^4 + 2*A*b^2 + 2*(5*B*b^2 - 7*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x)
)/(b^3*c*x^6 + b^4*x^4)
```

Sympy [F]

$$\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{A+Bx^2}{\sqrt{x}\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2)**(3/2)/x**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(sqrt(x)*(x**2*(b + c*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{x}\left(bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{Bx^2+A}{\left(cx^4+bx^2\right)^{\frac{3}{2}}\sqrt{x}}\,dx$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*sqrt(x)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+bx^2)^{\frac{3}{2}}\sqrt{x}}\,dx$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2)/x^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{x}\,(bx^2+cx^4)^{3/2}}\,dx = \int \frac{B\,x^2+A}{\sqrt{x}\,(c\,x^4+b\,x^2)^{3/2}}\,dx$$

```
[In] int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^(3/2)),x)
[Out] int((A + B*x^2)/(x^(1/2)*(b*x^2 + c*x^4)^(3/2)), x)
```

3.267 $\int \frac{A+Bx^2}{x^{3/2}(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 203

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}} + \frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5(7bB-9Ac)\sqrt{bx^2+cx^4}}{21b^3x^{5/2}} - \frac{5c^{3/4}(7bB-9Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{42b^{13/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/7*A/b/x^(5/2)/(c*x^4+b*x^2)^(1/2)+1/7*(-9*A*c+7*B*b)/b^2/x^(1/2)/(c*x^4+
b*x^2)^(1/2)-5/21*(-9*A*c+7*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/x^(5/2)-5/42*c^(3/
4)*(-9*A*c+7*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*
arctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(
1/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(
1/2)/b^(13/4)/(c*x^4+b*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used

$= \{2063, 2048, 2050, 2057, 335, 226\}$

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{5c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(7bB-9Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{42b^{13/4}\sqrt{bx^2+cx^4}} - \frac{5\sqrt{bx^2+cx^4}(7bB-9Ac)}{21b^3x^{5/2}} + \frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}}$$

```
[In] Int[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*A)/(7*b*x^(5/2)*Sqrt[b*x^2 + c*x^4]) + (7*b*B - 9*A*c)/(7*b^2*Sqrt[x]*S
qrt[b*x^2 + c*x^4]) - (5*(7*b*B - 9*A*c)*Sqrt[b*x^2 + c*x^4])/(21*b^3*x^(5/
2)) - (5*c^(3/4)*(7*b*B - 9*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(
Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2]
)/(42*b^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
```

```
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}}-\frac{\left(2\left(-\frac{7bB}{2}+\frac{9Ac}{2}\right)\right)\int\frac{\sqrt{x}}{(bx^2+cx^4)^{3/2}}\,dx}{7b}\\
&= -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}}+\frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}}+\frac{(5(7bB-9Ac))\int\frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx}{14b^2}\\
&= -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}}+\frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}}\\
&\quad -\frac{5(7bB-9Ac)\sqrt{bx^2+cx^4}}{21b^3x^{5/2}}-\frac{(5c(7bB-9Ac))\int\frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx}{42b^3}\\
&= -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}}+\frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}}-\frac{5(7bB-9Ac)\sqrt{bx^2+cx^4}}{21b^3x^{5/2}}\\
&\quad -\frac{\left(5c(7bB-9Ac)x\sqrt{b+cx^2}\right)\int\frac{1}{\sqrt{x}\sqrt{b+cx^2}}\,dx}{42b^3\sqrt{bx^2+cx^4}}\\
&= -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}}+\frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}}-\frac{5(7bB-9Ac)\sqrt{bx^2+cx^4}}{21b^3x^{5/2}}\\
&\quad -\frac{\left(5c(7bB-9Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+cx^4}}\,dx,x,\sqrt{x}\right)}{21b^3\sqrt{bx^2+cx^4}}
\end{aligned}
$$

$$= -\frac{2A}{7bx^{5/2}\sqrt{bx^2+cx^4}} + \frac{7bB-9Ac}{7b^2\sqrt{x}\sqrt{bx^2+cx^4}} - \frac{5(7bB-9Ac)\sqrt{bx^2+cx^4}}{21b^3x^{5/2}} - \frac{5c^{3/4}(7bB-9Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{42b^{13/4}\sqrt{bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.39

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{-6Ab+2(-7bB+9Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{1}{4},-\frac{cx^2}{b}\right)}{21b^2x^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-6*A*b + 2*(-7*b*B + 9*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-3/4
, 3/2, 1/4, -((c*x^2)/b)])/(21*b^2*x^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.25

method	result
default	$\frac{(c\,x^2+b)\left(45A\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)c\,x^3-35B\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\right.}{42(x^4c+b\,x^2)^{\frac{3}{2}}\sqrt{x}\,b^3}$
risch	$-\frac{2(c\,x^2+b)\left(-12Ac\,x^2+7bB\,x^2+3Ab\right)}{21b^3x^{\frac{5}{2}}\sqrt{x^2(c\,x^2+b)}}+\frac{c\left(-\frac{7Bb\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)}{c\sqrt{c\,x^3+bx}}+\frac{12A\sqrt{\right.}{}}{}$

```
[In] int((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/42/(c*x^4+b*x^2)^(3/2)/x^(1/2)*(c*x^2+b)*(45*A*(-b*c)^(1/2)*((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
),1/2*2^(1/2))*c*x^3-35*B*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1
/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*x^3+90
*A*c^2*x^4-70*x^4*B*b*c+36*A*b*c*x^2-28*b^2*B*x^2-12*b^2*A)/b^3
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.62

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{5\left((7\,Bbc-9\,Ac^2)x^7+(7\,Bb^2-9\,Abc)x^5\right)\sqrt{c}\mathrm{weierstrassPInverse}\left(-\frac{4\,b}{c},0,x\right)+(5\,(7\,Bbc-9\,Ac^2)x^4+6\,Ab}{21\,(b^3cx^7+b^4x^5)}$$

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/21*(5*((7*B*b*c - 9*A*c^2)*x^7 + (7*B*b^2 - 9*A*b*c)*x^5)*sqrt(c)*weiers
trassPInverse(-4*b/c, 0, x) + (5*(7*B*b*c - 9*A*c^2)*x^4 + 6*A*b^2 + 2*(7*B
*b^2 - 9*A*b*c)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^3*c*x^7 + b^4*x^5)
```

Sympy [F]

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{A+Bx^2}{x^{\frac{3}{2}}\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)/x**(3/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**(3/2)*(x**2*(b + c*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{Bx^2+A}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{Bx^2+A}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)/x^(3/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^{3/2}\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{B\,x^2+A}{x^{3/2}\,\left(c\,x^4+b\,x^2\right)^{3/2}}\,dx$$

```
[In] int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x)
[Out] int((A + B*x^2)/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)), x)
```

3.268 $\int \frac{A+Bx^2}{x^{5/2}(bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 405

$$\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx=-\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}}+\frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}}-\frac{7c^{3/2}(9bB-11Ac)x^{3/2}(b+cx^2)}{15b^4\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}}+\frac{7c(9bB-11Ac)\sqrt{bx^2+cx^4}}{15b^4x^{3/2}}+\frac{7c^{5/4}(9bB-11Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{15/4}\sqrt{bx^2+cx^4}}-\frac{7c^{5/4}(9bB-11Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{30b^{15/4}\sqrt{bx^2+cx^4}}$$

```
[Out] -2/9*A/b/x^(7/2)/(c*x^4+b*x^2)^(1/2)+1/9*(-11*A*c+9*B*b)/b^2/x^(3/2)/(c*x^4
+b*x^2)^(1/2)-7/15*c^(3/2)*(-11*A*c+9*B*b)*x^(3/2)*(c*x^2+b)/b^4/(b^(1/2)+x
*c^(1/2))/(c*x^4+b*x^2)^(1/2)-7/45*(-11*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/b^3/
x^(7/2)+7/15*c*(-11*A*c+9*B*b)*(c*x^4+b*x^2)^(1/2)/b^4/x^(3/2)+7/15*c^(5/4)
*(-11*A*c+9*B*b)*x*(cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*a
rctan(c^(1/4)*x^(1/2)/b^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1
/4))),1/2*2^(1/2))*(b^(1/2)+x*c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1
/2)/b^(15/4)/(c*x^4+b*x^2)^(1/2)-7/30*c^(5/4)*(-11*A*c+9*B*b)*x*(cos(2*arct
an(c^(1/4)*x^(1/2)/b^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))
)*EllipticF(sin(2*arctan(c^(1/4)*x^(1/2)/b^(1/4))),1/2*2^(1/2))*(b^(1/2)+x*
c^(1/2))*((c*x^2+b)/(b^(1/2)+x*c^(1/2))^2)^(1/2)/b^(15/4)/(c*x^4+b*x^2)^(1/
2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2063, 2048, 2050, 2057, 335, 311, 226, 1210}

$$\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = -\frac{7c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9bB-11Ac)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{30b^{15/4}\sqrt{bx^2+cx^4}} + \frac{7c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}(9bB-11Ac)E\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{15/4}\sqrt{bx^2+cx^4}} - \frac{7c^{3/2}x^{3/2}(b+cx^2)(9bB-11Ac)}{15b^4\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{7c\sqrt{bx^2+cx^4}(9bB-11Ac)}{15b^4x^{3/2}} - \frac{7\sqrt{bx^2+cx^4}(9bB-11Ac)}{45b^3x^{7/2}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} - \frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}}$$

```
[In] Int[(A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*A)/(9*b*x^(7/2)*Sqrt[b*x^2 + c*x^4]) + (9*b*B - 11*A*c)/(9*b^2*x^(3/2)*
Sqrt[b*x^2 + c*x^4]) - (7*c^(3/2)*(9*b*B - 11*A*c)*x^(3/2)*(b + c*x^2))/(15
*b^4*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (7*(9*b*B - 11*A*c)*Sqrt[
b*x^2 + c*x^4])/(45*b^3*x^(7/2)) + (7*c*(9*b*B - 11*A*c)*Sqrt[b*x^2 + c*x^4
])/(15*b^4*x^(3/2)) + (7*c^(5/4)*(9*b*B - 11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*S
qrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x
])/b^(1/4)], 1/2])/(15*b^(15/4)*Sqrt[b*x^2 + c*x^4]) - (7*c^(5/4)*(9*b*B -
11*A*c)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*E
llipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(30*b^(15/4)*Sqrt[b*x^2
 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2063

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Dist[(a*d*(m + j*p + 1) - b
```

```
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1)), Int[(e*x)^(m + n)*(a*x^j
 + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j +
 n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1]
 || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (G
tQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1,
 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} - \frac{\left(2\left(-\frac{9bB}{2}+\frac{11Ac}{2}\right)\right)\int \frac{1}{\sqrt{x}(bx^2+cx^4)^{3/2}}\,dx}{9b}\\
&= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} + \frac{(7(9bB-11Ac))\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx}{18b^2}\\
&= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}}\\
&\quad - \frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} - \frac{(7c(9bB-11Ac))\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx}{30b^3}\\
&= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} - \frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}}\\
&\quad + \frac{7c(9bB-11Ac)\sqrt{bx^2+cx^4}}{15b^4x^{3/2}} - \frac{(7c^2(9bB-11Ac))\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx}{30b^4}\\
&= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} - \frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}}\\
&\quad + \frac{7c(9bB-11Ac)\sqrt{bx^2+cx^4}}{15b^4x^{3/2}} - \frac{\left(7c^2(9bB-11Ac)x\sqrt{b+cx^2}\right)\int \frac{\sqrt{x}}{\sqrt{b+cx^2}}\,dx}{30b^4\sqrt{bx^2+cx^4}}\\
&= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}}\\
&\quad - \frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} + \frac{7c(9bB-11Ac)\sqrt{bx^2+cx^4}}{15b^4x^{3/2}}\\
&\quad - \frac{\left(7c^2(9bB-11Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15b^4\sqrt{bx^2+cx^4}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} \\
&\quad - \frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} + \frac{7c(9bB-11Ac)\sqrt{bx^2+cx^4}}{15b^4x^{3/2}} \\
&\quad - \frac{\left(7c^{3/2}(9bB-11Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15b^{7/2}\sqrt{bx^2+cx^4}} \\
&\quad + \frac{\left(7c^{3/2}(9bB-11Ac)x\sqrt{b+cx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{b}}}{\sqrt{b+cx^4}}\,dx, x, \sqrt{x}\right)}{15b^{7/2}\sqrt{bx^2+cx^4}} \\
&= -\frac{2A}{9bx^{7/2}\sqrt{bx^2+cx^4}} + \frac{9bB-11Ac}{9b^2x^{3/2}\sqrt{bx^2+cx^4}} - \frac{7c^{3/2}(9bB-11Ac)x^{3/2}(b+cx^2)}{15b^4\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} \\
&\quad - \frac{7(9bB-11Ac)\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} + \frac{7c(9bB-11Ac)\sqrt{bx^2+cx^4}}{15b^4x^{3/2}} \\
&\quad + \frac{7c^{5/4}(9bB-11Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{15/4}\sqrt{bx^2+cx^4}} \\
&\quad - \frac{7c^{5/4}(9bB-11Ac)x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{30b^{15/4}\sqrt{bx^2+cx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.20

$$
\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx = \frac{-10Ab+2(-9bB+11Ac)x^2\sqrt{1+\frac{cx^2}{b}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{2},-\frac{1}{4},-\frac{cx^2}{b}\right)}{45b^2x^{7/2}\sqrt{x^2\left(b+cx^2\right)}}
$$

```
[In] Integrate[(A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x]
[Out] (-10*A*b + 2*(-9*b*B + 11*A*c)*x^2*Sqrt[1 + (c*x^2)/b]*Hypergeometric2F1[-5
/4, 3/2, -1/4, -((c*x^2)/b)])/(45*b^2*x^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.11

method	result
default	$\frac{(c\,x^2+b)\left(462A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,E\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)b\,c^2x^4-231A\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{2}\,\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\,\sqrt{-\frac{xc}{\sqrt{-bc}}}\,F\right.}{}$
risch	$-\frac{2(c\,x^2+b)\,(93A\,c^2x^4-72x^4Bbc-16Abc\,x^2+9b^2B\,x^2+5b^2A)}{45b^4x^{\frac{7}{2}}\sqrt{x^2(c\,x^2+b)}}+\frac{c^2\left(\frac{(31Ac-24Bb)\sqrt{-bc}\sqrt{\frac{\left(x+\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-bc}}{c}\right)c}{\sqrt{-bc}}}\sqrt{-\frac{xc}{\sqrt{-bc}}}}{c\cdots}\right.}{}$

[In] int((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(3/2),x,method=_RETURNVERBOSE)

```
[Out] 1/90/(c*x^4+b*x^2)^(3/2)/x^(3/2)*(c*x^2+b)*(462*A*((c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c
)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*b*c^2*x^4-231*A*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+
(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2*x^4-378*B*((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2),1/2*2^(1/2))*b^2*c*x^4+189*B*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(
1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Ell
ipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c*x^4-462*A
*c^3*x^6+378*x^6*B*b*c^2-308*A*b*c^2*x^4+252*x^4*B*b^2*c+44*A*b^2*c*x^2-36*
b^3*B*x^2-20*b^3*A)/b^4
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.40

$$\int \frac{A+Bx^2}{x^{5/2}\,(bx^2+cx^4)^{3/2}}\,dx = \frac{21\,((9\,Bbc^2-11\,Ac^3)x^8+(9\,Bb^2c-11\,Abc^2)x^6)\sqrt{c}\text{weierstrassZeta}\left(-\frac{4\,b}{c},0,\text{wei}\cdots\right.}{}$$

[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="fricas")

```
[Out] 1/45*(21*((9*B*b*c^2 - 11*A*c^3)*x^8 + (9*B*b^2*c - 11*A*b*c^2)*x^6)*sqrt(c
)*weierstrassZeta(-4*b/c, 0, weierstrassPInverse(-4*b/c, 0, x)) + (21*(9*B*
b*c^2 - 11*A*c^3)*x^6 + 14*(9*B*b^2*c - 11*A*b*c^2)*x^4 - 10*A*b^3 - 2*(9*B
*b^3 - 11*A*b^2*c)*x^2)*sqrt(c*x^4 + b*x^2)*sqrt(x))/(b^4*c*x^8 + b^5*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^2}{x^{5/2}\left(bx^2 + cx^4\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**2+A)/x**(5/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + Bx^2}{x^{5/2}\left(bx^2 + cx^4\right)^{3/2}}\,dx = \int \frac{Bx^2 + A}{\left(cx^4 + bx^2\right)^{\frac{3}{2}} x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^(5/2)), x)
```

Giac [F]

$$\int \frac{A + Bx^2}{x^{5/2}\left(bx^2 + cx^4\right)^{3/2}}\,dx = \int \frac{Bx^2 + A}{\left(cx^4 + bx^2\right)^{\frac{3}{2}} x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)/x^(5/2)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2)^(3/2)*x^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^{5/2}\left(bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{B\,x^2+A}{x^{5/2}\,{\left(c\,x^4+b\,x^2\right)}^{3/2}} \,dx$$

```
[In] int((A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x)
[Out] int((A + B*x^2)/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)), x)
```

3.269 $\int x^m(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 96

$$\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx=\frac{Ab^3x^{7+m}}{7+m}+\frac{b^2(bB+3Ac)x^{9+m}}{9+m}+\frac{3bc(bB+Ac)x^{11+m}}{11+m}+\frac{c^2(3bB+Ac)x^{13+m}}{13+m}+\frac{Bc^3x^{15+m}}{15+m}$$

```
[Out] A*b^3*x^(7+m)/(7+m)+b^2*(3*A*c+B*b)*x^(9+m)/(9+m)+3*b*c*(A*c+B*b)*x^(11+m)/
(11+m)+c^2*(A*c+3*B*b)*x^(13+m)/(13+m)+B*c^3*x^(15+m)/(15+m)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx=\frac{Ab^3x^{m+7}}{m+7}+\frac{b^2x^{m+9}(3Ac+bB)}{m+9}+\frac{c^2x^{m+13}(Ac+3bB)}{m+13}+\frac{3bcx^{m+11}(Ac+bB)}{m+11}+\frac{Bc^3x^{m+15}}{m+15}$$

```
[In] Int[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (A*b^3*x^(7 + m))/(7 + m) + (b^2*(b*B + 3*A*c)*x^(9 + m))/(9 + m) + (3*b*c*
(b*B + A*c)*x^(11 + m))/(11 + m) + (c^2*(3*b*B + A*c)*x^(13 + m))/(13 + m)
+ (B*c^3*x^(15 + m))/(15 + m)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
```

```
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int x^{6+m}\left(A+Bx^2\right)\left(b+cx^2\right)^3\,dx$$

$$= \int \left(Ab^3x^{6+m}+b^2(bB+3Ac)x^{8+m}+3bc(bB+Ac)x^{10+m}+c^2(3bB+Ac)x^{12+m}+Bc^3x^{14+m}\right)\,dx$$

$$= \frac{Ab^3x^{7+m}}{7+m}+\frac{b^2(bB+3Ac)x^{9+m}}{9+m}+\frac{3bc(bB+Ac)x^{11+m}}{11+m}+\frac{c^2(3bB+Ac)x^{13+m}}{13+m}+\frac{Bc^3x^{15+m}}{15+m}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.93

$$\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)^3\,dx = x^{7+m}\left(\frac{Ab^3}{7+m}+\frac{b^2(bB+3Ac)x^2}{9+m}+\frac{3bc(bB+Ac)x^4}{11+m}+\frac{c^2(3bB+Ac)x^6}{13+m}+\frac{Bc^3x^8}{15+m}\right)$$

```
[In] Integrate[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] x^(7 + m)*((A*b^3)/(7 + m) + (b^2*(b*B + 3*A*c)*x^2)/(9 + m) + (3*b*c*(b*B
+ A*c)*x^4)/(11 + m) + (c^2*(3*b*B + A*c)*x^6)/(13 + m) + (B*c^3*x^8)/(15 +
 m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(96) = 192.

Time = 2.05 (sec) , antiderivative size = 474, normalized size of antiderivative = 4.94

method	result
gosper	$\frac{x^{7+m}(Bc^3m^4x^8+40Bc^3m^3x^8+Ac^3m^4x^6+3Bbc^2m^4x^6+590Bc^3m^2x^8+42Ac^3m^3x^6+126Bbc^2m^3x^6+3800mx^8Bc^3+3Abc^2}{}$
risch	$\frac{x^m(Bc^3m^4x^8+40Bc^3m^3x^8+Ac^3m^4x^6+3Bbc^2m^4x^6+590Bc^3m^2x^8+42Ac^3m^3x^6+126Bbc^2m^3x^6+3800mx^8Bc^3+3Abc^2m}{}$
parallelrisch	$\frac{3Bx^{13}x^mbc^2m^4+126Bx^{13}x^mbc^2m^3+3Ax^{11}x^mbc^2m^4+1932Bx^{13}x^mbc^2m^2+3Bx^{11}x^mb^2cm^4+132Ax^{11}x^mbc^2m^3+12834}{}$

[In] int(x^m*(B*x^2+A)*(c*x^4+b*x^2)^3,x,method=_RETURNVERBOSE)

[Out] x^(7+m)/(7+m)/(9+m)/(11+m)/(13+m)/(15+m)*(B*c^3*m^4*x^8+40*B*c^3*m^3*x^8+A*c^3*m^4*x^6+3*B*b*c^2*m^4*x^6+590*B*c^3*m^2*x^8+42*A*c^3*m^3*x^6+126*B*b*c^2*m^3*x^6+3800*B*c^3*m*x^8+3*A*b*c^2*m^4*x^4+644*A*c^3*m^2*x^6+3*B*b^2*c*m^4*x^4+1932*B*b*c^2*m^2*x^6+9009*B*c^3*x^8+132*A*b*c^2*m^3*x^4+4278*A*c^3*m*x^6+132*B*b^2*c*m^3*x^4+12834*B*b*c^2*m*x^6+3*A*b^2*c*m^4*x^2+2118*A*b*c^2*m^2*x^4+10395*A*c^3*x^6+B*b^3*m^4*x^2+2118*B*b^2*c*m^2*x^4+31185*B*b*c^2*x^6+138*A*b^2*c*m^3*x^2+14652*A*b*c^2*m*x^4+46*B*b^3*m^3*x^2+14652*B*b^2*c*m*x^4+A*b^3*m^4+2328*A*b^2*c*m^2*x^2+36855*A*b*c^2*x^4+776*B*b^3*m^2*x^2+36855*B*b^2*c*x^4+48*A*b^3*m^3+16998*A*b^2*c*m*x^2+5666*B*b^3*m*x^2+854*A*b^3*m^2+45045*A*b^2*c*x^2+15015*B*b^3*x^2+6672*A*b^3*m+19305*A*b^3)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(96) = 192$.

Time = 0.42 (sec) , antiderivative size = 381, normalized size of antiderivative = 3.97

$$\int x^m(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx = \frac{((Bc^3m^4+40\,Bc^3m^3+590\,Bc^3m^2+3800\,Bc^3m+9009\,Bc^3)x^{15}+((3\,Bbc^2+Ac^3)m^4+31185\,Bbc^2+10}{}$$

[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="fricas")

[Out] ((B*c^3*m^4 + 40*B*c^3*m^3 + 590*B*c^3*m^2 + 3800*B*c^3*m + 9009*B*c^3)*x^15 + ((3*B*b*c^2 + A*c^3)*m^4 + 31185*B*b*c^2 + 10395*A*c^3 + 42*(3*B*b*c^2 + A*c^3)*m^3 + 644*(3*B*b*c^2 + A*c^3)*m^2 + 4278*(3*B*b*c^2 + A*c^3)*m)*x^13 + 3*((B*b^2*c + A*b*c^2)*m^4 + 12285*B*b^2*c + 12285*A*b*c^2 + 44*(B*b^2*c + A*b*c^2)*m^3 + 706*(B*b^2*c + A*b*c^2)*m^2 + 4884*(B*b^2*c + A*b*c^2)*m)*x^11 + ((B*b^3 + 3*A*b^2*c)*m^4 + 15015*B*b^3 + 45045*A*b^2*c + 46*(B*b^3 + 3*A*b^2*c)*m^3 + 776*(B*b^3 + 3*A*b^2*c)*m^2 + 5666*(B*b^3 + 3*A*b^2*c)*m)*x^9 + (A*b^3*m^4 + 48*A*b^3*m^3 + 854*A*b^3*m^2 + 6672*A*b^3*m + 19305*A*b^3)*x^7)*x^m/(m^5 + 55*m^4 + 1190*m^3 + 12650*m^2 + 66009*m + 135135)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2077 vs. 2(87) = 174.

Time = 1.06 (sec) , antiderivative size = 2077, normalized size of antiderivative = 21.64

$$\int x^m (A + Bx^2) (bx^2 + cx^4)^3 \, dx = \text{Too large to display}$$

```
[In] integrate(x**m*(B*x**2+A)*(c*x**4+b*x**2)**3,x)
```

```
[Out] Piecewise((-A*b**3/(8*x**8) - A*b**2*c/(2*x**6) - 3*A*b*c**2/(4*x**4) - A*c
**3/(2*x**2) - B*b**3/(6*x**6) - 3*B*b**2*c/(4*x**4) - 3*B*b*c**2/(2*x**2)
+ B*c**3*log(x), Eq(m, -15)), (-A*b**3/(6*x**6) - 3*A*b**2*c/(4*x**4) - 3*A
*b*c**2/(2*x**2) + A*c**3*log(x) - B*b**3/(4*x**4) - 3*B*b**2*c/(2*x**2) +
3*B*b*c**2*log(x) + B*c**3*x**2/2, Eq(m, -13)), (-A*b**3/(4*x**4) - 3*A*b**
2*c/(2*x**2) + 3*A*b*c**2*log(x) + A*c**3*x**2/2 - B*b**3/(2*x**2) + 3*B*b*
*2*c*log(x) + 3*B*b*c**2*x**2/2 + B*c**3*x**4/4, Eq(m, -11)), (-A*b**3/(2*x
**2) + 3*A*b**2*c*log(x) + 3*A*b*c**2*x**2/2 + A*c**3*x**4/4 + B*b**3*log(x
) + 3*B*b**2*c*x**2/2 + 3*B*b*c**2*x**4/4 + B*c**3*x**6/6, Eq(m, -9)), (A*b
**3*log(x) + 3*A*b**2*c*x**2/2 + 3*A*b*c**2*x**4/4 + A*c**3*x**6/6 + B*b**3
*x**2/2 + 3*B*b**2*c*x**4/4 + B*b*c**2*x**6/2 + B*c**3*x**8/8, Eq(m, -7)),
(A*b**3*m**4*x**7*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m +
 135135) + 48*A*b**3*m**3*x**7*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**
2 + 66009*m + 135135) + 854*A*b**3*m**2*x**7*x**m/(m**5 + 55*m**4 + 1190*m*
*3 + 12650*m**2 + 66009*m + 135135) + 6672*A*b**3*m*x**7*x**m/(m**5 + 55*m*
*4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 19305*A*b**3*x**7*x**m/(m
**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 3*A*b**2*c*m**
4*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) +
138*A*b**2*c*m**3*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 6600
9*m + 135135) + 2328*A*b**2*c*m**2*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 +
12650*m**2 + 66009*m + 135135) + 16998*A*b**2*c*m*x**9*x**m/(m**5 + 55*m**4
 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 45045*A*b**2*c*x**9*x**m/(m
**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 3*A*b*c**2*m**
4*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) +
 132*A*b*c**2*m**3*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66
009*m + 135135) + 2118*A*b*c**2*m**2*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3
 + 12650*m**2 + 66009*m + 135135) + 14652*A*b*c**2*m*x**11*x**m/(m**5 + 55*
m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 36855*A*b*c**2*x**11*x*
*m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + A*c**3*m*
*4*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135)
+ 42*A*c**3*m**3*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 6600
9*m + 135135) + 644*A*c**3*m**2*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12
650*m**2 + 66009*m + 135135) + 4278*A*c**3*m*x**13*x**m/(m**5 + 55*m**4 + 1
190*m**3 + 12650*m**2 + 66009*m + 135135) + 10395*A*c**3*x**13*x**m/(m**5 +
 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + B*b**3*m**4*x**9*x*
```

```
*m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 46*B*b**3
*m**3*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135
) + 776*B*b**3*m**2*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66
009*m + 135135) + 5666*B*b**3*m*x**9*x**m/(m**5 + 55*m**4 + 1190*m**3 + 126
50*m**2 + 66009*m + 135135) + 15015*B*b**3*x**9*x**m/(m**5 + 55*m**4 + 1190
*m**3 + 12650*m**2 + 66009*m + 135135) + 3*B*b**2*c*m**4*x**11*x**m/(m**5 +
 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 132*B*b**2*c*m**3*x
**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 21
18*B*b**2*c*m**2*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 6600
9*m + 135135) + 14652*B*b**2*c*m*x**11*x**m/(m**5 + 55*m**4 + 1190*m**3 + 1
2650*m**2 + 66009*m + 135135) + 36855*B*b**2*c*x**11*x**m/(m**5 + 55*m**4 +
 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 3*B*b*c**2*m**4*x**13*x**m/(m
**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 126*B*b*c**2*m
**3*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135)
 + 1932*B*b*c**2*m**2*x**13*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 +
 66009*m + 135135) + 12834*B*b*c**2*m*x**13*x**m/(m**5 + 55*m**4 + 1190*m**
3 + 12650*m**2 + 66009*m + 135135) + 31185*B*b*c**2*x**13*x**m/(m**5 + 55*m
**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + B*c**3*m**4*x**15*x**m/(
m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) + 40*B*c**3*m**
3*x**15*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 66009*m + 135135) +
 590*B*c**3*m**2*x**15*x**m/(m**5 + 55*m**4 + 1190*m**3 + 12650*m**2 + 6600
9*m + 135135) + 3800*B*c**3*m*x**15*x**m/(m**5 + 55*m**4 + 1190*m**3 + 1265
0*m**2 + 66009*m + 135135) + 9009*B*c**3*x**15*x**m/(m**5 + 55*m**4 + 1190*
m**3 + 12650*m**2 + 66009*m + 135135), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.34

$$\int x^m(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx=\frac{Bc^3x^{m+15}}{m+15}+\frac{3\,Bbc^2x^{m+13}}{m+13}+\frac{Ac^3x^{m+13}}{m+13}+\frac{3\,Bb^2cx^{m+11}}{m+11}+\frac{3\,Abc^2x^{m+11}}{m+11}+\frac{Bb^3x^{m+9}}{m+9}+\frac{3\,Ab^2cx^{m+9}}{m+9}+\frac{Ab^3x^{m+7}}{m+7}$$

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="maxima")
```

```
[Out] B*c^3*x^(m + 15)/(m + 15) + 3*B*b*c^2*x^(m + 13)/(m + 13) + A*c^3*x^(m + 13
)/(m + 13) + 3*B*b^2*c*x^(m + 11)/(m + 11) + 3*A*b*c^2*x^(m + 11)/(m + 11)
+ B*b^3*x^(m + 9)/(m + 9) + 3*A*b^2*c*x^(m + 9)/(m + 9) + A*b^3*x^(m + 7)/(
m + 7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. 2(96) = 192.

Time = 0.29 (sec) , antiderivative size = 603, normalized size of antiderivative = 6.28

$$\int x^m(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx = \frac{Bc^3m^4x^{15}x^m+40\,Bc^3m^3x^{15}x^m+3\,Bbc^2m^4x^{13}x^m+Ac^3m^4x^{13}x^m+590\,Bc^3m^2x^{15}x^m+126\,Bbc^2m^3x^{13}x^m}{}$$

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^3,x, algorithm="giac")
[Out] (B*c^3*m^4*x^15*x^m + 40*B*c^3*m^3*x^15*x^m + 3*B*b*c^2*m^4*x^13*x^m + A*c^
3*m^4*x^13*x^m + 590*B*c^3*m^2*x^15*x^m + 126*B*b*c^2*m^3*x^13*x^m + 42*A*c
^3*m^3*x^13*x^m + 3800*B*c^3*m*x^15*x^m + 3*B*b^2*c*m^4*x^11*x^m + 3*A*b*c^
2*m^4*x^11*x^m + 1932*B*b*c^2*m^2*x^13*x^m + 644*A*c^3*m^2*x^13*x^m + 9009*
B*c^3*x^15*x^m + 132*B*b^2*c*m^3*x^11*x^m + 132*A*b*c^2*m^3*x^11*x^m + 1283
4*B*b*c^2*m*x^13*x^m + 4278*A*c^3*m*x^13*x^m + B*b^3*m^4*x^9*x^m + 3*A*b^2*
c*m^4*x^9*x^m + 2118*B*b^2*c*m^2*x^11*x^m + 2118*A*b*c^2*m^2*x^11*x^m + 311
85*B*b*c^2*x^13*x^m + 10395*A*c^3*x^13*x^m + 46*B*b^3*m^3*x^9*x^m + 138*A*b
^2*c*m^3*x^9*x^m + 14652*B*b^2*c*m*x^11*x^m + 14652*A*b*c^2*m*x^11*x^m + A*
b^3*m^4*x^7*x^m + 776*B*b^3*m^2*x^9*x^m + 2328*A*b^2*c*m^2*x^9*x^m + 36855*
B*b^2*c*x^11*x^m + 36855*A*b*c^2*x^11*x^m + 48*A*b^3*m^3*x^7*x^m + 5666*B*b
^3*m*x^9*x^m + 16998*A*b^2*c*m*x^9*x^m + 854*A*b^3*m^2*x^7*x^m + 15015*B*b^
3*x^9*x^m + 45045*A*b^2*c*x^9*x^m + 6672*A*b^3*m*x^7*x^m + 19305*A*b^3*x^7*
x^m)/(m^5 + 55*m^4 + 1190*m^3 + 12650*m^2 + 66009*m + 135135)
```

Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.03

$$\begin{aligned}\int x^m(A+Bx^2)\left(bx^2+cx^4\right)^3\,dx &= \frac{A\,b^3\,x^m\,x^7\,(m^4+48\,m^3+854\,m^2+6672\,m+19305)}{m^5+55\,m^4+1190\,m^3+12650\,m^2+66009\,m+135135}\\ &+\frac{B\,c^3\,x^m\,x^{15}\,(m^4+40\,m^3+590\,m^2+3800\,m+9009)}{m^5+55\,m^4+1190\,m^3+12650\,m^2+66009\,m+135135}\\ &+\frac{b^2\,x^m\,x^9\,(3\,A\,c+B\,b)\,(m^4+46\,m^3+776\,m^2+5666\,m+15015)}{m^5+55\,m^4+1190\,m^3+12650\,m^2+66009\,m+135135}\\ &+\frac{c^2\,x^m\,x^{13}\,(A\,c+3\,B\,b)\,(m^4+42\,m^3+644\,m^2+4278\,m+10395)}{m^5+55\,m^4+1190\,m^3+12650\,m^2+66009\,m+135135}\\ &+\frac{3\,b\,c\,x^m\,x^{11}\,(A\,c+B\,b)\,(m^4+44\,m^3+706\,m^2+4884\,m+12285)}{m^5+55\,m^4+1190\,m^3+12650\,m^2+66009\,m+135135}\end{aligned}$$

```
[In] int(x^m*(A + B*x^2)*(b*x^2 + c*x^4)^3,x)
```

```
[Out] (A*b^3*x^m*x^7*(6672*m + 854*m^2 + 48*m^3 + m^4 + 19305))/(66009*m + 12650*
m^2 + 1190*m^3 + 55*m^4 + m^5 + 135135) + (B*c^3*x^m*x^15*(3800*m + 590*m^2
 + 40*m^3 + m^4 + 9009))/(66009*m + 12650*m^2 + 1190*m^3 + 55*m^4 + m^5 + 1
35135) + (b^2*x^m*x^9*(3*A*c + B*b)*(5666*m + 776*m^2 + 46*m^3 + m^4 + 1501
5))/(66009*m + 12650*m^2 + 1190*m^3 + 55*m^4 + m^5 + 135135) + (c^2*x^m*x^1
3*(A*c + 3*B*b)*(4278*m + 644*m^2 + 42*m^3 + m^4 + 10395))/(66009*m + 12650
*m^2 + 1190*m^3 + 55*m^4 + m^5 + 135135) + (3*b*c*x^m*x^11*(A*c + B*b)*(488
4*m + 706*m^2 + 44*m^3 + m^4 + 12285))/(66009*m + 12650*m^2 + 1190*m^3 + 55
*m^4 + m^5 + 135135)
```

3.270 $\int x^m(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 71

$$\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx=\frac{Ab^2x^{5+m}}{5+m}+\frac{b(bB+2Ac)x^{7+m}}{7+m}+\frac{c(2bB+Ac)x^{9+m}}{9+m}+\frac{Bc^2x^{11+m}}{11+m}$$

```
[Out] A*b^2*x^(5+m)/(5+m)+b*(2*A*c+B*b)*x^(7+m)/(7+m)+c*(A*c+2*B*b)*x^(9+m)/(9+m)
+B*c^2*x^(11+m)/(11+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1598, 459}

$$\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx=\frac{Ab^2x^{m+5}}{m+5}+\frac{bx^{m+7}(2Ac+bB)}{m+7}+\frac{cx^{m+9}(Ac+2bB)}{m+9}+\frac{Bc^2x^{m+11}}{m+11}$$

```
[In] Int[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (A*b^2*x^(5 + m))/(5 + m) + (b*(b*B + 2*A*c)*x^(7 + m))/(7 + m) + (c*(2*b*B
 + A*c)*x^(9 + m))/(9 + m) + (B*c^2*x^(11 + m))/(11 + m)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^{4+m}\left(A+Bx^2\right)\left(b+cx^2\right)^2\,dx \\
&= \int \left(Ab^2x^{4+m}+b(bB+2Ac)x^{6+m}+c(2bB+Ac)x^{8+m}+Bc^2x^{10+m}\right)dx \\
&= \frac{Ab^2x^{5+m}}{5+m}+\frac{b(bB+2Ac)x^{7+m}}{7+m}+\frac{c(2bB+Ac)x^{9+m}}{9+m}+\frac{Bc^2x^{11+m}}{11+m}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)^2\,dx = x^{5+m}\left(\frac{Ab^2}{5+m}+\frac{b(bB+2Ac)x^2}{7+m}+\frac{c(2bB+Ac)x^4}{9+m}+\frac{Bc^2x^6}{11+m}\right)$$

```
[In] Integrate[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^2,x]
[Out] x^(5 + m)*((A*b^2)/(5 + m) + (b*(b*B + 2*A*c)*x^2)/(7 + m) + (c*(2*b*B + A*
c)*x^4)/(9 + m) + (B*c^2*x^6)/(11 + m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(71) = 142.

Time = 2.11 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.69

method	result
gosper	$\frac{x^{5+m}\left(Bc^2m^3x^6+21Bc^2m^2x^6+Ac^2m^3x^4+2Bbcm^3x^4+143mx^6Bc^2+23Ac^2m^2x^4+46Bbcm^2x^4+315Bc^2x^6+2Abcm^3x^2+1\right.}{}$
risch	$\frac{x^m\left(Bc^2m^3x^6+21Bc^2m^2x^6+Ac^2m^3x^4+2Bbcm^3x^4+143mx^6Bc^2+23Ac^2m^2x^4+46Bbcm^2x^4+315Bc^2x^6+2Abcm^3x^2+167\right.}{}$
parallelrisch	$\frac{2Bx^9x^mbcm^3+46Bx^9x^mbcm^2+25Bx^7x^mb^2m^2+Ax^5x^mb^2m^3+199Bx^7x^mb^2m+990Ax^7x^mbc+27Ax^5x^mb^2m^2+239Ax^5x}{}$

```
[In] int(x^m*(B*x^2+A)*(c*x^4+b*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x^(5+m)/(5+m)/(7+m)/(9+m)/(11+m)*(B*c^2*m^3*x^6+21*B*c^2*m^2*x^6+A*c^2*m^3*
x^4+2*B*b*c*m^3*x^4+143*B*c^2*m*x^6+23*A*c^2*m^2*x^4+46*B*b*c*m^2*x^4+315*B
*c^2*x^6+2*A*b*c*m^3*x^2+167*A*c^2*m*x^4+B*b^2*m^3*x^2+334*B*b*c*m*x^4+50*A
*b*c*m^2*x^2+385*A*c^2*x^4+25*B*b^2*m^2*x^2+770*B*b*c*x^4+A*b^2*m^3+398*A*b
*c*m*x^2+199*B*b^2*m*x^2+27*A*b^2*m^2+990*A*b*c*x^2+495*B*b^2*x^2+239*A*b^2
*m+693*A*b^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(71) = 142$.

Time = 0.31 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.06

$$\int x^m(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx$$
$$= \frac{((Bc^2m^3+21\,Bc^2m^2+143\,Bc^2m+315\,Bc^2)x^{11}+((2\,Bbc+Ac^2)m^3+770\,Bbc+385\,Ac^2+23\,(2\,Bbc+$$

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] ((B*c^2*m^3 + 21*B*c^2*m^2 + 143*B*c^2*m + 315*B*c^2)*x^11 + ((2*B*b*c + A*
c^2)*m^3 + 770*B*b*c + 385*A*c^2 + 23*(2*B*b*c + A*c^2)*m^2 + 167*(2*B*b*c
+ A*c^2)*m)*x^9 + ((B*b^2 + 2*A*b*c)*m^3 + 495*B*b^2 + 990*A*b*c + 25*(B*b^
2 + 2*A*b*c)*m^2 + 199*(B*b^2 + 2*A*b*c)*m)*x^7 + (A*b^2*m^3 + 27*A*b^2*m^2
 + 239*A*b^2*m + 693*A*b^2)*x^5)*x^m/(m^4 + 32*m^3 + 374*m^2 + 1888*m + 346
5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1051 vs. $2(63) = 126$.

Time = 0.67 (sec) , antiderivative size = 1051, normalized size of antiderivative = 14.80

$$\int x^m(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx$$
$$= \begin{cases} -\frac{Ab^2}{6x^6}-\frac{Abc}{2x^4}-\frac{Ac^2}{2x^2}-\frac{Bb^2}{4x^4}-\frac{Bbc}{x^2}+Bc^2\log{(x)} \\ -\frac{Ab^2}{4x^4}-\frac{Abc}{x^2}+Ac^2\log{(x)}-\frac{Bb^2}{2x^2}+2Bbc\log{(x)}+\frac{Bc^2x^2}{2} \\ -\frac{Ab^2}{2x^2}+2Abc\log{(x)}+\frac{Ac^2x^2}{2}+Bb^2\log{(x)}+Bbcx^2+\frac{Bc^2x^4}{4} \\ Ab^2\log{(x)}+Abcx^2+\frac{Ac^2x^4}{4}+\frac{Bb^2x^2}{2}+\frac{Bbcx^4}{2}+\frac{Bc^2x^6}{6} \\ \frac{Ab^2m^3x^5x^m}{m^4+32m^3+374m^2+1888m+3465}+\frac{27Ab^2m^2x^5x^m}{m^4+32m^3+374m^2+1888m+3465}+\frac{239Ab^2mx^5x^m}{m^4+32m^3+374m^2+1888m+3465}+\frac{693Ab^2x^5x^m}{m^4+32m^3+374m^2+188} \end{cases}$$

```
[In] integrate(x**m*(B*x**2+A)*(c*x**4+b*x**2)**2,x)
```

```
[Out] Piecewise((-A*b**2/(6*x**6) - A*b*c/(2*x**4) - A*c**2/(2*x**2) - B*b**2/(4*
x**4) - B*b*c/x**2 + B*c**2*log(x), Eq(m, -11)), (-A*b**2/(4*x**4) - A*b*c/
x**2 + A*c**2*log(x) - B*b**2/(2*x**2) + 2*B*b*c*log(x) + B*c**2*x**2/2, Eq
(m, -9)), (-A*b**2/(2*x**2) + 2*A*b*c*log(x) + A*c**2*x**2/2 + B*b**2*log(x
) + B*b*c*x**2 + B*c**2*x**4/4, Eq(m, -7)), (A*b**2*log(x) + A*b*c*x**2 + A
*c**2*x**4/4 + B*b**2*x**2/2 + B*b*c*x**4/2 + B*c**2*x**6/6, Eq(m, -5)), (A
*b**2*m**3*x**5*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 27*A*b**
2*m**2*x**5*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 239*A*b**2*m
*x**5*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 693*A*b**2*x**5*x*
*m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 2*A*b*c*m**3*x**7*x**m/(m*
*4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 50*A*b*c*m**2*x**7*x**m/(m**4 +
32*m**3 + 374*m**2 + 1888*m + 3465) + 398*A*b*c*m*x**7*x**m/(m**4 + 32*m**3
 + 374*m**2 + 1888*m + 3465) + 990*A*b*c*x**7*x**m/(m**4 + 32*m**3 + 374*m*
*2 + 1888*m + 3465) + A*c**2*m**3*x**9*x**m/(m**4 + 32*m**3 + 374*m**2 + 18
88*m + 3465) + 23*A*c**2*m**2*x**9*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m
 + 3465) + 167*A*c**2*m*x**9*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 346
5) + 385*A*c**2*x**9*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + B*b
**2*m**3*x**7*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 25*B*b**2*
m**2*x**7*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 199*B*b**2*m*x
**7*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 495*B*b**2*x**7*x**m
/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 2*B*b*c*m**3*x**9*x**m/(m**4
 + 32*m**3 + 374*m**2 + 1888*m + 3465) + 46*B*b*c*m**2*x**9*x**m/(m**4 + 32
*m**3 + 374*m**2 + 1888*m + 3465) + 334*B*b*c*m*x**9*x**m/(m**4 + 32*m**3 +
 374*m**2 + 1888*m + 3465) + 770*B*b*c*x**9*x**m/(m**4 + 32*m**3 + 374*m**2
 + 1888*m + 3465) + B*c**2*m**3*x**11*x**m/(m**4 + 32*m**3 + 374*m**2 + 188
8*m + 3465) + 21*B*c**2*m**2*x**11*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m
 + 3465) + 143*B*c**2*m*x**11*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 34
65) + 315*B*c**2*x**11*x**m/(m**4 + 32*m**3 + 374*m**2 + 1888*m + 3465), Tr
ue))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.28

$$\int x^m \left(A+Bx^2\right) \left(bx^2+cx^4\right)^2 \, dx = \frac{Bc^2x^{m+11}}{m+11} + \frac{2\,Bbcx^{m+9}}{m+9} + \frac{Ac^2x^{m+9}}{m+9} + \frac{Bb^2x^{m+7}}{m+7} + \frac{2\,Abcx^{m+7}}{m+7} + \frac{Ab^2x^{m+5}}{m+5}$$

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] B*c^2*x^(m + 11)/(m + 11) + 2*B*b*c*x^(m + 9)/(m + 9) + A*c^2*x^(m + 9)/(m
+ 9) + B*b^2*x^(m + 7)/(m + 7) + 2*A*b*c*x^(m + 7)/(m + 7) + A*b^2*x^(m + 5
)/(m + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(71) = 142$.

Time = 0.27 (sec) , antiderivative size = 340, normalized size of antiderivative = 4.79

$$\int x^m(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx = \frac{Bc^2m^3x^{11}x^m + 21\,Bc^2m^2x^{11}x^m + 2\,Bbcm^3x^9x^m + Ac^2m^3x^9x^m + 143\,Bc^2mx^{11}x^m + 46\,Bbcm^2x^9x^m + 23}{}$$

[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^2,x, algorithm="giac")

[Out] (B*c^2*m^3*x^11*x^m + 21*B*c^2*m^2*x^11*x^m + 2*B*b*c*m^3*x^9*x^m + A*c^2*m^3*x^9*x^m + 143*B*c^2*m*x^11*x^m + 46*B*b*c*m^2*x^9*x^m + 23*A*c^2*m^2*x^9*x^m + 315*B*c^2*x^11*x^m + B*b^2*m^3*x^7*x^m + 2*A*b*c*m^3*x^7*x^m + 334*B*b*c*m*x^9*x^m + 167*A*c^2*m*x^9*x^m + 25*B*b^2*m^2*x^7*x^m + 50*A*b*c*m^2*x^7*x^m + 770*B*b*c*x^9*x^m + 385*A*c^2*x^9*x^m + A*b^2*m^3*x^5*x^m + 199*B*b^2*m*x^7*x^m + 398*A*b*c*m*x^7*x^m + 27*A*b^2*m^2*x^5*x^m + 495*B*b^2*x^7*x^m + 990*A*b*c*x^7*x^m + 239*A*b^2*m*x^5*x^m + 693*A*b^2*x^5*x^m)/(m^4 + 32*m^3 + 374*m^2 + 1888*m + 3465)

Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.52

$$\int x^m(A+Bx^2)\left(bx^2+cx^4\right)^2\,dx = x^m\left(\frac{A\,b^2\,x^5\,(m^3+27\,m^2+239\,m+693)}{m^4+32\,m^3+374\,m^2+1888\,m+3465} + \frac{B\,c^2\,x^{11}\,(m^3+21\,m^2+143\,m+315)}{m^4+32\,m^3+374\,m^2+1888\,m+3465} + \frac{b\,x^7\,(2\,A\,c+B\,b)\,(m^3+25\,m^2+199\,m+495)}{m^4+32\,m^3+374\,m^2+1888\,m+3465} + \frac{c\,x^9\,(A\,c+2\,B\,b)\,(m^3+23\,m^2+167\,m+385)}{m^4+32\,m^3+374\,m^2+1888\,m+3465}\right)$$

[In] int(x^m*(A + B*x^2)*(b*x^2 + c*x^4)^2,x)

[Out] x^m*((A*b^2*x^5*(239*m + 27*m^2 + m^3 + 693))/(1888*m + 374*m^2 + 32*m^3 + m^4 + 3465) + (B*c^2*x^11*(143*m + 21*m^2 + m^3 + 315))/(1888*m + 374*m^2 + 32*m^3 + m^4 + 3465) + (b*x^7*(2*A*c + B*b)*(199*m + 25*m^2 + m^3 + 495))/(1888*m + 374*m^2 + 32*m^3 + m^4 + 3465) + (c*x^9*(A*c + 2*B*b)*(167*m + 23*m^2 + m^3 + 385))/(1888*m + 374*m^2 + 32*m^3 + m^4 + 3465))

3.271 $\int x^m(A+Bx^2)\left(bx^2+cx^4\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 45

$$\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx=\frac{Abx^{3+m}}{3+m}+\frac{(bB+Ac)x^{5+m}}{5+m}+\frac{Bcx^{7+m}}{7+m}$$

```
[Out] A*b*x^(3+m)/(3+m)+(A*c+B*b)*x^(5+m)/(5+m)+B*c*x^(7+m)/(7+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1598, 459}

$$\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx=\frac{x^{m+5}(Ac+bB)}{m+5}+\frac{Abx^{m+3}}{m+3}+\frac{Bcx^{m+7}}{m+7}$$

```
[In] Int[x^m*(A + B*x^2)*(b*x^2 + c*x^4),x]
```

```
[Out] (A*b*x^(3 + m))/(3 + m) + ((b*B + A*c)*x^(5 + m))/(5 + m) + (B*c*x^(7 + m))
/(7 + m)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^{2+m}\left(A+Bx^2\right)\left(b+cx^2\right)\,dx \\
&= \int \left(Abx^{2+m}+(bB+Ac)x^{4+m}+Bcx^{6+m}\right)\,dx \\
&= \frac{Abx^{3+m}}{3+m}+\frac{(bB+Ac)x^{5+m}}{5+m}+\frac{Bcx^{7+m}}{7+m}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)\,dx = x^{3+m}\left(\frac{Ab}{3+m}+\frac{(bB+Ac)x^2}{5+m}+\frac{Bcx^4}{7+m}\right)$$

```
[In] Integrate[x^m*(A + B*x^2)*(b*x^2 + c*x^4),x]
[Out] x^(3 + m)*((A*b)/(3 + m) + ((b*B + A*c)*x^2)/(5 + m) + (B*c*x^4)/(7 + m))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

method	result
norman	$\frac{(Ac+Bb)x^5\mathrm{e}^{m\ln(x)}}{5+m}+\frac{Ab\,x^3\mathrm{e}^{m\ln(x)}}{3+m}+\frac{Bc\,x^7\mathrm{e}^{m\ln(x)}}{7+m}$
gosper	$\frac{x^{3+m}\big(Bc\,m^2x^4+8Bcm\,x^4+Ac\,m^2x^2+Bb\,m^2x^2+15Bc\,x^4+10Acm\,x^2+10Bbm\,x^2+Ab\,m^2+21Ac\,x^2+21bB\,x^2+12Abm+35A}{(3+m)(5+m)(7+m)}$
risch	$\frac{x^m\left(Bc\,m^2x^4+8Bcm\,x^4+Ac\,m^2x^2+Bb\,m^2x^2+15Bc\,x^4+10Acm\,x^2+10Bbm\,x^2+Ab\,m^2+21Ac\,x^2+21bB\,x^2+12Abm+35Ab\right)x}{(7+m)(5+m)(3+m)}$
parallelrisch	$\frac{B\,x^7x^mc\,m^2+8B\,x^7x^mcm+A\,x^5x^mc\,m^2+15B\,x^7x^mc+B\,x^5x^mb\,m^2+10A\,x^5x^mcm+10B\,x^5x^mbm+21A\,x^5x^mc+A\,x^3x^mb\,m}{(7+m)(5+m)(3+m)}$

```
[In] int(x^m*(B*x^2+A)*(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] (A*c+B*b)/(5+m)*x^5*exp(m*ln(x))+A*b/(3+m)*x^3*exp(m*ln(x))+B*c/(7+m)*x^7*e
xp(m*ln(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(45) = 90$.

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.09

$$\int x^m (A + Bx^2) (bx^2 + cx^4) \, dx = \frac{((Bcm^2 + 8 Bcm + 15 Bc)x^7 + ((Bb + Ac)m^2 + 21 Bb + 21 Ac + 10 (Bb + Ac)m)x^5 + (Abm^2 + 12 Abm +}{m^3 + 15 m^2 + 71 m + 105}$$

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] ((B*c*m^2 + 8*B*c*m + 15*B*c)*x^7 + ((B*b + A*c)*m^2 + 21*B*b + 21*A*c + 10
*(B*b + A*c)*m)*x^5 + (A*b*m^2 + 12*A*b*m + 35*A*b)*x^3)*x^m/(m^3 + 15*m^2
+ 71*m + 105)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(37) = 74$.

Time = 0.40 (sec) , antiderivative size = 415, normalized size of antiderivative = 9.22

$$\int x^m (A + Bx^2) (bx^2 + cx^4) \, dx = \begin{cases} -\frac{Ab}{4x^4} - \frac{Ac}{2x^2} - \frac{Bb}{2x^2} + Bc \log(x) \\ -\frac{Ab}{2x^2} + Ac \log(x) + Bb \log(x) + \frac{Bcx^2}{2} \\ Ab \log(x) + \frac{Acx^2}{2} + \frac{Bbx^2}{2} + \frac{Bcx^4}{4} \\ \frac{Abm^2x^3x^m}{m^3+15m^2+71m+105} + \frac{12Abmx^3x^m}{m^3+15m^2+71m+105} + \frac{35Abx^3x^m}{m^3+15m^2+71m+105} + \frac{Acm^2x^5x^m}{m^3+15m^2+71m+105} + \frac{10Acmx^5x^m}{m^3+15m^2+71m+105} + \frac{21A}{m^3+15m} \end{cases}$$

```
[In] integrate(x**m*(B*x**2+A)*(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((-A*b/(4*x**4) - A*c/(2*x**2) - B*b/(2*x**2) + B*c*log(x), Eq(m,
-7)), (-A*b/(2*x**2) + A*c*log(x) + B*b*log(x) + B*c*x**2/2, Eq(m, -5)), (A
*b*log(x) + A*c*x**2/2 + B*b*x**2/2 + B*c*x**4/4, Eq(m, -3)), (A*b*m**2*x**
3*x**m/(m**3 + 15*m**2 + 71*m + 105) + 12*A*b*m*x**3*x**m/(m**3 + 15*m**2 +
 71*m + 105) + 35*A*b*x**3*x**m/(m**3 + 15*m**2 + 71*m + 105) + A*c*m**2*x*
*5*x**m/(m**3 + 15*m**2 + 71*m + 105) + 10*A*c*m*x**5*x**m/(m**3 + 15*m**2
+ 71*m + 105) + 21*A*c*x**5*x**m/(m**3 + 15*m**2 + 71*m + 105) + B*b*m**2*x
**5*x**m/(m**3 + 15*m**2 + 71*m + 105) + 10*B*b*m*x**5*x**m/(m**3 + 15*m**2
 + 71*m + 105) + 21*B*b*x**5*x**m/(m**3 + 15*m**2 + 71*m + 105) + B*c*m**2*
x**7*x**m/(m**3 + 15*m**2 + 71*m + 105) + 8*B*c*m*x**7*x**m/(m**3 + 15*m**2
 + 71*m + 105) + 15*B*c*x**7*x**m/(m**3 + 15*m**2 + 71*m + 105), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int x^m (A+Bx^2) \left(bx^2+cx^4\right) \, dx = \frac{Bcx^{m+7}}{m+7} + \frac{Bbx^{m+5}}{m+5} + \frac{Acx^{m+5}}{m+5} + \frac{Abx^{m+3}}{m+3}$$

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] B*c*x^(m + 7)/(m + 7) + B*b*x^(m + 5)/(m + 5) + A*c*x^(m + 5)/(m + 5) + A*b
*x^(m + 3)/(m + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(45) = 90$.

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 3.31

$$\int x^m (A+Bx^2) \left(bx^2+cx^4\right) \, dx$$
$$= \frac{Bcm^2x^7x^m + 8\,Bcmx^7x^m + Bbm^2x^5x^m + Acm^2x^5x^m + 15\,Bcx^7x^m + 10\,Bbmx^5x^m + 10\,Acmx^5x^m + Ab}{m^3+15\,m^2+71\,m+105}$$

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] (B*c*m^2*x^7*x^m + 8*B*c*m*x^7*x^m + B*b*m^2*x^5*x^m + A*c*m^2*x^5*x^m + 15
*B*c*x^7*x^m + 10*B*b*m*x^5*x^m + 10*A*c*m*x^5*x^m + A*b*m^2*x^3*x^m + 21*B
*b*x^5*x^m + 21*A*c*x^5*x^m + 12*A*b*m*x^3*x^m + 35*A*b*x^3*x^m)/(m^3 + 15*
m^2 + 71*m + 105)
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.16

$$\int x^m (A+Bx^2) \left(bx^2+cx^4\right) \, dx = x^m \left(\frac{x^5\,(A\,c+B\,b)\,\left(m^2+10\,m+21\right)}{m^3+15\,m^2+71\,m+105} + \frac{A\,b\,x^3\,\left(m^2+12\,m+35\right)}{m^3+15\,m^2+71\,m+105} + \frac{B\,c\,x^7\,\left(m^2+8\,m+15\right)}{m^3+15\,m^2+71\,m+105}\right)$$

```
[In] int(x^m*(A + B*x^2)*(b*x^2 + c*x^4),x)
```

```
[Out] x^m*((x^5*(A*c + B*b)*(10*m + m^2 + 21))/(71*m + 15*m^2 + m^3 + 105) + (A*b
*x^3*(12*m + m^2 + 35))/(71*m + 15*m^2 + m^3 + 105) + (B*c*x^7*(8*m + m^2 +
 15))/(71*m + 15*m^2 + m^3 + 105))
```

3.272 $\int \frac{x^m(A+Bx^2)}{bx^2+cx^4} dx$

Optimal result

Integrand size = 24, antiderivative size = 71

$$\int \frac{x^m(A+Bx^2)}{bx^2+cx^4} dx = -\frac{Bx^{-1+m}}{c(1-m)} + \frac{(bB-Ac)x^{-1+m} \text{Hypergeometric2F1}\left(1, \frac{1}{2}(-1+m), \frac{1+m}{2}, -\frac{cx^2}{b}\right)}{bc(1-m)}$$

```
[Out] -B*x^(-1+m)/c/(1-m)+(-A*c+B*b)*x^(-1+m)*hypergeom([1, -1/2+1/2*m],[1/2+1/2*
m],-c*x^2/b)/b/c/(1-m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 470, 371}

$$\int \frac{x^m(A+Bx^2)}{bx^2+cx^4} dx = \frac{x^{m-1}(bB-Ac) \text{Hypergeometric2F1}\left(1, \frac{m-1}{2}, \frac{m+1}{2}, -\frac{cx^2}{b}\right)}{bc(1-m)} - \frac{Bx^{m-1}}{c(1-m)}$$

```
[In] Int[(x^m*(A + B*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] -((B*x^(-1 + m))/(c*(1 - m))) + ((b*B - A*c)*x^(-1 + m)*Hypergeometric2F1[1
, (-1 + m)/2, (1 + m)/2, -((c*x^2)/b)])/(b*c*(1 - m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^{-2+m}(A+Bx^2)}{b+cx^2}\,dx\\ &= -\frac{Bx^{-1+m}}{c(1-m)} - \frac{(bB(-1+m)-Ac(-1+m))\int \frac{x^{-2+m}}{b+cx^2}\,dx}{c(-1+m)}\\ &= -\frac{Bx^{-1+m}}{c(1-m)} + \frac{(bB-Ac)x^{-1+m}\,{}_2F_1\left(1,\frac{1}{2}(-1+m);\frac{1+m}{2};-\frac{cx^2}{b}\right)}{bc(1-m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \frac{x^m(A+Bx^2)}{bx^2+cx^4}\,dx = \frac{x^{-1+m}\left(bB+(-bB+Ac)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-1+m),\frac{1+m}{2},-\frac{cx^2}{b}\right)\right)}{bc(-1+m)}$$

```
[In] Integrate[(x^m*(A + B*x^2))/(b*x^2 + c*x^4),x]
[Out] (x^(-1 + m)*(b*B + (-(b*B) + A*c)*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/
2, -((c*x^2)/b)]))/(b*c*(-1 + m))
```

Maple [F]

$$\int \frac{x^m (x^2 B + A)}{x^4 c + b\,x^2} dx$$

[In] int(x^m*(B*x^2+A)/(c*x^4+b*x^2),x)

[Out] int(x^m*(B*x^2+A)/(c*x^4+b*x^2),x)

Fricas [F]

$$\int \frac{x^m (A + Bx^2)}{bx^2 + cx^4}\, dx = \int \frac{(Bx^2 + A)x^m}{cx^4 + bx^2}\, dx$$

[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="fricas")

[Out] integral((B*x^2 + A)*x^m/(c*x^4 + b*x^2), x)

Sympy [F]

$$\int \frac{x^m (A + Bx^2)}{bx^2 + cx^4}\, dx = \int \frac{x^m (A + Bx^2)}{x^2 (b + cx^2)}\, dx$$

[In] integrate(x**m*(B*x**2+A)/(c*x**4+b*x**2),x)

[Out] Integral(x**m*(A + B*x**2)/(x**2*(b + c*x**2)), x)

Maxima [F]

$$\int \frac{x^m (A + Bx^2)}{bx^2 + cx^4}\, dx = \int \frac{(Bx^2 + A)x^m}{cx^4 + bx^2}\, dx$$

[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="maxima")

[Out] integrate((B*x^2 + A)*x^m/(c*x^4 + b*x^2), x)

Giac [F]

$$\int \frac{x^m(A+Bx^2)}{bx^2+cx^4}\,dx = \int \frac{(Bx^2+A)x^m}{cx^4+bx^2}\,dx$$

```
[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^m/(c*x^4 + b*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(A+Bx^2)}{bx^2+cx^4}\,dx = \int \frac{x^m\,(B\,x^2+A)}{c\,x^4+b\,x^2}\,dx$$

```
[In] int((x^m*(A + B*x^2))/(b*x^2 + c*x^4),x)
[Out] int((x^m*(A + B*x^2))/(b*x^2 + c*x^4), x)
```

3.273 $\int \frac{x^m(A+Bx^2)}{(bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 98

$$\int \frac{x^m(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = -\frac{(bB-Ac)x^{-3+m}}{2bc\,(b+cx^2)} + \frac{(bB(3-m)-Ac(5-m))x^{-3+m}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),-\frac{cx^2}{b}\right)}{2b^2c(3-m)}$$

```
[Out] -1/2*(-A*c+B*b)*x^(-3+m)/b/c/(c*x^2+b)+1/2*(b*B*(3-m)-A*c*(5-m))*x^(-3+m)*h
ypergeom([1, -3/2+1/2*m],[-1/2+1/2*m],-c*x^2/b)/b^2/c/(3-m)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1598, 468, 371}

$$\int \frac{x^m(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{x^{m-3}\left(\frac{bB}{c}-\frac{A(5-m)}{3-m}\right)\text{Hypergeometric2F1}\left(1,\frac{m-3}{2},\frac{m-1}{2},-\frac{cx^2}{b}\right)}{2b^2} - \frac{x^{m-3}(bB-Ac)}{2bc\,(b+cx^2)}$$

```
[In] Int[(x^m*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*((b*B - A*c)*x^(-3 + m))/(b*c*(b + c*x^2)) + (((b*B)/c - (A*(5 - m))/(
3 - m))*x^(-3 + m)*Hypergeometric2F1[1, (-3 + m)/2, (-1 + m)/2, -((c*x^2)/b
)])/(2*b^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{x^{-4+m}(A+Bx^2)}{(b+cx^2)^2}\,dx$$

$$= -\frac{(bB-Ac)x^{-3+m}}{2bc\,(b+cx^2)} + \frac{(-Ac(-5+m)+bB(-3+m))\int \frac{x^{-4+m}}{b+cx^2}\,dx}{2bc}$$

$$= -\frac{(bB-Ac)x^{-3+m}}{2bc\,(b+cx^2)} + \frac{\left(\frac{bB}{c}-\frac{A(5-m)}{3-m}\right)x^{-3+m}\,{}_2F_1\left(1,\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);-\frac{cx^2}{b}\right)}{2b^2}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{x^m(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \frac{x^{-3+m}\left(bB\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),-\frac{cx^2}{b}\right)+(-bB+Ac)\,\text{Hypergeometric2F1}\left(2\right.\right.}{b^2c(-3+m)}$$

```
[In] Integrate[(x^m*(A + B*x^2))/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^(-3 + m)*(b*B*Hypergeometric2F1[1, (-3 + m)/2, (-1 + m)/2, -((c*x^2)/b)]
 + (-(b*B) + A*c)*Hypergeometric2F1[2, (-3 + m)/2, (-1 + m)/2, -((c*x^2)/b)
]))/(b^2*c*(-3 + m))
```

Maple [F]

$$\int \frac{x^{m}\left(x^{2} B+A\right)}{\left(x^{4} c+b\,x^{2}\right)^{2}}d x$$

```
[In] int(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

```
[Out] int(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x)
```

Fricas [F]

$$\int \frac{x^m(A+Bx^2)}{(bx^2+cx^4)^2}\,dx=\int \frac{(Bx^2+A)x^m}{(cx^4+bx^2)^2}\,dx$$

```
[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)*x^m/(c^2*x^8 + 2*b*c*x^6 + b^2*x^4), x)
```

Sympy [F]

$$\int \frac{x^m(A+Bx^2)}{(bx^2+cx^4)^2}\,dx=\int \frac{x^m(A+Bx^2)}{x^4\,(b+cx^2)^2}\,dx$$

```
[In] integrate(x**m*(B*x**2+A)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Integral(x**m*(A + B*x**2)/(x**4*(b + c*x**2)**2), x)
```

Maxima [F]

$$\int \frac{x^m(A+Bx^2)}{(bx^2+cx^4)^2}\,dx=\int \frac{(Bx^2+A)x^m}{(cx^4+bx^2)^2}\,dx$$

```
[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^m/(c*x^4 + b*x^2)^2, x)
```

Giac [F]

$$\int \frac{x^m(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \int \frac{(Bx^2+A)x^m}{(cx^4+bx^2)^2}\,dx$$

```
[In] integrate(x^m*(B*x^2+A)/(c*x^4+b*x^2)^2,x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^m/(c*x^4 + b*x^2)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(A+Bx^2)}{(bx^2+cx^4)^2}\,dx = \int \frac{x^m\,(B\,x^2+A)}{(c\,x^4+b\,x^2)^2}\,dx$$

```
[In] int((x^m*(A + B*x^2))/(b*x^2 + c*x^4)^2,x)
[Out] int((x^m*(A + B*x^2))/(b*x^2 + c*x^4)^2, x)
```

3.274 $\int x^m(A+Bx^2)\left(bx^2+cx^4\right)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 140

$$\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)^p\,dx=\frac{Bx^{-1+m}\left(bx^2+cx^4\right)^{1+p}}{c(3+m+4p)}-\frac{(bB(1+m+2p)-Ac(3+m+4p))x^{1+m}\left(1+\frac{cx^2}{b}\right)^{-p}\left(bx^2+cx^4\right)^p\text{Hypergeometric2F1}\left(-p,\frac{1}{2}(1+m+\right.}{c(1+m+2p)(3+m+4p)}$$

```
[Out] B*x^(-1+m)*(c*x^4+b*x^2)^(p+1)/c/(3+m+4*p)-(b*B*(1+m+2*p)-A*c*(3+m+4*p))*x^
(1+m)*(c*x^4+b*x^2)^p*hypergeom([-p, 1/2+1/2*m+p],[3/2+1/2*m+p],-c*x^2/b)/c
/(1+m+2*p)/(3+m+4*p)/((1+c*x^2/b)^p)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2064, 2057, 372, 371}

$$\int x^m\left(A+Bx^2\right)\left(bx^2+cx^4\right)^p\,dx=x^{m+1}\left(\frac{cx^2}{b}+1\right)^{-p}\left(bx^2+cx^4\right)^p\left(\frac{A}{m+2p+1}-\frac{bB}{c(m+4p+3)}\right)\text{Hypergeometric2F1}\left(-p,\frac{1}{2}(m+2p+1),\frac{1}{2}(m+2p+3),-\frac{cx^2}{b}\right)+\frac{Bx^{m-1}\left(bx^2+cx^4\right)^{p+1}}{c(m+4p+3)}$$

```
[In] Int[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^p,x]
```

```
[Out] (B*x^(-1 + m)*(b*x^2 + c*x^4)^(1 + p))/(c*(3 + m + 4*p)) + ((A/(1 + m + 2*p
) - (b*B)/(c*(3 + m + 4*p)))*x^(1 + m)*(b*x^2 + c*x^4)^p*Hypergeometric2F1[
-p, (1 + m + 2*p)/2, (3 + m + 2*p)/2, -((c*x^2)/b)])/(1 + (c*x^2)/b)^p
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{Bx^{-1+m}(bx^2+cx^4)^{1+p}}{c(3+m+4p)} - \left(-A+\frac{bB(1+m+2p)}{c(3+m+4p)}\right)\int x^m\left(bx^2+cx^4\right)^p\,dx \\ &= \frac{Bx^{-1+m}(bx^2+cx^4)^{1+p}}{c(3+m+4p)} \\ &\quad -\left(\left(-A+\frac{bB(1+m+2p)}{c(3+m+4p)}\right)x^{-2p}\left(b+cx^2\right)^{-p}\left(bx^2+cx^4\right)^p\right)\int x^{m+2p}\left(b+cx^2\right)^p\,dx\end{aligned}$$

$$= \frac{Bx^{-1+m}(bx^2+cx^4)^{1+p}}{c(3+m+4p)} - \left(\left(-A+\frac{bB(1+m+2p)}{c(3+m+4p)}\right)x^{-2p}\left(1+\frac{cx^2}{b}\right)^{-p}(bx^2+cx^4)^p\right)\int x^{m+2p}\left(1+\frac{cx^2}{b}\right)^p dx$$

$$= \frac{Bx^{-1+m}(bx^2+cx^4)^{1+p}}{c(3+m+4p)} + \left(\frac{A}{1+m+2p}-\frac{bB}{c(3+m+4p)}\right)x^{1+m}\left(1+\frac{cx^2}{b}\right)^{-p}(bx^2+cx^4)^p\,{}_2F_1\left(-p,\frac{1}{2}(1+m+2p);\frac{1}{2}(3+m+2p);-\frac{cx^2}{b}\right)$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

$$\int x^m(A+Bx^2)(bx^2+cx^4)^p\,dx$$

$$= \frac{x^{1+m}(x^2(b+cx^2))^p\left(1+\frac{cx^2}{b}\right)^{-p}\left(A(3+m+2p)\,\text{Hypergeometric2F1}\left(-p,\frac{1}{2}(1+m+2p),\frac{1}{2}(3+m+2p),\right.\right.}{(1+m+2p)(3+m}$$

```
[In] Integrate[x^m*(A + B*x^2)*(b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^(1 + m)*(x^2*(b + c*x^2))^p*(A*(3 + m + 2*p)*Hypergeometric2F1[-p, (1 +
m + 2*p)/2, (3 + m + 2*p)/2, -((c*x^2)/b)] + B*(1 + m + 2*p)*x^2*Hypergeome
tric2F1[-p, (3 + m + 2*p)/2, (5 + m + 2*p)/2, -((c*x^2)/b)]))/((1 + m + 2*p
)*(3 + m + 2*p)*(1 + (c*x^2)/b)^p)
```

Maple [F]

$$\int x^m\left(x^2B+A\right)\left(x^4c+b\,x^2\right)^p dx$$

```
[In] int(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x)
```

```
[Out] int(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x)
```

Fricas [F]

$$\int x^m (A+Bx^2) (bx^2+cx^4)^p \, dx = \int (Bx^2+A)(cx^4+bx^2)^p x^m \, dx$$

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x, algorithm="fricas")
[Out] integral((B*x^2 + A)*(c*x^4 + b*x^2)^p*x^m, x)
```

Sympy [F]

$$\int x^m (A+Bx^2) (bx^2+cx^4)^p \, dx = \int x^m (x^2(b+cx^2))^p (A+Bx^2) \, dx$$

```
[In] integrate(x**m*(B*x**2+A)*(c*x**4+b*x**2)**p,x)
[Out] Integral(x**m*(x**2*(b + c*x**2))**p*(A + B*x**2), x)
```

Maxima [F]

$$\int x^m (A+Bx^2) (bx^2+cx^4)^p \, dx = \int (Bx^2+A)(cx^4+bx^2)^p x^m \, dx$$

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*(c*x^4 + b*x^2)^p*x^m, x)
```

Giac [F]

$$\int x^m (A+Bx^2) (bx^2+cx^4)^p \, dx = \int (Bx^2+A)(cx^4+bx^2)^p x^m \, dx$$

```
[In] integrate(x^m*(B*x^2+A)*(c*x^4+b*x^2)^p,x, algorithm="giac")
[Out] integrate((B*x^2 + A)*(c*x^4 + b*x^2)^p*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m (A + Bx^2) (bx^2 + cx^4)^p \, dx = \int x^m \left(B\,x^2 + A\right) \left(c\,x^4 + b\,x^2\right)^p \, dx$$

```
[In] int(x^m*(A + B*x^2)*(b*x^2 + c*x^4)^p,x)
[Out] int(x^m*(A + B*x^2)*(b*x^2 + c*x^4)^p, x)
```

3.275 $\int x^{-1+n-jp}(c+dx^n)\left(ax^j+bx^{j+n}\right)^p\,dx$

Optimal result

Integrand size = 32, antiderivative size = 95

$$\int x^{-1+n-jp}(c+dx^n)\left(ax^j+bx^{j+n}\right)^p\,dx=-\frac{(ad-bc(2+p))x^{-j(1+p)}\left(ax^j+bx^{j+n}\right)^{1+p}}{b^2n(1+p)(2+p)}+\frac{dx^{n-j(1+p)}\left(ax^j+bx^{j+n}\right)^{1+p}}{bn(2+p)}$$

```
[Out] -(a*d-b*c*(2+p))*(a*x^j+b*x^(j+n))^(p+1)/b^2/n/(p^2+3*p+2)/(x^(j*(p+1)))+d*
x^(n-j*(p+1))*(a*x^j+b*x^(j+n))^(p+1)/b/n/(2+p)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2064, 2039}

$$\int x^{-1+n-jp}(c+dx^n)\left(ax^j+bx^{j+n}\right)^p\,dx=\frac{dx^{n-j(p+1)}\left(ax^j+bx^{j+n}\right)^{p+1}}{bn(p+2)}-\frac{x^{-j(p+1)}(ad-bc(p+2))\left(ax^j+bx^{j+n}\right)^{p+1}}{b^2n(p+1)(p+2)}$$

```
[In] Int[x^(-1 + n - j*p)*(c + d*x^n)*(a*x^j + b*x^(j + n))^p,x]
```

```
[Out] -(((a*d - b*c*(2 + p))*(a*x^j + b*x^(j + n))^(1 + p))/(b^2*n*(1 + p)*(2 + p
)*x^(j*(1 + p)))) + (d*x^(n - j*(1 + p))*(a*x^j + b*x^(j + n))^(1 + p))/(b*
n*(2 + p))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2064

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Dist[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1)), Int[(e*x
)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x
] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p*
(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Rubi steps

$$\text{integral} = \frac{dx^{n-j(1+p)}\left(ax^j+bx^{j+n}\right)^{1+p}}{bn(2+p)} - \left(-c+\frac{ad}{b(2+p)}\right)\int x^{-1+n-jp}\left(ax^j+bx^{j+n}\right)^p\,dx$$
$$= \frac{\left(c-\frac{ad}{b(2+p)}\right)x^{-j(1+p)}\left(ax^j+bx^{j+n}\right)^{1+p}}{bn(1+p)} + \frac{dx^{n-j(1+p)}\left(ax^j+bx^{j+n}\right)^{1+p}}{bn(2+p)}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

$$\int x^{-1+n-jp}(c+dx^n)\left(ax^j+bx^{j+n}\right)^p\,dx = \frac{x^{-jp}(a+bx^n)\left(x^j(a+bx^n)\right)^p\left(-ad+bc(2+p)+bd(1+p)x^n\right)}{b^2n(1+p)(2+p)}$$

```
[In] Integrate[x^(-1 + n - j*p)*(c + d*x^n)*(a*x^j + b*x^(j + n))^p,x]
[Out] ((a + b*x^n)*(x^j*(a + b*x^n))^p*(-(a*d) + b*c*(2 + p) + b*d*(1 + p)*x^n))/
(b^2*n*(1 + p)*(2 + p)*x^(j*p))
```

Maple [F]

$$\int x^{-jp+n-1}(c+d\,x^n)\left(a\,x^j+b\,x^{j+n}\right)^p dx$$

```
[In] int(x^(-j*p+n-1)*(c+d*x^n)*(a*x^j+b*x^(j+n))^p,x)
```

```
[Out] int(x^(-j*p+n-1)*(c+d*x^n)*(a*x^j+b*x^(j+n))^p,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.47

$$\int x^{-1+n-jp}(c+dx^n)\left(ax^j+bx^{j+n}\right)^p\,dx = \frac{((b^2dp+b^2d)xx^{-jp+n-1}x^{2\,n}+(2\,b^2c+(b^2c+abd)p)xx^{-jp+n-1}x^n+(abcp+2\,abc-a^2d)xx^{-jp+n-1})\left(\frac{(bx^n+\cdots}{\cdots}\right.}{(b^2np^2+3\,b^2np+2\,b^2n)x^n}$$

```
[In] integrate(x^(-j*p+n-1)*(c+d*x^n)*(a*x^j+b*x^(j+n))^p,x, algorithm="fricas")
```

```
[Out] ((b^2*d*p + b^2*d)*x*x^(-j*p + n - 1)*x^(2*n) + (2*b^2*c + (b^2*c + a*b*d)*
p)*x*x^(-j*p + n - 1)*x^n + (a*b*c*p + 2*a*b*c - a^2*d)*x*x^(-j*p + n - 1))
*((b*x^n + a)*x^(j + n)/x^n)^p/((b^2*n*p^2 + 3*b^2*n*p + 2*b^2*n)*x^n)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1+n-jp}(c+dx^n)\left(ax^j+bx^{j+n}\right)^p\,dx = \text{Timed out}$$

```
[In] integrate(x**(-j*p+n-1)*(c+d*x**n)*(a*x**j+b*x**(j+n))**p,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.18

$$\int x^{-1+n-jp}(c+dx^n)\left(ax^j+bx^{j+n}\right)^p\,dx = \frac{(bx^n+a)ce^{\left(-jp\log(x)+p\log(bx^n+a)+p\log(x^j)\right)}}{bn(p+1)} + \frac{(b^2(p+1)x^{2\,n}+abpx^n-a^2)de^{\left(-jp\log(x)+p\log(bx^n+a)+p\log(x^j)\right)}}{(p^2+3\,p+2)b^2n}$$

```
[In] integrate(x^(-j*p+n-1)*(c+d*x^n)*(a*x^j+b*x^(j+n))^p,x, algorithm="maxima")
```

```
[Out] (b*x^n + a)*c*e^(-j*p*log(x) + p*log(b*x^n + a) + p*log(x^j))/(b*n*(p + 1))
 + (b^2*(p + 1)*x^(2*n) + a*b*p*x^n - a^2)*d*e^(-j*p*log(x) + p*log(b*x^n +
 a) + p*log(x^j))/((p^2 + 3*p + 2)*b^2*n)
```

Giac [F]

$$\int x^{-1+n-jp}(c+dx^n)\left(ax^j+bx^{j+n}\right)^p\,dx = \int (dx^n+c)\left(bx^{j+n}+ax^j\right)^p x^{-jp+n-1}\,dx$$

```
[In] integrate(x^(-j*p+n-1)*(c+d*x^n)*(a*x^j+b*x^(j+n))^p,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)*(b*x^(j + n) + a*x^j)^p*x^(-j*p + n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+n-jp}(c+dx^n)\left(ax^j+bx^{j+n}\right)^p\,dx = \int x^{n-j\,p-1}\,\left(a\,x^j+b\,x^{j+n}\right)^p\,\left(c+d\,x^n\right)\,dx$$

```
[In] int(x^(n - j*p - 1)*(a*x^j + b*x^(j + n))^p*(c + d*x^n),x)
```

```
[Out] int(x^(n - j*p - 1)*(a*x^j + b*x^(j + n))^p*(c + d*x^n), x)
```

3.276 $\int (ex)^m (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^p \, dx$

Optimal result

Integrand size = 30, antiderivative size = 113

$$\int (ex)^m (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^p \, dx$$
$$= \frac{x(ex)^m \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \left(ax^j+bx^{j+n}\right)^p \operatorname{AppellF1}\left(\frac{1+m+jp}{n},-p,-q,\frac{1+m+n+jp}{n},-\frac{bx^n}{a},-\frac{d}{\ }\right.}{1+m+jp}$$

```
[Out] x*(e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p*AppellF1((j*p+m+1)/n,-p,-q,(j*p+m
+n+1)/n,-b*x^n/a,-d*x^n/c)/(j*p+m+1)/((1+b*x^n/a)^p)/((1+d*x^n/c)^q)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2067, 525, 524}

$$\int (ex)^m (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^p \, dx$$
$$= \frac{x(ex)^m \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^q \left(\frac{dx^n}{c}+1\right)^{-q} \left(ax^j+bx^{j+n}\right)^p \operatorname{AppellF1}\left(\frac{m+jp+1}{n},-p,-q,\frac{m+n+jp+1}{n},-\frac{bx^n}{a},-\frac{d}{\ }\right.}{jp+m+1}$$

```
[In] Int[(e*x)^m*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^p,x]
```

```
[Out] (x*(e*x)^m*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^p*AppellF1[(1 + m + j*p)/n,
-p, -q, (1 + m + n + j*p)/n, -((b*x^n)/a), -((d*x^n)/c)])/((1 + m + j*p)*(1
 + (b*x^n)/a)^p*(1 + (d*x^n)/c)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
```

```
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2067

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p])), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
&& NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Rubi steps

$$\text{integral} = \left(x^{-m-jp}(ex)^m\left(a+bx^n\right)^{-p}\left(ax^j+bx^{j+n}\right)^p\right)\int x^{m+jp}(a+bx^n)^p\left(c+dx^n\right)^q\,dx$$

$$= \left(x^{-m-jp}(ex)^m\left(1+\frac{bx^n}{a}\right)^{-p}\left(ax^j+bx^{j+n}\right)^p\right)\int x^{m+jp}\left(1+\frac{bx^n}{a}\right)^p\left(c+dx^n\right)^q\,dx$$

$$= \left(x^{-m-jp}(ex)^m\left(1+\frac{bx^n}{a}\right)^{-p}\left(c+dx^n\right)^q\left(1+\frac{dx^n}{c}\right)^{-q}\left(ax^j+bx^{j+n}\right)^p\right)\int x^{m+jp}\left(1+\frac{bx^n}{a}\right)^p\left(1+\frac{dx^n}{c}\right)^q\,dx$$

$$= \frac{x(ex)^m\left(1+\frac{bx^n}{a}\right)^{-p}\left(c+dx^n\right)^q\left(1+\frac{dx^n}{c}\right)^{-q}\left(ax^j+bx^{j+n}\right)^p F_1\left(\frac{1+m+jp}{n};-p,-q;\frac{1+m+n+jp}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{1+m+jp}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.98

$$\int (ex)^m (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^p \, dx$$
$$=\frac{x(ex)^m (x^j(a+bx^n))^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \text{AppellF1}\left(\frac{1+m+jp}{n},-p,-q,\frac{1+m+n+jp}{n},-\frac{bx^n}{a},-\frac{d}{}\right.}{1+m+jp}$$

```
[In] Integrate[(e*x)^m*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^p,x]
```

```
[Out] (x*(e*x)^m*(x^j*(a + b*x^n))^p*(c + d*x^n)^q*AppellF1[(1 + m + j*p)/n, -p,
-q, (1 + m + n + j*p)/n, -((b*x^n)/a), -((d*x^n)/c)])/((1 + m + j*p)*(1 + (
b*x^n)/a)^p*(1 + (d*x^n)/c)^q)
```

Maple [F]

$$\int (ex)^m (c+d\,x^n)^q \left(a\,x^j+b\,x^{j+n}\right)^p dx$$

```
[In] int((e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p,x)
```

```
[Out] int((e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p,x)
```

Fricas [F]

$$\int (ex)^m (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^p \, dx = \int \left(bx^{j+n}+ax^j\right)^p (dx^n+c)^q (ex)^m \, dx$$

```
[In] integrate((e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^(j + n) + a*x^j)^p*(d*x^n + c)^q*(e*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^p \, dx = \text{Timed out}$$

```
[In] integrate((e*x)**m*(c+d*x**n)**q*(a*x**j+b*x**(j+n))**p,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (ex)^m (c+dx^n)^q \left(ax^j + bx^{j+n}\right)^p \, dx = \int \left(bx^{j+n} + ax^j\right)^p (dx^n + c)^q (ex)^m \, dx$$

```
[In] integrate((e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p,x, algorithm="maxima")
[Out] integrate((b*x^(j + n) + a*x^j)^p*(d*x^n + c)^q*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m (c+dx^n)^q \left(ax^j + bx^{j+n}\right)^p \, dx = \int \left(bx^{j+n} + ax^j\right)^p (dx^n + c)^q (ex)^m \, dx$$

```
[In] integrate((e*x)^m*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^p,x, algorithm="giac")
[Out] integrate((b*x^(j + n) + a*x^j)^p*(d*x^n + c)^q*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m (c+dx^n)^q \left(ax^j + bx^{j+n}\right)^p \, dx = \int \left(a\,x^j + b\,x^{j+n}\right)^p (e\,x)^m \left(c + d\,x^n\right)^q dx$$

```
[In] int((a*x^j + b*x^(j + n))^p*(e*x)^m*(c + d*x^n)^q,x)
[Out] int((a*x^j + b*x^(j + n))^p*(e*x)^m*(c + d*x^n)^q, x)
```

3.277 $\int (ex)^{7/4} (c + dx^n)^q \left(ax^j + bx^{j+n}\right)^{5/3} dx$

Optimal result

Integrand size = 34, antiderivative size = 129

$$\int (ex)^{7/4} (c + dx^n)^q \left(ax^j + bx^{j+n}\right)^{5/3} dx = \frac{12aex^{2+j}(ex)^{3/4}(c + dx^n)^q \left(1 + \frac{dx^n}{c}\right)^{-q} \left(ax^j + bx^{j+n}\right)^{2/3} \text{AppellF1}\left(\frac{33+20j}{12n}, -\frac{5}{3}, -q, \frac{33+20j+1}{12n}\right.}{(33 + 20j)\left(1 + \frac{bx^n}{a}\right)^{2/3}}$$

```
[Out] 12*a*e*x^(2+j)*(e*x)^(3/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(2/3)*AppellF1(1/1
2*(33+20*j)/n,-5/3,-q,1+(11/4+5/3*j)/n,-b*x^n/a,-d*x^n/c)/(33+20*j)/(1+b*x^
n/a)^(2/3)/((1+d*x^n/c)^q)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {2067, 525, 524}

$$\int (ex)^{7/4} (c + dx^n)^q \left(ax^j + bx^{j+n}\right)^{5/3} dx = \frac{12ae(ex)^{3/4}x^{j+2}\left(ax^j + bx^{j+n}\right)^{2/3} (c + dx^n)^q \left(\frac{dx^n}{c} + 1\right)^{-q} \text{AppellF1}\left(\frac{20j+33}{12n}, -\frac{5}{3}, -q, \frac{20j+12n+}{12n}\right.}{(20j + 33)\left(\frac{bx^n}{a} + 1\right)^{2/3}}$$

```
[In] Int[(e*x)^(7/4)*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^(5/3),x]
```

```
[Out] (12*a*e*x^(2 + j)*(e*x)^(3/4)*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^(2/3)*App
ellF1[(33 + 20*j)/(12*n), -5/3, -q, (33 + 20*j + 12*n)/(12*n), -((b*x^n)/a)
, -((d*x^n)/c)])/((33 + 20*j)*(1 + (b*x^n)/a)^(2/3)*(1 + (d*x^n)/c)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2067

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p])), Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; F
reeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
&& NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Rubi steps

$$\text{integral} = \frac{\left(ex^{-\frac{3}{4}-\frac{2j}{3}}(ex)^{3/4}\left(ax^j+bx^{j+n}\right)^{2/3}\right)\int x^{\frac{7}{4}+\frac{5j}{3}}\left(a+bx^n\right)^{5/3}\left(c+dx^n\right)^q\,dx}{\left(a+bx^n\right)^{2/3}}$$

$$= \frac{\left(aex^{-\frac{3}{4}-\frac{2j}{3}}(ex)^{3/4}\left(ax^j+bx^{j+n}\right)^{2/3}\right)\int x^{\frac{7}{4}+\frac{5j}{3}}\left(1+\frac{bx^n}{a}\right)^{5/3}\left(c+dx^n\right)^q\,dx}{\left(1+\frac{bx^n}{a}\right)^{2/3}}$$

$$= \frac{\left(aex^{-\frac{3}{4}-\frac{2j}{3}}(ex)^{3/4}\left(c+dx^n\right)^q\left(1+\frac{dx^n}{c}\right)^{-q}\left(ax^j+bx^{j+n}\right)^{2/3}\right)\int x^{\frac{7}{4}+\frac{5j}{3}}\left(1+\frac{bx^n}{a}\right)^{5/3}\left(1+\frac{dx^n}{c}\right)^q\,dx}{\left(1+\frac{bx^n}{a}\right)^{2/3}}$$

$$= \frac{12aex^{2+j}(ex)^{3/4}\left(c+dx^n\right)^q\left(1+\frac{dx^n}{c}\right)^{-q}\left(ax^j+bx^{j+n}\right)^{2/3}F_1\left(\frac{33+20j}{12n};-\frac{5}{3},-q;\frac{33+20j+12n}{12n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(33+20j)\left(1+\frac{bx^n}{a}\right)^{2/3}}$$

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.63

$$\int (ex)^{7/4} (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^{5/3} dx = \frac{12x^{1+j}(ex)^{7/4} \left(x^j(a+bx^n)\right)^{2/3} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \left(a(33+20j+12n) \operatorname{AppellF1}\left(\frac{33+20}{12n}\right.\right.}{(33+20j)(3}$$

```
[In] Integrate[(e*x)^(7/4)*(c + d*x^n)^q*(a*x^j + b*x^(j + n))^(5/3),x]
```

```
[Out] (12*x^(1 + j)*(e*x)^(7/4)*(x^j*(a + b*x^n))^(2/3)*(c + d*x^n)^q*(a*(33 + 20
*j + 12*n)*AppellF1[(33 + 20*j)/(12*n), -2/3, -q, (11/4 + (5*j)/3 + n)/n, -
((b*x^n)/a), -((d*x^n)/c)] + b*(33 + 20*j)*x^n*AppellF1[(33 + 20*j + 12*n)/
(12*n), -2/3, -q, (33 + 20*j + 24*n)/(12*n), -((b*x^n)/a), -((d*x^n)/c)]))/
((33 + 20*j)*(33 + 20*j + 12*n)*(1 + (b*x^n)/a)^(2/3)*(1 + (d*x^n)/c)^q)
```

Maple [F]

$$\int (ex)^{\frac{7}{4}} (c+d\,x^n)^q \left(a\,x^j+b\,x^{j+n}\right)^{\frac{5}{3}} dx$$

```
[In] int((e*x)^(7/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(5/3),x)
```

```
[Out] int((e*x)^(7/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(5/3),x)
```

Fricas [F]

$$\int (ex)^{7/4} (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^{5/3} dx = \int \left(bx^{j+n}+ax^j\right)^{\frac{5}{3}} (ex)^{\frac{7}{4}} (dx^n+c)^q \, dx$$

```
[In] integrate((e*x)^(7/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(5/3),x, algorithm="fri
cas")
```

```
[Out] integral((b*e*x*x^(j + n) + a*e*x*x^j)*(b*x^(j + n) + a*x^j)^(2/3)*(e*x)^(3
/4)*(d*x^n + c)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{7/4} (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^{5/3} dx = \text{Timed out}$$

```
[In] integrate((e*x)**(7/4)*(c+d*x**n)**q*(a*x**j+b*x**(j+n))**(5/3),x)
[Out] Timed out
```

Maxima [F]

$$\int (ex)^{7/4} (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^{5/3} dx = \int \left(bx^{j+n}+ax^j\right)^{\frac{5}{3}} (ex)^{\frac{7}{4}} (dx^n+c)^q dx$$

```
[In] integrate((e*x)^(7/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(5/3),x, algorithm="max
ima")
[Out] integrate((b*x^(j + n) + a*x^j)^(5/3)*(e*x)^(7/4)*(d*x^n + c)^q, x)
```

Giac [F]

$$\int (ex)^{7/4} (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^{5/3} dx = \int \left(bx^{j+n}+ax^j\right)^{\frac{5}{3}} (ex)^{\frac{7}{4}} (dx^n+c)^q dx$$

```
[In] integrate((e*x)^(7/4)*(c+d*x^n)^q*(a*x^j+b*x^(j+n))^(5/3),x, algorithm="gia
c")
[Out] integrate((b*x^(j + n) + a*x^j)^(5/3)*(e*x)^(7/4)*(d*x^n + c)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{7/4} (c+dx^n)^q \left(ax^j+bx^{j+n}\right)^{5/3} dx = \int \left(a\,x^j+b\,x^{j+n}\right)^{5/3} (e\,x)^{7/4} (c+d\,x^n)^q\, dx$$

```
[In] int((a*x^j + b*x^(j + n))^(5/3)*(e*x)^(7/4)*(c + d*x^n)^q,x)
[Out] int((a*x^j + b*x^(j + n))^(5/3)*(e*x)^(7/4)*(c + d*x^n)^q, x)
```

3.278 $\int \frac{4+3x^4}{5x+2x^5} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{4+3x^4}{5x+2x^5} \, dx = \frac{4\log(x)}{5}+\frac{7}{40}\log\left(5+2x^4\right)$$

```
[Out] 4/5*ln(x)+7/40*ln(2*x^4+5)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1607, 457, 78}

$$\int \frac{4+3x^4}{5x+2x^5} \, dx = \frac{7}{40}\log\left(2x^4+5\right)+\frac{4\log(x)}{5}$$

```
[In] Int[(4 + 3*x^4)/(5*x + 2*x^5),x]
```

```
[Out] (4*Log[x])/5 + (7*Log[5 + 2*x^4])/40
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{4+3x^4}{x\left(5+2x^4\right)}\,dx \\
&= \frac{1}{4}\text{Subst}\left(\int \frac{4+3x}{x(5+2x)}\,dx, x, x^4\right) \\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{4}{5x}+\frac{7}{5(5+2x)}\right)\,dx, x, x^4\right) \\
&= \frac{4\log(x)}{5}+\frac{7}{40}\log\left(5+2x^4\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{4+3x^4}{5x+2x^5}\,dx = \frac{4\log(x)}{5}+\frac{7}{40}\log\left(5+2x^4\right)$$

```
[In] Integrate[(4 + 3*x^4)/(5*x + 2*x^5),x]
[Out] (4*Log[x])/5 + (7*Log[5 + 2*x^4])/40
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$\frac{4\ln(x)}{5}+\frac{7\ln\left(x^4+\frac{5}{2}\right)}{40}$	14
default	$\frac{4\ln(x)}{5}+\frac{7\ln(2x^4+5)}{40}$	16
norman	$\frac{4\ln(x)}{5}+\frac{7\ln(2x^4+5)}{40}$	16
risch	$\frac{4\ln(x)}{5}+\frac{7\ln(2x^4+5)}{40}$	16
meijerg	$\frac{4\ln(x)}{5}+\frac{\ln(2)}{5}-\frac{\ln(5)}{5}+\frac{7\ln\left(1+\frac{2x^4}{5}\right)}{40}$	24

```
[In] int((3*x^4+4)/(2*x^5+5*x),x,method=_RETURNVERBOSE)
[Out] 4/5*ln(x)+7/40*ln(x^4+5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{4+3x^4}{5x+2x^5}\,dx=\frac{7}{40}\,\log\left(2\,x^4+5\right)+\frac{4}{5}\,\log\left(x\right)$$

```
[In] integrate((3*x^4+4)/(2*x^5+5*x),x, algorithm="fricas")
[Out] 7/40*log(2*x^4 + 5) + 4/5*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{4+3x^4}{5x+2x^5}\,dx=\frac{4\log\left(x\right)}{5}+\frac{7\log\left(2x^4+5\right)}{40}$$

```
[In] integrate((3*x**4+4)/(2*x**5+5*x),x)
[Out] 4*log(x)/5 + 7*log(2*x**4 + 5)/40
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{4+3x^4}{5x+2x^5}\,dx = \frac{7}{40}\,\log\left(2\,x^4+5\right)+\frac{4}{5}\,\log\left(x\right)$$

```
[In] integrate((3*x^4+4)/(2*x^5+5*x),x, algorithm="maxima")
[Out] 7/40*log(2*x^4 + 5) + 4/5*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{4+3x^4}{5x+2x^5}\,dx = \frac{7}{40}\,\log\left(2\,x^4+5\right)+\frac{1}{5}\,\log\left(x^4\right)$$

```
[In] integrate((3*x^4+4)/(2*x^5+5*x),x, algorithm="giac")
[Out] 7/40*log(2*x^4 + 5) + 1/5*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{4+3x^4}{5x+2x^5}\,dx = \frac{7\,\ln\left(x^4+\frac{5}{2}\right)}{40}+\frac{4\,\ln\left(x\right)}{5}$$

```
[In] int((3*x^4 + 4)/(5*x + 2*x^5),x)
[Out] (7*log(x^4 + 5/2))/40 + (4*log(x))/5
```

3.279 $\int \frac{1+x^6}{x-x^7} dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{1+x^6}{x-x^7} dx = \log(x) - \frac{1}{3}\log\left(1-x^6\right)$$

```
[Out] ln(x)-1/3*ln(-x^6+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1607, 457, 78}

$$\int \frac{1+x^6}{x-x^7} dx = \log(x) - \frac{1}{3}\log\left(1-x^6\right)$$

```
[In] Int[(1 + x^6)/(x - x^7),x]
```

```
[Out] Log[x] - Log[1 - x^6]/3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1+x^6}{x\left(1-x^6\right)}\,dx \\
&= \frac{1}{6}\text{Subst}\left(\int \frac{1+x}{(1-x)x}\,dx, x, x^6\right) \\
&= \frac{1}{6}\text{Subst}\left(\int \left(-\frac{2}{-1+x}+\frac{1}{x}\right)\,dx, x, x^6\right) \\
&= \log(x)-\frac{1}{3}\log\left(1-x^6\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1+x^6}{x-x^7}\,dx = \log(x)-\frac{1}{3}\log\left(1-x^6\right)$$

```
[In] Integrate[(1 + x^6)/(x - x^7),x]
[Out] Log[x] - Log[1 - x^6]/3
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
risch	$\ln(x)-\frac{\ln(x^6-1)}{3}$	12
meijerg	$-\frac{\ln(-x^6+1)}{3}+\ln(x)+\frac{i\pi}{6}$	18
default	$\ln(x)-\frac{\ln(-1+x)}{3}-\frac{\ln(1+x)}{3}-\frac{\ln(x^2+x+1)}{3}-\frac{\ln(x^2-x+1)}{3}$	36
norman	$\ln(x)-\frac{\ln(-1+x)}{3}-\frac{\ln(1+x)}{3}-\frac{\ln(x^2+x+1)}{3}-\frac{\ln(x^2-x+1)}{3}$	36
parallelrisch	$\ln(x)-\frac{\ln(-1+x)}{3}-\frac{\ln(1+x)}{3}-\frac{\ln(x^2+x+1)}{3}-\frac{\ln(x^2-x+1)}{3}$	36

```
[In] int((x^6+1)/(-x^7+x),x,method=_RETURNVERBOSE)
[Out] ln(x)-1/3*ln(x^6-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1+x^6}{x-x^7}\,dx=-\frac{1}{3}\log\left(x^6-1\right)+\log(x)$$

```
[In] integrate((x^6+1)/(-x^7+x),x, algorithm="fricas")
[Out] -1/3*log(x^6 - 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1+x^6}{x-x^7}\,dx=\log(x)-\frac{\log(x^6-1)}{3}$$

```
[In] integrate((x**6+1)/(-x**7+x),x)
[Out] log(x) - log(x**6 - 1)/3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(13) = 26$.

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \frac{1+x^6}{x-x^7}\,dx = -\frac{1}{3}\log\left(x^2+x+1\right) - \frac{1}{3}\log\left(x^2-x+1\right) - \frac{1}{3}\log\left(x+1\right) - \frac{1}{3}\log\left(x-1\right) + \log\left(x\right)$$

```
[In] integrate((x^6+1)/(-x^7+x),x, algorithm="maxima")
[Out] -1/3*log(x^2 + x + 1) - 1/3*log(x^2 - x + 1) - 1/3*log(x + 1) - 1/3*log(x -
 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{1+x^6}{x-x^7}\,dx = \frac{1}{6}\log\left(x^6\right) - \frac{1}{3}\log\left(\left|x^6-1\right|\right)$$

```
[In] integrate((x^6+1)/(-x^7+x),x, algorithm="giac")
[Out] 1/6*log(x^6) - 1/3*log(abs(x^6 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1+x^6}{x-x^7}\,dx = \ln\left(x\right) - \frac{\ln\left(x^6-1\right)}{3}$$

```
[In] int((x^6 + 1)/(x - x^7),x)
[Out] log(x) - log(x^6 - 1)/3
```

3.280 $\int \frac{8+5x^{10}}{2x-x^{11}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 17

$$\int \frac{8+5x^{10}}{2x-x^{11}}\,dx = 4\log(x) - \frac{9}{10}\log\left(2-x^{10}\right)$$

```
[Out] 4*ln(x)-9/10*ln(-x^10+2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1607, 457, 78}

$$\int \frac{8+5x^{10}}{2x-x^{11}}\,dx = 4\log(x) - \frac{9}{10}\log\left(2-x^{10}\right)$$

```
[In] Int[(8 + 5*x^10)/(2*x - x^11),x]
```

```
[Out] 4*Log[x] - (9*Log[2 - x^10])/10
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{8+5x^{10}}{x\left(2-x^{10}\right)}\,dx\\ &= \frac{1}{10}\text{Subst}\left(\int \frac{8+5x}{(2-x)x}\,dx, x, x^{10}\right)\\ &= \frac{1}{10}\text{Subst}\left(\int \left(-\frac{9}{-2+x}+\frac{4}{x}\right)\,dx, x, x^{10}\right)\\ &= 4\log(x)-\frac{9}{10}\log\left(2-x^{10}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{8+5x^{10}}{2x-x^{11}}\,dx = 4\log(x)-\frac{9}{10}\log\left(2-x^{10}\right)$$

```
[In] Integrate[(8 + 5*x^10)/(2*x - x^11),x]
[Out] 4*Log[x] - (9*Log[2 - x^10])/10
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$4\ln(x)-\frac{9\ln(x^{10}-2)}{10}$	14
norman	$4\ln(x)-\frac{9\ln(x^{10}-2)}{10}$	14
risch	$4\ln(x)-\frac{9\ln(x^{10}-2)}{10}$	14
parallelrisch	$4\ln(x)-\frac{9\ln(x^{10}-2)}{10}$	14
meijerg	$4\ln(x)-\frac{2\ln(2)}{5}+\frac{2i\pi}{5}-\frac{9\ln\left(1-\frac{x^{10}}{2}\right)}{10}$	24

```
[In] int((5*x^10+8)/(-x^11+2*x),x,method=_RETURNVERBOSE)
[Out] 4*ln(x)-9/10*ln(x^10-2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{8+5x^{10}}{2x-x^{11}}\,dx=-\frac{9}{10}\,\log\left(x^{10}-2\right)+4\,\log\left(x\right)$$

```
[In] integrate((5*x^10+8)/(-x^11+2*x),x, algorithm="fricas")
[Out] -9/10*log(x^10 - 2) + 4*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{8+5x^{10}}{2x-x^{11}}\,dx=4\log\left(x\right)-\frac{9\log\left(x^{10}-2\right)}{10}$$

```
[In] integrate((5*x**10+8)/(-x**11+2*x),x)
[Out] 4*log(x) - 9*log(x**10 - 2)/10
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{8+5x^{10}}{2x-x^{11}}\,dx = -\frac{9}{10}\,\log\left(x^{10}-2\right)+4\,\log\left(x\right)$$

```
[In] integrate((5*x^10+8)/(-x^11+2*x),x, algorithm="maxima")
[Out] -9/10*log(x^10 - 2) + 4*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{8+5x^{10}}{2x-x^{11}}\,dx = \frac{2}{5}\,\log\left(x^{10}\right)-\frac{9}{10}\,\log\left(\left|x^{10}-2\right|\right)$$

```
[In] integrate((5*x^10+8)/(-x^11+2*x),x, algorithm="giac")
[Out] 2/5*log(x^10) - 9/10*log(abs(x^10 - 2))
```

Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{8+5x^{10}}{2x-x^{11}}\,dx = 4\,\ln\left(x\right)-\frac{9\,\ln\left(x^{10}-2\right)}{10}$$

```
[In] int((5*x^10 + 8)/(2*x - x^11),x)
[Out] 4*log(x) - (9*log(x^10 - 2))/10
```

3.281 $\int \frac{-3+2x}{-x^2+x^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \frac{-3+2x}{-x^2+x^3} \, dx = -\frac{3}{x} - \log(1-x) + \log(x)$$

```
[Out] -3/x-ln(1-x)+ln(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1607, 78}

$$\int \frac{-3+2x}{-x^2+x^3} \, dx = -\frac{3}{x} - \log(1-x) + \log(x)$$

```
[In] Int[(-3 + 2*x)/(-x^2 + x^3),x]
```

```
[Out] -3/x - Log[1 - x] + Log[x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{-3+2x}{(-1+x)x^2}\,dx\\ &= \int \left(\frac{1}{1-x}+\frac{3}{x^2}+\frac{1}{x}\right)dx\\ &= -\frac{3}{x}-\log(1-x)+\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{-3+2x}{-x^2+x^3}\,dx = -\frac{3}{x}-\log(1-x)+\log(x)$$

```
[In] Integrate[(-3 + 2*x)/(-x^2 + x^3),x]
[Out] -3/x - Log[1 - x] + Log[x]
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\ln(x)-\frac{3}{x}-\ln(-1+x)$	15
norman	$\ln(x)-\frac{3}{x}-\ln(-1+x)$	15
risch	$\ln(x)-\frac{3}{x}-\ln(-1+x)$	15
parallelrisch	$\frac{\ln(x)x-\ln(-1+x)x-3}{x}$	18
meijerg	$-\frac{3}{x}+\ln(x)+i\pi-\ln(1-x)$	21

```
[In] int((-3+2*x)/(x^3-x^2),x,method=_RETURNVERBOSE)
[Out] ln(x)-3/x-ln(-1+x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{-3+2x}{-x^2+x^3}\,dx = -\frac{x\log(x-1)-x\log(x)+3}{x}$$

```
[In] integrate((-3+2*x)/(x^3-x^2),x, algorithm="fricas")
[Out] -(x*log(x - 1) - x*log(x) + 3)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{-3+2x}{-x^2+x^3}\,dx = \log(x)-\log(x-1)-\frac{3}{x}$$

```
[In] integrate((-3+2*x)/(x**3-x**2),x)
[Out] log(x) - log(x - 1) - 3/x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-3+2x}{-x^2+x^3}\,dx = -\frac{3}{x}-\log(x-1)+\log(x)$$

```
[In] integrate((-3+2*x)/(x^3-x^2),x, algorithm="maxima")
[Out] -3/x - log(x - 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{-3+2x}{-x^2+x^3}\,dx = -\frac{3}{x}-\log(|x-1|)+\log(|x|)$$

```
[In] integrate((-3+2*x)/(x^3-x^2),x, algorithm="giac")
[Out] -3/x - log(abs(x - 1)) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-3+2x}{-x^2+x^3}\,dx = 2\,\mathrm{atanh}(2\,x-1)-\frac{3}{x}$$

```
[In] int(-(2*x - 3)/(x^2 - x^3),x)
[Out] 2*atanh(2*x - 1) - 3/x
```

3.282 $\int \frac{ax^m+bx^n}{cx^m+dx^n}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 54

$$\int \frac{ax^m+bx^n}{cx^m+dx^n}\,dx=\frac{ax}{c}+\frac{(bc-ad)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{m-n},1+\frac{1}{m-n},-\frac{cx^{m-n}}{d}\right)}{cd}$$

```
[Out] a*x/c+(-a*d+b*c)*x*hypergeom([1, 1/(m-n)],[1+1/(m-n)],-c*x^(m-n)/d)/c/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1607, 1598, 396, 251}

$$\int \frac{ax^m+bx^n}{cx^m+dx^n}\,dx=\frac{x(bc-ad)\,\text{Hypergeometric2F1}\left(1,\frac{1}{m-n},1+\frac{1}{m-n},-\frac{cx^{m-n}}{d}\right)}{cd}+\frac{ax}{c}$$

```
[In] Int[(a*x^m + b*x^n)/(c*x^m + d*x^n),x]
```

```
[Out] (a*x)/c + ((b*c - a*d)*x*Hypergeometric2F1[1, (m - n)^(-1), 1 + (m - n)^(-1
), -((c*x^(m - n))/d)])/(c*d)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^n(b+ax^{m-n})}{cx^m+dx^n}\,dx\\ &= \int \frac{b+ax^{m-n}}{d+cx^{m-n}}\,dx\\ &= \frac{ax}{c}-\frac{(-bc+ad)\int \frac{1}{d+cx^{m-n}}\,dx}{c}\\ &= \frac{ax}{c}+\frac{(bc-ad)x\,{}_2F_1\left(1,\frac{1}{m-n};1+\frac{1}{m-n};-\frac{cx^{m-n}}{d}\right)}{cd}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{ax^m+bx^n}{cx^m+dx^n}\,dx = \frac{x\left(ad+(bc-ad)\,\text{Hypergeometric2F1}\left(1,\frac{1}{m-n},1+\frac{1}{m-n},-\frac{cx^{m-n}}{d}\right)\right)}{cd}$$

```
[In] Integrate[(a*x^m + b*x^n)/(c*x^m + d*x^n),x]
[Out] (x*(a*d + (b*c - a*d)*Hypergeometric2F1[1, (m - n)^(-1), 1 + (m - n)^(-1),
-((c*x^(m - n))/d)]))/(c*d)
```

Maple [F]

$$\int \frac{x^m a + b\,x^n}{c\,x^m + d\,x^n}dx$$

```
[In] int((x^m*a+b*x^n)/(c*x^m+d*x^n),x)
[Out] int((x^m*a+b*x^n)/(c*x^m+d*x^n),x)
```

Fricas [F]

$$\int \frac{ax^m + bx^n}{cx^m + dx^n}\,dx = \int \frac{ax^m + bx^n}{cx^m + dx^n}\,dx$$

```
[In] integrate((a*x^m+b*x^n)/(c*x^m+d*x^n),x, algorithm="fricas")
[Out] integral((a*x^m + b*x^n)/(c*x^m + d*x^n), x)
```

Sympy [F]

$$\int \frac{ax^m + bx^n}{cx^m + dx^n}\,dx = \int \frac{ax^m + bx^n}{cx^m + dx^n}\,dx$$

```
[In] integrate((a*x**m+b*x**n)/(c*x**m+d*x**n),x)
[Out] Integral((a*x**m + b*x**n)/(c*x**m + d*x**n), x)
```

Maxima [F]

$$\int \frac{ax^m + bx^n}{cx^m + dx^n}\,dx = \int \frac{ax^m + bx^n}{cx^m + dx^n}\,dx$$

```
[In] integrate((a*x^m+b*x^n)/(c*x^m+d*x^n),x, algorithm="maxima")
[Out] -(b*c - a*d)*integrate(x^m/(c*d*x^m + d^2*x^n), x) + b*x/d
```

Giac [F]

$$\int \frac{ax^m + bx^n}{cx^m + dx^n} \, dx = \int \frac{ax^m + bx^n}{cx^m + dx^n} \, dx$$

```
[In] integrate((a*x^m+b*x^n)/(c*x^m+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((a*x^m + b*x^n)/(c*x^m + d*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ax^m + bx^n}{cx^m + dx^n} \, dx = \int \frac{a\,x^m + b\,x^n}{c\,x^m + d\,x^n} \, dx$$

```
[In] int((a*x^m + b*x^n)/(c*x^m + d*x^n),x)
```

```
[Out] int((a*x^m + b*x^n)/(c*x^m + d*x^n), x)
```

3.283 $\int x^m(a+bx^n)^p\left(a(1+m+q)x^q+b(1+m+n(1+$

Optimal result

Integrand size = 39, antiderivative size = 18

$$\int x^m(a+bx^n)^p\left(a(1+m+q)x^q+b(1+m+n(1+p)+q)x^{n+q}\right)\,dx=x^{1+m+q}(a+bx^n)^{1+p}$$

```
[Out] x^(1+m+q)*(a+b*x^n)^(p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {1598, 460}

$$\int x^m(a+bx^n)^p\left(a(1+m+q)x^q+b(1+m+n(1+p)+q)x^{n+q}\right)\,dx=x^{m+q+1}(a+bx^n)^{p+1}$$

```
[In] Int[x^m*(a + b*x^n)^p*(a*(1 + m + q)*x^q + b*(1 + m + n*(1 + p) + q)*x^(n + q)),x]
[Out] x^(1 + m + q)*(a + b*x^n)^(1 + p)
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))), x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{m+q}(a+bx^n)^p\left(a(1+m+q)+b(1+m+n(1+p)+q)x^n\right)\,dx\\ &= x^{1+m+q}(a+bx^n)^{1+p}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.46 (sec) , antiderivative size = 116, normalized size of antiderivative = 6.44

$$\int x^m(a+bx^n)^p\left(a(1+m+q)x^q+b(1+m+n(1+p)+q)x^{n+q}\right)\,dx = x^{1+m+q}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(a\,\text{Hypergeometric2F1}\left(-p,\frac{1+m+q}{n},\frac{1+m+n+q}{n},-\frac{bx^n}{a}\right)+\frac{b(1+m+n+np+q)x^n\,\text{Hypergeometric2F1}\left(-p,\frac{1+m+n+q}{n},\frac{1+m+2n+q}{n},-\frac{bx^n}{a}\right)}{1+m+n+q}\right)$$

```
[In] Integrate[x^m*(a + b*x^n)^p*(a*(1 + m + q)*x^q + b*(1 + m + n*(1 + p) + q)*
x^(n + q)),x]
```

```
[Out] (x^(1 + m + q)*(a + b*x^n)^p*(a*Hypergeometric2F1[-p, (1 + m + q)/n, (1 + m
 + n + q)/n, -((b*x^n)/a)] + (b*(1 + m + n + n*p + q)*x^n*Hypergeometric2F1
[-p, (1 + m + n + q)/n, (1 + m + 2*n + q)/n, -((b*x^n)/a)])/(1 + m + n + q)
))/(1 + (b*x^n)/a)^p
```

Maple [F]

$$\int x^m(a+b\,x^n)^p\left(a(1+m+q)\,x^q+b(1+m+n(1+p)+q)\,x^{n+q}\right)dx$$

```
[In] int(x^m*(a+b*x^n)^p*(a*(1+m+q)*x^q+b*(1+m+n*(1+p)+q)*x^(n+q)),x)
```

```
[Out] int(x^m*(a+b*x^n)^p*(a*(1+m+q)*x^q+b*(1+m+n*(1+p)+q)*x^(n+q)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.39

$$\int x^m(a+bx^n)^p\left(a(1+m+q)x^q+b(1+m+n(1+p)+q)x^{n+q}\right)\,dx$$
$$=\left(bxx^mx^{n+q}+axx^mx^q\right)\left(\frac{bx^{n+q}+ax^q}{x^q}\right)^p$$

```
[In] integrate(x^m*(a+b*x^n)^p*(a*(1+m+q)*x^q+b*(1+m+n*(1+p)+q)*x^(n+q)),x, algo
rithm="fricas")
[Out] (b*x*x^m*x^(n + q) + a*x*x^m*x^q)*((b*x^(n + q) + a*x^q)/x^q)^p
```

Sympy [F(-1)]

Timed out.

$$\int x^m(a+bx^n)^p\left(a(1+m+q)x^q+b(1+m+n(1+p)+q)x^{n+q}\right)\,dx=\text{Timed out}$$

```
[In] integrate(x**m*(a+b*x**n)**p*(a*(1+m+q)*x**q+b*(1+m+n*(1+p)+q)*x**(n+q)),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(18) = 36$.

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int x^m(a+bx^n)^p\left(a(1+m+q)x^q+b(1+m+n(1+p)+q)x^{n+q}\right)\,dx$$
$$=\left(axx^m+bxe^{(m\log(x)+n\log(x))}\right)e^{(p\log(bx^n+a)+q\log(x))}$$

```
[In] integrate(x^m*(a+b*x^n)^p*(a*(1+m+q)*x^q+b*(1+m+n*(1+p)+q)*x^(n+q)),x, algo
rithm="maxima")
[Out] (a*x*x^m + b*x*e^(m*log(x) + n*log(x)))*e^(p*log(b*x^n + a) + q*log(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(18) = 36$.

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.67

$$\int x^m(a+bx^n)^p\left(a(1+m+q)x^q+b(1+m+n(1+p)+q)x^{n+q}\right)\,dx = (bx^n+a)^p bxx^n e^{(m\log(x)+q\log(x))} + (bx^n+a)^p axe^{(m\log(x)+q\log(x))}$$

```
[In] integrate(x^m*(a+b*x^n)^p*(a*(1+m+q)*x^q+b*(1+m+n*(1+p)+q)*x^(n+q)),x, algo
rithm="giac")
[Out] (b*x^n + a)^p*b*x*x^n*e^(m*log(x) + q*log(x)) + (b*x^n + a)^p*a*x*e^(m*log(
x) + q*log(x))
```

Mupad [F(-1)]

Timed out.

$$\int x^m(a+bx^n)^p\left(a(1+m+q)x^q+b(1+m+n(1+p)+q)x^{n+q}\right)\,dx = \int x^m\,\left(a\,x^q\,(m+q+1)+b\,x^{n+q}\,(m+q+n\,(p+1)+1)\right)\,{\left(a+b\,x^n\right)}^p\,dx$$

```
[In] int(x^m*(a*x^q*(m + q + 1) + b*x^(n + q)*(m + q + n*(p + 1) + 1))*(a + b*x^
n)^p,x)
[Out] int(x^m*(a*x^q*(m + q + 1) + b*x^(n + q)*(m + q + n*(p + 1) + 1))*(a + b*x^
n)^p, x)
```

3.284 $\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 64

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx}\,dx=\frac{\left(a+\frac{b}{x}\right)^n\left(1+\frac{b}{ax}\right)^{-n}x^m\,\mathrm{AppellF1}\left(-m,-n,1,1-m,-\frac{b}{ax},-\frac{c}{dx}\right)}{dm}$$

```
[Out] (a+b/x)^n*x^m*AppellF1(-m,-n,1,1-m,-b/a/x,-c/d/x)/d/m/((1+b/a/x)^n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {528, 511, 140, 138}

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx}\,dx=\frac{x^m\left(a+\frac{b}{x}\right)^n\left(\frac{b}{ax}+1\right)^{-n}\mathrm{AppellF1}\left(-m,-n,1,1-m,-\frac{b}{ax},-\frac{c}{dx}\right)}{dm}$$

```
[In] Int[((a + b/x)^n*x^m)/(c + d*x),x]
```

```
[Out] ((a + b/x)^n*x^m*AppellF1[-m, -n, 1, 1 - m, -(b/(a*x)), -(c/(d*x))])/(d*m*(
1 + b/(a*x))^n)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

Rule 511

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Dist[(-(e*x)^m)*(x^(-1))^m, Subst[Int[(a + b/x^n)^p*((
c + d/x^n)^q/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q},
 x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && !RationalQ[m]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] || !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\left(a+\frac{b}{x}\right)^n x^{-1+m}}{d+\frac{c}{x}}\,dx\\
&= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-1-m}(a+bx)^n}{d+cx}\,dx,x,\frac{1}{x}\right)\right)\\
&= -\left(\left(\left(a+\frac{b}{x}\right)^n\left(1+\frac{b}{ax}\right)^{-n}\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-1-m}\left(1+\frac{bx}{a}\right)^n}{d+cx}\,dx,x,\frac{1}{x}\right)\right)\\
&= \frac{\left(a+\frac{b}{x}\right)^n\left(1+\frac{b}{ax}\right)^{-n} x^m F_1\left(-m;-n,1;1-m;-\frac{b}{ax},-\frac{c}{dx}\right)}{dm}
\end{aligned}$$

Mathematica [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx}\,dx$$

```
[In] Integrate[((a + b/x)^n*x^m)/(c + d*x),x]
[Out] Integrate[((a + b/x)^n*x^m)/(c + d*x), x]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{dx+c}dx$$

```
[In] int((a+b/x)^n*x^m/(d*x+c),x)
[Out] int((a+b/x)^n*x^m/(d*x+c),x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n x^m}{dx+c}\,dx$$

```
[In] integrate((a+b/x)^n*x^m/(d*x+c),x, algorithm="fricas")
[Out] integral(x^m*((a*x + b)/x)^n/(d*x + c), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx}\,dx = \int \frac{x^m\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx$$

```
[In] integrate((a+b/x)**n*x**m/(d*x+c),x)
[Out] Integral(x**m*(a + b/x)**n/(c + d*x), x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n x^m}{dx+c}\,dx$$

```
[In] integrate((a+b/x)^n*x^m/(d*x+c),x, algorithm="maxima")
[Out] integrate((a + b/x)^n*x^m/(d*x + c), x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{dx+c}\,dx$$

```
[In] integrate((a+b/x)^n*x^m/(d*x+c),x, algorithm="giac")
[Out] integrate((a + b/x)^n*x^m/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{c+dx}\,dx=\int \frac{x^m\,\left(a+\frac{b}{x}\right)^n}{c+d\,x}\,dx$$

```
[In] int((x^m*(a + b/x)^n)/(c + d*x),x)
[Out] int((x^m*(a + b/x)^n)/(c + d*x), x)
```

3.285 $\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 195

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx} \, dx = -\frac{(2ac+bd(1-n))\left(a+\frac{b}{x}\right)^{1+n} x}{2a^2d^2} + \frac{\left(a+\frac{b}{x}\right)^{1+n} x^2}{2ad} - \frac{c^3\left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^3(ac-bd)(1+n)} + \frac{(2a^2c^2-2abcdn-b^2d^2(1-n)n)\left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b}{ax}\right)}{2a^3d^3(1+n)}$$

```
[Out] -1/2*(2*a*c+b*d*(1-n))*(a+b/x)^(1+n)*x/a^2/d^2+1/2*(a+b/x)^(1+n)*x^2/a/d-c^
3*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/d^3/(a*c-b*d)
/(1+n)+1/2*(2*a^2*c^2-2*a*b*c*d*n-b^2*d^2*(1-n)*n)*(a+b/x)^(1+n)*hypergeom(
[1, 1+n],[2+n],1+b/a/x)/a^3/d^3/(1+n)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used

$= \{528, 457, 105, 156, 162, 67, 70\}$

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx}\,dx = -\frac{x\left(a+\frac{b}{x}\right)^{n+1}(2ac+bd(1-n))}{2a^2d^2} + \frac{\left(a+\frac{b}{x}\right)^{n+1}\left(2a^2c^2-2abcdn-b^2d^2(1-n)n\right)\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b}{ax}+1\right)}{2a^3d^3(n+1)} - \frac{c^3\left(a+\frac{b}{x}\right)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^3(n+1)(ac-bd)} + \frac{x^2\left(a+\frac{b}{x}\right)^{n+1}}{2ad}$$

```
[In] Int[((a + b/x)^n*x^2)/(c + d*x),x]
```

```
[Out] -1/2*((2*a*c + b*d*(1 - n))*(a + b/x)^(1 + n)*x)/(a^2*d^2) + ((a + b/x)^(1
+ n)*x^2)/(2*a*d) - (c^3*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 +
n, (c*(a + b/x))/(a*c - b*d)])/(d^3*(a*c - b*d)*(1 + n)) + ((2*a^2*c^2 - 2*
a*b*c*d*n - b^2*d^2*(1 - n)*n)*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n
, 2 + n, 1 + b/(a*x)])/(2*a^3*d^3*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
```

```
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\left(a+\frac{b}{x}\right)^n x}{d+\frac{c}{x}}\,dx \\
&= -\text{Subst}\left(\int \frac{(a+bx)^n}{x^3(d+cx)}\,dx, x, \frac{1}{x}\right) \\
&= \frac{\left(a+\frac{b}{x}\right)^{1+n} x^2}{2ad} + \frac{\text{Subst}\left(\int \frac{(a+bx)^n(2ac+bd(1-n)+bc(1-n)x)}{x^2(d+cx)}\,dx, x, \frac{1}{x}\right)}{2ad} \\
&= -\frac{(2ac+bd(1-n))\left(a+\frac{b}{x}\right)^{1+n} x}{2a^2d^2} + \frac{\left(a+\frac{b}{x}\right)^{1+n} x^2}{2ad} \\
&\quad - \frac{\text{Subst}\left(\int \frac{(a+bx)^n(2a^2c^2-2abcdn-b^2d^2(1-n)n-bc(2ac+bd(1-n))nx)}{x(d+cx)}\,dx, x, \frac{1}{x}\right)}{2a^2d^2}
\end{aligned}
$$

$$= -\frac{(2ac+bd(1-n))\left(a+\frac{b}{x}\right)^{1+n}x}{2a^2d^2} + \frac{\left(a+\frac{b}{x}\right)^{1+n}x^2}{2ad} + \frac{c^3\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{d^3} - \frac{(2a^2c^2-2abcdn-b^2d^2(1-n)n)\,\text{Subst}\left(\int \frac{(a+bx)^n}{x}\,dx, x, \frac{1}{x}\right)}{2a^2d^3}$$

$$= -\frac{(2ac+bd(1-n))\left(a+\frac{b}{x}\right)^{1+n}x}{2a^2d^2} + \frac{\left(a+\frac{b}{x}\right)^{1+n}x^2}{2ad} - \frac{c^3\left(a+\frac{b}{x}\right)^{1+n}{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^3(ac-bd)(1+n)} + \frac{(2a^2c^2-2abcdn-b^2d^2(1-n)n)\left(a+\frac{b}{x}\right)^{1+n}{}_2F_1\left(1,1+n;2+n;1+\frac{b}{ax}\right)}{2a^3d^3(1+n)}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx}\,dx = \frac{\left(a+\frac{b}{x}\right)^n(b+ax)\left(-2a^3c^3\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+(ac-bd)\left(ad(1+n)x(bd(-1\right.\right.}{2a^3d^3(ac-}$$

```
[In] Integrate[((a + b/x)^n*x^2)/(c + d*x),x]
```

```
[Out] ((a + b/x)^n*(b + a*x)*(-2*a^3*c^3*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a
 + b/x))/(a*c - b*d)] + (a*c - b*d)*(a*d*(1 + n)*x*(b*d*(-1 + n) + a*(-2*c
+ d*x)) + (2*a^2*c^2 - 2*a*b*c*d*n + b^2*d^2*(-1 + n)*n)*Hypergeometric2F1[
1, 1 + n, 2 + n, 1 + b/(a*x)])))/(2*a^3*d^3*(a*c - b*d)*(1 + n)*x)
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{dx+c}dx$$

```
[In] int((a+b/x)^n*x^2/(d*x+c),x)
```

```
[Out] int((a+b/x)^n*x^2/(d*x+c),x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx} \, dx = \int \frac{\left(a+\frac{b}{x}\right)^n x^2}{dx+c} \, dx$$

```
[In] integrate((a+b/x)^n*x^2/(d*x+c),x, algorithm="fricas")
[Out] integral(x^2*((a*x + b)/x)^n/(d*x + c), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx} \, dx = \int \frac{x^2\left(a+\frac{b}{x}\right)^n}{c+dx} \, dx$$

```
[In] integrate((a+b/x)**n*x**2/(d*x+c),x)
[Out] Integral(x**2*(a + b/x)**n/(c + d*x), x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx} \, dx = \int \frac{\left(a+\frac{b}{x}\right)^n x^2}{dx+c} \, dx$$

```
[In] integrate((a+b/x)^n*x^2/(d*x+c),x, algorithm="maxima")
[Out] integrate((a + b/x)^n*x^2/(d*x + c), x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx} \, dx = \int \frac{\left(a+\frac{b}{x}\right)^n x^2}{dx+c} \, dx$$

```
[In] integrate((a+b/x)^n*x^2/(d*x+c),x, algorithm="giac")
[Out] integrate((a + b/x)^n*x^2/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{c+dx}\,dx=\int \frac{x^2\,\left(a+\frac{b}{x}\right)^n}{c+d\,x}\,dx$$

```
[In] int((x^2*(a + b/x)^n)/(c + d*x),x)
[Out] int((x^2*(a + b/x)^n)/(c + d*x), x)
```

3.286 $\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 131

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx} \, dx = \frac{\left(a+\frac{b}{x}\right)^{1+n} x}{ad} + \frac{c^2\left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(ac-bd)(1+n)} - \frac{(ac-bdn)\left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b}{ax}\right)}{a^2d^2(1+n)}$$

```
[Out] (a+b/x)^(1+n)*x/a/d+c^2*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(a
*c-b*d))/d^2/(a*c-b*d)/(1+n)-(-b*d*n+a*c)*(a+b/x)^(1+n)*hypergeom([1, 1+n],
[2+n],1+b/a/x)/a^2/d^2/(1+n)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {528, 382, 105, 162, 67, 70}

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx} \, dx = -\frac{\left(a+\frac{b}{x}\right)^{n+1}(ac-bdn) \text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b}{ax}+1\right)}{a^2d^2(n+1)} + \frac{c^2\left(a+\frac{b}{x}\right)^{n+1} \text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(n+1)(ac-bd)} + \frac{x\left(a+\frac{b}{x}\right)^{n+1}}{ad}$$

```
[In] Int[((a + b/x)^n*x)/(c + d*x),x]
```

```
[Out] ((a + b/x)^(1 + n)*x)/(a*d) + (c^2*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1
 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(d^2*(a*c - b*d)*(1 + n)) - ((a*c
- b*d*n)*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + b/(a*x)])
/(a^2*d^2*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
```

```
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\left(a+\frac{b}{x}\right)^n}{d+\frac{c}{x}}\,dx \\
&= -\text{Subst}\left(\int \frac{(a+bx)^n}{x^2(d+cx)}\,dx, x, \frac{1}{x}\right) \\
&= \frac{\left(a+\frac{b}{x}\right)^{1+n}x}{ad} + \frac{\text{Subst}\left(\int \frac{(a+bx)^n(ac-bdn-bcnx)}{x(d+cx)}\,dx, x, \frac{1}{x}\right)}{ad} \\
&= \frac{\left(a+\frac{b}{x}\right)^{1+n}x}{ad} - \frac{c^2\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(ac-bdn)\text{Subst}\left(\int \frac{(a+bx)^n}{x}\,dx, x, \frac{1}{x}\right)}{ad^2} \\
&= \frac{\left(a+\frac{b}{x}\right)^{1+n}x}{ad} + \frac{c^2\left(a+\frac{b}{x}\right)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(ac-bd)(1+n)} \\
&\quad - \frac{(ac-bdn)\left(a+\frac{b}{x}\right)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{b}{ax}\right)}{a^2d^2(1+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx}\,dx$$

$$= \frac{\left(a+\frac{b}{x}\right)^n(b+ax)\left(a^2c^2\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+(ac-bd)\left(ad(1+n)x+(-ac+\right.\right.}{a^2d^2(ac-bd)(1+n)x}$$

```
[In] Integrate[((a + b/x)^n*x)/(c + d*x),x]
```

```
[Out] ((a + b/x)^n*(b + a*x)*(a^2*c^2*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a +
b/x))/(a*c - b*d)] + (a*c - b*d)*(a*d*(1 + n)*x + (-(a*c) + b*d*n)*Hypergeo
metric2F1[1, 1 + n, 2 + n, 1 + b/(a*x)])))/(a^2*d^2*(a*c - b*d)*(1 + n)*x)
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{dx+c}dx$$

```
[In] int((a+b/x)^n*x/(d*x+c),x)
[Out] int((a+b/x)^n*x/(d*x+c),x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n x}{dx+c}\,dx$$

```
[In] integrate((a+b/x)^n*x/(d*x+c),x, algorithm="fricas")
[Out] integral(x*((a*x + b)/x)^n/(d*x + c), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx}\,dx = \int \frac{x\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx$$

```
[In] integrate((a+b/x)**n*x/(d*x+c),x)
[Out] Integral(x*(a + b/x)**n/(c + d*x), x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n x}{dx+c}\,dx$$

```
[In] integrate((a+b/x)^n*x/(d*x+c),x, algorithm="maxima")
[Out] integrate((a + b/x)^n*x/(d*x + c), x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n x}{dx+c}\,dx$$

```
[In] integrate((a+b/x)^n*x/(d*x+c),x, algorithm="giac")
[Out] integrate((a + b/x)^n*x/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{c+dx}\,dx = \int \frac{x\left(a+\frac{b}{x}\right)^n}{c+d\,x}\,dx$$

```
[In] int((x*(a + b/x)^n)/(c + d*x),x)
[Out] int((x*(a + b/x)^n)/(c + d*x), x)
```

3.287 $\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 101

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx} \, dx = -\frac{c\left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(ac-bd)(1+n)} + \frac{\left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b}{ax}\right)}{ad(1+n)}$$

```
[Out] -c*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/d/(a*c-b*d)/
(1+n)+(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],1+b/a/x)/a/d/(1+n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {445, 457, 88, 67, 70}

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx} \, dx = \frac{\left(a+\frac{b}{x}\right)^{n+1} \text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b}{ax}+1\right)}{ad(n+1)} - \frac{c\left(a+\frac{b}{x}\right)^{n+1} \text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(n+1)(ac-bd)}$$

```
[In] Int[(a + b/x)^n/(c + d*x),x]
```

```
[Out] -((c*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*
c - b*d)])/(d*(a*c - b*d)*(1 + n))) + ((a + b/x)^(1 + n)*Hypergeometric2F1[
1, 1 + n, 2 + n, 1 + b/(a*x)])/(a*d*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 445

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)x}\,dx\\
&= -\text{Subst}\left(\int \frac{(a+bx)^n}{x(d+cx)}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{(a+bx)^n}{x}\,dx, x, \frac{1}{x}\right)}{d}+\frac{c\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{d}\\
&= -\frac{c\left(a+\frac{b}{x}\right)^{1+n}\ {}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(ac-bd)(1+n)}+\frac{\left(a+\frac{b}{x}\right)^{1+n}\ {}_2F_1\left(1,1+n;2+n;1+\frac{b}{ax}\right)}{ad(1+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx$$
$$=\frac{\left(a+\frac{b}{x}\right)^n(b+ax)\left(ac\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+(-ac+bd)\,\text{Hypergeometric2F1}\,(1\ldots\right.}{ad(-ac+bd)(1+n)x}$$

```
[In] Integrate[(a + b/x)^n/(c + d*x),x]
```

```
[Out] ((a + b/x)^n*(b + a*x)*(a*c*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x)
)/(a*c - b*d)] + (-(a*c) + b*d)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + b/(a
*x)]))/(a*d*(-(a*c) + b*d)*(1 + n)*x)
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{dx+c}dx$$

```
[In] int((a+b/x)^n/(d*x+c),x)
```

```
[Out] int((a+b/x)^n/(d*x+c),x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{dx+c}\,dx$$

```
[In] integrate((a+b/x)^n/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^n/(d*x + c), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx$$

```
[In] integrate((a+b/x)**n/(d*x+c),x)
```

```
[Out] Integral((a + b/x)**n/(c + d*x), x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{dx+c}\,dx$$

```
[In] integrate((a+b/x)^n/(d*x+c),x, algorithm="maxima")
[Out] integrate((a + b/x)^n/(d*x + c), x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{dx+c}\,dx$$

```
[In] integrate((a+b/x)^n/(d*x+c),x, algorithm="giac")
[Out] integrate((a + b/x)^n/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{c+d\,x}\,dx$$

```
[In] int((a + b/x)^n/(c + d*x),x)
[Out] int((a + b/x)^n/(c + d*x), x)
```

3.288 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 54

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)}\,dx=\frac{\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)(1+n)}$$

```
[Out] (a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/(a*c-b*d)/(1+n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {528, 455, 70}

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)}\,dx=\frac{\left(a+\frac{b}{x}\right)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(n+1)(ac-bd)}$$

```
[In] Int[(a + b/x)^n/(x*(c + d*x)),x]
```

```
[Out] ((a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c -
b*d)])/((a*c - b*d)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\text{integral} = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)x^2}\,dx$$

$$= -\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)(1+n)}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)}\,dx = \frac{\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)(1+n)}$$

```
[In] Integrate[(a + b/x)^n/(x*(c + d*x)),x]
```

```
[Out] ((a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c -
b*d)])/((a*c - b*d)*(1 + n))
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x\,(dx+c)}dx$$

```
[In] int((a+b/x)^n/x/(d*x+c),x)
[Out] int((a+b/x)^n/x/(d*x+c),x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x}\,dx$$

```
[In] integrate((a+b/x)^n/x/(d*x+c),x, algorithm="fricas")
[Out] integral(((a*x + b)/x)^n/(d*x^2 + c*x), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{x\,(c+dx)}\,dx$$

```
[In] integrate((a+b/x)**n/x/(d*x+c),x)
[Out] Integral((a + b/x)**n/(x*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x}\,dx$$

```
[In] integrate((a+b/x)^n/x/(d*x+c),x, algorithm="maxima")
[Out] integrate((a + b/x)^n/((d*x + c)*x), x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x}\,dx$$

```
[In] integrate((a+b/x)^n/x/(d*x+c),x, algorithm="giac")
[Out] integrate((a + b/x)^n/((d*x + c)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{x\,\left(c+d\,x\right)}\,dx$$

```
[In] int((a + b/x)^n/(x*(c + d*x)),x)
[Out] int((a + b/x)^n/(x*(c + d*x)), x)
```

3.289 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 84

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}\,dx = -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc(1+n)} - \frac{d\left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(ac-bd)(1+n)}$$

```
[Out] -(a+b/x)^(1+n)/b/c/(1+n)-d*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)
/(a*c-b*d))/c/(a*c-b*d)/(1+n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {528, 457, 81, 70}

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}\,dx = -\frac{d\left(a+\frac{b}{x}\right)^{n+1} \text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(n+1)(ac-bd)} - \frac{\left(a+\frac{b}{x}\right)^{n+1}}{bc(n+1)}$$

```
[In] Int[(a + b/x)^n/(x^2*(c + d*x)),x]
```

```
[Out] -((a + b/x)^(1 + n)/(b*c*(1 + n))) - (d*(a + b/x)^(1 + n)*Hypergeometric2F1
[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(c*(a*c - b*d)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)x^3}\,dx \\
&= -\text{Subst}\left(\int \frac{x(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc(1+n)} + \frac{d\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{c} \\
&= -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc(1+n)} - \frac{d\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(ac-bd)(1+n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}\,dx = \frac{\left(a+\frac{b}{x}\right)^n (b+ax)\left(ac-bd+bd\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)\right)}{bc(-ac+bd)(1+n)x}$$

```
[In] Integrate[(a + b/x)^n/(x^2*(c + d*x)),x]
[Out] ((a + b/x)^n*(b + a*x)*(a*c - b*d + b*d*Hypergeometric2F1[1, 1 + n, 2 + n,
(c*(a + b/x))/(a*c - b*d)]))/(b*c*(-(a*c) + b*d)*(1 + n)*x)
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2\,(dx+c)}dx$$

```
[In] int((a+b/x)^n/x^2/(d*x+c),x)
[Out] int((a+b/x)^n/x^2/(d*x+c),x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^2}\,dx$$

```
[In] integrate((a+b/x)^n/x^2/(d*x+c),x, algorithm="fricas")
[Out] integral(((a*x + b)/x)^n/(d*x^3 + c*x^2), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{x^2\,(c+dx)}\,dx$$

```
[In] integrate((a+b/x)**n/x**2/(d*x+c),x)
[Out] Integral((a + b/x)**n/(x**2*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^2}\,dx$$

```
[In] integrate((a+b/x)^n/x^2/(d*x+c),x, algorithm="maxima")
[Out] integrate((a + b/x)^n/((d*x + c)*x^2), x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^2}\,dx$$

```
[In] integrate((a+b/x)^n/x^2/(d*x+c),x, algorithm="giac")
[Out] integrate((a + b/x)^n/((d*x + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{x^2\,\left(c+d\,x\right)}\,dx$$

```
[In] int((a + b/x)^n/(x^2*(c + d*x)),x)
[Out] int((a + b/x)^n/(x^2*(c + d*x)), x)
```

3.290 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 115

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}\,dx = \frac{(ac+bd)\left(a+\frac{b}{x}\right)^{1+n}}{b^2c^2(1+n)} - \frac{\left(a+\frac{b}{x}\right)^{2+n}}{b^2c(2+n)} + \frac{d^2\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(ac-bd)(1+n)}$$

```
[Out] (a*c+b*d)*(a+b/x)^(1+n)/b^2/c^2/(1+n)-(a+b/x)^(2+n)/b^2/c/(2+n)+d^2*(a+b/x)
^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/c^2/(a*c-b*d)/(1+n)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {528, 457, 90, 70}

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}\,dx = \frac{(ac+bd)\left(a+\frac{b}{x}\right)^{n+1}}{b^2c^2(n+1)} - \frac{\left(a+\frac{b}{x}\right)^{n+2}}{b^2c(n+2)} + \frac{d^2\left(a+\frac{b}{x}\right)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(n+1)(ac-bd)}$$

```
[In] Int[(a + b/x)^n/(x^3*(c + d*x)),x]
```

```
[Out] ((a*c + b*d)*(a + b/x)^(1 + n))/(b^2*c^2*(1 + n)) - (a + b/x)^(2 + n)/(b^2*
c*(2 + n)) + (d^2*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(
a + b/x))/(a*c - b*d)])/(c^2*(a*c - b*d)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)x^4}\,dx\\
&= -\text{Subst}\left(\int \frac{x^2(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \left(\frac{(-ac-bd)(a+bx)^n}{bc^2}+\frac{(a+bx)^{1+n}}{bc}+\frac{d^2(a+bx)^n}{c^2(d+cx)}\right)dx, x, \frac{1}{x}\right)\\
&= \frac{(ac+bd)\left(a+\frac{b}{x}\right)^{1+n}}{b^2c^2(1+n)}-\frac{\left(a+\frac{b}{x}\right)^{2+n}}{b^2c(2+n)}-\frac{d^2\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{c^2}\\
&= \frac{(ac+bd)\left(a+\frac{b}{x}\right)^{1+n}}{b^2c^2(1+n)}-\frac{\left(a+\frac{b}{x}\right)^{2+n}}{b^2c(2+n)}+\frac{d^2\left(a+\frac{b}{x}\right)^{1+n}{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(ac-bd)(1+n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}\,dx =$$
$$-\frac{\left(a+\frac{b}{x}\right)^n(b+ax)\left((ac-bd)(-bc(1+n)+acx+bd(2+n)x)+b^2d^2(2+n)x\,\text{Hypergeometric2F1}\left(1,1\right.\right.}{b^2c^2(-ac+bd)(1+n)(2+n)x^2}$$

```
[In] Integrate[(a + b/x)^n/(x^3*(c + d*x)),x]
```

```
[Out] -(((a + b/x)^n*(b + a*x)*((a*c - b*d)*(-(b*c*(1 + n)) + a*c*x + b*d*(2 + n)
*x) + b^2*d^2*(2 + n)*x*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a
*c - b*d)]))/(b^2*c^2*(-(a*c) + b*d)*(1 + n)*(2 + n)*x^2))
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3\,(dx+c)}dx$$

```
[In] int((a+b/x)^n/x^3/(d*x+c),x)
```

```
[Out] int((a+b/x)^n/x^3/(d*x+c),x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^3}\,dx$$

```
[In] integrate((a+b/x)^n/x^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^n/(d*x^4 + c*x^3), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{x^3\,(c+dx)}\,dx$$

```
[In] integrate((a+b/x)**n/x**3/(d*x+c),x)
```

```
[Out] Integral((a + b/x)**n/(x**3*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^3}\,dx$$

```
[In] integrate((a+b/x)^n/x^3/(d*x+c),x, algorithm="maxima")
[Out] integrate((a + b/x)^n/((d*x + c)*x^3), x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^3}\,dx$$

```
[In] integrate((a+b/x)^n/x^3/(d*x+c),x, algorithm="giac")
[Out] integrate((a + b/x)^n/((d*x + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3\,\left(c+d\,x\right)}\,dx$$

```
[In] int((a + b/x)^n/(x^3*(c + d*x)),x)
[Out] int((a + b/x)^n/(x^3*(c + d*x)), x)
```

3.291 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 207

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx = \frac{(ac+bd)\left(a^2c^2+b^2d^2\right)\left(a+\frac{b}{x}\right)^{1+n}}{b^4c^4(1+n)} - \frac{\left(3a^2c^2+2abcd+b^2d^2\right)\left(a+\frac{b}{x}\right)^{2+n}}{b^4c^3(2+n)} + \frac{(3ac+bd)\left(a+\frac{b}{x}\right)^{3+n}}{b^4c^2(3+n)} - \frac{\left(a+\frac{b}{x}\right)^{4+n}}{b^4c(4+n)} + \frac{d^4\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^4(ac-bd)(1+n)}$$

```
[Out] (a*c+b*d)*(a^2*c^2+b^2*d^2)*(a+b/x)^(1+n)/b^4/c^4/(1+n)-(3*a^2*c^2+2*a*b*c*
d+b^2*d^2)*(a+b/x)^(2+n)/b^4/c^3/(2+n)+(3*a*c+b*d)*(a+b/x)^(3+n)/b^4/c^2/(3
+n)-(a+b/x)^(4+n)/b^4/c/(4+n)+d^4*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*
(a+b/x)/(a*c-b*d))/c^4/(a*c-b*d)/(1+n)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

= {528, 457, 90, 70}

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx = \frac{(ac+bd)\left(a^2c^2+b^2d^2\right)\left(a+\frac{b}{x}\right)^{n+1}}{b^4c^4(n+1)} - \frac{\left(3a^2c^2+2abcd+b^2d^2\right)\left(a+\frac{b}{x}\right)^{n+2}}{b^4c^3(n+2)} + \frac{(3ac+bd)\left(a+\frac{b}{x}\right)^{n+3}}{b^4c^2(n+3)} - \frac{\left(a+\frac{b}{x}\right)^{n+4}}{b^4c(n+4)} + \frac{d^4\left(a+\frac{b}{x}\right)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^4(n+1)(ac-bd)}$$

```
[In] Int[(a + b/x)^n/(x^5*(c + d*x)),x]
```

```
[Out] ((a*c + b*d)*(a^2*c^2 + b^2*d^2)*(a + b/x)^(1 + n))/(b^4*c^4*(1 + n)) - ((3
*a^2*c^2 + 2*a*b*c*d + b^2*d^2)*(a + b/x)^(2 + n))/(b^4*c^3*(2 + n)) + ((3*
a*c + b*d)*(a + b/x)^(3 + n))/(b^4*c^2*(3 + n)) - (a + b/x)^(4 + n)/(b^4*c*
(4 + n)) + (d^4*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a
+ b/x))/(a*c - b*d)])/(c^4*(a*c - b*d)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)x^6}\,dx \\
&= -\text{Subst}\left(\int \frac{x^4(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right) \\
&= -\text{Subst}\left(\int \left(\frac{(ac+bd)\left(-a^2c^2-b^2d^2\right)(a+bx)^n}{b^3c^4} + \frac{\left(3a^2c^2+2abcd+b^2d^2\right)(a+bx)^{1+n}}{b^3c^3} + \frac{(-3ac-bd)(a+bx)^{2+n}}{b^2c^2} + \frac{(a+bx)^{3+n}}{b^3c} + \frac{d^4(a+bx)^n}{c^4(d+cx)}\right) dx, x, \frac{1}{x}\right) \\
&= \frac{(ac+bd)\left(a^2c^2+b^2d^2\right)\left(a+\frac{b}{x}\right)^{1+n}}{b^4c^4(1+n)} - \frac{\left(3a^2c^2+2abcd+b^2d^2\right)\left(a+\frac{b}{x}\right)^{2+n}}{b^4c^3(2+n)} + \frac{(3ac+bd)\left(a+\frac{b}{x}\right)^{3+n}}{b^4c^2(3+n)} - \frac{\left(a+\frac{b}{x}\right)^{4+n}}{b^4c(4+n)} - \frac{d^4\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{c^4} \\
&= \frac{(ac+bd)\left(a^2c^2+b^2d^2\right)\left(a+\frac{b}{x}\right)^{1+n}}{b^4c^4(1+n)} - \frac{\left(3a^2c^2+2abcd+b^2d^2\right)\left(a+\frac{b}{x}\right)^{2+n}}{b^4c^3(2+n)} + \frac{(3ac+bd)\left(a+\frac{b}{x}\right)^{3+n}}{b^4c^2(3+n)} - \frac{\left(a+\frac{b}{x}\right)^{4+n}}{b^4c(4+n)} + \frac{d^4\left(a+\frac{b}{x}\right)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^4(ac-bd)(1+n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.89

$$
\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx = \frac{\left(a+\frac{b}{x}\right)^{1+n}\left(\frac{(ac+bd)\left(a^2c^2+b^2d^2\right)}{b^4(1+n)} - \frac{c\left(3a^2c^2+2abcd+b^2d^2\right)\left(a+\frac{b}{x}\right)}{b^4(2+n)} + \frac{c^2(3ac+bd)\left(a+\frac{b}{x}\right)^2}{b^4(3+n)} - \frac{c^3\left(a+\frac{b}{x}\right)^3}{b^4(4+n)} + \frac{d^4\,\text{Hypergeometric2F1}\left(1\right.}{(ac-bd)(}\right.}{c^4}
$$

```
[In] Integrate[(a + b/x)^n/(x^5*(c + d*x)),x]
[Out] ((a + b/x)^(1 + n)*(((a*c + b*d)*(a^2*c^2 + b^2*d^2))/(b^4*(1 + n)) - (c*(3
*a^2*c^2 + 2*a*b*c*d + b^2*d^2)*(a + b/x))/(b^4*(2 + n)) + (c^2*(3*a*c + b*
d)*(a + b/x)^2)/(b^4*(3 + n)) - (c^3*(a + b/x)^3)/(b^4*(4 + n)) + (d^4*Hype
rgeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/((a*c - b*d)*(1
+ n))))/c^4
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5\left(dx+c\right)}dx$$

```
[In] int((a+b/x)^n/x^5/(d*x+c),x)
[Out] int((a+b/x)^n/x^5/(d*x+c),x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^5}\,dx$$

```
[In] integrate((a+b/x)^n/x^5/(d*x+c),x, algorithm="fricas")
[Out] integral(((a*x + b)/x)^n/(d*x^6 + c*x^5), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{x^5\left(c+dx\right)}\,dx$$

```
[In] integrate((a+b/x)**n/x**5/(d*x+c),x)
[Out] Integral((a + b/x)**n/(x**5*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^5}\,dx$$

```
[In] integrate((a+b/x)^n/x^5/(d*x+c),x, algorithm="maxima")
[Out] integrate((a + b/x)^n/((d*x + c)*x^5), x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)x^5}\,dx$$

```
[In] integrate((a+b/x)^n/x^5/(d*x+c),x, algorithm="giac")
[Out] integrate((a + b/x)^n/((d*x + c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5(c+dx)}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{x^5\,\left(c+d\,x\right)}\,dx$$

```
[In] int((a + b/x)^n/(x^5*(c + d*x)),x)
[Out] int((a + b/x)^n/(x^5*(c + d*x)), x)
```

3.292 $\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 73

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2} \, dx = -\frac{\left(a+\frac{b}{x}\right)^n \left(1+\frac{b}{ax}\right)^{-n} x^{-1+m} \text{AppellF1}\left(1-m,-n,2,2-m,-\frac{b}{ax},-\frac{c}{dx}\right)}{d^2(1-m)}$$

```
[Out] -(a+b/x)^n*x^(-1+m)*AppellF1(1-m,-n,2,2-m,-b/a/x,-c/d/x)/d^2/(1-m)/((1+b/a/
x)^n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {528, 511, 140, 138}

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2} \, dx = -\frac{x^{m-1}\left(a+\frac{b}{x}\right)^n \left(\frac{b}{ax}+1\right)^{-n} \text{AppellF1}\left(1-m,-n,2,2-m,-\frac{b}{ax},-\frac{c}{dx}\right)}{d^2(1-m)}$$

```
[In] Int[((a + b/x)^n*x^m)/(c + d*x)^2,x]
```

```
[Out] -(((a + b/x)^n*x^(-1 + m)*AppellF1[1 - m, -n, 2, 2 - m, -(b/(a*x)), -(c/(d*
x))])/(d^2*(1 - m)*(1 + b/(a*x))^n))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
```

```
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 511

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Dist[(-(e*x)^m)*(x^(-1))^m, Subst[Int[(a + b/x^n)^p*((
c + d/x^n)^q/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q},
 x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\left(a+\frac{b}{x}\right)^n x^{-2+m}}{\left(d+\frac{c}{x}\right)^2}\,dx\\
&= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-m}(a+bx)^n}{(d+cx)^2}\,dx,x,\frac{1}{x}\right)\right)\\
&= -\left(\left(\left(a+\frac{b}{x}\right)^n\left(1+\frac{b}{ax}\right)^{-n}\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-m}\left(1+\frac{bx}{a}\right)^n}{(d+cx)^2}\,dx,x,\frac{1}{x}\right)\right)\\
&= -\frac{\left(a+\frac{b}{x}\right)^n\left(1+\frac{b}{ax}\right)^{-n}x^{-1+m}F_1\left(1-m;-n,2;2-m;-\frac{b}{ax},-\frac{c}{dx}\right)}{d^2(1-m)}
\end{aligned}$$

Mathematica [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2}\,dx$$

```
[In] Integrate[((a + b/x)^n*x^m)/(c + d*x)^2,x]
[Out] Integrate[((a + b/x)^n*x^m)/(c + d*x)^2, x]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{\left(dx+c\right)^2}dx$$

```
[In] int((a+b/x)^n*x^m/(d*x+c)^2,x)
[Out] int((a+b/x)^n*x^m/(d*x+c)^2,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{\left(dx+c\right)^2}\,dx$$

```
[In] integrate((a+b/x)^n*x^m/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(x^m*((a*x + b)/x)^n/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2}\,dx=\int \frac{x^m\left(a+\frac{b}{x}\right)^n}{\left(c+dx\right)^2}\,dx$$

```
[In] integrate((a+b/x)**n*x**m/(d*x+c)**2,x)
[Out] Integral(x**m*(a + b/x)**n/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(dx+c)^2}\,dx$$

```
[In] integrate((a+b/x)^n*x^m/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((a + b/x)^n*x^m/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(dx+c)^2}\,dx$$

```
[In] integrate((a+b/x)^n*x^m/(d*x+c)^2,x, algorithm="giac")
[Out] integrate((a + b/x)^n*x^m/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^m}{(c+dx)^2}\,dx = \int \frac{x^m \left(a+\frac{b}{x}\right)^n}{(c+d\,x)^2}\,dx$$

```
[In] int((x^m*(a + b/x)^n)/(c + d*x)^2,x)
[Out] int((x^m*(a + b/x)^n)/(c + d*x)^2, x)
```

3.293 $\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 202

$$
\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2}\,dx
&= \frac{c(2ac-bd)\left(a+\frac{b}{x}\right)^{1+n}}{ad^2(ac-bd)\left(d+\frac{c}{x}\right)}+\frac{\left(a+\frac{b}{x}\right)^{1+n}x}{ad\left(d+\frac{c}{x}\right)} \\
&\quad+\frac{c^2(2ac-bd(2-n))\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^3(ac-bd)^2(1+n)} \\
&\quad-\frac{(2ac-bdn)\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b}{ax}\right)}{a^2d^3(1+n)}
\end{aligned}
$$

```
[Out] c*(2*a*c-b*d)*(a+b/x)^(1+n)/a/d^2/(a*c-b*d)/(d+c/x)+(a+b/x)^(1+n)*x/a/d/(d+
c/x)+c^2*(2*a*c-b*d*(2-n))*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)
/(a*c-b*d))/d^3/(a*c-b*d)^2/(1+n)-(-b*d*n+2*a*c)*(a+b/x)^(1+n)*hypergeom([1
, 1+n],[2+n],1+b/a/x)/a^2/d^3/(1+n)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used

$= \{528, 382, 105, 156, 162, 67, 70\}$

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2}\,dx$$
$$= -\frac{\left(a+\frac{b}{x}\right)^{n+1}(2ac-bdn)\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b}{ax}+1\right)}{a^2d^3(n+1)} + \frac{c^2\left(a+\frac{b}{x}\right)^{n+1}(2ac-bd(2-n))\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^3(n+1)(ac-bd)^2} + \frac{c(2ac-bd)\left(a+\frac{b}{x}\right)^{n+1}}{ad^2\left(\frac{c}{x}+d\right)(ac-bd)} + \frac{x\left(a+\frac{b}{x}\right)^{n+1}}{ad\left(\frac{c}{x}+d\right)}$$

```
[In] Int[((a + b/x)^n*x^2)/(c + d*x)^2,x]
```

```
[Out] (c*(2*a*c - b*d)*(a + b/x)^(1 + n))/(a*d^2*(a*c - b*d)*(d + c/x)) + ((a + b
/x)^(1 + n)*x)/(a*d*(d + c/x)) + (c^2*(2*a*c - b*d*(2 - n))*(a + b/x)^(1 +
n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(d^3*(a*c
 - b*d)^2*(1 + n)) - ((2*a*c - b*d*n)*(a + b/x)^(1 + n)*Hypergeometric2F1[1
, 1 + n, 2 + n, 1 + b/(a*x)])/(a^2*d^3*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2}\,dx \\
&= -\text{Subst}\left(\int \frac{(a+bx)^n}{x^2(d+cx)^2}\,dx, x, \frac{1}{x}\right) \\
&= \frac{\left(a+\frac{b}{x}\right)^{1+n}x}{ad\left(d+\frac{c}{x}\right)}+\frac{\text{Subst}\left(\int \frac{(a+bx)^n(2ac-bdn+bc(1-n)x)}{x(d+cx)^2}\,dx, x, \frac{1}{x}\right)}{ad} \\
&= \frac{c(2ac-bd)\left(a+\frac{b}{x}\right)^{1+n}}{ad^2(ac-bd)\left(d+\frac{c}{x}\right)}+\frac{\left(a+\frac{b}{x}\right)^{1+n}x}{ad\left(d+\frac{c}{x}\right)}+\frac{\text{Subst}\left(\int \frac{(a+bx)^n((ac-bd)(2ac-bdn)-bc(2ac-bd)nx)}{x(d+cx)}\,dx, x, \frac{1}{x}\right)}{ad^2(ac-bd)}
\end{aligned}$$

$$= \frac{c(2ac-bd)\left(a+\frac{b}{x}\right)^{1+n}}{ad^2(ac-bd)\left(d+\frac{c}{x}\right)} + \frac{\left(a+\frac{b}{x}\right)^{1+n}x}{ad\left(d+\frac{c}{x}\right)} - \frac{(c^2(2ac-bd(2-n)))\,\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{d^3(ac-bd)} + \frac{(2ac-bdn)\text{Subst}\left(\int \frac{(a+bx)^n}{x}\,dx, x, \frac{1}{x}\right)}{ad^3}$$

$$= \frac{c(2ac-bd)\left(a+\frac{b}{x}\right)^{1+n}}{ad^2(ac-bd)\left(d+\frac{c}{x}\right)} + \frac{\left(a+\frac{b}{x}\right)^{1+n}x}{ad\left(d+\frac{c}{x}\right)} + \frac{c^2(2ac-bd(2-n))\left(a+\frac{b}{x}\right)^{1+n}{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^3(ac-bd)^2(1+n)} - \frac{(2ac-bdn)\left(a+\frac{b}{x}\right)^{1+n}{}_2F_1\left(1,1+n;2+n;1+\frac{b}{ax}\right)}{a^2d^3(1+n)}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2}\,dx$$

$$= \frac{\left(a+\frac{b}{x}\right)^{1+n}\left(acd(ac-bd)(2ac-bd)(1+n)x + ad^2(ac-bd)^2(1+n)x^2 + (c+dx)\left(a^2c^2(2ac+bd(-2+n\right.\right.}{a^2d^3(ac}$$

```
[In] Integrate[((a + b/x)^n*x^2)/(c + d*x)^2,x]
```

```
[Out] ((a + b/x)^(1 + n)*(a*c*d*(a*c - b*d)*(2*a*c - b*d)*(1 + n)*x + a*d^2*(a*c
- b*d)^2*(1 + n)*x^2 + (c + d*x)*(a^2*c^2*(2*a*c + b*d*(-2 + n))*Hypergeome
tric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)] - (a*c - b*d)^2*(2*a*c
- b*d*n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + b/(a*x)])))/(a^2*d^3*(a*c -
 b*d)^2*(1 + n)*(c + d*x))
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(dx+c)^2}dx$$

```
[In] int((a+b/x)^n*x^2/(d*x+c)^2,x)
```

```
[Out] int((a+b/x)^n*x^2/(d*x+c)^2,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(dx+c)^2}\,dx$$

```
[In] integrate((a+b/x)^n*x^2/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(x^2*((a*x + b)/x)^n/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2}\,dx=\int \frac{x^2\left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx$$

```
[In] integrate((a+b/x)**n*x**2/(d*x+c)**2,x)
[Out] Integral(x**2*(a + b/x)**n/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(dx+c)^2}\,dx$$

```
[In] integrate((a+b/x)^n*x^2/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((a + b/x)^n*x^2/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(dx+c)^2}\,dx$$

```
[In] integrate((a+b/x)^n*x^2/(d*x+c)^2,x, algorithm="giac")
[Out] integrate((a + b/x)^n*x^2/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x^2}{(c+dx)^2}\,dx = \int \frac{x^2\,\left(a+\frac{b}{x}\right)^n}{\left(c+d\,x\right)^2}\,dx$$

```
[In] int((x^2*(a + b/x)^n)/(c + d*x)^2,x)
[Out] int((x^2*(a + b/x)^n)/(c + d*x)^2, x)
```

3.294 $\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 150

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2}\,dx &= -\frac{c\left(a+\frac{b}{x}\right)^{1+n}}{d(ac-bd)\left(d+\frac{c}{x}\right)} \\ &\quad -\frac{c(ac-bd(1-n))\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(ac-bd)^2(1+n)} \\ &\quad +\frac{\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b}{ax}\right)}{ad^2(1+n)}\end{aligned}$$

```
[Out] -c*(a+b/x)^(1+n)/d/(a*c-b*d)/(d+c/x)-c*(a*c-b*d*(1-n))*(a+b/x)^(1+n)*hyperg
eom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/d^2/(a*c-b*d)^2/(1+n)+(a+b/x)^(1+n)
*hypergeom([1, 1+n],[2+n],1+b/a/x)/a/d^2/(1+n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{528, 457, 105, 162, 67, 70\}$

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2}\,dx = -\frac{c\left(a+\frac{b}{x}\right)^{n+1}(ac-bd(1-n))\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(n+1)(ac-bd)^2} - \frac{c\left(a+\frac{b}{x}\right)^{n+1}}{d\left(\frac{c}{x}+d\right)(ac-bd)} + \frac{\left(a+\frac{b}{x}\right)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b}{ax}+1\right)}{ad^2(n+1)}$$

```
[In] Int[((a + b/x)^n*x)/(c + d*x)^2,x]
```

```
[Out] -((c*(a + b/x)^(1 + n))/(d*(a*c - b*d)*(d + c/x))) - (c*(a*c - b*d*(1 - n))
*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c -
b*d)])/(d^2*(a*c - b*d)^2*(1 + n)) + ((a + b/x)^(1 + n)*Hypergeometric2F1[1
, 1 + n, 2 + n, 1 + b/(a*x)])/(a*d^2*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2 x}\,dx\\
&= -\text{Subst}\left(\int \frac{(a+bx)^n}{x(d+cx)^2}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{c\left(a+\frac{b}{x}\right)^{1+n}}{d(ac-bd)\left(d+\frac{c}{x}\right)}-\frac{\text{Subst}\left(\int \frac{(a+bx)^n(ac-bd-bcnx)}{x(d+cx)}\,dx,x,\frac{1}{x}\right)}{d(ac-bd)}\\
&= -\frac{c\left(a+\frac{b}{x}\right)^{1+n}}{d(ac-bd)\left(d+\frac{c}{x}\right)}-\frac{\text{Subst}\left(\int \frac{(a+bx)^n}{x}\,dx,x,\frac{1}{x}\right)}{d^2}\\
&\quad+\frac{(c(ac-bd(1-n)))\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx,x,\frac{1}{x}\right)}{d^2(ac-bd)}\\
&= -\frac{c\left(a+\frac{b}{x}\right)^{1+n}}{d(ac-bd)\left(d+\frac{c}{x}\right)}-\frac{c(ac-bd(1-n))\left(a+\frac{b}{x}\right)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d^2(ac-bd)^2(1+n)}\\
&\quad+\frac{\left(a+\frac{b}{x}\right)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{b}{ax}\right)}{ad^2(1+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2}\,dx = \frac{\left(a+\frac{b}{x}\right)^{1+n}\left(-\frac{cdx}{(ac-bd)(c+dx)}-\frac{c(ac+bd(-1+n))\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)^2(1+n)}+\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{b}{ax}\right)}{a(1+n)}\right)}{d^2}$$

```
[In] Integrate[((a + b/x)^n*x)/(c + d*x)^2,x]
[Out] ((a + b/x)^(1 + n)*(-((c*d*x)/((a*c - b*d)*(c + d*x))) - (c*(a*c + b*d*(-1
+ n))*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/((a*c
- b*d)^2*(1 + n)) + Hypergeometric2F1[1, 1 + n, 2 + n, 1 + b/(a*x)]/(a*(1 +
 n))))/d^2
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{\left(dx+c\right)^2}dx$$

```
[In] int((a+b/x)^n*x/(d*x+c)^2,x)
[Out] int((a+b/x)^n*x/(d*x+c)^2,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n x}{\left(dx+c\right)^2}\,dx$$

```
[In] integrate((a+b/x)^n*x/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(x*((a*x + b)/x)^n/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2}\,dx = \int \frac{x\left(a+\frac{b}{x}\right)^n}{\left(c+dx\right)^2}\,dx$$

```
[In] integrate((a+b/x)**n*x/(d*x+c)**2,x)
[Out] Integral(x*(a + b/x)**n/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n x}{\left(dx+c\right)^2}\,dx$$

```
[In] integrate((a+b/x)^n*x/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((a + b/x)^n*x/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n x}{\left(dx+c\right)^2}\,dx$$

```
[In] integrate((a+b/x)^n*x/(d*x+c)^2,x, algorithm="giac")
[Out] integrate((a + b/x)^n*x/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n x}{(c+dx)^2}\,dx = \int \frac{x\,\left(a+\frac{b}{x}\right)^n}{\left(c+d\,x\right)^2}\,dx$$

```
[In] int((x*(a + b/x)^n)/(c + d*x)^2,x)
[Out] int((x*(a + b/x)^n)/(c + d*x)^2, x)
```

3.295 $\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 56

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2} dx = -\frac{b\left(a+\frac{b}{x}\right)^{1+n} \text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)^2(1+n)}$$

```
[Out] -b*(a+b/x)^(1+n)*hypergeom([2, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/(a*c-b*d)^2/
(1+n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {445, 455, 70}

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2} dx = -\frac{b\left(a+\frac{b}{x}\right)^{n+1} \text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(n+1)(ac-bd)^2}$$

```
[In] Int[(a + b/x)^n/(c + d*x)^2,x]
```

```
[Out] -((b*(a + b/x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (c*(a + b/x))/(a*
c - b*d)])/((a*c - b*d)^2*(1 + n)))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 445

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2x^2}\,dx\\&=-\text{Subst}\left(\int\frac{(a+bx)^n}{(d+cx)^2}\,dx,x,\frac{1}{x}\right)\\&=-\frac{b\left(a+\frac{b}{x}\right)^{1+n}{}_2F_1\left(2,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)^2(1+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int\frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx=-\frac{b\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(2,1+n,2+n,-\frac{c\left(a+\frac{b}{x}\right)}{-ac+bd}\right)}{(-ac+bd)^2(1+n)}$$

```
[In] Integrate[(a + b/x)^n/(c + d*x)^2,x]
```

```
[Out] -((b*(a + b/x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, -((c*(a + b/x))/(
-(a*c) + b*d))])/((-(a*c) + b*d)^2*(1 + n)))
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{\left(dx+c\right)^2}dx$$

```
[In] int((a+b/x)^n/(d*x+c)^2,x)
[Out] int((a+b/x)^n/(d*x+c)^2,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(dx+c\right)^2}\,dx$$

```
[In] integrate((a+b/x)^n/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(((a*x + b)/x)^n/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(c+dx\right)^2}\,dx$$

```
[In] integrate((a+b/x)**n/(d*x+c)**2,x)
[Out] Integral((a + b/x)**n/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(dx+c\right)^2}\,dx$$

```
[In] integrate((a+b/x)^n/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((a + b/x)^n/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)^2}\,dx$$

[In] integrate((a+b/x)^n/(d*x+c)^2,x, algorithm="giac")

[Out] integrate((a + b/x)^n/(d*x + c)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^n}{(c+d\,x)^2}\,dx$$

[In] int((a + b/x)^n/(c + d*x)^2,x)

[Out] int((a + b/x)^n/(c + d*x)^2, x)

3.296 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 105

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2}\,dx = -\frac{d\left(a+\frac{b}{x}\right)^{1+n}}{c(ac-bd)\left(d+\frac{c}{x}\right)} + \frac{(ac-bd(1+n))\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(ac-bd)^2(1+n)}$$

```
[Out] -d*(a+b/x)^(1+n)/c/(a*c-b*d)/(d+c/x)+(a*c-b*d*(1+n))*(a+b/x)^(1+n)*hypergeo
m([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/c/(a*c-b*d)^2/(1+n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {528, 457, 79, 70}

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2}\,dx = \frac{\left(a+\frac{b}{x}\right)^{n+1}(ac-bd(n+1))\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(n+1)(ac-bd)^2} - \frac{d\left(a+\frac{b}{x}\right)^{n+1}}{c\left(\frac{c}{x}+d\right)(ac-bd)}$$

```
[In] Int[(a + b/x)^n/(x*(c + d*x)^2),x]
```

```
[Out] -((d*(a + b/x)^(1 + n))/(c*(a*c - b*d)*(d + c/x))) + ((a*c - b*d*(1 + n))*(
a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*
d)])/(c*(a*c - b*d)^2*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\text{integral} = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2 x^3}\,dx$$

$$= -\text{Subst}\left(\int \frac{x(a+bx)^n}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{d\left(a+\frac{b}{x}\right)^{1+n}}{c(ac-bd)\left(d+\frac{c}{x}\right)} - \frac{(ac-bd(1+n))\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{c(ac-bd)}$$

$$= -\frac{d\left(a+\frac{b}{x}\right)^{1+n}}{c(ac-bd)\left(d+\frac{c}{x}\right)} + \frac{(ac-bd(1+n))\left(a+\frac{b}{x}\right)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(ac-bd)^2(1+n)}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2} dx = \frac{\left(a+\frac{b}{x}\right)^{1+n}\left(\frac{d(-ac+bd)x}{c+dx} + \frac{(ac-bd(1+n))\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{1+n}\right)}{c(ac-bd)^2}$$

```
[In] Integrate[(a + b/x)^n/(x*(c + d*x)^2),x]
[Out] ((a + b/x)^(1 + n)*((d*(-(a*c) + b*d)*x)/(c + d*x) + ((a*c - b*d*(1 + n))*H
ypergeometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(1 + n)))/(c*
(a*c - b*d)^2)
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x\left(dx+c\right)^2} dx$$

```
[In] int((a+b/x)^n/x/(d*x+c)^2,x)
[Out] int((a+b/x)^n/x/(d*x+c)^2,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2} dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(dx+c\right)^2 x} dx$$

```
[In] integrate((a+b/x)^n/x/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(((a*x + b)/x)^n/(d^2*x^3 + 2*c*d*x^2 + c^2*x), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{x\left(c+dx\right)^2}\,dx$$

```
[In] integrate((a+b/x)**n/x/(d*x+c)**2,x)
[Out] Integral((a + b/x)**n/(x*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(dx+c\right)^2 x}\,dx$$

```
[In] integrate((a+b/x)^n/x/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((a + b/x)^n/((d*x + c)^2*x), x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(dx+c\right)^2 x}\,dx$$

```
[In] integrate((a+b/x)^n/x/(d*x+c)^2,x, algorithm="giac")
[Out] integrate((a + b/x)^n/((d*x + c)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{x\left(c+d\,x\right)^2}\,dx$$

```
[In] int((a + b/x)^n/(x*(c + d*x)^2),x)
[Out] int((a + b/x)^n/(x*(c + d*x)^2), x)
```

3.297 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 133

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2} dx = -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc^2(1+n)}+\frac{d^2\left(a+\frac{b}{x}\right)^{1+n}}{c^2(ac-bd)\left(d+\frac{c}{x}\right)} - \frac{d(2ac-bd(2+n))\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(ac-bd)^2(1+n)}$$

```
[Out] -(a+b/x)^(1+n)/b/c^2/(1+n)+d^2*(a+b/x)^(1+n)/c^2/(a*c-b*d)/(d+c/x)-d*(2*a*c
-b*d*(2+n))*(a+b/x)^(1+n)*hypergeom([1, 1+n],[2+n],c*(a+b/x)/(a*c-b*d))/c^2
/(a*c-b*d)^2/(1+n)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {528, 457, 91, 81, 70}

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2}\,dx$$
$$= \frac{d^2\left(a+\frac{b}{x}\right)^{n+1}}{c^2\left(\frac{c}{x}+d\right)(ac-bd)} - \frac{d\left(a+\frac{b}{x}\right)^{n+1}(2ac-bd(n+2))\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(n+1)(ac-bd)^2} - \frac{\left(a+\frac{b}{x}\right)^{n+1}}{bc^2(n+1)}$$

```
[In] Int[(a + b/x)^n/(x^2*(c + d*x)^2),x]
```

```
[Out] -((a + b/x)^(1 + n)/(b*c^2*(1 + n))) + (d^2*(a + b/x)^(1 + n))/(c^2*(a*c -
b*d)*(d + c/x)) - (d*(2*a*c - b*d*(2 + n))*(a + b/x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(c^2*(a*c - b*d)^2*(1 + n)
)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
```

```
+ 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
(EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
1])))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\text{integral} = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2 x^4}\,dx$$

$$= -\text{Subst}\left(\int \frac{x^2(a+bx)^n}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{d^2\left(a+\frac{b}{x}\right)^{1+n}}{c^2(ac-bd)\left(d+\frac{c}{x}\right)} - \frac{\text{Subst}\left(\int \frac{(a+bx)^n(-d(ac-bd(1+n))+c(ac-bd)x)}{d+cx}\,dx, x, \frac{1}{x}\right)}{c^2(ac-bd)}$$

$$= -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc^2(1+n)} + \frac{d^2\left(a+\frac{b}{x}\right)^{1+n}}{c^2(ac-bd)\left(d+\frac{c}{x}\right)} + \frac{(d(2ac-bd(2+n)))\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{c^2(ac-bd)}$$

$$= -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc^2(1+n)} + \frac{d^2\left(a+\frac{b}{x}\right)^{1+n}}{c^2(ac-bd)\left(d+\frac{c}{x}\right)} - \frac{d(2ac-bd(2+n))\left(a+\frac{b}{x}\right)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^2(ac-bd)^2(1+n)}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.94

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2}\,dx = -\frac{\left(a+\frac{b}{x}\right)^n(b+ax)\left((ac-bd)(ac(c+dx)-bd(c+d(2+n)x))+bd(2ac-bd(2+n))(c+dx)\,\text{Hyperge}\right.}{bc^2(ac-bd)^2(1+n)x(c+dx)}$$

```
[In] Integrate[(a + b/x)^n/(x^2*(c + d*x)^2),x]
```

```
[Out] -(((a + b/x)^n*(b + a*x)*((a*c - b*d)*(a*c*(c + d*x) - b*d*(c + d*(2 + n)*x
)) + b*d*(2*a*c - b*d*(2 + n))*(c + d*x)*Hypergeometric2F1[1, 1 + n, 2 + n,
 (c*(a + b/x))/(a*c - b*d)]))/(b*c^2*(a*c - b*d)^2*(1 + n)*x*(c + d*x)))
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2\left(dx+c\right)^2}dx$$

```
[In] int((a+b/x)^n/x^2/(d*x+c)^2,x)
```

```
[Out] int((a+b/x)^n/x^2/(d*x+c)^2,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(dx+c\right)^2x^2}\,dx$$

```
[In] integrate((a+b/x)^n/x^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^n/(d^2*x^4 + 2*c*d*x^3 + c^2*x^2), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{x^2\left(c+dx\right)^2}\,dx$$

```
[In] integrate((a+b/x)**n/x**2/(d*x+c)**2,x)
```

```
[Out] Integral((a + b/x)**n/(x**2*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)^2x^2}\,dx$$

```
[In] integrate((a+b/x)^n/x^2/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((a + b/x)^n/((d*x + c)^2*x^2), x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{(dx+c)^2x^2}\,dx$$

```
[In] integrate((a+b/x)^n/x^2/(d*x+c)^2,x, algorithm="giac")
[Out] integrate((a + b/x)^n/((d*x + c)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^2(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{x^2\,(c+d\,x)^2}\,dx$$

```
[In] int((a + b/x)^n/(x^2*(c + d*x)^2),x)
[Out] int((a + b/x)^n/(x^2*(c + d*x)^2), x)
```

3.298 $\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 217

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2} \, dx = -\frac{\left(a+\frac{b}{x}\right)^{1+n}\left(d(bd(2+n)(ac+bd(3+n))-ac(ac+bd(5+3n)))-\frac{c(ac-bd)(ac+bd(3+n))}{x}\right)}{b^2c^3(ac-bd)(1+n)(2+n)\left(d+\frac{c}{x}\right)} - \frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc(2+n)\left(d+\frac{c}{x}\right)x^2} + \frac{d^2(3ac-bd(3+n))\left(a+\frac{b}{x}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^3(ac-bd)^2(1+n)}$$

```
[Out] -(a+b/x)^(1+n)*(d*(b*d*(2+n)*(a*c+b*d*(3+n))-a*c*(a*c+b*d*(5+3*n)))-c*(a*c-
b*d)*(a*c+b*d*(3+n))/x)/b^2/c^3/(a*c-b*d)/(1+n)/(2+n)/(d+c/x)-(a+b/x)^(1+n)
/b/c/(2+n)/(d+c/x)/x^2+d^2*(3*a*c-b*d*(3+n))*(a+b/x)^(1+n)*hypergeom([1, 1+
n],[2+n],c*(a+b/x)/(a*c-b*d))/c^3/(a*c-b*d)^2/(1+n)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {528, 457, 102, 151, 70}

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\,dx$$
$$= -\frac{\left(a+\frac{b}{x}\right)^{n+1}\left(d(bd(n+2)(ac+bd(n+3))-ac(ac+bd(3n+5)))-\frac{c(ac-bd)(ac+bd(n+3))}{x}\right)}{b^2c^3(n+1)(n+2)\left(\frac{c}{x}+d\right)(ac-bd)}$$
$$+\frac{d^2\left(a+\frac{b}{x}\right)^{n+1}(3ac-bd(n+3))\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^3(n+1)(ac-bd)^2}$$
$$-\frac{\left(a+\frac{b}{x}\right)^{n+1}}{bc(n+2)x^2\left(\frac{c}{x}+d\right)}$$

```
[In] Int[(a + b/x)^n/(x^3*(c + d*x)^2),x]
```

```
[Out] -(((a + b/x)^(1 + n)*(d*(b*d*(2 + n)*(a*c + b*d*(3 + n)) - a*c*(a*c + b*d*(
5 + 3*n))) - (c*(a*c - b*d)*(a*c + b*d*(3 + n)))/x))/(b^2*c^3*(a*c - b*d)*(
1 + n)*(2 + n)*(d + c/x))) - (a + b/x)^(1 + n)/(b*c*(2 + n)*(d + c/x)*x^2)
+ (d^2*(3*a*c - b*d*(3 + n))*(a + b/x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
2 + n, (c*(a + b/x))/(a*c - b*d)])/(c^3*(a*c - b*d)^2*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
```

```
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)
*(c + d*x)^(n + 1), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(d+\frac{c}{x}\right)^2 x^5}\,dx \\
&= -\text{Subst}\left(\int \frac{x^3(a+bx)^n}{(d+cx)^2}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc(2+n)\left(d+\frac{c}{x}\right)x^2} - \frac{\text{Subst}\left(\int \frac{x(a+bx)^n(-2ad+(-ac-bd(3+n))x)}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)}{bc(2+n)} \\
&= -\frac{\left(a+\frac{b}{x}\right)^{1+n}\left(d(bd(2+n)(ac+bd(3+n))-ac(ac+bd(5+3n)))-\frac{c(ac-bd)(ac+bd(3+n))}{x}\right)}{b^2c^3(ac-bd)(1+n)(2+n)\left(d+\frac{c}{x}\right)} \\
&\quad -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc(2+n)\left(d+\frac{c}{x}\right)x^2} - \frac{\left(d^2(3ac-bd(3+n))\right)\text{Subst}\left(\int \frac{(a+bx)^n}{d+cx}\,dx, x, \frac{1}{x}\right)}{c^3(ac-bd)} \\
&= -\frac{\left(a+\frac{b}{x}\right)^{1+n}\left(d(bd(2+n)(ac+bd(3+n))-ac(ac+bd(5+3n)))-\frac{c(ac-bd)(ac+bd(3+n))}{x}\right)}{b^2c^3(ac-bd)(1+n)(2+n)\left(d+\frac{c}{x}\right)} \\
&\quad -\frac{\left(a+\frac{b}{x}\right)^{1+n}}{bc(2+n)\left(d+\frac{c}{x}\right)x^2} + \frac{d^2(3ac-bd(3+n))\left(a+\frac{b}{x}\right)^{1+n}{}_2F_1\left(1,1+n;2+n;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c^3(ac-bd)^2(1+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\,dx = \frac{\left(a+\frac{b}{x}\right)^{1+n}\left(-\frac{1}{x(c+dx)}+\frac{-a^2c^2(c+dx)+b^2d^2(3+n)(c+d(2+n)x)-abcd(c(2+n)+d(3+2n)x)}{bc^2(-ac+bd)(1+n)(c+dx)}-\frac{bd^2(2+n)(-3ac+bd(3+n))\ \text{Hypergeome}}{c^2(ac-bd)^2(1}\right.}{bc(2+n)}$$

```
[In] Integrate[(a + b/x)^n/(x^3*(c + d*x)^2),x]
```

```
[Out] ((a + b/x)^(1 + n)*(-(1/(x*(c + d*x))) + (-(a^2*c^2*(c + d*x)) + b^2*d^2*(3
 + n)*(c + d*(2 + n)*x) - a*b*c*d*(c*(2 + n) + d*(3 + 2*n)*x))/(b*c^2*(-(a*
c) + b*d)*(1 + n)*(c + d*x)) - (b*d^2*(2 + n)*(-3*a*c + b*d*(3 + n))*Hyperg
eometric2F1[1, 1 + n, 2 + n, (c*(a + b/x))/(a*c - b*d)])/(c^2*(a*c - b*d)^2
*(1 + n))))/(b*c*(2 + n))
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3\left(dx+c\right)^2}dx$$

```
[In] int((a+b/x)^n/x^3/(d*x+c)^2,x)
```

```
[Out] int((a+b/x)^n/x^3/(d*x+c)^2,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(dx+c\right)^2x^3}\,dx$$

```
[In] integrate((a+b/x)^n/x^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^n/(d^2*x^5 + 2*c*d*x^4 + c^2*x^3), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{x^3\left(c+dx\right)^2}\,dx$$

```
[In] integrate((a+b/x)**n/x**3/(d*x+c)**2,x)
[Out] Integral((a + b/x)**n/(x**3*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(dx+c\right)^2x^3}\,dx$$

```
[In] integrate((a+b/x)^n/x^3/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((a + b/x)^n/((d*x + c)^2*x^3), x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{\left(dx+c\right)^2x^3}\,dx$$

```
[In] integrate((a+b/x)^n/x^3/(d*x+c)^2,x, algorithm="giac")
[Out] integrate((a + b/x)^n/((d*x + c)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^n}{x^3(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^n}{x^3\left(c+d\,x\right)^2}\,dx$$

```
[In] int((a + b/x)^n/(x^3*(c + d*x)^2),x)
[Out] int((a + b/x)^n/(x^3*(c + d*x)^2), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(4,m1)    #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(5,m1)    #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(6,m1)    #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```